A game theory approach towards real-time pricing in future electricity markets

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Abstract: Pricing can be challenging when it comes to energy, as energy pricing models need to consider various different parameters, interactions and decisions. This study aims at introducing a real-time pricing algorithm which attempts to model the interaction between companies and customers as a Stackelberg game, the competition among companies as a non-cooperative game, while the decision-making process of the customers is represented by an evolutionary process. The customer's knowledge of the company production capacities and the companies' knowledge of the customer utility functions are the key assumptions of the algorithm. For the purpose of this research, customers are also assumed to have the right to choose any company that they desire in an hourly basis.

Keywords: pricing model; Stackelberg game; non-cooperative game; evolutionary process.

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1 Introduction

Pricing has been a major concern for electricity companies over time and despite hard efforts, no universally accepted optimal pricing strategy has been proposed, due to the complexity of the issue. Factors such as demand prediction, estimation of production costs, transportation costs, as well as the unknown incentives of consumers and companies result in high uncertainty. Additionally, external parameters like regulations and the different structure of each specific market also add to the complexity of the issue. Plenty of pricing models have been proposed, (Wei et al., 2015; Belgana et al., 2015; Hobbs and Kelly, 1992; Jeon et al., 2018; Chen et al., 2018; Abaza and Azmy, 2013; Yu et al., 2015; Ma et al., 2017; Han et al., 2017; Cavraro and Badia, 2013) to name just a few, but each one of them addresses only one of the issues, set under specific assumptions.

The main aim of this research is the development and implementation of pricing algorithms that could be potentially adopted by the electricity market as a new pricing technique using a game theory approach. Specifically, by expanding the already existing literature on Stackelberg game theory (Maharjan et al., 2013; Chai et al., 2014; Dai et al., 2017) a new pricing model under real-time pricing assumptions is proposed. For this study, the interaction between companies and customers is modelled as a Stackelberg game, the competition among companies as a non-cooperative game, while the decision-making process of the customers is represented by an evolutionary process. The customer's knowledge of the company's production capacities and the companies' knowledge of the customer's utility functions are the key assumptions of the algorithm. For the purpose of this research it is also assumed that customers have the right to choose any company they desire in an hourly basis. Finally, it is assumed that all the producers own one unit producing energy and that the overall costs of each producer are equal to the production cost of their power plants.

Furthermore, advanced models that take aspects such as customer profiles into account are also included, and for that purpose, demographics are considered. Moreover, in order to verify the practical value of these models, various simulations have been made. Before the implementation of the simulations, the parameters of the theoretical model have been chosen by hyperparameter optimisation methods.

Lastly, the methodology is implemented in a hypothetical scenario, as a theoretical game on an electricity market, since real-time pricing has not yet been widely used as a pricing scheme. In the following sections, the relevant literature and the said methodology are presented along with the characteristics of the market that supports the future scenario and the results deriving from the proposed scheme.

2 Existing Stackelberg and evolutionary game theory literature

A Stackelberg game (Etro, 2013) is a strategic game in which the players make their decisions sequentially. Therefore, the decisions of each player depend on the decisions the previous players make. The number of players can vary, but most of the times two players participate, the 'leader' and the 'follower'. When it comes to electricity pricing, the players are the companies and the customers. Depending on the exact nature of the game, either the first or the last player enjoys an advantage, which however diminishes as the number of rounds increases. The main advantage of game theory models over traditional approaches such as Bertrand, Cournot and Edgeworth is that the former enables a more accurate and dynamic modelling approach to the problem, without the need of making bold assumptions.

In evolutionary game theory (Ortmann and Weibull, 1997), there is the assumption that during the first decision round, a population of N beings that belong to K species exists. Each species has a particular 'fitness', which is determined by a set of rules. After the first decision round is over, the species with the highest fitness acquire more members, while the others lose members. N beings in total will still exist, but the distribution among the K species is different. This process continues, until the set termination condition is satisfied. In this case, the beings are customers and the species are companies.

A variety of related Stackelberg game models have already been proposed (Dai and Gao, 2015a; Chen et al., 2011; Stamtsis and Doukas, 2018; Dai and Gao, 2014; Chen et al., 2012; Maloney, 2001), examining concepts such as consumer utility, company profits, company probability, net utility, and average utility. These concepts are briefly described below.

The models in Maharjan et al. (2013), Chai et al. (2014), Dai and Gao (2015a), Dai et al. (2017), Fujiwara-Greve (2015) and Chen et al. (2012) examine the case of only one operating company ($K = 1$). They cannot be implemented in the problem of this study, but the concepts of consumer and company utility functions they propose were considered.

According Maharjan et al. (2013), companies are supposed to know the behaviour of the consumers. Additionally, the consumers have the right to choose more than one producer at the same time. Furthermore, the production costs are completely ignored, as it is assumed that companies produce as much energy as possible, while each consumer has a maximum disposable income. The customer utility and the company utility produced by Maharjan et al. (2013), calculate the optimal demand and the optimal supply by addressing an optimisation problem under constraints.

This has received criticism by Dai et al. (2017) as it does not consider certain aspects, for instance that utility increases only up to a point. As for Chai et al. (2014), customers have a specific utility function and a net utility, and these are used to measure their fitness in the evolutionary process. The companies strive to maximise their profits by adjusting their prices and production, depending on the relationship between the current production quantity and their demand.

The model of Dai et al. (2017) is much similar to that of Chai et al. (2014). The main difference is that instead of trying to optimise the production quantity, it is assumed that the companies have already produced/bought a certain amount of energy, so the only problem that the companies face is the choice of the optimal price.

However, it is taken for granted that the optimal strategy for each company is to have a demand equal to its available energy. Dai et al. (2017) claim that when the demand exceeds this quantity, prices should be increased and when the opposite holds, prices should be decreased.

3 The proposed model

3.1 Contribution of the proposed model

In contrast to the models of Dai et al. (2017) and Chai et al. (2014), the model proposed in this study does not rely on the demand-supply equilibrium. The prices are in this case adjusted depending on the income. The optimal supply quantity will be indirectly specified, as a function of the optimal price. That way, there is no longer need for the assumption that the available energy is already known, as in Chai et al. (2014).

3.2 Theoretical description of the proposed model

Just like the models analysed in the previous section, the behaviour of the customers has been modelled as an evolutionary process while the company's behaviour has been modelled as a non-cooperative game. A Stackelberg game between the customers and the companies exists.

To be more precise, the main idea is that:

- There are two players: the customers and the companies.
- The companies and the customers wish to maximise their utility. The companies strive to maximise their profit, while the customers desire a value-for-money option.
- The companies announce their prices, the customers decide depending on the prices, then the companies adjust their prices depending on the demand. This process continues, until no change is left to be made.

It is important to narrow down the differences among decision makers' thinking, reasoning, representation and computing (Doukas, 2013). In this respect, we can argue that this approach, each customer is initially assigned to one company. Once the companies announce their initial prices to the customers, the customers estimate their net utilities. Depending on these utilities, some customers decide to change company, if their net utility is lower than the average. Then the customers estimate their net utilities once again and change companies if needed. This process is repeated until the average net utility rises to a satisfactory level or until many iterations have occurred. Then the evolutionary process comes to an end and it is claimed that the costumers have made their decisions. After that, the companies find out the customers' decisions and adjust their prices one by one, by examining their potential profits in case of a slight price decrease, a slight price increase, or in case the price is stable. It is assumed that the companies can predict the behaviour of the customers in advance. Each time the companies modify their prices, the evolutionary process of the customers is repeated. Once no company has an incentive to change its price, that means that an equilibrium has been reached.

The theoretical contribution of this new model can be summarised to the following points:

- The non-cooperation game among the companies is different. Instead of checking if demand is higher than supply or supply is lower than demand, three other calculations are made:
  1. Val_0 = income with current price
  2. Val_1 = income with slightly higher price
  3. Val_2 = income with slightly lower price.

  After calculating Val_0, Val_1 and Val_2, companies compare the three options and choose the one that yields the highest income. If Val_0 is the maximum value, the price remains unchanged. If Val_1 is the maximum value, the price increases by e_2. If Val_2 is the maximum value, the price decreases by e_2.
- The convergence condition is also different. The game ends when no company has modified its price. By focusing on income instead of the demand-supply equilibrium, there is no longer the need to have a predefined production quantity. Additionally, in the proposed game, instead of simultaneous price modifications, companies modify their prices one by one and the decision of each company depends not only on the consumers but also on the previous decisions of the rest of the companies. This modification makes the game more realistic.

3.2.1 Consumer analysis

3.2.1.1 Price and demand (p and q)

Let $p_1, p_2, ..., p_k$ be the prices of the companies and $q_{11}, q_{12}, ..., q_{1k}, q_{21}, q_{22}, ..., q_{2k}, q_{n1}, q_{n2}, ..., q_{nk}$ be the demand of consumer *i* in case this consumer chooses company *j*. Each consumer must buy all the desired quantity from a single company for every particular hour. It is not possible to have many producers within this particular hour.

Utility and welfare functions (U and W)

The same functions proposed by Dai et al. (2017) have been used, as they adequately describe the fact that consumers desire as much energy as possible, but only up to a specific quantity. Moreover, the desire to obtain energy is greater that the dissatisfaction from paying money, provided that the right parameter values have been chosen.

Regarding the utility function:

$$U_i = \begin{cases} b_i.q_{ij} - \dfrac{a_i}{2}.\left(q_{ij}\right)^2 & \text{for} \quad q_{ij} \le \dfrac{b_i}{a_i} \\ \dfrac{b_i^2}{(2.a_i)} & \text{for} \quad q_{ij} > \dfrac{b_i}{a_i} \end{cases}$$

Note that the function is continuous. Therefore, it can also be claimed that:

$$U_i = b_i.q_{ij} - \frac{a_i}{2}.\left(q_{ij}\right)^2, \; q_{ij\min} \le q_{ij} \le q_{ij\max}$$

where $q_{ij\max} = b_i / a_i$ and $q_{ij\min}$ a chosen minimum.

Concerning welfare, there is:

$$W_i = U_i - p_j . q_{ij}$$

$$W_i = b_i . q_{ij} - \frac{a_i}{2} . (q_{ij})^2 - p_j * q_{ij}, \; q_{ij\min} \le q_{ij} \le q_{ij\max}$$

where j is the company the consumer chose.

Parameters a and b are unique for each customer but differ even for the same customer per hour, depending on factors such as weather conditions.

3.2.1.2 Optimal consumer demand (Q)

By finding the maximum of the previous function, we get:

$$Q_{ij} = \begin{cases} q_{ij\min} & \text{for } \frac{(b_i - p_j)}{a_i} \le q_{ij\min} \\ \frac{(b_i - p_j)}{a_i} & \text{for } q_{ij\min} \le \frac{(b_i - p_j)}{a_i} \le q_{ij\max} \\ q_{ij\max} & \text{for } q_{ij\max} \le \frac{(b_i - p_j)}{a_i} \end{cases}$$

where Q is the optimal demand for each consumer.

3.2.1.3 Company probability (pr)

The probability of each company j to be chosen by a consumer is pr_j.

It is known that: $\sum_{j=1}^{K} pr_j = 1$ and $0 \le pr_j \le 1$.

Note that the probability is the same for all customers.

The customers will be gradually able to revaluate these probabilities on their own, considering the prices of the companies as well as their own ability to deliver the requested amount of energy. The exact process will be further described below.

3.2.1.4 Total company demand and company production capacity (D κɑι power)

The total demand of company j is defined as:

$$D_j = pr_j . \sum_{i=1}^{N} Q_{ij}$$

and the production capacity as $Power_j$.

Companies are not always in position to provide the requested amount of energy. When $D_j > Power_j$, only a fraction $Power_j / D_j$, can be delivered to the customers. That means that in case the demand exceeds the production capacity, the consumers will only get a fraction of the energy they requested.

Consumer net utility

Net utility is the utility that consumers receive in total from a particular company. It is used in order to modify the probabilities in a way that will be described later. When the consumers as a total receive greater welfare from company A compared to company *B*, that means that consumers have an incentive to choose company *A* instead of *B*. Therefore, the probability of *A* should be higher and the probability of *B* lower in the next round. The usage of net utility and probabilities are necessary because otherwise all consumers would simply choose the cheapest company and the evolutionary process would never end.

The net utility that the customers receive from company j is:

If $D_j < Power_j$, company j can fully satisfy demand and therefore:

$$N_j = \sum_{i=1}^{N} W_i = \sum_{i=1}^{N}\left(b_i.Q_{ij} - \frac{a_i}{2}.\left(Q_{ij}\right)^2 - p_j.Q_{ij}\right), Power_j \geq D_j$$

$$N_j = \sum_{i=1}^{N}\left(Q_{ij}.\left(b_i - p_j\right) - \frac{a_i}{2}.\left(Q_{ij}\right)^2\right), Power_j \geq D_j$$

$$N_j = \sum_{i=1}^{N}\left(a_i.\left(Q_{ij}\right)^2 - \frac{a_i}{2}.\left(Q_{ij}\right)^2\right) = \sum_{i=1}^{N}\left(\frac{a_i}{2}.\left(Q_{ij}\right)^2\right), Power_j \geq D_j$$

If $D_j > Power_j$, then each customer receives only a fraction of $Power_j / D_j$ and therefore:

$$N_j = \sum_{i=1}^{N} W_i = \sum_{i=1}^{N}\left(b_i.Q_{ij}.ratio - \frac{a_i}{2}.\left(Q_{ij}.ratio\right)^2 - p_j.Q_{ij}.ratio\right), Power_j < D_j$$

$$N_j = \sum_{i=1}^{N}\left(Q_{ij}.ratio.\left(b_i - p_j\right) - \frac{a_i}{2}.\left(Q_{ij}.ratio\right)^2\right), Power_j < D_j$$

$$N_j = \sum_{i=1}^{N}\left(a_i.ratio.\left(Q_{ij}\right)^2 - \frac{a_i}{2}.\left(Q_{ij}.ratio\right)^2\right), Power_j < D_j$$

$$N_j = \left(ratio - \frac{ratio^2}{2}\right).\sum_{i=1}^{N}\left(a_i.\left(Q_{ij}\right)^2\right), Power_j < D_j$$

$$N_j = \left(\frac{Power_j}{D_j} - \frac{\left(\frac{Power_j}{D_j}\right)^2}{2}\right).\sum_{i=1}^{N}\left(a_i.\left(Q_{ij}\right)^2\right), Power_j < D_j$$

To sum up,

$$N_j = \begin{cases} \sum_{i=1}^{N} \left(\frac{a_i}{2} . (Q_{ij})^2 \right) & \text{for} \quad Power_j \geq D_j \\ \left(\frac{Power_j}{D_j} - \frac{\left(\frac{Power_j}{D_j} \right)^2}{2} \right) . \sum_{i=1}^{N} \left(a_i . (Q_{ij})^2 \right) & \text{for} \quad Power_j < D_j \end{cases}$$

3.2.1.5 Average net utility (N_{avg})

We define as average net utility:

$$N_{avg} = \sum_{j=1}^{K} \left(pr_j . N_j \right)$$

3.2.2 Consumer evolutionary process

How the consumers make their decisions is given below:

Step 1 Initialise pr_j and learn current p_j.

Step 2 For every customer i, for every company j calculate Q_{ij}.

Step 3 For every company j, calculate D_j.

Step 4 For every company j, calculate N_j.

Step 5 Calculate $N_{(avg)}$.

Step 6 If for every company j $|N_j - N_{(avg)}| < e_1$, then the process ends. Otherwise $pr_j = pr_j + sigma_1 * (N_j - N_{avg})$ is updated for all companies that do not satisfy the condition and we return to *Step 3*, e_1 and $sigma_1$ are positive values that represent the convergence limit and the convergence speed respectively.

3.2.3 Company analysis

The income of each company is $p_j * s_j$, with $s_j = \min(D_j, Power_j)$. The costs of each company are $s_j * c_j + A_j$, with c representing the variable production cost and A the fixed costs.

The cost function, for each producer is assumed to be equal to the production costs of their unit, thus resulting to a linear form. Also, each company desires to sell as much energy as possible, since *Marginal_Income* > *Marginal_Cost* for the usual prices.

The company profits are:

$$R_j = p_j . s_j - c_j . s_j - A_j$$

$$R_j = \begin{cases} (p_j - c_j) . pr_j . \sum_{i=1}^{N} \left(\frac{(b_i - p_j)}{a_i} \right) - A_j & \text{for} \quad Power_j \geq D_j \\ (p_j - c_j) . Power_j - A_j & \text{for} \quad Power_j < D_j \end{cases}$$

3.2.4 Company pricing algorithm

Step 1 Initial prices for every company are chosen followed by running the evolutionary process, so that the results can be checked.

Step 2 For every company j, the price is updated in the following way:

Step 2a The evolutionary process of the customers is run, by using the new prices of the previous companies, the current prices of the following companies and the current price of company j. $snew_j$ is calculated which is the new sale quantity for company j, and $Income_{oj} = p_{j.snew_j}$.

Step 2b The evolutionary process of the customers is ran by using the new prices of the previous companies, the current prices of the following companies and $p_j = p_j + e_2$ as price of company j. $snew_j$ is calculated which is the new sale quantity for company j, and $Income_{1j} = (p_j + e_2).snew_j$.

Step 2c The evolutionary process of the customers is also ran by using the new prices of the previous companies, the current prices of the following companies and $p_j = p_j - e_2$ as price of company j, $snew_j$ is calculated which is the new sale quantity for company j, and $Income_{2j} = (p_j - e_2).snew_j$.

Step 2d $income_0$, $income_1$ and $income_2$ are compared. If max ($income_0$, $income_1$, $income_2$) = $income_0$, then the price does not change. If max ($income_0$, $income_1$, $income_2$) = $income_1$, then $p_j = p_j + e_2$. If max ($income_0$, $income_1$, $income_2$) = $income_2$, then $p_j = p_j - e_2$.

Step 3 If any price was modified during Step 2, then Step 2 is revisited. Otherwise, the final prices are announced.

4 Choice of parameters – implementation

4.1 Problem description

During the theoretical presentation of the model section, various parameters such as the customer profiles, the company profiles, the production capacity, the initial probability distribution and parameters a, b, e and sigma were not specified. Another parameter that needs to be specified is q. Information about q is provided in the section below. Thus, in this section the process that was followed, in order to choose parameter values that are appropriate for the hypothetical market in the use case scenario, is described.

4.2 Choice of demand data

The validity of the dataset remains a challenge (Doukas et al., 2007), so the historical data of the first semester (January–June) of the year 2016 were chosen for our simulations. Therefore, there are (31 + 29 + 31 + 30 + 31 + 30) * 24 = 4,368 hours to examine. The main reason for choosing year 2016 is that that year was the last year that was included in the research of Tyralis et al. (2017a). Analytical data regarding demand, temperature and GDP are also provided in Tyralis et al. (2017b).

4.3 Customer and company profiles

4.3.1 Customer profiles

Regarding the customer profiles for the theoretical game that the methodology will be implemented on, it is going to be assumed that all the customers have the exact same behaviour. Such a simplification is going to allow as to reduce the complexity of the algorithm and take into account an average consumer that could be later on customised. Such a simplification does not impede with the main objective of this study and significantly reduces the complexity of the algorithm, from $O(C1 * C2 * K * N)$ to $O(C1 * C2 * K)$, where K is the number of companies, N the number of customers, $C1$ the number of the non-cooperative game iterations and $C2$ the number of the evolutionary process iterations.

4.3.2 Company profiles

For the proposed model to resemble a real electricity market where one of the main players with the largest market share is a public energy producer and supplier, eight participants are taken into account. One of them is a public energy producer and supplier, and six of them are private energy producers and supplier.

Apart from the seven participants mentioned above, for this theoretical use case, a number of small companies was also considered, without however holding a considerable market share. For the purpose of this study, all the small companies are represented by five companies with a small but considerable market share equal to the total market share. This iteration was made since there are several small companies in real electricity markets so the five random companies will be the 8th participant in running this algorithm. The participation of the small players in the analysis is important since without them the rest of the companies would appear overpowered and would not depict the actual situation taking place in several markets across Europe. Also, if the small companies were not grouped but were included separately, the analysis would get more complicated without adding any qualitative benefit.

4.4 Production capacities

Regarding the production capacities, it is worth mentioning that due to the assumed existing electricity pool, companies have access to more energy than they can produce on their own. This is something that happens for instance in the Greek electricity market. However, the way the marginal price is determined (the price should be at least equal to the highest of the production costs), buying energy from another company is considered unprofitable. Purchasing large quantities of energy from abroad is also not feasible most of the times. It is thus safe to assume that the available energy of each company is roughly equal to its production capacity.

For the use case of this study the production capacity is 17,528 MW. 12,760 MW are owned by the public energy producer that also holds the biggest market share, 1,200 MW by one of the private companies with the largest market share (second to the public corporation), 820 MW by another private company that holds the third biggest market share, 582 by the fourth player in terms of market share in the electricity market, and the rest is RES capacity (which enters the market by priority). The production capacity of the remaining companies is unknown, and they may even obtain energy solely from the electricity pool. In any case, the total remaining capacity was distributed to the remaining companies in accordance to their market share as presented in Table 1.

Table 1 Production capacity of the companies in use case electricity market

Companies	*Production capacity (MW)*	*Price (euro)*
Public corporation (largest market share in the market)	12,760	0.15
Private company 1 (largest market share among the private companies, 2nd in general)	1,200	0.092
Private company 2 (3rd in market share)	820	0.09
Private company 3 (4th in market share)	582	0.089
Private company 4	250	0.085
Private company 5	201	0.09
Private company 6	200	0.086
Random 1	75	0.084
Random 2	75	0.084
Random 3	100	0.084
Random 4	100	0.083
Random 5	100	0.083

4.5 Parameters e and sigma

Parameters *e* and sigma were chosen by trial and error method, before *a* and *b* were optimised. The run time of the program and the accuracy of the results were considered.

4.6 Optimisation of parameters a and b

4.6.1 The proposed method

If all customers have the same behaviour, the total demand for each hour is:

$$Q = \frac{(b - P)}{a}, Q_{\min} \leq Q \leq q_{\max}$$

Note that $1/a$ represents the price elasticity.

The independent operator provides demand predictions in an hourly rate for the following day. Taking advantage of that fact, by considering the hourly prediction as Q_{avg}. Then $Q_{avg} = (b - P_{avr}) / a$. By knowing Q_{avg}, we can express *b* as a function of *a*, P_{avg} and Q_{avg}, so only *a* and P_{avg} remains to be found.

In order to find optimal values for *a* and P_{avg}, it is possible to compare the initial market share that the models predict for various values of *a* and P_{avg} with the real marker shares. In that way, we will be able to choose the best values of *a* and P_{avg}.

There are many ways to measure the discrepancy between the calculated and the real market shares, one of them is the following error value:

If MS_j is the real market share of company j and ms_j the calculated initial market share of that company, then:

$$x_j = \begin{cases} \dfrac{ms_j}{MS_j} - 1 & \text{for} \quad MS_j \le ms_j \\ \dfrac{MS_j}{ms_j} - 1 & \text{for} \quad MS_j > ms_j \end{cases}$$

and

$$error = \sum_{j=1}^{K} \left(x_j\right)^2$$

With that choice, it is ensured that small companies will not have an initial market share that is a multiple or a submultiple of the respective real market share.

In order to systematically examine different values of a and P_{avg}, grid search was applied. Since $1/a$ represents price elasticity, $1/a$ receives values in the interval [10,000, 45,000], which means that a receives values in the interval [0.0000222, 0.0001]. The values of P_{avg}, belong to the interval [0.05, 0.15], considering the production costs and the maximum prices that were noticed. Grid search might have a reputation of being slow but in our case, the 'curse of dimensionality' does not hold, as we have only two dimensions (Keogh and Mueen, 2017). This problem is also 'embarrassingly parallel' (Régin et al., 2013), so the process can be speeded up by utilising parallel programming.

To be more precise, optimisation was achieved in two steps:

During the first step, the values of $1/a$ were increased by 5,000 each time, while the values of P_{avg}, were increased by 0.01. During the second step, the new intervals were [$1/a\_opt$ – 5,000, $1/a\_opt$ + 5,000] and [$Pavg\_opt$ – 0.005, $Pavg\_opt$ + 0.005], while the steps were 1,000 and 0.001 respectively.

4.7 The proposed model

For the purpose of this study the total population that receives energy from the electricity market of the proposed use case is about 10 million people. It is assumed that 6.5 million are active and make their own decisions since the rest are either children or dependent members within households. We assume that 60% remains loyal to the public provider.

Note that the goal of the proposed model is just to significantly improve the simple model, not to be absolutely realistic. By making the simple assumption that 60% of the consumers always remain loyal to the public corporation and only the rest 40% of the consumers follow the evolutionary process, we obtain our proposed model.

5 Simulation results

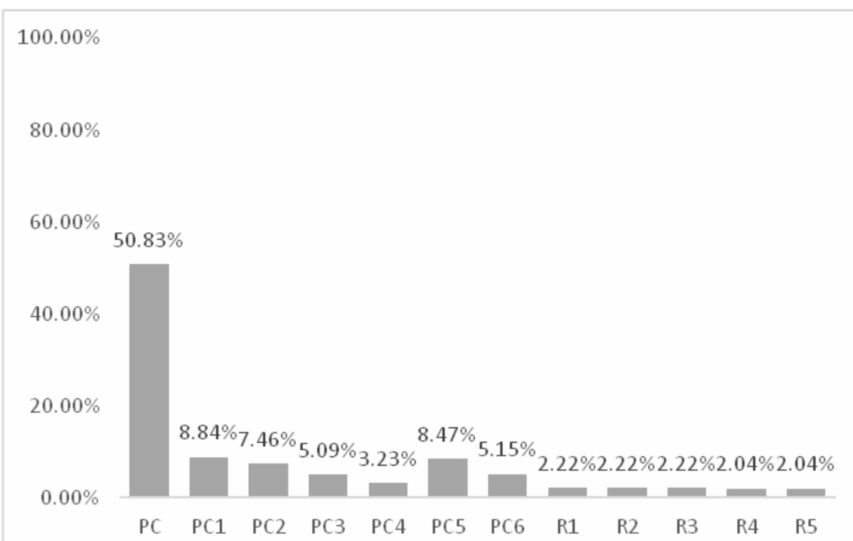
5.1 Purpose of the simulation

In order to test the results of our model, the profits that the simple model calculates under standard conditions are compared to the calculated profits under 'no retaliation' conditions and to the profits under 'total cooperation' conditions. No retaliation conditions mean that each company takes into account the possibility that its competitors will, likewise, decrease their prices, and only if it is still profitable, it proceeds in actually decreasing the price. Total cooperation conditions mean that all companies cooperate, in order to maximise their total profit. If our model is valid from a business perspective, then the total cooperation profits should be the highest and the standard condition profits the lowest. In addition, the calculated market shares of the first iteration (before any price is modified) for the simple and the proposed model were compared, in order to test if marketing aspects can be incorporated correctly.

5.1.1 Simple model, market share results

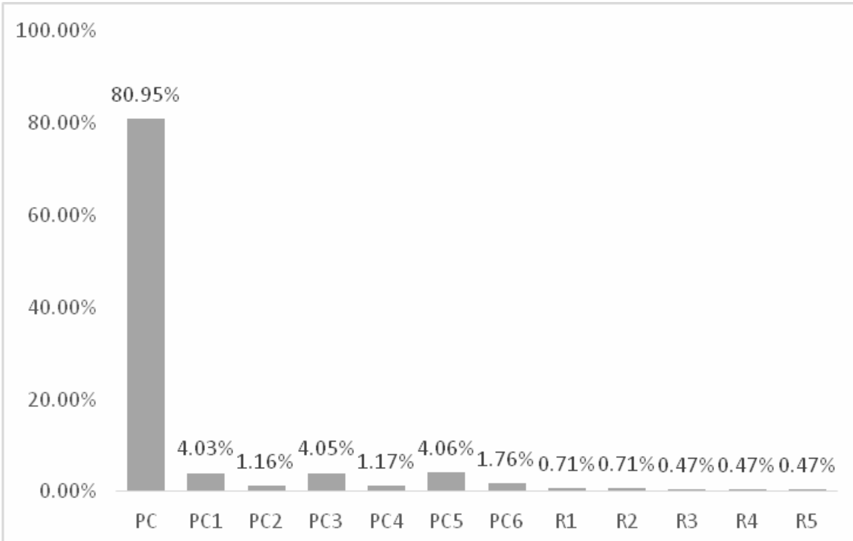
By adding the results for each particular hour of the semester, we obtained the total market shares, as they are presented in Figure 1, where PC is the public corporation, PC1-PC6 are the private companies, and R1-R5 are the smallest companies modelled as the 8th participant in the algorithm.

Figure 1 Initial market shares by running the algorithm



In Figure 2, we can see the different results that the proposed model yields. It is evident that the customer loyalty was incorporated correctly, as the market share of the public corporation significantly increased.

Figure 2 Market shares with the improved proposed model



5.1.2 Profits comparison

After the market shares, the profits are compared for the two scenarios.

Figure 3 Final incomes under standard conditions

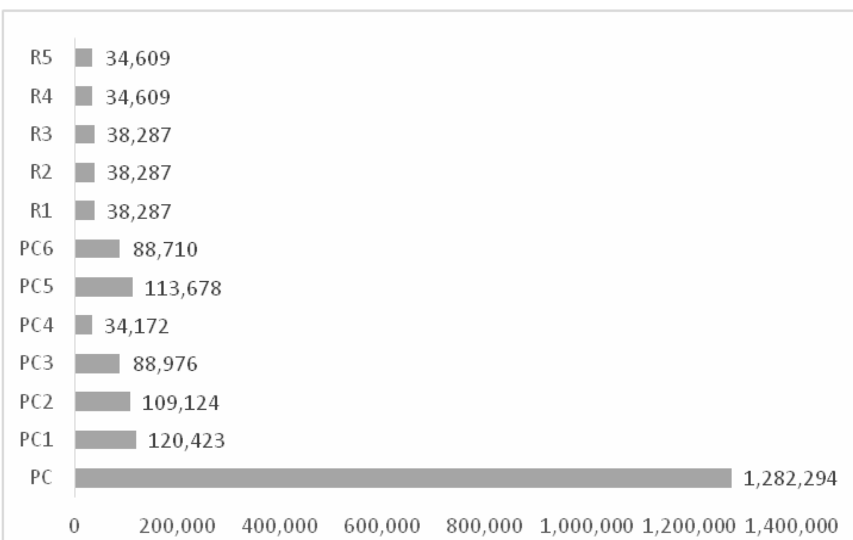


Figure 4 Final income under no retaliation conditions

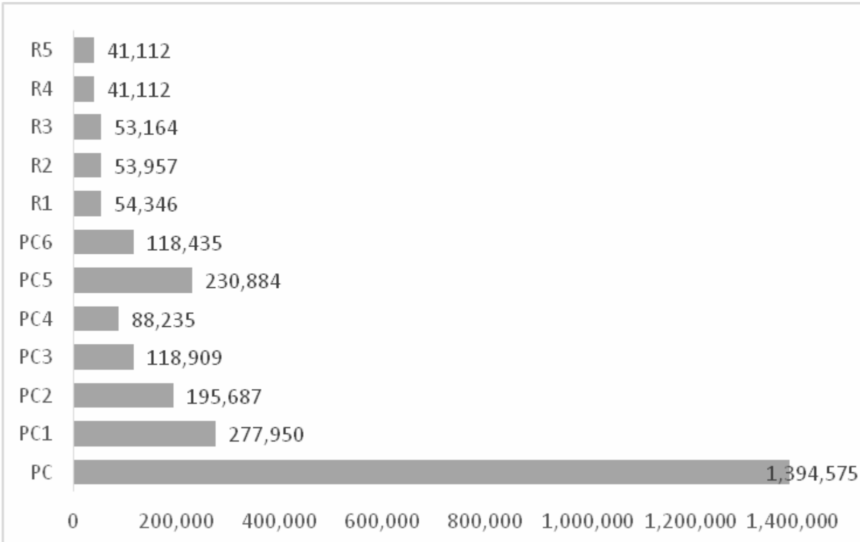
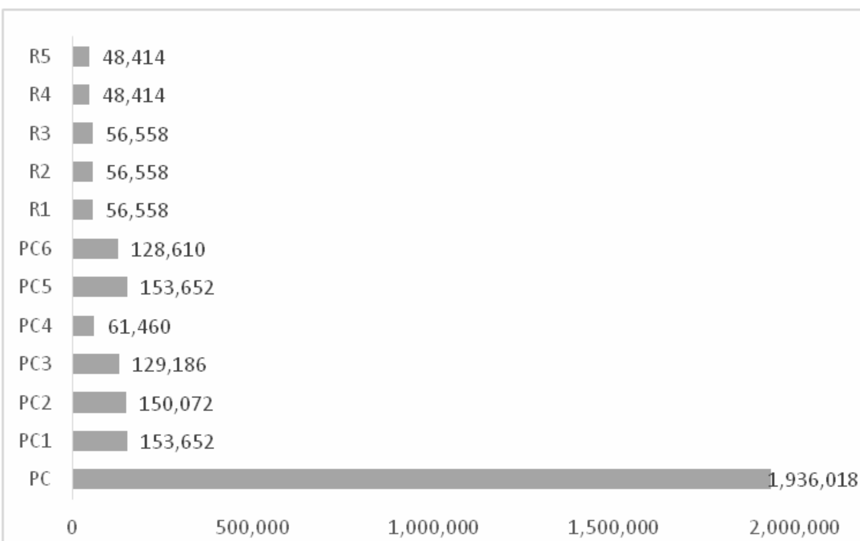


Figure 5 Final income under total cooperation conditions



It can be observed that the profits are greater for the non-retaliation scenario and even greater for the total cooperation (cartel) scenario. These results are expected, as by avoiding price wars, companies can increase their profits.

6 Conclusions

This study attempts to produce a new algorithm for the electricity market taking into account the preferences of both the customers and the producers and suppliers. In contrast to countries such as China where various pricing models already exist (Meng et al., 2018), similar models for European markets are scarce. Thus, this study explores a rather unexplored field. Furthermore, in contrast to most of the existing models that are limited to a strict game theoretical approach with no apparent business value, the proposed model includes aspects of marketing as well. As it can be observed in the previous section, even a very simple improvement can lead to more accurate results. With proper market segmentation and with more accurate demographic data, it could probably be implemented by companies. At a theoretical level, the proposed alternative Stackelberg models tackle some of the problems that the existing models face. Opportunities for further research include to explore the effect on the algorithm of the fact that the available energy of companies may surpass their production capacities. Additionally, some assumptions regarding the demographics and the market shares of the companies could customised to specific country needs for more accurate results. The real utility functions of customers can include numerous additional parameters such as the customer's willingness to change energy producer. Finally, the algorithm can be adjusted to the changes that are expected to be implemented in the market.

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