A fuzzy economic order quantity model with credibility induced demand and shortages

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Abstract: The purpose of this study is to explore the research for a decision-maker to minimise the total cost of the newly launched products of good quality and to control the inventory introduced in the market. This research provides the modification way of the stock to regulate the inventory management of the retailer. Due to the uncertainty in the marketplace, various parameters like a shortage, ordering, and deteriorating cost not fixed in nature. So, it is difficult to obtain the proper estimate of such costs. In this paper, we develop a traditional economic order quantity model with a fuzzy approach and provide a suitable structure to handle such uncertain parameters, improving the exactness and computational efficiency of the inventory system. The optimal solution of the problem is done with the method of graded mean integration. The model illustration is demonstrated through the appropriate numerical and sensitivity analysis.

Keywords: graded mean integration technique; credibility induced demand; procurement time; order quantity; deterioration; uncertainty; optimisation.

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1 Introduction

In general, inventory management indecisions may be depend on several relevant costs, the demand rate and lead-time. Although, in predictable inventory-models, uncertainties considered as randomness, which is handled by stochastic-techniques. Many decision-making problems become too complicated due to uncertainty in the form of vagueness. The fuzzy set approach (FSA) is a useful tool to control uncertainty. FSA helps to incorporate unpredictability in decision-making process. Nowadays, FSA used in many fields of science and technology. In particular, researchers take different parameters of an inventory model for the expansion of the economic order quantity models either as constant or probabilistic in nature. However, these parameters can have minor deviations from the accurate value in real-life situations. So the probability distribution is not suitable in such types of conditions. But in this situation, the parameters can be treated with fuzzy, and it will become more realistic, and these types of problems handled by using an appropriate fuzzy technique, and the solution procedure may be achieved as usual. Due to uncertainty in various mathematical formulations in the inventory model, researchers introduced the fuzzy inventory model by using different fuzzy numbers. A fuzzy number is an amount whose values are vague, rather than exact. Zadeh's Extension Principle introduced the normal arithmetic operations with real numbers, which extended to fuzzy numbers. Due to fault in measuring method, machine fault. Several records in our observation cannot be correctly found. This leads to a new type of Fuzzy number called the pentagonal fuzzy numbers (PFN's) being introduced. A fuzzy pentagonal number (PFN) is a 5-tuple subset with five points of a real number R.

This research comprises a fuzzy inventory model under shortages with a constant deterioration rate of the product, whereas; demand is directly proportional to the time and Credibility factor. In order to optimise the standard total cost (STC), shortage-cost, deterioration-cost, ordering-cost, and holding costs are fuzzified by using a PFN, and graded mean technique is used. Further, the sensitivity analysis helps to analysis the effect of unalike parameters on STC in both fuzzy and crisp sense.

The structure of the paper as follows: the literature review presented in Section 2. Some crucial definitions and preliminaries related to this research work described in Section 3. Section 4 introduced the notations and assumption related to the paper. The mathematical fuzzy inventory model developed in Section 5. A computation algorithm descried in Section 6. The computational analysis through a numerical example with a comparison graph between the crisp and fuzzy inventory model presented in Section 7, and sensitivity analysis showing the effect of various parameters in Section 8, and the conclusion is in the last section.

2 Literature review

Chang et al. (1998) presented the economic order and backorder quantity model, Chauhan and Singh (2014) investigate optimal replenishments and ordering policy for the time-dependent demand rate. Chauhan and Singh (2015) developed a replenishment policy of the integrated demand pattern. Goyal et al. (2015) discussed the EPQ model for selling price-dependent demand. Several inventory models for decaying goods with different characteristics of the inventory organisations have been described; it is not feasible to list them individually. For an extensive study of inventory models with fading products, concerned researchers may pass on to review articles by Raafat (1991), and Janssen et al. (2016). Zadeh (1965) introduced the concept of fuzzy numbers in 1965 and gives an extension principle for arithmetic operations of fuzzy numbers. Further study of operations of fuzzy numbers is done by Dubois and Prade (1978). Due to uncertainty, they used fuzziness in their inventory model. Then Park (1987) introduced the EOQ technique with a fuzzy environment. Chang et al. (1998) presented the economic order and backorder quantity with the fuzzy approach, the model in the fuzzy environment; and observed that total cost is more significant than with the crisp model; on the other hand, it permits better use of the economic fuzzy quantities arising with changes in sales, deliveries, and orders. An optimal total cost for healthcare industries for three types of variable demand with fuzzy parameters considered by Rajput et al. (2019). Kazemi et al. (2017) established a fuzzy model in which, fuzzified the input parameters, and used a grated mean integration technique. Lee and Yun (2014) define the fuzzy pentagonal sets and simplify the addition, subtraction, multiplication, and division for two fuzzy pentagonal sets. Panda and Pal (2015) introduced a PFN with its arithmetic operations. They also described a particular type of pentagonal fuzzy matrices. Raj and Karthik (2016) use a PFN to choose the best machine for a job by feed-forward neural network. A model with a shortage that completely backlogged is introduced by Rajalakshmi and Rosario (2017) with a fuzzy approach and compared the total cost using triangular, trapezoidal, PFN's, and Graded mean method used for defuzzification. Kamble (2017) described different operations of a canonical PFN. The arithmetical operations of a particular type of PNF's described by Mondal and Mandal's (2017) research work, also carried out pentagonal fuzzy solutions for the fuzzy equation. Shekarian et al. (2017) studied the Comprehensive review of the fuzzy inventory models.

3 Definition and preliminaries

Definition 3.1 (Fuzzy set (Rajput et al., 2019))**:** A set $\tilde{N}$ is a fuzzy set, if his membership function, from the domain U mapped into [0, 1], i.e., $\tilde{N} = \{u, \mu_{\tilde{N}}(u) : u \in U\}$. Here the membership function is $\mu_{\tilde{N}} : \tilde{N} \to [0,1]$ and the grade value of $u \in U$ is $\mu_{\tilde{N}}(u)$ in a fuzzy set $\tilde{N}$.

Definition 3.2 (Fuzzy number (Rajput et al., 2019))**:** Let $\tilde{N}$ is a subset of real set R, with the membership function $\mu_{\tilde{N}}$ is said to be a fuzzy number if

- $\mu_{\tilde{N}}(u)$, piecewise continuous in its domain.
- $\tilde{N}$ (normal), i.e., $\mu_{\tilde{N}}(u_0) = 1$ for $u_0 \in \tilde{N}$.
- $\tilde{N}$ (convex), i.e., $\mu_{\tilde{N}}(\lambda u_1 + (1-\lambda)u_2) \geq \min(\mu_{\tilde{N}}(u_1), \mu_{\tilde{N}}(u_1))\ \forall\ u_1,\ u_2 \in U$.

Definition 3.3 (Pentagonal fuzzy number (PNF's) (Mondal and Mandal, 2017))**:** The following conditions should be satisfied for a fuzzy pentagonal number $\tilde{N} = (\eta_1,\ \eta_2,\ \eta_3,\ \eta_4,\ \eta_5)$

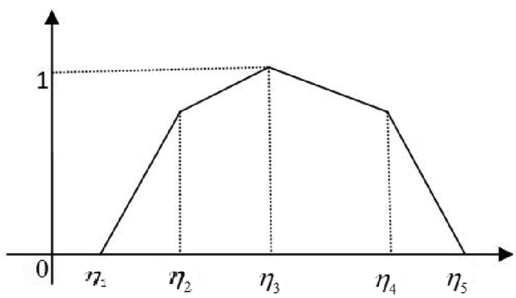
- $\mu_{\tilde{N}}(x)$, a continuous function in the interval [0,1].
- $\mu_{\tilde{N}}(x)$, strictly increasing in $[\eta_1, \eta_2]$ and $[\eta_2,\ \eta_3]$.
- $\mu_{\tilde{N}}(x)$, strictly decreasing in $[\eta_3,\ \eta_4]$ and $[\eta_4,\ \eta_5]$.

The membership function of a PFN $\tilde{N} = (\eta_1,\ \eta_2,\ \eta_3,\ \eta_4,\ \eta_5)$ is of the form

$$\mu_{\tilde{N}}(u;v_1,v_2)=\begin{cases} v_1\dfrac{u-\eta_1}{\eta_2-\eta_1}, & \text{if } \eta_1\le u\le\eta_2 \\ 1-(1-v_1)\dfrac{u-\eta_2}{\eta_3-\eta_2}, & \text{if } \eta_2\le u\le\eta_3 \\ 1, & \text{if } u=\eta_3 \\ 1\text{-}\ (1-v_2)\dfrac{u-\eta_3}{\eta_4-\eta_3}, & \text{if } \eta_3\le u\le\eta_4 \\ v_2\dfrac{u-\eta_5}{\eta_4-\eta_5}, & \text{if } \eta_4\le u\le\eta_5 \\ 0, & \text{if } \ u<\eta_1\ \&\ u>\eta_5 \end{cases}$$

In the case of PFN, Figure 1, the membership value is obtained for a particular range.

Figure 1 Graphical representation of PFN's



Definition 3.4 (α-Cut of PFN's (Mondal and Mandal, 2017))**:** Let PFN $\tilde{N}=(\eta_1,\ \eta_2,\ \eta_3,\ \eta_4,\ \eta_5)$ then the *α-Cut* operation on PFN's is defined as

$$L_L(\alpha)=[2\alpha(\eta_2-\eta_1)+\eta_1,\ \ -2\alpha(\eta_5-\eta_4)+\eta_5]$$

$$L_R(\alpha)=[2\alpha(\eta_3-\eta_2)+2\eta_2,\ \ 2\alpha(\eta_4-\eta_3)-\eta_4+2\eta_3].$$

Definition 3.5 (Graded mean method (GMI) (Rajput et al., 2019))**:** Let $\tilde{N}=(\eta_1,\eta_2,\eta_3,\eta_4,\eta_5)$ be a PFN, then GMI representation of $\tilde{N}$ as

$$G(\tilde{N})=\frac{\frac{1}{2}\int_0^1\alpha[L_L(\alpha)+L_R(\alpha)]d\alpha}{\int_0^1\alpha d\alpha}$$

Definition 3.6 (Fuzzy arithmetical operations for PFN's (Rajput et al., 2019))**:** Let $\tilde{\tau}=(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5)$ and $\tilde{\omega}=(\omega_1,\omega_2,\omega_3,\omega_4,\omega_5)$ are two PFN's where $\tau_1,\tau_2,\tau_3,\tau_4,\tau_5$, $\omega_1,\omega_2,\omega_3,\omega_4\ ,\omega_5$ are positive **real** number. Note that every PFN is associated with two weights: v_1 and v_2.

- $\tilde{\tau}\oplus\tilde{\omega}$=$(\tau_1+\omega_1,\tau_2+\omega_2,\tau_3+\omega_3,\tau_4+\omega_4,\tau_5+\omega_5)$,
  with $v_{i(\tau+\omega)}\ge\max(v_{i\tau},v_{i\omega})$ for $i=1,2$

- $\tilde{\tau} \otimes \tilde{\omega} = (\tau_1\omega_1, \tau_2\omega_2, \tau_3\omega_3, \tau_4\omega_4, \tau_5\omega_5)$, with $\nu_{i(\tau\omega)} \geq \max(\nu_{i\tau}, \nu_{i\omega})$ for $i = 1, 2$
- $\tilde{\tau} \Theta \tilde{\omega} = (\tau_1 - m_4,\ \tau_2 - m_3,\ \tau_3 - m_2,\ \tau_4 - m_1,\ \tau_5 - \omega_5)$,
  with $\nu_{i(\tau-\omega)} \geq \max(\nu_{i\tau}, \nu_{i\omega})$ for $i = 1, 2$
- $\tilde{\tau} \div \tilde{\omega} = \left(\frac{\tau_1}{\omega_5}, \frac{\tau_2}{\omega_4}, \frac{\tau_3}{\omega_3}, \frac{\tau_4}{\omega_2}, \frac{\tau_1}{\omega_1} \right)$
- $\alpha\tilde{\tau} = (\alpha\tau_1, \alpha\tau_2, \alpha\tau_3, \alpha\tau_4, \alpha\tau_5)$, where α is any positive real number.

4 Notation and assumption

Assumptions

- The system has a finite-time horizon.
- The model deals with a single type of product.
- The replenishment time of items is instantaneous and finite.
- Demand influenced by time as well as credibility, i.e., $D(t) = \rho t \kappa^t$ here ρ and κ are the demand parameter.
- Deterioration rate is assumed to be constant, and no replacement provision for such item.
- Shortages completely backlogged.
- Ordering, holding, shortage, and deterioration cost are considered as fuzzy parameters.
- The lead time (delivery time) is negligible.

Notations

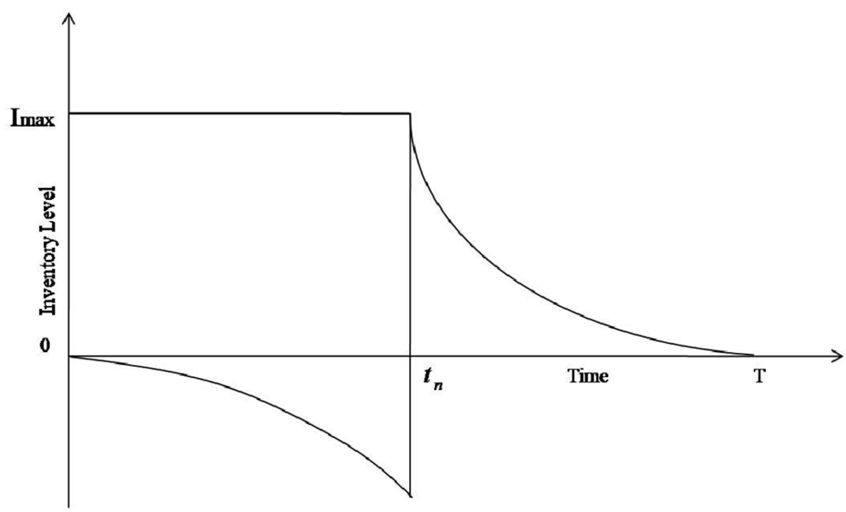
- $I(t)$: Inventory level at any time t in the interval $[0, T]$.
- $I_{\max}$: The greatest level of inventory.
- T: Fixed duration for the ordering-cycle.
- I_0: Initial ordering-quantity.
- $\tilde{\delta}$: Fuzzy shortage cost per unit time.
- $\tilde{\beta}$: Fuzzy deterioration cost per unit time.
- t_n: Procurement time.
- $\tilde{\gamma}$: Fuzzy holding cost per unit time.
- $\tilde{\alpha}$: Fuzzy ordering-cost per order.
- ρ: Shape parameter of demand factor.

- κ: Credibility factor of item for the inventory system, i.e., κ of perfect item present in the inventory.
- ϕ: Deterioration rate; $(0 < \phi < 1)$
- STC (t_n): Standard total cost per unit time.
- t_n^* : Optimal procurement time.
- STC* (t_n^*): Optimal standard total cost per unit time.

5 Formulation fuzzy inventory model

Initially, there is no inventory to the retailer, there are only advance booking of item. Due to the continuous effect of demand, the shortages occur in $[0, t_n]$. At time t_n, the inventory is replenished. Then, the deterioration of the product will start in $[t_n, T]$. Finally, the inventory point reaches zero at T, shown in Figure 2.

Figure 2 Inventory level with respect to time t_n



Governing differential equations are

$$\frac{dI(t)}{dt} = -\rho t \kappa^t, \quad 0 \le t \le t_n \tag{1}$$

$$\frac{dI(t)}{dt} + \phi I(t) = -\rho t \kappa^t, \; t_n < t \le T \tag{2}$$

Subject to a boundary conditions are $I(0) = 0$ and $I(T) = 0$. Solution of the equations (1) and (2) we obtain:

$$I(t) = \frac{-\rho\{\kappa^t (t \log[\kappa] - 1) + 1\}}{\log[\kappa]^2} \tag{3}$$

$$I(t)=\frac{\rho\{(\kappa^{T}+\kappa^{t})-(\log[\kappa]+\phi)(t\kappa^{t}+T\kappa^{T})\}}{(\log[\kappa]+\phi)^{2}} \tag{4}$$

at $t = t_n$ we get the maximum inventory level ($I_{\max}$) for each ordering rotation, i.e.,

$$I_{\max}=\frac{\rho((\kappa^{t_n}+\kappa^{T})-(t_n\kappa^{t_n}+T\kappa^{T})(\phi+\log[\kappa]))}{(\log[\kappa]+\phi)^{2}} \tag{5}$$

$$\text{Backlogged demand } (b)=\int_0^{t_n}\rho t\kappa^{t}\,dt=\frac{\rho}{\log[\kappa]^{2}}[\kappa^{t_n}(-1+t_n\ \log[\kappa])+1] \tag{6}$$

$$\begin{aligned}\text{Initial inventory}\quad I_0=I_{\max}+b&=\frac{\rho\{(\kappa^{t_n}+\kappa^{T})-(\phi+\log[\kappa])(t_n\kappa^{t_n}+T\kappa^{T})\}}{(\phi+\log[\kappa])^{2}}\\&+\frac{\rho\{\kappa^{t_n}(t_n\log[\kappa]-1)+1\}}{(\log[\kappa])^{2}}\end{aligned} \tag{7}$$

The relevant costs associated with model are such as:

- The ordering cost (OC):

$$\text{OC}=\tilde{a} \tag{8}$$

- The deteriorating cost (DC) for the interval $[t_n, T]$

$$\begin{aligned}&=\widetilde{\beta}\otimes\left\{\frac{\rho\{(\kappa^{t_n}+\kappa^{T})-(\phi+\log[\kappa])(T\kappa^{T}+t_n\kappa^{t_n})\}}{(\phi+\log[\kappa])^{2}}-\int_{t_n}^{T}\rho t\kappa^{t}dt\right\}\\&=\widetilde{\beta}\otimes\left\{\frac{\rho\{(\kappa^{t_n}+\kappa^{T})-(\phi+\log[\kappa])(t_n\kappa^{t_n}+T\kappa^{T})\}}{(\phi+\log[\kappa])^{2}}\right.\\&\quad\left.+\frac{\rho(\kappa^{t_n}+\kappa^{T})-\rho\ \log[\kappa](t_n\kappa^{t_n}+T\kappa^{T})}{\log[\kappa]^{2}}\right\}\end{aligned} \tag{9}$$

- The inventory holding cost (HC) for the interval $[t_n, T]$

$$\begin{aligned}\tilde{\gamma}\otimes\int_{t_n}^{T}I(t)\,\mathrm{d}t&=\tilde{\gamma}\otimes\int_{t_n}^{T}\frac{\rho\{(\kappa^{t}+\kappa^{T})-(\phi+\log[\kappa])(t\kappa^{t}+T\kappa^{T})\}}{(\phi+\log[\kappa])^{2}}\mathrm{d}t\\&=\frac{\tilde{\gamma}\otimes\rho}{(\phi+\log[\kappa])^{2}}\left(T\kappa^{T}+\frac{\kappa^{T}-\kappa^{t_n}}{\log[\kappa]}-\kappa^{t_n}\right.\\&\quad\left.+\frac{(\phi+\log[\kappa])(\kappa^{t_n}(\log[\kappa]t_n-1)+\kappa^{T}(1-T\log[\kappa](1+T\log[\kappa])+T\log[\kappa]^{2}t_n))}{(\log[\kappa])^{2}}\right)\end{aligned} \tag{10}$$

- The shortage cost (SC) in $[0, t_n]$

$$=\widetilde{\delta}\otimes\int_0^{t_n}\frac{\rho\left\{\kappa^{t}(t\ \log[\kappa]-1)+1\right\}}{\log[\kappa]^{2}}\mathrm{d}t=\widetilde{\delta}\otimes\rho\left\{\frac{2-2\kappa^{t_n}+(1+\kappa^{t_n})\log[\kappa]t_n}{\log[\kappa]^{3}}\right\} \tag{11}$$

By using equations (8), (9), (10) and (11) we get the standard total cost (STC).

Standard total cost $\mathrm{STC}(t_n)$

$$
\begin{aligned}
\mathrm{STC}(t_n) = \frac{1}{12}\Bigg(& \frac{\alpha_1 + 3\alpha_2 + 4\alpha_3 + 3\alpha_4 + \alpha_5}{T} + \frac{(\beta_1 + 3\beta_2 + 4\beta_3 + 3\beta_4 + \beta_5)\rho}{T}\Bigg(\frac{(\kappa^T + \kappa^{t_n})}{(\phi + \log[\kappa])^2} \\
& - \frac{(\phi + \log[\kappa])(T\kappa^T + t_n\kappa^{t_n})}{(\phi + \log[\kappa])^2} + \frac{(\phi^T + \phi^{t_n}) - \rho\left(T\phi^T + t_n\phi^{t_n}\right)\log[\phi]}{(\log[\phi])^2}\Bigg) \\
& + \frac{(\gamma_1 + 3\gamma_2 + 4\gamma_3 + 3\gamma_4 + \gamma_5)\ a}{(\theta + \log[\phi])^2\ T}\Bigg(T\phi^T + \frac{\phi^T - \phi^{t_n}}{\log[\phi]} \\
& - \frac{\phi^{t_n} t_n\left(\left(\log[\phi] + \theta\right)\left(\phi^{t_n}(\log[\phi]t_n - 1) + \phi^T\left(1 - T\log[\phi] + \left(T\log[\phi]\right)^2 + T\log[\phi]^2 t_n\right)\right)\right)}{\left(\log[\phi]\right)^2}\Bigg) \\
& + \frac{(\delta_1 + 3\delta_2 + 4\delta_3 + 3\delta_4 + \delta_5)}{T}\left(\frac{\rho(2 - 2\phi^{t_p} + (1 + \phi^{t_n})\log[\phi]t_n)}{(\log[\phi])^3}\right)\Bigg)
\end{aligned}
\tag{12}
$$

6 Computational algorithm for optimal criterion

To get the optimal value of the standard total cost STC (t_n) with respect to t_n the following steps are used to minimise STC (t_n).

Step 1: Start with the first derivative of STC (t_n) of equation (12) and equate it to zero and get the critical point t_n.

Step 2: Take second derivative of STC (t_n), i.e., $\partial^2\mathrm{STC}\ (t_n) / \partial t_n{}^2$.

Step 3: Use a critical point in step 2 we get the second derivative of total average cost will be positive at critical point, i.e., $\partial^2\mathrm{STC}\ (t_n) / \partial t_n{}^2 > 0,$ and hence STC (t_n) is minimum at a critical point t_n. Shown in Appendix.

Step 4: Find the minimum value of STC (t_n) by putting the value t_n in equation (12).

7 Computational analysis

The advance booking of mobiles is started before it launched. After a fixed duration of time, the mobile phone comes to the market to replenish. The buyer wants the durability of the product for a long time. If mobile store for a long time, both system parameters like holding and ordering cost have highly impacted the order to customer service and the retailer's profit. So we study the credibility with uncertain parameters such as holding-cost, deterioration-cost, ordering-cost, and shortage-cost. The numeric data result from a mobile retailer shop obtained from different resources in both cases as follows: the demand rate does not only depend on time but also on the credibility of the mobile quality. Therefore, the demand rate is taken as $D(t) = \rho t\kappa^t$, where let $\rho = 0.01$.

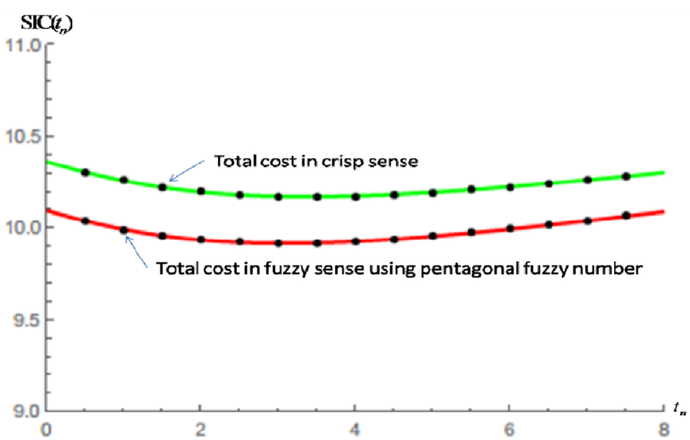
Thus, let a new product be presented in the marketplace, of which 80% are of acceptable quality. The ordering cost is \$20 per unit per year. To hold the item, the retailer spends \$2 per unit per year. The item undergoes deterioration with time at the rate of 1%, which costs \$10 per unit per year. During shortage, part of the customers wait for item to come to in the market with cost \$5 per unit. The total system cycle is considering for four years.

Table 1 Computational result and comparison of standard total cost (STC)

Input parameters	*Outcome under crips*	*Outcome under fuzzy*
$\alpha = 20$, $\beta = 10$, $\gamma = 2$, $\delta = 5$, $\rho = 0.01$, $T = 4$, $\kappa = 0.5$, $\phi = 0.01$ $\tilde{\alpha} = (18, 19, 19.5, 20, 21)$ $\tilde{\beta} = (8.5, 9, 9.2, 9.4, 9.9)$ $\tilde{\gamma} = (1.9, 2, 2.3, 2.6, 2.7)$ $\tilde{\delta} = (3, 4, 6, 8, 9)$	Optimal procurement time $t_n^* = 3.3638$ and optimal standard total cost $(\text{STC}^*) = \$10.1716$	Optimal procurement time $t_n^* = 3.0769$ and optimal standard fuzzy total cost $(\text{SFTC}^*) = \$9.9189$

By computational algorithm, the optimum standard total cost (STC^*) for crisp model and fuzzy model obtained shown in Table 1, by using graded mean integration technique from equation (12), we get Optimal procurement time $t_n^* = 3.3638$ and optimal standard total cost $(\text{STC}^*) = \$10.1716$ with crisp model while Optimal procurement time $t_n^* = 3.0769$ and optimal standard fuzzy total cost $(\text{SFTC}^*) = \$9.9189$ for fuzzy. Moreover, the behaviour of the STC for both models shown in Figure 3.

Figure 3 Behaviour of standard total cost (TC) in crisp and fuzzy (see online version for colours)



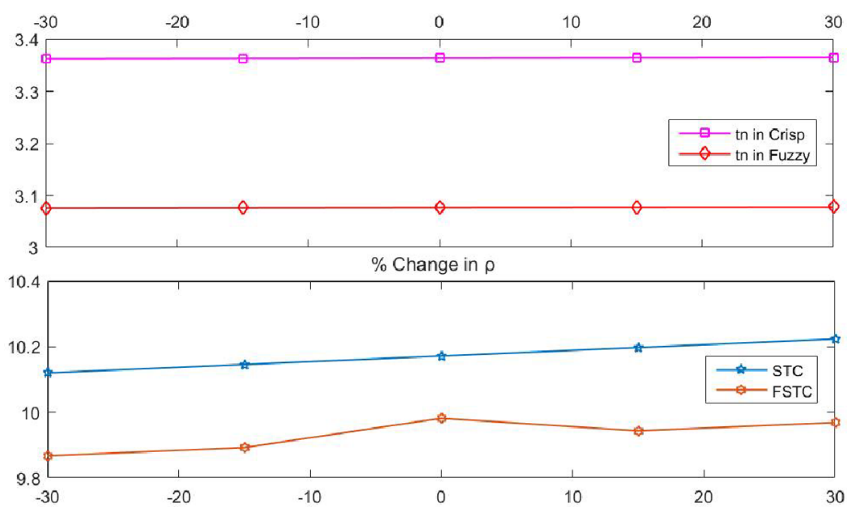
8 Sensitivity analysis

From Table 2 and Figure 4, the effect of demand factor ρ, deterioration parameter, and procurement time t_n with STC examine and observed that as demand fluctuating, STC and procurement time t_n would be relatively changed for both models. So, the model is affected by demand. The variation of the credibility factor κ, procurement time t_n, and STC will also change for both crisp and fuzzy models. Credibility factor has an impact on model. Procurement time t_n, and STC moderately affected by the deterioration factor ϕ. Although, we observed STC is lesser in fuzzy than that of the crisp model.

Table 2 Effect of different parameter on standard total cost

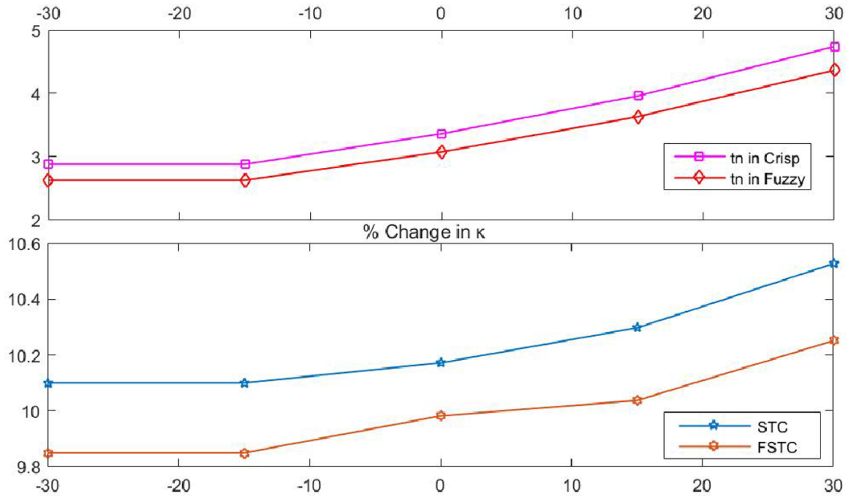
		(a) Under crips model		*(b) Under fuzzy model*	
Parameter	*% change in parameter*	*Procurement time t_n*	*STC*	*Procurement time t_n*	*FSTC*
Demand factor ρ	30%	3.3650	10.2233	3.0778	9.9688
	15%	3.3643	10.1974	3.0773	9.9435
	0%	**3.3638**	**10.1716**	**3.0769**	**9.9819**
	–15%	3.3631	10.1458	3.0763	9.8928
	–30%	3.3625	10.1200	3.0758	9.8676
Credibility factor κ	30%	4.7381	10.5265	4.3655	10.2511
	15%	3.9604	10.2968	3.6335	10.0370
	0%	**3.3638**	**10.1716**	**3.0769**	**9.9819**
	–15%	2.8816	10.0998	2.6305	9.8489
	–30%	2.8816	10.0998	2.6305	9.8489
Deterioration rate ϕ	30%	3.3705	10.1721	3.0832	9.9186
	15%	3.3671	10.1718	3.0800	9.9184
	0%	**3.3638**	**10.1716**	**3.0769**	**9.9819**
	–15%	3.3603	10.1713	3.0736	9.9179
	–30%	3.3570	10.1711	3.0704	9.9177

Figure 4 Effect of parameter on STC and FSTC: (a) demand factor; (b) credibility factor and (c) deterioration rate (see online version for colours)

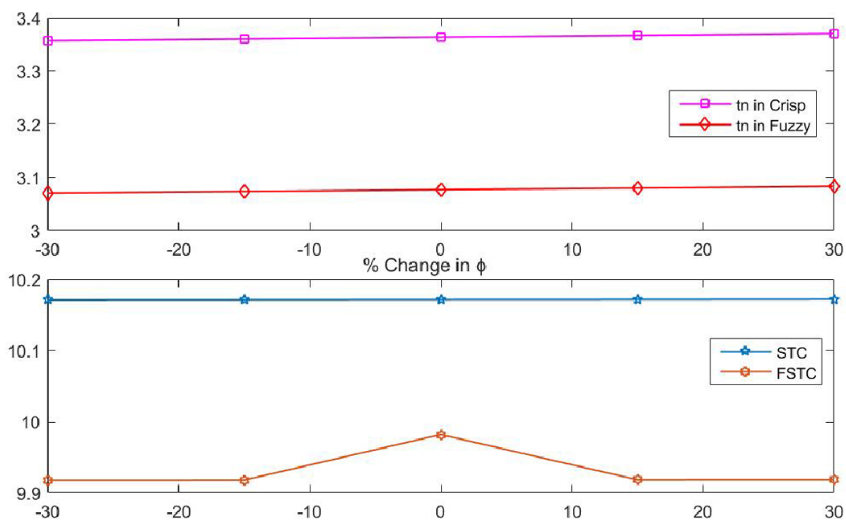


(a)

Figure 4 Effect of parameter on STC and FSTC: (a) demand factor; (b) credibility factor and (c) deterioration rate (see online version for colours) (continued)



(b)



(c)

9 Conclusion

This research paper is established for an economic order quantity model with uncertainty in cost factors. The concept of PFN introduced for inventory model due to uncertainty, and we observed that optimal standard total cost (FSTC*) under uncertain environment is a lesser than optimal standard total cost (STC*) in the crisp environment. We determined that uncertainty in decision variables is encouraging the decision maker to minimise the total cost.

We also concluded that the model is affected and worthwhile by the credibility factor in the demand. The study motivates us, how the retailers get additional profit in the inventory system with critical conditions like uncertain demand rate, cost, and the other factors. For, future prospect model will be extended with various factors like inflation, different demand pattern, channel financing with trade credit policy, it gives more advantages for suppliers and manufacturers.

References

Chang, S.C., Yao, J.S. and Lee, H.M. (1998) 'Economic reorder point for fuzzy backorder quantity', *European Journal of Operational Research*, Vol. 109, No. 1, pp.183–202.

Chauhan, A. and Singh, A.P. (2014) 'Optimal replenishment and ordering policy for time-dependent demand and deterioration with discounted cash flow analysis', *International Journal of Mathematics in Operational Research*, Vol. 6, No. 4, pp.407–436.

Chauhan, A. and Singh, A.P. (2015) 'A note on the inventory models for deteriorating items with Verhulst's model type demand rate', *International Journal of Operational Research*, Vol. 22, No. 2, pp.243–261.

Dubois, D. and Prade, H. (1978) 'Operation on fuzzy numbers', *International Journal of Systems Science*, Vol. 9, No. 6, pp.613–626.

Goyal, A., Singh, S. and Chauhan, A. (2015) 'An EPQ inventory model with stock and selling price dependent demand rate, partial backlogging and variables ordering cost', *International Journal of Agricultural and Statistical Sciences*, Vol. 11, No. 2, pp.441–447.

Janssen, L., Claus, T. and Sauer, J. (2016) 'Literature review of deteriorating inventory models by key topics from 2012 to 2015', *Int. J. Production Economics*, Vol. 182, pp.86–112.

Kamble, A.J. (2017) 'Some notes on Pentagonal fuzzy numbers', *International Journal of fuzzy Mathematical Archive*, Vol. 13, No. 2, pp.113–121.

Kazemi, N., Ehsani, E. and Jaber, M.Y. (2010) 'An inventory model with backorders with fuzzy parameters and decision variables', *International Journal of Approximate Reasoning*, Vol. 51, pp.964–972.

Lee, B.J. and Yun, Y.S. (2014) 'The pentagonal fuzzy numbers', *Journal of Chungcheong Mathematical Society*, Vol. 27, No. 2, pp.277–286.

Mondal, S.P. and Mandal, M. (2017) 'Pentagonal fuzzy number its properties and application in fuzzy equation', *Future Computing and Informatics Journal*, Vol. 2, pp.110–117.

Panda, A. and Pal, M. (2015) 'A study on pentagonal fuzzy number and its corresponding matrices', *Pacific Science Review B: Humanities and Social Sciences*, Vol. 1, pp.131–139.

Park, K.S. (1987) 'Fuzzy set theoretical interpretation of economic order quantity', *IEEE Transactions on Systems, Man and Cybernetics Society*, Vol. 17, No. 6, pp.1082–1084.

Raafat, F. (1991) 'Survey of literature on continuously deteriorating inventory models', *Journal of the Operational Research Society*, Vol. 42, pp.27–37.

Raj, A.V. and Karthik, S. (2016) 'Application of pentagonal fuzzy numbers in neural network', *International Journal of Mathematics and Its Applications*, Vol. 4, No. 4, pp.149–154.

Rajalakshmi, R.M. and Rosario, G.M. (2017) 'A fuzzy inventory model with allowable shortage using different fuzzy numbers', *International Journal of Computational and Applied Mathematics*, Vol. 12, No. 1, pp.227–239, ISSN 1819-4966.

Rajput, N., Pandey, R.K., Singh, A.P. and Chauhan, A. (2019) 'An optimisation of fuzzy EOQ model in healthcare industries with three different demand pattern using graded mean technique', *Mathematics in Engineering, Science and Aerospace*, Vol. 10, No. 2, pp.205–218.

Shekarian, E., Kazemi, N., Rashid, S.H.A. and Olugu, E.U. (2017) 'Fuzzy inventory models: a comprehensive review', *Applied Soft Computing*, Vol. 55, pp.588–621.

Zadeh, L.A. (1965) 'Fuzzy set', *Information and Control*, Vol. 8, pp.338–353.

Appendix

STC(t_n) is minimum at critical point

Necessary condition

$$\frac{\partial \mathrm{STC}(t_n)}{\partial t_n}=\frac{\rho}{12T}\left(-\frac{\phi^{t_n}(\beta_1+3\beta_2+4\beta_3+3\beta_4+\beta_5)((-1+\rho)\theta^2+(-1+2a)\theta\log[\phi]}{\log[\phi]\left(\log[\phi]+\theta\right)^2}\right.$$
$$+\frac{\left(\log[\phi]\right)^2(\rho-1)+t_n\log[\phi]\left(\log[\phi]+\theta\right)\left(\rho\theta+\rho\log[\phi]+\log[\phi]\right)}{\log[\phi]\left(\log[\phi]+\theta\right)^2}$$
$$+\frac{(\gamma_1+3\gamma_2+4\gamma_3+3\gamma_4+\gamma_5)(T\phi^T(\theta+\log[\phi])+\phi^{t_n}(-2+\theta t_n))}{\left(\log[\phi]+\theta\right)^2}$$
$$\left.+\frac{(\delta_1+3\delta_2+4\delta_3+3\delta_4+\delta_5)(1+\phi^{t_n}(-1+\log[\phi]t_n))}{\log[\phi]^2}\right)=0.$$

As the given consideration $t_n \geq 0$.

Sufficient condition

$$\frac{\partial^2 \mathrm{STC}(t_n)}{\partial {t_n}^2}=\frac{\rho}{12T}\left(-\frac{\phi^{t_n}(\rho\theta+(1+\rho)\log[\phi])(\beta_1+3\beta_2+4\beta_3+3\beta_4+\beta_5)}{\left(\log[\phi]+\theta\right)}\right.$$
$$-\frac{\phi^{t_n}(\beta_1+3\beta_2+4\beta_3+3\beta_4+\beta_5)((\rho-1)\theta^2+(2\rho-1)\theta\log[\phi]}{\left(\log[\phi]+\theta\right)^2}$$
$$-\frac{(\rho-1)\log[\phi]^2+\log[\phi](\theta+\log[\phi])(\rho\theta+\log[\phi]+\rho\log[\phi])t_p}{\left(\log[\phi]+\theta\right)^2}$$
$$+\frac{(\gamma_1+3\gamma_2+4\gamma_3+3\gamma_4+\gamma_5)(\theta\phi^{t_n}+\phi^{t_n}\log[\phi](-2+\theta t_n))}{\left(\log[\phi]+\theta\right)^2}$$
$$\left.+\frac{(\delta_1+3\delta_2+4\delta_3+3\delta_4+\delta_5)(\phi^{t_n}\log[\phi]+\phi^{t_n}\log[\phi](-1+\log[\phi]t_n))}{\log[\phi]^2}\right)>0$$

is exits if $0<t_n<\dfrac{-7+2\sqrt{11}}{5\log[\phi]}$ and $0<\theta<\dfrac{1}{2}(-3\mathrm{Log}[\phi]+\sqrt{5}\log[\phi])$ where $0<\phi<1$.