Resource distribution and performance of complex systems

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Abstract: This paper considers an entropic approach to solving the problem of finding the optimal resource distribution within a complex system and its sub-systems. We consider the indicator of uneven distribution of resources in a complex economic system and the adaptation potential based on the approximation of the Lorentz diagrams with one and two-parameter families of functions. The dependence of the adaptation potential on the values of the uneven distribution of resources is investigated. The entropic approach allows to not only monitor the optimality of resource distribution, but also develop recommendations on distribution readjustment in adaptive mode and dynamic conditions.

Keywords: multi-agent systems; efficiency; entropic approach; adaptation potential; variability; adaptation potential maximisation principle; balance state; stability; system condition monitoring.

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1 Introduction

The evolution of modern economic systems is characterised by continual flux of the number of interacting segments (agents), their functional orientation and complexity of relations\associations\inter-connections. Evolution of a modern economic system is characterised by continual change of interacting subsystems (agents[1, 2]), their functional orientation and inter-relational complexity. The constant change in the economic system naturally leads to a change in the efficiency[3] of its work as a whole. Managerial decisions that are aimed at amending previous changes for the purpose of restoring the efficiency of an economic system often lead to a contrary result. Specifically, managerial solutions that implicate simplified interrelations or decrease in the number of agents lead to a reduction of system operational efficiency. Thus, a fundamental task of determining the viability and promptness of solution implementation arises. The solution of the task at hand can be attained within the framework of entropic approach for selection of a managerial solution (Antoniou et al., 2002, 2004; Aoki, 1998; Liiv, 1998; Haritonov 1999; Haritonov et al., 2008; Kryanev et al., 2010; Panchenkov, 1999, 2007; Prangishvili, 2000, 2003; Vlasova, 2006; Maisseu, 2016, 2017). Fundamentally, the entropic approach for developing optimal managerial solutions is based on the theory and experiments that determined the interconnection between the objective function of a multi-agent system and the distribution of resources consumed by an economic system during its activities.

2 Distribution of resources in a complex economic system

The efficiency of system activity may be measured by attaining the values of some target function of a given system. Specific expression for evaluating the values of a target function may vary. For instance, the consolidated profit may be chosen as a target function of a multi-agent business structure. For this purpose, the consolidated profit is calculated as the difference between the revenues and expenses of all in-system agents for a fixed period of time. Premised on this definition of target function, arises a temptation to multiply profits by means of engaging more agents. However, the multiplication of agents does not necessarily lead to a corresponding increase in profits. Namely, the increase in the number of agents causes a non-linear escalation of system complexity which conditions the engagement of additional resources. Eventually, there comes a point at which the increase of number of agents does not contribute to the integral efficiency of a system. An alternate route consists of simplifying the managerial process at the expense of agent reduction and an inevitable resulting decrease in integral profits. Naturally, in this case the business structure is tending to an agent set and a resource distribution at which the maximum value of the system's integral operational efficiency is reached. At first glance the introduced example demonstrates that at the creation and the subsequent operation of a system a certain resource distribution is projected and sustained. Actual resource distribution of a complex system is contained in the range within two extreme distribution types, specifically, uniform resource distribution and the distribution involving the allocation of a resource to one agent only. The question is how one should determine the 'golden proportion' of resource distribution among parts of a system that establishes maximal efficiency and optimal integral continuity of a system. An answer to this big question has been proposed within the entropic approach (Antoniou et al., 2002, 2004; Panchenkov, 2007; Haritinov et al., 2008). The present paper introduces the notion of an adaptation potential of a system, maximisation of which will contribute to sustainability of a complex system in the long term.

3 Commensuration of resource distribution

3.1 Lorenz curves and the corresponding analytic approximations

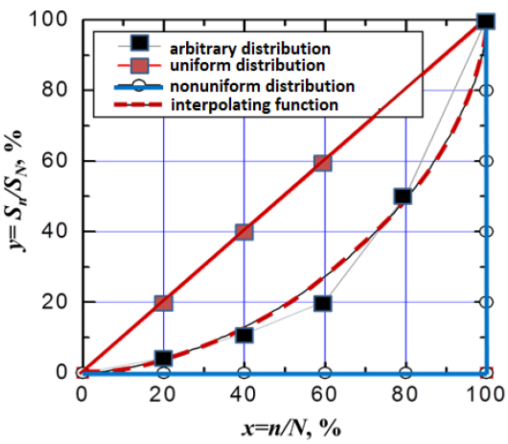
Let indices $i = 1,..,n,...,N$ denote all agents of an economic system, among which the resources G_i are distributed. To draw the Lorenz curve (Kryanev et al., 2010;

Imamutdinov, 2014) of resource distribution $\{G_i,\ i=1,\ldots,N\}$ one ranges the input series in question $G_1 \le G_2 \le \ldots \le G_N$. Next the accumulated sums

$$S_n = \sum_{i=1}^{n} G_i \tag{1}$$

are calculated and plotted on a plane graph with axes $x = n/N$ and $y = S_n/S_N$ (the continuous polygonal line in Figure 1).

Figure 1 Resource distribution of a complex system on a Lorenz diagram



For the purpose of applying analytical methods we approximate the piecewise linear Lorenz curve throughout the interval $x \in (0,1)$ with a continuous curve (the dashed line in Figure 1). We propose that the class of functions y = *y* (*x, a*), given by expression

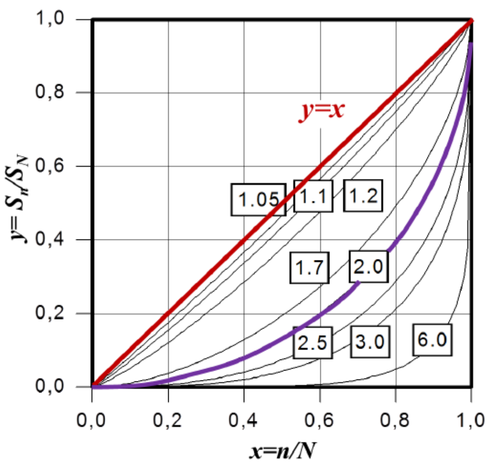
$$y(x,\alpha) = 1 - \left(1 - x^{\alpha}\right)^{1/\alpha}, \tag{2}$$

may serve as continuous approximants of the Lorenz curve (see Figure 2). The parameter α $(1 \le \alpha < \infty)$ acts as a proportionality (degree of irregularity) indicator of share distribution and hence is the counterpart of Gini index K_G, used extensively in economics as an indice of resource distribution inequality (Haritonov et al., 2008; Kryanev et al., 2010).

$$K_G = 1 - 2\int_0^1 y(x,\alpha)\,dx, 0 \le K_G \le 1. \tag{3}$$

From (2) at α =1 it follows that $y(x,\alpha) = x$, which corresponds to the uniform resource share distribution while Jini Index K_G = 0, whereas at $\alpha \to \infty$ the share distribution approaches the extremely irregular distribution while Jini Index $K_G \to 1$ (see Figure 1). It should be noted that Jini Index is equal to the ratio of the area of the figure formed by curves $y = x$ and $y = y(x,\alpha)$ to half of the area of the square depicted in Figure 1. We name the functions *y* (*x, α*) control functions (Kryanev et al., 1998).

Figure 2 Plots of control function $y\,(x,\,\alpha)$ for different values of α



4 Frequency distribution of resource shares

In regards to complex systems, analysis works (Prigogine, 1962, p.6; Nicolis and Prigogine, 1977; Kondepudi and Prigogine, 1998) state: "In the usual presentation of mechanics the essential quantities are the coordinates and momenta. Here, however, the basic quantity is the statistical distribution function *p*, from which the average values of all functions of coordinates and momenta may be computed. Thus we may say that a knowledge of *p* implies complete knowledge of the "state" of the system. In this development, the "state" of the system is given by the correlations and in homogeneities, and the evolution of the system becomes a dynamics of correlations, governed by the Liouville operator."

Alternately, it may be asserted that every control function $y = y(x,\alpha)$ has a corresponding statistical density function $\rho(\mathrm{x},\alpha)$, that may be found according to the following procedure. By virtue of the fact that the function $y = y(x,\alpha)$ is defined on the plane (x, y) over the range $L=\{(x, y);\ 0 \le x \le 1;\ 0 \le y \le 1,\ y \le x\}$ and is a monotonically increasing, continuously differentiable, downward convex function that takes on values $y = y(0,\alpha) = 0, y = y(1,\alpha) = 1$ at $1 < \alpha < \infty$, its derivative is given by

$$\frac{dy(x,\alpha)}{dx} = \frac{x^{\alpha-1}}{\left(1-x^{\alpha}\right)^{\frac{\alpha-1}{\alpha}}} \equiv g\left(x,a\right) \text{at } \alpha>1 \tag{4}$$

Within the range $0 \le x \le 1$ the function $g(x,\alpha)$ increases monotonically from $g(0,\ \alpha) = 0$ to $g(1,\alpha) \to \infty$. It should be noted that the meaning of the expression (4) becomes clear if we conduct a reciprocal substitution of variables *x* and *y* with initial values at $N >> 1$ (Kryanev et al., 1998; Prangishvili, 2000):

$$g = \frac{dy(x,\alpha)}{dx} = \frac{d\left(S_n/S_N\right)}{d(n/N)} = \frac{dS_n}{dn} \cdot \frac{N}{S_N} = \frac{G_n}{\overline{G}} \tag{5}$$

From (5) it follows that the deduced derivative function *g* is a share of a contribution, reduced in relation to the average contribution size $\bar{G} = S_N / N.$ Equation (4) allows us to find the inverse function (at $\alpha > 1$)

$$x(g,\alpha) = \frac{g^{\frac{1}{\alpha-1}}}{\left(1+g^{\frac{\alpha}{\alpha-1}}\right)^{1/\alpha}} \tag{6}$$

This implies that when *g* runs from 0 to ∞ the function $x(g,\alpha)$ runs from $x(0,\alpha)=0$ to $x(\infty,\alpha)=1.$ Hence, the function $x(g,\alpha)$ acts as the distribution function of a positive random variable $g > 0$, whereas its derivative $\rho = dx/dg$ acts as the density distribution function of the input *g*:

$$\rho(g,\alpha) = \frac{dx(g,\alpha)}{dg} = \frac{1}{\alpha-1} \frac{g^{\frac{2-\alpha}{\alpha-1}}}{\left(1-g^{\frac{\alpha}{\alpha-1}}\right)^{\frac{\alpha+1}{\alpha}}} \tag{7}$$

As it appears from Figures 2 and 3, the function $y(g,2)$, that corresponds to circumference on the Lorenz diagram ($\alpha = 2$), acts as a border that separates the family of densities $\rho(g,\alpha)$ into two sets. Moreover, $\rho(g,2)$ is the only function of the family $\rho(g,\alpha)$, that takes on a bounded non-zero value at $g = 0$.

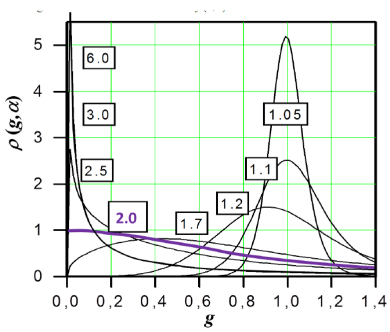
In addition to the one-parameter family of functions (2), we use a two-parameter family of functions

$$y(x,\alpha,\beta) = 1-\left(1-x^{\alpha}\right)^{\frac{1}{\beta}}, 0 \le \alpha, \beta \le \infty \tag{8}$$

which makes it possible to approximate the Lorentz diagrams with greater accuracy in the case of a violation of its symmetry relative to the second diagonal of the Lorentz square $0 \le y, x \le 1$ (see Figure 1).

The two-parameter function (8), by analogy with the probability density (7), found a two-parameter probability density $\rho(g,\alpha,\beta)$ (Antoniou et al., 2004), see also (Rasche et al., 1980; Chotikapanich, 2008).

Figure 3 Plots of density function $\rho(g,\alpha)$ of input *g* for different values of coefficient *α*



5 Entropy as an indicator of a system's adaptation potential

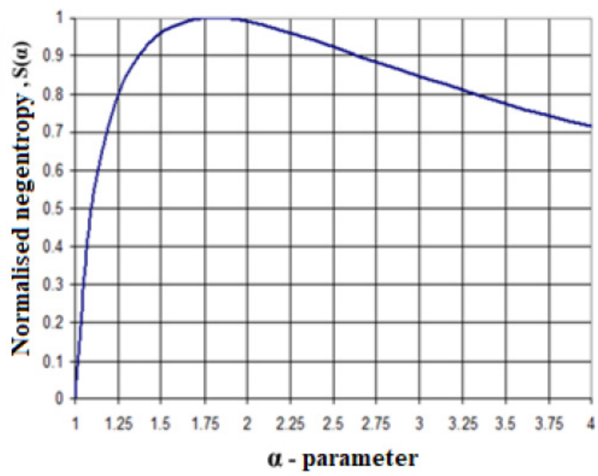
For known density function $\rho(g,\alpha)$ the integral

$$S(\alpha) = -\int_0^\infty \rho(g,\alpha)\ln\left[\rho(g,\alpha)\right]dg \tag{9}$$

is aligned with entropy. It has previously been named adaptation potential in works (Antoniou et al., 2002; Haritonov et al., 2008; Kryanev et al., 2010) and has been used for economic systems analysis. Numerical results of computing the values of adaptation potential are presented in Figure 4.

As is evident, the adaptation potential calculated by (8) reaches its maximal value at $\alpha_0 \approx 1,84.$ This value corresponds to the discovered commensuration of the agents' input shares to the potential of the system at large. In other words, the distribution at $\alpha_0 \approx 1.84$ corresponds to the 'golden ratio' between two extreme cases: uniform distribution at α = 1, and an absolutely non-uniform distribution is attained at $\alpha \to \infty.$ The latter case denotes a situation when a rigid vertical of control is present, and all the resources are redistributed into one segment of a system.

Figure 4 Plot of the adaptation potential as a normalised function $S(\alpha)$



The adaptive indicator $S(\alpha, \beta)$ for a two-parameter family of functions (8) (Antoniou et al., 2004)

$$S(\alpha,\beta) = -\int_0^g \rho\left(g,\alpha,\beta\right)\ln\left(\rho\left(g,\alpha,\beta\right)\right)dg \tag{10}$$

According to Ashby's Law of Requisite Varity, the entropy of a controllable system is indicative of the state manifold degree of a controllable system and, thereby, the possibility of a state transition (Ashby, 1961). Adaptation potential is a measure of the variety of a considered system and is alike to Ashby's entropy measure of a controllable system.

Indeed, when a system's state is typified by the maximal value of the adaptation potential, then if redistribution of resources is required for a transition to a new state with a different value of the non-uniformity indicator, then generally the amount to be transferred is minimal in comparison to any other system state. Besides, if the system is characterised by maximal value of the adaptation potential, this guarantees minimal variation of the adaptation level due to level changes of the degree of resource distribution irregularity within sub-systems of a complex system.

Hence, when the distribution of resources is similar to uniform (α = 1) or to maximally non-uniform ($\alpha \to \infty$), a system has a lower capacity for adaptation to the varied external conditions. This follows from the minimal local values of adaptation potential for these two distribution types (see Figure 4 and Haritonov et al., 2008; Kryanev et al., 2010).

It is notable that in the range $\alpha < \alpha_0$ that corresponds to 'anarchistic' control tendencies, the drop of the adaptation potential relatively to its maximal value is more pronounced in comparison to the constrained decrease in the range $\alpha > \alpha_0$, which is characterised by the tendency of strengthening the 'chain of command'.

A viable functional economic system is characterised by minimal resource costs and the shortest adaptation times. The use of share distribution with the maximal value of the adaptation potential allows for more freedom in choosing within the range of viable managerial decision and consequently the optimal capacity to adapt to ongoing changes.

In the process of system operation the real input share distribution within individual units of a complex system may differ from the projected optimal distribution that corresponds to the maximal value of the adaptation potential. What is more, the exceedance of the real share in relation to the projected share that corresponds to the 'golden ratio' of a subset is a signal of its high performance. Conversely, a reduction in the real share relatively to the projected share of a subset indicates its weakness. Therefore, continual time monitoring of real share values and the value of the adaptation potential indicator are a measure of the current system state. This allows for time efficient managerial decision making aimed at optimising the structure and the functionality of the system.

The entropic approach can be used to analyse various data with regards to variability and the capacity to adapt to change. Below is a result of an implementation of the entropic approach for analysis of energy production in selected world regions. For this purpose, recent data (BP, 2017) on primary energy consumption by source has been processed with the intent to determine any possible trends in fuel-power complex development. The present distribution of primary energy consumption by the six sources: oil, natural gas, coal, nuclear energy, hydroelectricity, and renewables, has been depicted by the Lorenz curves, presented below (Figures 5–8). The empirical shares of the energy market, allocated to the various power sources, were approximated by the single-parameter and the two-parameter approximants. The results of applying the procedure proposed above for determining the parameter α and β. The value of the adaptation potential $S(a)$ and $S(\alpha, \beta)$ are presented below.

Thus, symmetry breaking in the Lorentz diagram leads to a decrease in the adaptation of the economic system under consideration.

The single-parameter approximating curves are not always proximate enough to the empiric Lorenz curves. In that case a two-parameter approximant is advisable and allows for a better fit as is depicted in Figure 5. Accuracy of approximation is measured by the parameter – sum of squared differences between the given data and the results of the approximation.

In some cases, however, the single parameter approximation is feasible as the underlying data result in a symmetric (with regards to the second diagonal of the Lorenz square) distribution. Such is the case for the energy source distributions in EU, Russia and China, which are reasonably well fitted by the single parameter approximants (see Figures 6–8).

Figure 5 The one-parameter and the two-parameter approximations of the Lorenz curve of energy consumption in the World $\left(S_{2014}(\alpha,\beta)=0.75; S_{2015}(\alpha,\beta)=0.74; S_{2016}(\alpha,\beta)=0.73\right)$

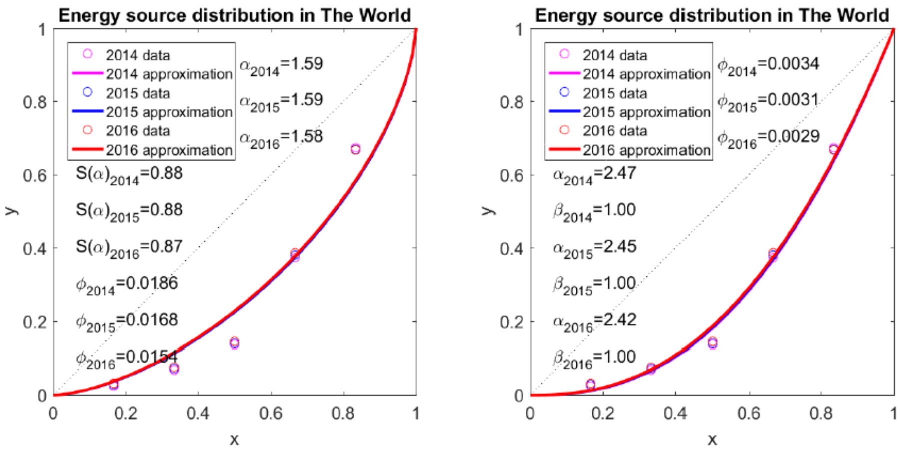


Figure 6 The one-parameter and the two-parameter approximations of the Lorenz curve of energy consumption in the EU

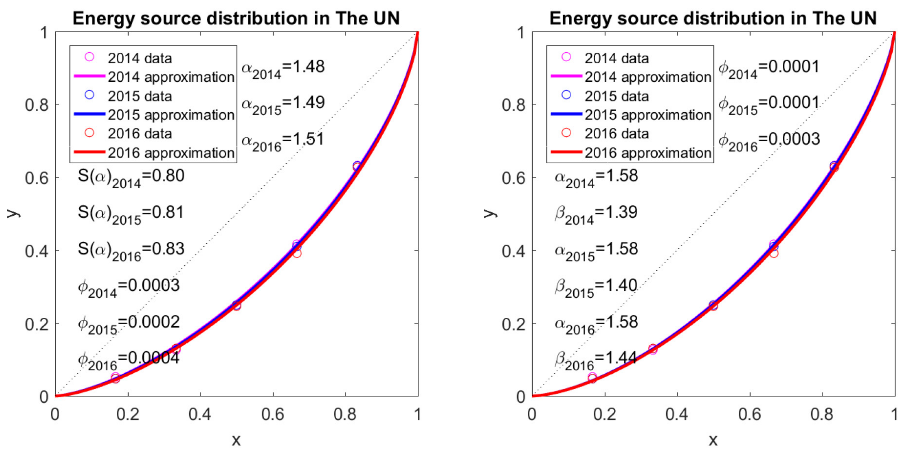


Figure 7 The one-parameter and the two-parameter approximations of the Lorenz curve of energy consumption in Russia

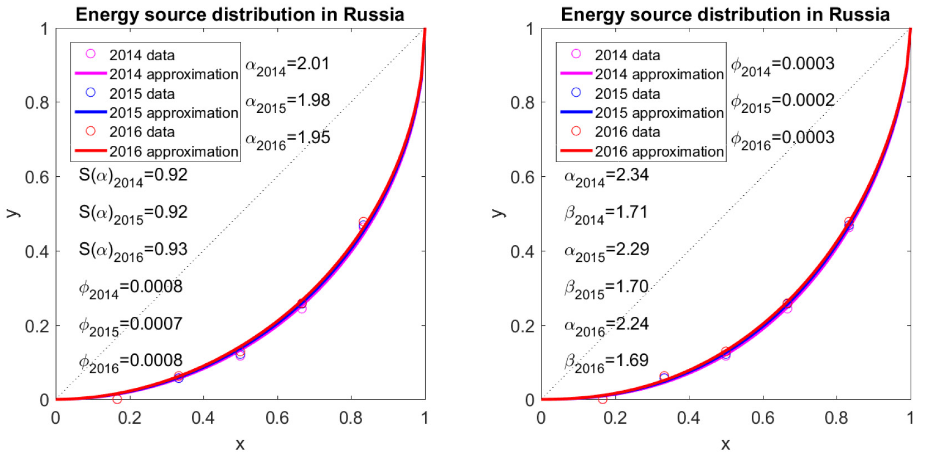
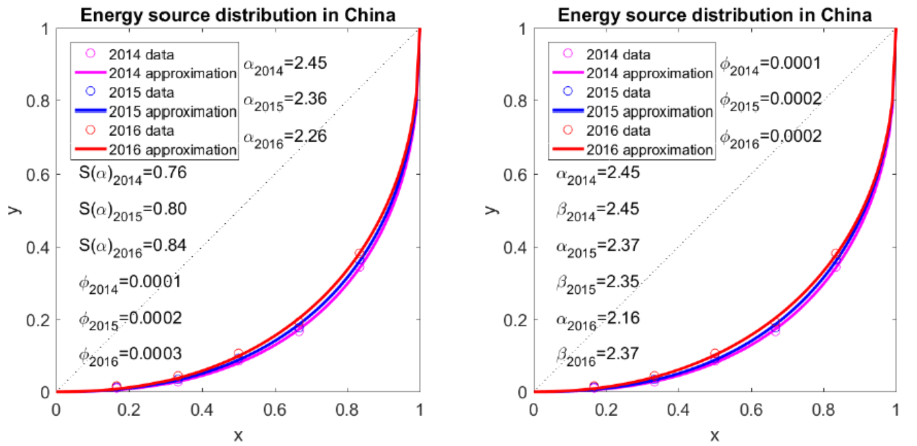


Figure 8 The one-parameter and the two-parameter approximations of the Lorenz curve of energy consumption in China



It may be noted that the annual change in the parameters α and β and the value of the adaptation potential is gradual, but overall the values α and β seem to tend to the values that correspond to the sustainable distribution $\rho(g,\alpha)$ and the optimal value of the adaptation potential $S(a)$.

Thus, it follows from Figures 6–8 that for the EU, Russia and China, the values of the adaptation potential remain almost constant, at least for 2014–2016. This shows a sufficiently large degree of adaptive stability for these three economic systems. The previously described regularities of the adaptation potential of a complex system are in accord with the recently developed 'swarm technologies' that simulate 'Swarm Intelligence' (Imamutdinov, 2014; Rzevski and Skobelev, 2014; Waud, 1990). Indeed, the aim of 'swarm technologies' is to optimise the process of controlling a complex system or to redistribute the resources of a system among its subsystems in such a way as to increase the attained value of the target function. An example of an implementation of swarm technologies, based on the optimal resource distribution among independent portions of a complex economic system, is the work activity management of a taxi company in London (Imamutdinov, 2014).

6 Conclusion

Studying complex systems is premised on two approaches to effective operation with regard to the developmental stage of a system. If a system is currently in the process of formation, it is necessary to construct its structure and the interaction of its sub-systems in such a way as to secure its sustainability. If a system is currently in a state of dynamic operation, it is necessary to sustain its optimal structure, which is responsible for the greatest possible stability and resistance, taking into account any changes in system performance. The procedure, based on the adaptation potential that was proposed in this paper, allows finding the solutions to the two previously stated problems. Examples of the application of the proposed scheme in the article for the analysis of energy consumption in such countries and regions of the world as the European Union, Russia, China and the world as a whole are considered.

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Notes

1 Agent – part of a system, that acts independently or on one's behalf.

2 A multi agent system consists of autonomous agents, capable of percepting the present situation and acting, interacting with peers, continuously competing and cooperating.

3 Efficiency reads as a measure of reaching a specified value of any target function characteristic of considered system in relation to the task at hand (revenue maximisation, increase in profitability, output expansion, etc.)