A new approach to find optimal solution of fuzzy assignment problem using penalty method for hendecagonal fuzzy number

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Abstract: Fuzzy assignment problem (FAP) is a well-known topic and it is used to solve many real-life problems. In many situations, the parameters are used to characterise the uncertainty either triangular or trapezoidal fuzzy number. But the representation of fuzzy number is not always a triangular or trapezoidal fuzzy number. In this paper, a new generalisation of fuzzy number called hendecagonal fuzzy number (HDFN) is introduced with its arithmetic operations. This research aims to introduce a new penalty approach in order to solve fuzzy assignment problem such that the solution is optimum. The cost of assignment is represented by trapezoidal fuzzy number (TrapFN) and HDFN. The proposed method overcomes the limitations of existing method and HDFN is more optimal than TrapFN.

Keywords: hendecagonal fuzzy number; HDFN; alpha cut; fuzzy arithmetic; fuzzy assignment problem; FAP; penalty approach; ranking method; fuzzy optimal solution; trapezoidal fuzzy number; TrapFN; fuzzy cost; graphical representation.

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1 Introduction

Assignment problem is a special type of linear programming problem in which objective is to assign 'n' number of jobs to 'n' number of persons at a minimum cost. In real life situation, the parameters of assignment problem are uncertain instead of fixed ones, because the time/cost for doing a job by a facility always vary for many situations. A fuzzy number is a quantity whose values are imprecise, rather than exact as in the case with single-valued function. The generalisation of a real number is the main concept of fuzzy number. In real-world applications, all the parameters may not be known precisely due to uncontrollable factors.

Zadeh (1965) introduced the concept of fuzzy set to deal with uncertainty and vagueness in real life situations. The application of fuzzy set is extended by Dubois and Prade (1980). Chen (1985a) proposed a fuzzy assignment model that considers all persons to have the same skills. Khun (1955) and Wang (1987) also solved a similar model. Chen (1985b) introduced operations on fuzzy numbers. The arithmetic operations, alpha cut and ranking function are already introduced for existing fuzzy numbers by Yager (1986) and Liou and Wang (1992).

In recent years, FAP has received much attention. Lin and Wen (2004) solved assignment problem with fuzzy number costs by a labelling algorithm. Vasant (2005) solved fuzzy linear programming problems with modified S-curve membership function. Abbasbandy and Hajjari (2009) proposed ranking technique for Trapezoidal fuzzy number. Mukherjee and Basu (2010) proposed a new method for solving fuzzy assignment problem. Kumar and Gupta (2011) used different membership functions to solve Fuzzy assignment problem. Kaur and Kumar (2012) used non-normal generalised trapezoidal fuzzy numbers for solving transportation problems in fuzzy environment. Sagaya and Henry (2012) used trapezoidal fuzzy numbers to represent the fuzzy cost or fuzzy times in the fuzzy assignment problem. Rathi et al. (2015) introduced heptagonal fuzzy number to solve fuzzy assignment problem. Garai et al. (2016) used fuzzy technique in multi-objective optimisation. Revathi et al. (2017) introduced hendecagonal fuzzy number for optimisation problem using row penalty method.

There are several papers in the literature for triangular or trapezoidal fuzzy numbers in order to characterise the vague parameters that arise in real life problems. Rathi and Balamohan (2014, 2016) developed arithmetic nature and ranking technique for heptagonal fuzzy number. It is not possible to restrict the membership function to be either triangular or trapezoidal. The degree of confidence is based on the expert's judgment. Because of this, the proposed fuzzy number is more suitable and realistic.

In this paper a new type of generalisation of fuzzy number named HDFN is defined with its membership function. The arithmetic operations, alpha cut and ranking procedure for HDFN are also proposed. The proposed method is easy to understand and the assignment is tested using the penalty approach for optimality.

The paper is organised as: in Section 2, basic definitions are discussed. In Section 3, generalisation of hendecagonal fuzzy number and its graphical representation are introduced. In Section 4, arithmetic operations and ranking of HDFN are defined. In Section 5, algorithm for the proposed method is presented. In Section 6, numerical example has given and it is shown that the proposed method offers an effective way for handling FAP using TrapFN and HDFN. Section 7 presents the concluding remarks.

2 Preliminary

In this section, some basic definitions of fuzzy set theory and fuzzy numbers are reviewed.

Definition 2.1 (Membership grade): The membership grade corresponds to the degree to which an element is compatible with the concept represented by fuzzy set.

Definition 2.2 (Membership function): Let X denotes a universal set. Then the characteristic function which assigns certain values or a membership grade to the elements of this universal set within a specified range [0, 1] is known as membership function.

Definition 2.3 (Fuzzy set): A fuzzy set is characterised by a membership function mapping the elements of a domain space or universe of discourse X to the unit interval [0, 1], i.e., $\mu_{\tilde{A}} : X \to [0,1]$.

Definition 2.4 (Support of fuzzy set): The support of a fuzzy set $\tilde{A}$ in the universal set X is the set $Supp(\tilde{A}) = \{x \in X \mid \mu_{\tilde{A}}(x) > 0\}$ that contains all the elements of X that has a non-zero membership grade in $\tilde{A}$.

Definition 2.5 (Core of fuzzy set): The core of a fuzzy set $\tilde{A}$ in the universal set X is the set $Core(\tilde{A}) = \{x \in X \mid \mu_{\tilde{A}}(x) = 1\}$ that contains all the elements of X that exhibit a unit level of membership in $\tilde{A}$.

Definition 2.6 (α-cut): An α-cut of a fuzzy set A is a crispest A^{α} that contains all the elements of the universal set X that have a membership grade in A greater or equal to specified value of α. Thus $A^{\alpha} = \{x \in X, \mu_A(x) \geq \alpha\}$, $0 \leq \alpha \leq 1$.

Definition 2.7 (Convex fuzzy set): A fuzzy set $\tilde{A}$ is a convex fuzzy set if and only if each of its *α-cuts* A^{α} is a convex set.

Definition 2.8 (Fuzzy number): A fuzzy set $\tilde{A}$ is a fuzzy number iff:

1 For all $\alpha \in (0, 1]$ the *α-cut* sets A_{α} is a convex set

2 $\mu_{\tilde{A}}$ is an upper semi continuous function

3 $Supp(\tilde{A})$ is a bounded set in R

4 the height of $\tilde{A} = \max_{x \in X} \mu_{\tilde{A}}(x) = \omega > 0$.

3 Hendecagonal fuzzy number

Definition 3.1 (Trapezoidal fuzzy number): A fuzzy number $\tilde{A} = (a, b, c, d; \omega)$ is said to be a generalised trapezoidal fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \omega\left(\dfrac{x-a}{b-a}\right), & a \le x \le b \\ \omega, & b \le x \le c \\ \omega\left(\dfrac{x-d}{d-c}\right), & c \le x \le d \\ 0, & \text{otherwise} \end{cases}$$

Definition 3.2 (Hendecagonal fuzzy number): A fuzzy number $\tilde{HD} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}, h_{11}; u, v, \omega)$ is said to be a hendecagonal fuzzy number if its membership function is given by

$$\mu_{\tilde{HD}}(x) = \begin{cases} u\left(\dfrac{x-h_1}{h_2-h_1}\right) & \text{for } h_1 \le x \le h_2 \\ u & \text{for } h_2 \le x \le h_3 \\ u+(v-u)\left(\dfrac{x-h_3}{h_4-h_3}\right) & \text{for } h_3 \le x \le h_4 \\ v & \text{for } h_4 \le x \le h_5 \\ v+(\omega-v)\left(\dfrac{x-h_5}{h_6-h_5}\right) & \text{for } h_5 \le x \le h_6 \\ v+(\omega-v)\left(\dfrac{h_7-x}{h_7-h_6}\right) & \text{for } h_6 \le x \le h_7 \\ v & \text{for } h_7 \le x \le h_8 \\ u+(v-u)\left(\dfrac{h_9-x}{h_9-h_8}\right) & \text{for } h_8 \le x \le h_9 \\ u & \text{for } h_9 \le x \le h_{10} \\ u\left(\dfrac{h_{11}-x}{h_{11}-h_{10}}\right) & \text{for } h_{10} \le x \le h_{11} \\ 0 & \text{otherwise} \end{cases}$$

where $0 < u < v \le \omega \le 1$.

Definition 3.3 (α-cut of HDFN): For $\alpha \in (0, 1]$, the α-cut of HDFN $\tilde{HD} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}, h_{11}; u, v, \omega)$ is defined as:

$$[\tilde{H}D]_\alpha = \begin{cases} \left[HD_{1\alpha}^L, HD_{1\alpha}^R\right] & \text{for } \alpha \in [0, u] \\ \left[HD_{2\alpha}^L, HD_{2\alpha}^R\right] & \text{for } \alpha \in (u, v] \\ \left[HD_{3\alpha}^L, HD_{3\alpha}^R\right] & \text{for } \alpha \in (v, \omega] \end{cases}$$

where

$$HD_{1\alpha}^L = h_1 + \frac{\alpha}{u}(h_2 - h_1), \quad HD_{1\alpha}^R = h_{11} - \frac{\alpha}{u}(h_{11} - h_{10}),$$

$$HD_{2\alpha}^L = h_3 + \left(\frac{\alpha - u}{v - u}\right)(h_4 - h_3), \quad HD_{2\alpha}^R = h_9 - \left(\frac{\alpha - u}{v - u}\right)(h_9 - h_8),$$

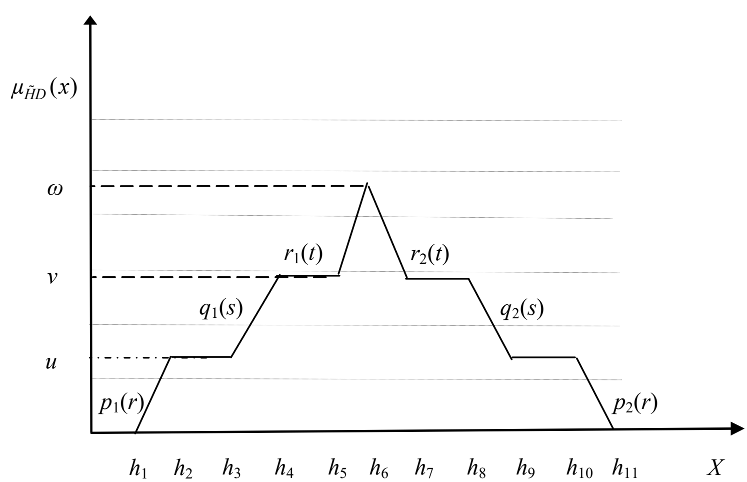
$$HD_{3\alpha}^L = h_5 + \left(\frac{\alpha - v}{\omega - v}\right)(h_6 - h_5), \quad HD_{3\alpha}^R = h_7 - \left(\frac{\alpha - v}{\omega - v}\right)(h_7 - h_6).$$

Remark 3.1:

1 The increasing part of the membership function of HDFN are $p_1(x) = u\left(\frac{x - h_1}{h_2 - h_1}\right)$, $q_1(x) = u + (v - u)\left(\frac{x - h_3}{h_4 - h_3}\right)$ and $r_1(x) = v + (\omega - v)\left(\frac{x - h_5}{h_6 - h_5}\right)$, which are bounded left continuous non-decreasing functions over $[0, \omega_1]$, $[u, \omega_2]$ and $[v, \omega_3]$ respectively with $0 \le \omega_1 \le u$, $u < \omega_2 \le v$, $v < \omega_3 \le \omega$.

2 The decreasing part of the membership function of HDFN are $r_2(x) = v + (\omega - v)\left(\frac{h_7 - x}{h_7 - h_6}\right)$, $q_2(x) = u + (v - u)\left(\frac{h_9 - x}{h_9 - h_8}\right)$ and $p_2(x) = u\left(\frac{h_{11} - x}{h_{11} - h_{10}}\right)$, which are bounded right continuous non-increasing functions over $[0, \omega_1]$, $[u, \omega_2]$ and $[v, \omega_3]$ respectively with $0 \le \omega_1 \le u$, $u < \omega_2 \le v$, $v < \omega_3 \le \omega$.

Figure 1 Graphical representation of hendecagonal fuzzy number



Remark 3.2:

1 The hendecagonal fuzzy number becomes normal if $\omega = 1$.

2 If $u = v = \omega$ then the hendecagonal fuzzy number reduced to trapezoidal fuzzy number, if $u = v$ then it becomes heptagonal fuzzy number.

4 Arithmetic operations on hendecagonal fuzzy numbers

In this section, arithmetic operations between two hendecagonal fuzzy numbers are defined on universal set of real numbers R.

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}; u_A, v_A, \omega_A)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}; u_B, v_B, \omega_B)$ be two hendecagonal fuzzy number then

1 Addition of two HDFN

$$\begin{aligned}\tilde{A} \oplus \tilde{B} = (&a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8, \\ &a_9 + b_9, a_{10} + b_{10}, a_{11} + b_{11}; \min\{u_A, u_B\}, \min\{v_A, v_B\}, \min\{\omega_A, \omega_B\})\end{aligned}$$

2 Scalar multiplication of HDFN

$$\lambda\tilde{A} = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4, \lambda a_5, \lambda a_6, \lambda a_7, \lambda a_8, \lambda a_9, \lambda a_{10}, \lambda a_{11}; u_A, v_A, \omega_A) & \text{if } \lambda > 0 \\ (\lambda a_{11}, \lambda a_{10}, \lambda a_9, \lambda a_8, \lambda a_7, \lambda a_6, \lambda a_5, \lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; u_A, v_A, \omega_A) & \text{if } \lambda < 0 \end{cases}$$

3 Subtraction of two HDFN

$$\begin{aligned}\tilde{A} \ominus \tilde{B} &= \tilde{A} \oplus (-\tilde{B}) \\ &= (a_1 - b_{11}, a_2 - b_{10}, a_3 - b_9, a_4 - b_8, a_5 - b_7, a_6 - b_6, a_7 - b_5, a_8 - b_4, \\ &\quad a_9 - b_3, a_{10} - b_2, a_{11} - b_1; \min\{u_A, u_B\}, \min\{v_A, v_B\}, \min\{\omega_A, \omega_B\})\end{aligned}$$

4 Ranking of HDFN

The ranking function $r : F(R) \to R$ where $F(R)$ is a set of fuzzy number defined on set of real numbers, which maps each fuzzy number into the real line, where the natural order exists, i.e.,

a $\tilde{A} > \tilde{B}$ iff $r(\tilde{A}) > r(\tilde{B})$

b $\tilde{A} < \tilde{B}$ iff $r(\tilde{A}) < r(\tilde{B})$

c $\tilde{A} = \tilde{B}$ iff $r(\tilde{A}) = r(\tilde{B})$.

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers then

$$r(\tilde{A}) = \frac{a_1 + a_2 + a_3 + a_4}{4} \text{ and } r(\tilde{B}) = \frac{b_1 + b_2 + b_3 + b_4}{4}$$

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11})$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11})$ be two hendecagonal fuzzy numbers then

$$r(\tilde{A}) = \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11}}{11}$$

and

$$r(\tilde{B}) = \frac{b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8 + b_9 + b_{10} + b_{11}}{11}$$

5 Fuzzy assignment problem

In this section, the mathematical formulation of fuzzy assignment problem has given and a direct penalty approach is proposed to solve FAP for optimal solution. The method is applicable for all optimisation problems.

5.1 Formulation of fuzzy assignment problem

Let there be m tasks and n workers. If $m = n$ then FAP is balanced otherwise unbalanced. Let $\tilde{C}_{ij}$ be the cost of assigning i^{th} worker to the j^{th} task and the uncertainty in cost is here represented as trapezoidal fuzzy number and Hendecagonal fuzzy numbers. Let X_{ij} be the decision variable. Then the fuzzy assignment problem is mathematically stated as follows:

$$\text{Minimise } z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} x_{ij},$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = 1, \qquad j = 1, 2, \ldots, n$$

$$x_{ij} = \begin{cases} 1, & \text{if the } i^{\text{th}} \text{ person is assigned to } j^{\text{th}} \text{ job} \\ 0, & \text{otherwise} \end{cases}$$

5.2 Algorithm for the proposed method

Step 1 Form the fuzzy cost table for the given problem. The number of rows need not be equal to the number of columns as the method.

Step 2 Calculate the respective ranking value of each trapezoidal fuzzy cost and hendecagonal fuzzy cost and form it as a separate table.

Step 3 For each row and column, find the difference between minimum and next minimum of the ranking value for the fuzzy cost (penalty). In the extended row and column, list out the penalties in the same table.

Step 4 Locate the maximum penalty and make assignment to the cell having minimum fuzzy cost ranking value in the row or column that continues in the same manner until all the assignment are made.

Step 5 Write the ranking value of the assigned fuzzy cost at the last row of the table and subtract assigned cost from each element in the column.

Step 6 In the resultant table identify the negative penalty and form 2×2 matrixes in such a way that one corner contains negative penalty and the remaining two corners are allocated with assigned fuzzy costs. Find the sum of unassigned diagonal $D_{ij} < 0$ and is applicable both balanced and unbalanced FAP. Identify the most negative value and exchange the assigned cell. Continue the process until all negative penalties are resolved.

Step 7 Based on the optimal fuzzy assignment obtained calculate the total optimal fuzzy cost.

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} x_{ij}.$$

6 Numerical Example

In this section, numerical examples are given to illustrate the proposed method and it is shown that the proposed method offers an effective way of handling FAP.

6.1 Example 1

A manufacturing company manufactures a certain type of spare parts with five different machines. The company official has to execute five jobs with three machines. The information about the cost of assignment is imprecise and here Trapezoidal fuzzy number and Hendecagonal fuzzy number are used to represent the fuzzy cost. The problem is to find the optimal assignment so that the assignment minimises the total fuzzy cost.

Solution

Step 1 The fuzzy assignment problem is given in Tables 1 and 7.

Step 2 Calculate the ranking value of each fuzzy cost $\tilde{C}_{ij}$ which is given in Tables 2 and 8.

Step 3 For each row and column, find the difference between minimum and next minimum of the ranking value of the fuzzy cost (penalty). In the extended row and column, list out the penalties of Tables 2 and 8.

Step 4 Locate the maximum penalty and make assignment to the cell having minimum fuzzy cost ranking value in the row or column which is assigned in Tables 3 and 9.

Step 5 The assigned fuzzy cost ranking values are given at the bottom of Tables 4 and 10 and subtract every element by the assigned cost of ranking values.

Step 6 The values of D_{ij} using negative penalties are calculated in Tables 5 and 11. Here the most negative value is –1 which corresponds to the first element of the first row, so make assignment to that cell and interchange the third row assignment. The resultant table has given in Tables 6 and 12, which is optimal assignment since all $D_{ij} > 0$ for the resultant table fuzzy cost.

Step 7 The optimal assignment is $J1 \rightarrow M1$, $J2 \rightarrow M4$, $J3 \rightarrow M5$, $J4 \rightarrow M3$, $J5 \rightarrow M2$

The total optimal assignment cost for HDFN is (19, 29, 36, 48, 62, 93, 111, 124, 139, 148, 156; 0.2, 0.5, 0.9) and ranking value is 87.5.

TrapFN is (19, 29, 148, 156: 0.9) and ranking value is 88.

HDFN is more optimal than TrapFN.

Table 1 Fuzzy assignment problem with trapezoidal fuzzy cost of example 1

		MACHINES				
		M1	*M2*	*M3*	*M4*	*M5*
JOBS	J1	(3,5,28,30;*0.9*)	(5,8,38,39;*1*)	(4,5,22,24;*0.8*)	(3,6,38,39;*1*)	(6,10,41,43;*1*)
	J2	(5,6,23,25;*0.8*)	(3,4,21,23;*0.8*)	(4,6,,29,31;*0.9*)	(2,3,20,22;*0.9*)	(3,6,36,37;*0.9*)
	J3	(2,3,34,35;*0.7*)	(3,6,38,39;*1*)	(3,6,36,37;*0.9*)	(4,6,29,31;*0.9*)	(3,6,38,39;*1*)
	J4	(7,11,42,44;*0.9*)	(10,14,45,47;*0.9*)	(5,8,38,39;*1*)	(13,18,49,51;*1*)	(12,16,47,49;*0.9*)
	J5	(2,5,35,36;*0.9*)	(6,7,24,26;*0.9*)	(4,6,29,31;*0.9*)	(3,5,28,30;*0.9*)	(2,5,35,36;*0.9*)

Table 2 Ranking value of Table 1 and its penalty value

		MACHINES					*Penalty*
		M1	*M2*	*M3*	*M4*	*M5*	
JOBS	J1	$r(\tilde{c}_{11}) = 16.5$	$r(\tilde{c}_{12}) = 22.5$	$r(\tilde{c}_{13}) = 13.75$	$r(\tilde{c}_{14}) = 21.5$	$r(\tilde{c}_{15}) = 25$	2.75 2.75 2.75 -
	J2	$r(\tilde{c}_{21}) = 14.75$	$r(\tilde{c}_{22}) = 12.75$	$r(\tilde{c}_{23}) = 17.5$	$r(\tilde{c}_{24}) = 11.75$	$r(\tilde{c}_{25}) = 20.5$	1 - - -
	J3	$r(\tilde{c}_{31}) = 18.5$	$r(\tilde{c}_{32}) = 21.5$	$r(\tilde{c}_{33}) = 20.5$	$r(\tilde{c}_{34}) = 17.5$	$r(\tilde{c}_{35}) = 21.5$	1 2 2 3
	J4	$r(\tilde{c}_{41}) = 26$	$r(\tilde{c}_{42}) = 29$	$r(\tilde{c}_{43}) = 22.5$	$r(\tilde{c}_{44}) = 32.75$	$r(\tilde{c}_{45}) = 31$	3.5 3.5 3.5 5
	J5	$r(\tilde{c}_{51}) = 19.5$	$r(\tilde{c}_{52}) = 15.75$	$r(\tilde{c}_{53}) = 17.5$	$r(\tilde{c}_{54}) = 16.5$	$r(\tilde{c}_{55}) = 19.5$	1 1.75 - -
Penalty		1.75 2 2 7.5	3 5.75 - -	3.75 3.75 6.75 -	4.75 - - -	1 2 3.5 9.5	

Table 3 Assigned fuzzy cost using maximum penalty with minimum fuzzy cost

		MACHINES				
		M1	*M2*	*M3*	*M4*	*M5*
JOBS	J1	(3,5,28,30;*0.9*)	(5,8,38,39;*1*)	(4,5,22,24;*0.8*)	(3,6,38,39;*1*)	(6,10,41,43;*1*)
	J2	(5,6,23,25;*0.8*)	(3,4,21,23;*0.8*)	(4,6,,29,31;*0.9*)	(2,3,20,22;*0.9*)	(3,6,36,37;*0.9*)
	J3	(2,3,34,35;*0.7*)	(3,6,38,39;*1*)	(3,6,36,37;*0.9*)	(4,6,29,31;*0.9*)	(3,6,38,39;*1*)
	J4	(7,11,42,44;*0.9*)	(10,14,45,47;*0.9*)	(5,8,38,39;*1*)	(13,18,49,51;*1*)	(12,16,47,49;*0.9*)
	J5	(2,5,35,36;*0.9*)	(6,7,24,26;*0.9*)	(4,6,29,31;*0.9*)	(3,5,28,30;*0.9*)	(2,5,35,36;*0.9*)

Table 4 Negative penalty calculation

		MACHINES				
		M1	*M2*	*M3*	*M4*	*M5*
JOBS	J1	–9.5	6.75	0	9.75	3.5
	J2	–11.25	–3	3.75	0	–1
	J3	–7.5	5.75	6.75	5.75	0
	J4	0	13.25	8.75	21	9.5
	J5	–6.5	0	3.75	4.75	–2
Assignment		*26*	*15.75*	*13.75*	*11.75*	*21.5*

Table 5 D_{ij} calculation

D_{11}	–9.5	0	–0.75
	0	8.75	
D_{21}	–11.25	0	9.75
	0	21	
D_{22}	–3	0	1.75
	0	4.75	
D_{25}	0	–1	4.75
	5.75	0	
D_{31}	–7.5	0	2
	0	9.5	
D_{51}	0	13.25	6.75
	–6.5	0	
D_{55}	5.75	0	3.75
	0	–2	

Table 6 Fuzzy cost assignment

		MACHINES				
		M1	*M2*	*M3*	*M4*	*M5*
JOBS	J1	(3,5,28,30;*0.9*)	(5,8,38,39;*1*)	(4,5,22,24;*0.8*)	(3,6,38,39;*1*)	(6,10,41,43;*1*)
	J2	(5,6,23,25;*0.8*)	(3,4,21,23;*0.8*)	(4,6,,29,31;*0.9*)	(2,3,20,22;*0.9*)	(3,6,36,37;*0.9*)
	J3	(2,3,34,35;*0.7*)	(3,6,38,39;*1*)	(3,6,36,37;*0.9*)	(4,6,29,31;*0.9*)	(3,6,38,39;*1*)
	J4	(7,11,42,44;*0.9*)	(10,14,45,47;*0.9*)	(5,8,38,39;*1*)	(13,18,49,51;*1*)	(12,16,47,49;*0.9*)
	J5	(2,5,35,36;*0.9*)	(6,7,24,26;*0.9*)	(4,6,29,31;*0.9*)	(3,5,28,30;*0.9*)	(2,5,35,36;*0.9*)

Table 7 Fuzzy assignment problem with hendecagonal fuzzy cost of example 1

		MACHINES				
		M1	*M2*	*M3*	*M4*	*M5*
JOBS	J1	(3,5,6,10,14,17, 20,23,26,28,30; *0.2,0.5,0.9*)	(5,8,9,11,14,24, 30,34,36,38,39; *0.4,0.8,1*)	(4,5,6,8,11,15,1 6,18,20,22,24; *0.5,0.6,0.8*)	(3,6,8,10,13,24, 29,32,35,38,39; *0.4,0.7,1*)	(6,10,12,14,20, 27,32,37,39,41, 43;*0.3,0.8,1*)
	J2	(5,6,7,9,12,15, 17,19,22,23,25; *0.4,0.6,0.8*)	(3,4,5,7,10,12, 15,18,20,21,23; *0.4,0.6,0.8*)	(4,6,7,10,14,20, 21,24,27,29,31; *0.2,0.6,0.9*)	(2,3,4,6,8,12,15, 16,19,20,22; *0.2,0.6,0.9*)	(3,6,7,9,12,22, 28,32,34,36,37; *0.3,0.6,0.9*)
	J3	(2,3,5,8,11,19, 25,30,32,34,35; *0.3,0.5,0.7*)	(3,6,8,10,13,24, 29,32,35,38,39; *0.4,0.7,1*)	(3,6,7,9,12,22, 28,32,34,36,37; *0.3,0.6,0.9*)	(4,6,7,10,14,20, 21,24,27,29,31; *0.2,0.6,0.9*)	(3,6,8,10,13,24, 29,32,35,38,39; *0.4,0.7,1*)
	J4	(7,11,14,17,19, 26,34,38,40,42, 44;*0.4,0.8,0.9*)	(10,14,17,20,22, 28,38,41,43,45, 47;*0.3,0.8,0.9*)	(5,8,9,11,14,24, 30,34,36,38,39; *0.4,0.8,1*)	(13,18,22,24,26, 33,41,45,47,49, 51;*0.3,0.7,1*)	(12,16,19,22,26, 31,37,43,45,47, 49;*0.2,0.7,0.9*)
	J5	(2,5,6,9,12,19, 27,31,33,35,36; *0.2,0.6,0.9*)	(6,7,9,11,13,16, 17,19,23,24,26; *0.3,0.7,0.9*)	(4,6,7,10,14,20, 21,24,27,29,31; *0.2,0.6,0.9*)	(3,5,6,10,14,17, 20,23,26,28,30; *0.2,0.5,0.9*)	(2,5,6,9,12,19, 27,31,33,35,36; *0.2,0.6,0.9*)

Table 8 Ranking value of Table 7 and its penalty value

		MACHINES					*Penalty*
		M1	*M2*	*M3*	*M4*	*M5*	
JOBS	J1	$r(\tilde{c}_{11})=16.5$	$r(\tilde{c}_{12})=22.5$	$r(\tilde{c}_{13})=13.5$	$r(\tilde{c}_{14})=21.5$	$r(\tilde{c}_{14})=25.5$	3 3 3 -
	J2	$r(\tilde{c}_{21})=14.5$	$r(\tilde{c}_{22})=12.5$	$r(\tilde{c}_{23})=17.5$	$r(\tilde{c}_{24})=11.5$	$r(\tilde{c}_{25})=20.5$	1 - - -
	J3	$r(\tilde{c}_{31})=18.5$	$r(\tilde{c}_{32})=21.5$	$r(\tilde{c}_{33})=20.5$	$r(c_{34})=17.5$	$r(\tilde{c}_{35})=21.5$	1 2 2 3
	J4	$r(\tilde{c}_{41})=26.5$	$r(\tilde{c}_{42})=29.5$	$r(\tilde{c}_{43})=22.5$	$r(\tilde{c}_{44})=33.5$	$r(\tilde{c}_{45})=31.5$	4 4 4 5
	J5	$r(\tilde{c}_{51})=19.5$	$r(\tilde{c}_{52})=15.5$	$r(\tilde{c}_{53})=17.5$	$r(\tilde{c}_{54})=16.5$	$r(\tilde{c}_{55})=19.5$	1 2 - -
Penalty		2 2 2 8	3 *6* - -	4 4 *7* -	*5* - - -	2 3 4 *10*	

Table 9 Assigned fuzzy cost using maximum penalty with minimum fuzzy cost

		MACHINES				
		M1	*M2*	*M3*	*M4*	*M5*
JOBS	J1	(3,5,6,10,14,17, 20,23,26,28,30; *0.2,0.5,0.9*)	(5,8,9,11,14,24, 30,34,36,38,39; *0.4,0.8,1*)	(4,5,6,8,11,15, 16,18,20,22,24; *0.5,0.6,0.8*)	(3,6,8,10,13,24, 29,32,35,38,39; *0.4,0.7,1*)	(6,10,12,14,20, 27,32,37,39,41, 43;*0.3,0.8,1*)
	J2	(5,6,7,9,12,15, 17,19,22,23,25; *0.4,0.6,0.8*)	(3,4,5,7,10,12, 15,18,20,21,23; *0.4,0.6,0.8*)	(4,6,7,10,14,20, 21,24,27,29,31; *0.2,0.6,0.9*)	(2,3,4,6,8,12,15, 16,19,20,22; *0.2,0.6,0.9*)	(3,6,7,9,12,22, 28,32,34,36,37; *0.3,0.6,0.9*)
	J3	(2,3,5,8,11,19, 25,30,32,34,35; *0.3,0.5,0.7*)	(3,6,8,10,13,24, 29,32,35,38,39; *0.4,0.7,1*)	(3,6,7,9,12,22, 28,32,34,36,37; *0.3,0.6,0.9*)	(4,6,7,10,14,20, 21,24,27,29,31; *0.2,0.6,0.9*)	(3,6,8,10,13,24, 29,32,35,38,39; *0.4,0.7,1*)
	J4	(7,11,14,17,19, 26,34,38,40,42, 44;*0.4,0.8,0.9*)	(10,14,17,20,22, 28,38,41,43,45, 47;*0.3,0.8,0.9*)	(5,8,9,11,14,24, 30,34,36,38,39; *0.4,0.8,1*)	(13,18,22,24,26, 33,41,45,47,49, 51;*0.3,0.7,1*)	(12,16,19,22,26, 31,37,43,45,47, 49;*0.2,0.7,0.9*)
	J5	(2,5,6,9,12,19, 27,31,33,35,36; *0.2,0.6,0.9*)	(6,7,9,11,13,16, 17,19,23,24,26; *0.3,0.7,0.9*)	(4,6,7,10,14,20, 21,24,27,29,31; *0.2,0.6,0.9*)	(3,5,6,10,14,17, 20,23,26,28,30; *0.2,0.5,0.9*)	(2,5,6,9,12,19, 27,31,33,35,36; *0.2,0.6,0.9*)

Table 10 Negative penalty calculation

		MACHINES				
		M1	*M2*	*M3*	*M4*	*M5*
JOBS	J1	–10	7	0	10	4
	J2	–12	–3	4	0	–1
	J3	–8	6	7	6	0
	J4	0	14	9	22	10
	J5	–7	0	4	5	-3
Assignment		*25*	*26.5*	*15.5*	*13.5*	*11.5*

Table 11 D_{ij} calculation

D_{11}	–10	0	–1
	0	9	
D_{21}	–12	0	10
	0	22	
D_{22}	–3	0	2
	0	5	
D_{25}	0	–1	5
	6	0	
D_{31}	–8	0	2
	0	10	
D_{51}	0	14	7
	–7	0	
D_{55}	6	0	3
	0	-3	

Table 12 Fuzzy cost assignment

		MACHINES				
		M1	*M2*	*M3*	*M4*	*M5*
JOBS	J1	(3,5,6,10,14,17, 20,23,26,28,30; *0.2,0.5,0.9*)	(5,8,9,11,14,24, 30,34,36,38,39; *0.4,0.8,1*)	(4,5,6,8,11,15, 16,18,20,22,24; *0.5,0.6,0.8*)	(3,6,8,10,13,24, 29,32,35,38,39; *0.4,0.7,1*)	(6,10,12,14,20, 27,32,37,39,41, 43;*0.3,0.8,1*)
	J2	(5,6,7,9,12,15, 17,19,22,23,25; *0.4,0.6,0.8*)	(3,4,5,7,10,12, 15,18,20,21,23; *0.4,0.6,0.8*)	(4,6,7,10,14,20, 21,24,27,29,31; *0.2,0.6,0.9*)	(2,3,4,6,8,12,15, 16,19,20,22; *0.2,0.6,0.9*)	(3,6,7,9,12,22, 28,32,34,36,37; *0.3,0.6,0.9*)
	J3	(2,3,5,8,11,19, 25,30,32,34,35; *0.3,0.5,0.7*)	(3,6,8,10,13,24, 29,32,35,38,39; *0.4,0.7,1*)	(3,6,7,9,12,22, 28,32,34,36,37; *0.3,0.6,0.9*)	(4,6,7,10,14,20, 21,24,27,29,31; *0.2,0.6,0.9*)	(3,6,8,10,13,24, 29,32,35,38,39; *0.4,0.7,1*)
	J4	(7,11,14,17,19, 26,34,38,40,42, 44;*0.4,0.8,0.9*)	(10,14,17,20,22, 28,38,41,43,45, 47;*0.3,0.8,0.9*)	(5,8,9,11,14,24, 30,34,36,38,39; *0.4,0.8,1*)	(13,18,22,24,26, 33,41,45,47,49, 51;*0.3,0.7,1*)	(12,16,19,22,26, 31,37,43,45,47, 49;*0.2,0.7,0.9*)
	J5	(2,5,6,9,12,19, 27,31,33,35,36; *0.2,0.6,0.9*)	(6,7,9,11,13,16, 17,19,23,24,26; *0.3,0.7,0.9*)	(4,6,7,10,14,20, 21,24,27,29,31; *0.2,0.6,0.9*)	(3,5,6,10,14,17, 20,23,26,28,30; *0.2,0.5,0.9*)	(2,5,6,9,12,19, 27,31,33,35,36; *0.2,0.6,0.9*)

7 Conclusions and future enhancement

In this paper, a direct penalty approach has been developed for solving FAP with Trapezoidal fuzzy cost and Hendecagonal fuzzy cost. The optimal solution to FAP obtained by the proposed method is same as that of the optimal solution obtained by the existing method for HDFN optimisation. The proposed method is simpler, easy to understand and it takes few steps for obtaining the fuzzy optimal solution. Numerical example shows that the proposed method offers an effective tool for handling the fuzzy assignment problem for existing fuzzy number. In future, the generalisation of hendecagonal fuzzy number can be developed with its similarity measure for decision analysis.

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