Non-fragile event-triggered control of positive switched systems

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Abstract: This paper presents a non-fragile event-triggered control approach to positive switched systems without/with input saturation. First, a 1-norm based on event-triggered mechanism is established for positive switched systems. By using the matrix decomposition technique, a non-fragile controller based on event-triggered mechanism is designed for positive switched systems without saturation. Then, the presented non-fragile event-triggered control design approach is developed to positive switched systems with saturation. The saturation term is transformed into interval form and a controller is designed by means of the linear programming. The positivity and stability of the closed-loop systems are guaranteed under the designed controllers. Finally, a numerical example is given to illustrate the effectiveness of the design.

Keywords: event-triggered mechanism; non-fragile control; positive switched systems; linear programming; actuator faults; input saturation; average dwell time switching.

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1 Introduction

Positive systems, whose states and outputs remain nonnegative when the initial states and inputs are nonnegative, become a hot research topic during last two decades (Farina and Rinaldi, 2011; Lam et al., 2019). Indeed, such systems are extensive encountered in many real world applications, including electronic circuits, chemical processes, storage systems and so on. Positive switched systems, consisting a family of positive subsystems and a switching law, are a special class of hybrid systems. The study of positive switched systems involves many typical control problems such as stability analysis (Mason and Shorten, 2007; Gurvits et al., 2007; Liu and Dang, 2011), stabilisation design (Benzaouia and Tadeo, 2008; Blanchini et al., 2008; Briat, 2017), L_1-gain analysis (Xiang and Xiang, 2013), delayed transaction (Aleksandrov and Mason, 2018; Liu et al., 2018) and so on. The literature Colaneri et al. (2014) considered the optimal control problem of positive switched systems. An L_1 finite-time controller was designed for positive switched delayed systems under mode-dependent average dwell time constraint in Liu et al. (2015a). By using Lyapunov-Krasovskii functions, the asynchronously switched control was discussed for discrete impulsive positive switched systems and some sufficient conditions were derived to ensure considered systems were finite-time stable (Liu et al., 2015b). In Zhang et al. (2017), Zhang et al. proposed a novel computation method called gain matrix decomposition for positive switched systems, which is more practical and simpler. It should be mentioned that in practical applications, it is hard to meet the actual requirements of an accurate controller owing to errors or uncertainties in the process of controller execution. Moreover, actuator faults and input saturation are two important reasons that bring negative effects to the system, such as performance decrease and even instability (Jin et al., 2014; Sakthivel et al., 2017). Research about actuator faults and saturation of various systems has accumulated some results (Zuo et al., 2010; Selvaraj et al., 2017, 2018). The stabilisation problem was considered for positive switched systems in the presence of actuator saturation and the convex hull technique was employed to solve the saturation problem in Wang and Zhao (2016). A non-fragile controller was designed for positive Markovian jump systems with faults and saturation in Zhang et al. (2019a). In Park et al (2018), the saturation control was studied for single input positive Markovian jump systems. The literature Qi et al. (2017) addressed the problem of stabilisation for positive Markovian jump systems subject to actuator saturation by using convex analysis, then the positivity and stochastic stability of considered system were proved. In order to eliminate the negative effect of actuator faults and saturation on the system, a non-fragile reliable controller design approach is proposed, which can tolerate admissible gain variations and failures. In Zhang et al. (2014), the saturation control of positive interval systems was investigated. The literature Zhang and Raïssi (2019) presented an appropriate control approach for nonlinear switched systems on the basis of positive systems and the control synthesis of nonlinear switched systems in the presence of input saturation. With the aid of a proper stochastic co-positive Lyapunov function, the continuous-time and discrete-time were both considered for positive Markovian jump systems (Yang et al., 2019).

In the literature mentioned above, the sampling scheme involve a time-periodic decision ruling, which may results in heavy communication burden and waste of communication resources. To overcome this issue, the author proposed a new control method called event-triggered control in Dorf et al. (1962). Compare with the time-triggered mechanism, the event-triggered sampling mechanism has more flexibility selection of sampling time. Due to the remarkable advantages of event-triggered control, it has accumulated plentiful achievements (Heemels et al., 2008; Qi et al., 2019). An event-triggered predictive control was considered for multi-agent systems in Zou et al. (2019). In the work Yin et al. (2018), the author addressed the event-triggered stabilisation problem of positive systems subject to saturation by constructing an event-triggered state feedback law. In Xiao et al. (2019), a dynamic event-triggered scheme was proposed for positive Markovian systems with the consideration of networked fault. By means of linear Lyapunov functions integrated with linear programming, an event-triggered controller was designed for positive switched systems in Liu et al. (2019). In Yang et al. (2019), a communication network was modelled via switched systems, for which the communication network was divided into busy-time and idle-time models. Noting the fact that the number of data package is nonnegative, the considered communication network can be regarded as a typical positive switched system. Event-triggered control approach is effective for reducing the waste of network resources and the design cost of controller. On the other hand, actuator failure and saturation usually occur in practice owing to the frequent use of components. They may cause the performance deterioration of systems and even lead to the instability of systems. Therefore, it is necessary to design a non-fragile event-triggered controller of positive switched systems. However, as far as we known, few results are concerned with the mentioned topic. Then, some questions naturally arise: how to design a non-fragile controller for positive switched systems based on event-triggered mechanism, and how to deal with the non-fragile event-triggered controller of positive switched systems? These motivate us to carry out this work.

This paper aims to design a non-fragile control of positive switched systems based on event-triggered mechanism. First, an event-triggered mechanism is introduced for positive switched systems without saturation. Then, by means of the linear co-positive Lyapunov function incorporated with matrix decomposition technique, the non-fragile event-triggered control is proposed. Finally, the presented approaches are extended to saturation situation for positive switched systems. Under the designed controller, the closed-loop system is positive and stable. The main contribution of this paper is as follows:

- i a non-fragile event-triggered controller is designed for positive switched systems
- ii a cone invariant set is constructed for dealing with the saturation issue of positive switched systems
- iii a linear programming approach is employed for solving all conditions.

Through compared with existing results, the non-fragile event-triggered control presented has more applications. The layout of this paper is as follows: Section 2 presents the preliminaries. Main results are addressed in Section 3. An example is given in Section 4 to illustrate the effectiveness of the controller. Section 5 concludes this paper.

Notation. Let $\mathbb{R}$, $\mathbb{R}^n$ (or $\mathbb{R}^n_+$), $\mathbb{R}^{n\times r}$ be the sets of real numbers, n-dimensional vectors (or nonnegative), and $n \times r$ matrices with real entries, respectively. $\mathbb{N}$ and $\mathbb{N}^+$ denote the sets of nonnegative and positive integers, respectively. x_i implies the ith elements of vector $x = (x_1, \dots, x_n)^T$. For $A = [a_{ij}] \in \mathbb{R}^{n\times n}$, $A \succeq 0$ ($\succ 0$) means that $a_{ij} \geq 0$ ($a_{ij} > 0$) $\forall i, j = 1, \dots, n$. $A \preceq 0$ ($\prec 0$) means that $a_{ij} \leq 0$ ($a_{ij} < 0$) $\forall i, j = 1, \dots, n$. Similarly, $A \succeq B$ ($A \preceq B$) means that $a_{ij} \geq b_{ij} (a_{ij} \leq b_{ij}) \forall i, j = 1, \dots, n$. Define I as a identity matrix of an appropriate dimension. Given a matrix F, F^T is its transpose, and $|F|$ denotes the elements of F take the absolute values of its corresponding elements. $\|\cdot\|$ indicates Euclidean norm. The 1-norm $\|x\|_1$ and infinite-norm of a vector $x \in \mathbb{R}^n$ are defined as $\|x\|_1 = \Sigma_{i=1}^n |x_i|$ and $\|x\|_\infty = \max(|x_1|, |x_2|, \dots, |x_n|)$, respectively. Define

$\mathbf{1}_r = (\underbrace{1, \ldots, 1}_{r})^T$, $\mathbf{1}_r^{(\iota)} = (\underbrace{0, \ldots, 0}_{\iota-1}, 1, \underbrace{0, \ldots, 0}_{r-\iota})^T$, and let $\mathbf{1}_{n\times n}$ be the $n \times n$ matrix with all elements 1. $co\{\cdot\}$ denotes the convex hull. Matrix A is called Metzler matrix if its off-diagonal elements are all nonnegative real numbers.

2 Preliminaries

Consider switched continuous systems with actuator faults:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u^f_{\sigma(t)}(t), \tag{1}$$

and switched systems subject to actuator faults and input saturation:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}\text{sat}(u^f_{\sigma(t)}(t)), \tag{2}$$

where $x(t) \in \mathbb{R}^n$ and $u^f_{\sigma(t)}(t) \in \mathbb{R}^r$ are system state and control input with actuator fault, respectively. The function $\sigma(t)$ denotes the switching law, and it takes values on the finite set $S = \{1, 2, \ldots, N\}$, $N \in \mathbb{N}^+$. The function sat$(\cdot)$: $\mathbb{R}^r \to \mathbb{R}^r$ is said to be the saturation function, that is, $\text{sat}(u) = [\text{sat}(u_1), \text{sat}(u_2), \ldots, \text{sat}(u_r)]^T$, where $\text{sat}(u_i) =$ sgn$(u_i) \min\{|u_i|, 1\}$, $i = 1, 2, \ldots, r$. In order to simplify the symbol, we assume that the ith subsystem is activated when $\sigma(t) = i$. Throughout this paper, assume that A_i are Metzler matrices with $A_i \in \mathbb{R}^{n\times n}$ and $B_i \succeq 0$ with $B_i \in \mathbb{R}^{n\times r}$.

Next, we introduce some definitions and lemmas.

Definition 1 (Farina and Rinaldi, 2011; Lam et al., 2019)**:** A system is said to be positive if its states and outputs are nonnegative for any initial condition $x(t_0) \succeq 0$ and control input $u(t) \succeq 0$.

Lemma 1 (Farina and Rinaldi, 2011; Lam et al., 2019)**:** *Systems (1) and (2) are positive if and only if matrix A_i are Metzler matrices and $B_i \succeq 0$.*

Lemma 2 (Farina and Rinaldi, 2011; Lam et al., 2019)**:** *If A is a Metzler matrix with $A \in \mathbb{R}^{n\times n}$, the following conditions are equivalent:*

(i) The matrix A is a Hurwitz matrix.

(ii) There exists a vector $v \succ 0$ such that $Av \prec 0$.

Definition 2 (Hespanha and Morse, 1999)**:** Define $N_\sigma(t, t_0)$ as the number of times the switched system switches in the time interval $[t_0, t]$, where $0 \leq t_0 \leq t$. If

$$N_\sigma(t_0, t) \leq N_0 + \frac{t - t_0}{\tau^*},$$

where N_0 is a positive constant and $\tau^* > 0$ stands for the average dwell time.

Lemma 3 (Farina and Rinaldi, 2011; Lam et al., 2019)**:** *Matrix A is Metzler if and only if there is a positive constant δ such that $A + \delta I \succeq 0$.*

Throughout this paper, we introduce a non-fragile controller:

$$u_i(t) = (F_i + \Delta F_i)x(t), \tag{3}$$

where $F_i \in \mathbb{R}^{r\times n}$ *are normal gain matrices,* $\Delta F_i = E_i H_i$ *are gain perturbation matrices,* $H_i \in \mathbb{R}^{r\times n}$ *are decision variables to be designed and* $E_i \in \mathbb{R}^{r\times r}$ *are known nonnegative matrices satisfying*

$$\theta_1 I \preceq E_i \preceq \theta_2 I \tag{4}$$

for $0 < \theta_1 < \theta_2 < 1$*. The controller with actuator fault can be defined as*

$$u_i^f(t) = L_i u_i(t), \tag{5}$$

where $L_i =$ *diag*$(l_{i1}, l_{i2}, \ldots, l_{ir})$ *are uncertainty fault matrices but bounded:*

$$0 \preceq L_{di} \preceq L_i \preceq L_{ui} \preceq \gamma L_{di}, \tag{6}$$

where $\gamma \geq 1$*,* L_{di} *and* L_{ui} *are given matrices satisfying* $L_{di} =$ *diag*$(l_{di1}, l_{di2}, \ldots, l_{dir})$ *and* $L_{ui} =$ *diag*$(l_{ui1}, l_{ui2}, \ldots, l_{uir})$*, respectively.*

Lemma 4 (Hu et al., 2002)**:** *Given vectors* $u \in \mathbb{R}^r$ *and* $v \in \mathbb{R}^r$*, let* $\|v\|_\infty < 1$*. Then*

$$sat(u) \in co\{G_1 u + G_1^- v, \ldots, G_{2^r} u + G_{2^r}^- v\},$$

where $g = 1, 2, \ldots, 2^r$*,* G_g *is an* $r \times r$ *diagonal matrix with either 0 or 1 elements and* $G_g + G_g^- = I$*.*

From equation (8), we have

$$sat(u) = \sum_{g=1}^{2^r} \ell_g (G_g u + G_g^- v),$$

where the constant ℓ_g *satisfies* $0 \leq \ell_g \leq 1$ *and* $\sum_{g=1}^{2^r} \ell_g = 1$*. A cone domain* $\mathbf{\Omega}(v_i, 1)$ *is defined as*

$$\mathbf{\Omega}(v_i, 1) = \{x \in \mathbb{R}_+^n \mid x^T v_i \leq 1\},$$

where $v_i \succ 0$ *with* $v_i \in \mathbb{R}^r$*. Set* $M_i \prec 0$ *with* $M_i \in \mathbb{R}^{r\times n}$ *and define a polyhedron:*

$$T(M_i) := \{x \in \mathbb{R}_+^n \mid |M_{ij} x| \leq 1\},$$

where $j = 1, 2, \ldots, r$*,* $i \in S$ *and* M_{ij} *is the* j*th row of* M_i*.*

3 Main results

In this section, the non-fragile event-triggered controllers are designed for systems (1) and (2) by using matrix decomposition technique and sufficient conditions are proposed in terms of linear programming. First, we consider the non-fragile event-triggered control of positive switched systems without actuator saturation.

Considering the non-fragile control law (3), the non-fragile event-triggered feedback control law can be expressed as:

$$u_i(t) = (F_i + \Delta F_i)\hat{x}(t),\ t \in [t_p, t_{p+1}),$$

where $p \in \mathbb{N}$, $t_0 = 0$, t_p represents the moment when the pth event is triggered and $\hat{x}(t) = x(t_p)$ is the sampling state.

The event-triggered condition is given as:

$$\|x_e(t)\|_1 > \alpha\|x(t)\|_1, \tag{7}$$

where $0 < \alpha < 1$ and $x_e(t) = \hat{x}(t) - x(t)$ is the sample error. Combined with form of controller given in equation (5), the non-fragile event-triggered controller with actuator fault can be written as follows:

$$u_i^f(t) = L_i(F_i + E_iH_i)\hat{x}(t). \tag{8}$$

Then, the closed-loop system (1) is

$$\dot{x}(t) = (A_i + B_iL_iF_i + B_iL_iE_iH_i)x(t) + (B_iL_iF_i + B_iL_iE_iH_i)x_e(t). \tag{9}$$

Theorem 1: *If there exist constants $\delta_i > 0$, $\mu > 0$, $\lambda > 1$, $\gamma \geq 1$ and vectors $v_i \succ 0$ with $v_i \in \mathbb{R}^n$, $\xi_{i\iota}^+ \succ 0$ with $\xi_{i\iota}^+ \in \mathbb{R}^n$, $\xi_{i\iota}^- \prec 0$ with $\xi_{i\iota}^- \in \mathbb{R}^n$, $\xi_i^+ \succ 0$ with $\xi_i^+ \in \mathbb{R}^n$, $\xi_i^- \prec 0$ with $\xi_i^- \in \mathbb{R}^n$, $\zeta_{i\iota}^+ \succ 0$ with $\zeta_{i\iota}^+ \in \mathbb{R}^n$, $\zeta_{i\iota}^- \prec 0$ with $\zeta_{i\iota}^- \in \mathbb{R}^n$, $\zeta_i^+ \succ 0$ with $\zeta_i^+ \in \mathbb{R}^n$, $\zeta_i^- \prec 0$ with $\zeta_i^- \in \mathbb{R}^n$ such that*

$$\begin{aligned} &\chi_{is}A_i + \Gamma_{is1}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{+T}\Psi + \Gamma_{is2}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{-T}\Upsilon \\ &\quad + \Gamma_{is3}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{+T}\Psi + \Gamma_{is4}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{-T}\Upsilon \\ &\quad + \delta_i I \succeq 0, \end{aligned} \tag{10a}$$

$$\begin{aligned} &A_i^T v_i + \Upsilon\xi_i^+ + \Psi\xi_i^- + \theta_2\Upsilon\zeta_i^+ \\ &\quad + \theta_1\Psi\zeta_i^- + \mu v_i \prec 0, \end{aligned} \tag{10b}$$

$$v_i \preceq \lambda v_j, \tag{10c}$$

$$\xi_{i\iota}^+ \preceq \xi_i^+,\ \xi_{i\iota}^- \preceq \xi_i^-,\ \iota = 1, \ldots, r, \tag{10d}$$

$$\zeta_{i\iota}^+ \preceq \zeta_i^+,\ \zeta_{i\iota}^- \preceq \zeta_i^-,\ \iota = 1, \ldots, r, \tag{10e}$$

hold $\forall(i,j)\in S\times S,\ i\neq j,\ s=1,\ldots,r,\ \chi_{is}=\mathbf{1}_r^T L_{di}^T B_i^T v_i,\ \Gamma_{is1}=\frac{1}{\gamma}B_iL_{di},\ \Gamma_{is2}=\gamma B_iL_{di},\ \Gamma_{is3}=\frac{1}{\gamma}B_iL_{di}E_i,\ \Gamma_{is4}=\gamma B_iL_{di}E_i,\ \Psi=I-\alpha\mathbf{1}_{n\times n},\ \Upsilon=I+\alpha\mathbf{1}_{n\times n}$, *then under the control law (8) with* $F_i=F_i^++F_i^-$, $H_i=H_i^++H_i^-$, *and*

$$\begin{aligned}
F_i^+&=\frac{1}{\mathbf{1}_r^T L_{ui}^T B_i^T v_i}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{+T},\\
F_i^-&=\frac{1}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{-T},\\
H_i^+&=\frac{1}{\mathbf{1}_r^T L_{ui}^T B_i^T v_i}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{+T},\\
H_i^-&=\frac{1}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{-T},
\end{aligned}\tag{11}$$

the resulting closed-loop system (9) is positive and stable with the average dwell time switching satisfying

$$\tau^*\geq\frac{\ln\lambda}{\mu}.\tag{12}$$

Proof: By $\mathbf{1}_r\succ 0$, $L_{di}\succ 0$, $B_i\succeq 0$ and $v_i\succ 0$, we have $\mathbf{1}_r^T L_{di}^T B_i^T v_i>0$. This together with $\xi_{i\iota}^+\succ 0,\ \xi_{i\iota}^-\prec 0,\ \zeta_{i\iota}^+\succ 0$ and $\zeta_{i\iota}^-\prec 0$, we can get $F_i^+\succ 0$ $F_i^-\prec 0$, $H_i^+\succ 0$ and $H_i^-\prec 0$. Then, we have

$$\begin{aligned}
&L_{di}F_i^++L_{ui}F_i^-+L_{di}E_iH_i^++L_{ui}E_iH_i^-\\
&\preceq L_iF_i^++L_iF_i^-+L_iE_iH_i^++L_iE_iH_i^-\\
&\preceq L_{ui}F_i^++L_{di}F_i^-+L_{ui}E_iH_i^++L_{di}E_iH_i^-.
\end{aligned}\tag{13}$$

Furthermore,

$$\begin{aligned}
&A_i+B_iL_{di}F_i^++B_iL_{ui}F_i^-+B_iL_{di}E_iH_i^++B_iL_{ui}E_iH_i^-\\
&\preceq A_i+B_iL_iF_i^++B_iL_iF_i^-+B_iL_iE_iH_i^++B_iL_iE_iH_i^-\\
&\preceq A_i+B_iL_{ui}F_i^++B_iL_{di}F_i^-+B_iL_{ui}E_iH_i^++B_iL_{di}E_iH_i^-.
\end{aligned}\tag{14}$$

Given any initial state $x(t_0)\succeq 0$, we can obtain from equation (7) that $\parallel x_e(t_0)\parallel_1\leq\alpha\parallel x(t_0)\parallel_1=\alpha\mathbf{1}_n^T x(t_0)$. Thus,

$$-\alpha\mathbf{1}_{n\times n}x(t_0)\preceq x_e(t_0)\preceq\alpha\mathbf{1}_{n\times n}x(t_0).\tag{15}$$

Substituting $F_i=F_i^++F_i^-$, $H_i=H_i^++H_i^-$, and equation (15) into equation (9), it follows that

$$\begin{aligned}
\dot{x}(t_0)\succeq{}&(A_i+B_iL_iF_i^+\Psi+B_iL_iF_i^-\Upsilon\\
&+B_iL_iE_iH_i^+\Psi+B_iL_iE_iH_i^-\Upsilon)x(t_0).
\end{aligned}$$

Using equations (6), (11) and (14), we have

$$\dot{x}(t_0) \succeq (A_i + \frac{1}{\gamma}\frac{B_i L_{di}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{+T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}\Psi + \gamma\frac{B_i L_{di}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}\Upsilon$$
$$+ \frac{1}{\gamma}\frac{B_i L_{di} E_i\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{+T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}\Psi + \gamma\frac{B_i L_{di} E_i\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}\Upsilon)x(t_0).$$

From equation (10a), it is easy to obtain

$$\begin{aligned}
& A_i + \frac{\Gamma_{is1}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{+T}}{\chi_{is}}\Psi + \frac{\Gamma_{is2}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{-T}}{\chi_{is}}\Upsilon \\
& + \frac{\Gamma_{is3}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{+T}}{\chi_{is}}\Psi + \frac{\Gamma_{is4}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{-T}}{\chi_{is}}\Upsilon + \frac{\delta_i}{\chi_{is}}I \\
& = \frac{1}{\gamma}\frac{B_i L_{di}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{+T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}\Psi + \gamma\frac{B_i L_{di}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}\Upsilon \\
& + \frac{1}{\gamma}\frac{B_i L_{di} E_i\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{+T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}\Psi + \frac{\delta_i}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}I \\
& + \gamma\frac{B_i L_{di} E_i\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di} B_i^T v_i}\Upsilon + A_i \succeq 0.
\end{aligned}$$

Thus, $A_i + B_i L_{di} F_i^+ \Psi + B_i L_{ui} F_i^- \Upsilon + B_i L_{di} E_i H_i^+ \Psi + B_i L_{ui} E_i H_i^- \Upsilon + \frac{\delta_i}{\mathbf{1}_m^T L_{di} B_i^T v_i} I \succeq 0$. This together with Lemma 3 gives that $A_i + B_i L_{di} F_i^+ \Psi + B_i L_{ui} F_i^- \Upsilon + B_i L_{di} E_i H_i^+ \Psi + B_i L_{ui} E_i H_i^- \Upsilon$ is a Metzler matrix. This implies that $A_i + B_i L_i F_i^+ \Psi + B_i L_i F_i^- \Upsilon + B_i L_i E_i H_i^+ \Psi + B_i L_i E_i H_i^- \Upsilon$ is Metzler. Define a class of indicators: $\Im := \{i : x_i(t) = 0\}$. Then, for some $p \in S$, we obtain $\dot{x}_i(t) \geq \sum_{j \notin \Im} \Theta_p^{(ij)} x_j(t)$ where $i \in \Im$, $\Lambda_p = A_p + B_p L_p F_p^+ \Psi + B_p L_p F_p^- \Upsilon + B_p L_p E_p H_p^+ \Psi + B_p L_p E_p H_p^- \Upsilon$ and $\Theta_p^{(ij)}$ is the ith row jth column element of Λ_p. Since Λ_p is a Metzler matrix, it is easy to get $\Theta_p^{(ij)} \geq 0$ for $i \neq j$. Thus, we have $\dot{x}_i(t_0) \geq 0$, that is, $x_i(t_0) \geq 0$. We obtain $x(t) \succeq 0$ by using recursive derivation for any initial state $x(t_0) \succeq 0$. Then, the resulting closed-loop system (15) is positive by Definition 1.

Choose a linear co-positive Lyapunov function as

$$V(x(t)) = x^T(t) v_i. \tag{16}$$

Give a switching interval $[t_s, t_{s+1})$ and the event-triggered interval $[t_p, t_{p+1})$. Assume that the switched system is switched from i to j, $\forall i, j \in S$ at the switching instant t_s. The stability of the system (9) is discussed in two cases.

Case 1: Assume that there is no any event-triggered time instant in the interval $[t_s, t_{s+1})$, that is, $t_p \leq t_s$ and $t_{p+1} \geq t_{s+1}$. Then

$$\begin{aligned}
\dot{V}(x(t)) = {} & ((A_i + B_i L_i F_i + B_i L_i E_i H_i)x(t) \\
& + (B_i L_i F_i + B_i L_i E_i H_i)x_e(t))^T v_i,
\end{aligned} \tag{17}$$

where $t \in [t_s, t_{s+1})$. Using equations (4), (10d), (10e), (14) and (15), we obtain

$$\begin{aligned}\dot{V}(x(t)) &\leq x(t)^T(A_i + \Upsilon\frac{\xi_i^{+T}B_iL_{ui}\mathbf{1}_r}{\mathbf{1}_r^TL_{ui}^TB_i^Tv_i} + \Psi\frac{\xi_i^{-T}B_iL_{di}\mathbf{1}_r}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} + \Upsilon\frac{\zeta_i^{+T}B_iL_{ui}E_i\mathbf{1}_r}{\mathbf{1}_r^TL_{ui}^TB_i^Tv_i} \\ &\quad + \Psi\frac{\zeta_i^{-T}B_iL_{di}E_i\mathbf{1}_r}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i})^Tv_i \\ &\leq x(t)^T(A_i^Tv_i + \Upsilon\xi_i^+ + \Psi\xi_i^- + \theta_2\Upsilon\zeta_i^+ + \theta_1\Psi\zeta_i^-).\end{aligned} \tag{18}$$

Connecting equations (10b) and (18), we have $\dot{V}(x(t)) \leq -\mu V_{\sigma(t_s)}(x(t_s))$. Taking integral for both sides of it gives

$$V(x(t)) \leq e^{-\mu(t-t_s)}V_{\sigma(t_s)}(x(t_s)), t \in [t_s, t_{s+1}). \tag{19}$$

Case 2: Assume that there are some event-triggered time instants in $[t_s, t_{s+1})$. Denote the switching sequence by $t_p \leq t_s < t_{p+1} < t_{p+2} < \ldots < t_{p+l} \leq t_{s+1}$, $\forall l \in \mathbb{N}$. Combining equations (17) and (18), it follows that

$$\dot{V}(x(t)) \leq -\mu V_{\sigma(t_{p+n})}(x(t_{p+l})),\ t \in [t_{p+l}, t_{s+1}).$$

Taking integration both sides of it from t_{p+l} to t yields that $V(x(t)) \leq e^{-\mu(t-t_{p+n})}V_{\sigma(t_{p+l})}(x(t_{p+l}))$. Then, we have

$$V(x(t)) \leq e^{-\mu(t-t_{p+l-1})}V_{\sigma(t_{p+l-1})}(x(t_{p+l-1})), \tag{20}$$

where $t \in [t_{p+l-1}, t_{p+l})$. Next, we can obtain similar inequalities in the form of equation (19) for each subinterval as well. With the fact $\sigma(t_s) = \sigma(t_{p+1}) = \ldots = \sigma(t_{p+l})$, it is easy to get a similar result to equation (19). Using equations (10c) and (19), we have $V(x(t)) \leq \lambda e^{-\mu(t-t_s)}V_{\sigma(t_{s-1})}(x(t_s))$. Suppose that $0 = t_0 < t_1 < t_2 < \ldots < t_s = t_{N_\sigma(t_0,t)} < t$ is the switching time sequences of $\sigma(t)$ in the interval $[0, t)$. The rest of the proof can be obtained using a similar method used in Zhang and Raïssi (2019). Hence, the resulting closed-loop system (9) is stable. □

Remark 1: The event-triggered control of positive systems is different from the one of general systems (Heemels et al., 2008; Qi et al., 2019; Zou et al., 2019). Under the event-triggering condition, the error term in general systems can be transformed into the term related to the state. Thus, the stability is achieved under Lyapunov functions. For positive systems, the first step is to guarantee the positivity of the closed-loop systems, which contain the error term. Therefore, the positivity criterion (e.g. Lemma 1) cannot be directly used. To solve this issue, Theorem 1 transforms the original system into an interval uncertain system, which only contains the state term. Thus, the positivity of the systems can be obtained by considering the lower bound system of the interval system.

Remark 2: Theorem 1 considers the fault problem of the actuator and the influence from the fluctuation of the actuator parameters. Suppose that $\Delta F_i = 0$ and $L_i = I$, the controller in Theorem 1 becomes an event-triggered controller of positive switched systems (Liu et al., 2019). Furthermore, the controller in Theorem 1 is a state-feedback controller of positive switched systems (Xiang and Xiang, 2013; Liu et al., 2015a, 2018; Zhang et al.,

2017) provided the event-triggering condition is removed. Therefore, the event-triggered controller in Theorem 1 is more practical and general than existing ones in literature.

Next, the positivity and stability of positive switched systems with actuator saturation is discussed in Theorem 2.

Theorem 2: *If there exist constants* $\delta_i > 0, \mu > 0, \lambda > 1$*, and vectors* $v_i \succ 0$ *with* $v_i \in \mathbb{R}^n$*.* $\xi_{i\iota}^+ \succ 0$ *with* $\xi_{i\iota}^+ \in \mathbb{R}^n$*,* $\xi_{i\iota}^- \prec 0$ *with* $\xi_{i\iota}^- \in \mathbb{R}^n$*,* $\xi_i^+ \succ 0$ *with* $\xi_i^+ \in \mathbb{R}^n$*,* $\xi_i^- \prec 0$ *with* $\xi_i^- \in \mathbb{R}^n$*,* $\zeta_{i\iota}^+ \succ 0$ *with* $\zeta_{i\iota}^+ \in \mathbb{R}^n$*,* $\zeta_{i\iota}^- \prec 0$ *with* $\zeta_{i\iota}^- \in \mathbb{R}^n$*,* $\zeta_i^+ \succ 0$ *with* $\zeta_i^+ \in \mathbb{R}^n$*,* $\zeta_i^- \prec 0$ *with* $\zeta_i^- \in \mathbb{R}^n$ *such that*

$$\begin{aligned} &\aleph_{is}A_i + \Im_{is1}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{+T}\Psi + \Im_{is2}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{-T}\Upsilon \\ &\quad + \Im_{is3}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{+T}\Psi + \Im_{is4}\sum_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{-T}\Upsilon \\ &\quad + \aleph_{is}\Im_{is5}\Upsilon + \delta_i I \succeq 0, \end{aligned} \tag{21a}$$

$$\begin{aligned} &A_i^T v_i + \Upsilon\xi_i^+ + \Psi\xi_i^- + \theta_2\Upsilon\zeta_i^+ + \theta_1\Psi\zeta_i^- \\ &\quad + \Psi M_i^T G_{ig}^{-T} B_i^T v_i + \mu v_i \prec 0, G_{ig} \neq I, G_{ig} \neq 0, \end{aligned} \tag{21b}$$

$$\begin{aligned} &A_i^T v_i + \Upsilon\xi_i^+ + \Psi\xi_i^- + \theta_2\Upsilon\zeta_i^+ + \theta_1\Psi\zeta_i^- \\ &\quad + \mu v_i \prec 0, G_{ig} = I, \end{aligned} \tag{21c}$$

$$A_i^T v_i + \Psi M_i^T G_{ig}^{-T} B_i^T v_i + \mu v_i \prec 0, G_{ig} = 0, \tag{21d}$$

$$v_i \preceq \lambda v_j, \tag{21e}$$

$$\xi_{i\iota}^+ \preceq \xi_i^+, \xi_{i\iota}^- \preceq \xi_i^-, \ \iota = 1, \ldots, r, \tag{21f}$$

$$\zeta_{i\iota}^+ \preceq \zeta_i^+, \zeta_{i\iota}^- \preceq \zeta_i^-, \ \iota = 1, \ldots, r, \tag{21g}$$

$$v_i + M_{is}^T \succeq 0, \tag{21h}$$

hold $\forall (i,j) \in S \times S, \ i \neq j, \ s = 1, \ldots, r, \ \aleph_{is} = \mathbf{1}_r^T L_{di}^T G_{ig}^T B_i^T v_i, \ \Im_{is1} = \frac{1}{\gamma} B_i G_{ig} L_{di},$ $\Im_{is2} = \gamma B_i G_{ig} L_{di}, \Im_{is3} = \frac{1}{\gamma} B_i G_{ig} L_{di} E_i, \Im_{is4} = \gamma B_i G_{ig} L_{di} E_i, \Im_{is5} = B_i G_{ig}^- M_i$*, and* $\Psi = I - \alpha \mathbf{1}_{n\times n}$*,* $\Upsilon = I + \alpha \mathbf{1}_{n\times n}$*, then under the control law (8) with* $F_i = F_i^+ + F_i^-$*,* $H_i = H_i^+ + H_i^-$*, and*

$$\begin{aligned} F_i^+ &= \frac{1}{\mathbf{1}_r^T L_{ui}^T G_{ig}^T B_i^T v_i} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{+T}, \\ F_i^- &= \frac{1}{\mathbf{1}_r^T L_{di}^T G_{ig}^T B_i^T v_i} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{-T}, \\ H_i^+ &= \frac{1}{\mathbf{1}_r^T L_{ui}^T G_{ig}^T B_i^T v_i} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T}, \\ H_i^- &= \frac{1}{\mathbf{1}_r^T L_{di}^T G_{ig}^T B_i^T v_i} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T}, \end{aligned} \tag{22}$$

the resulting closed-loop system (2) is positive and stable with the average dwell time switching law (12). In addition, the system states starting from $x(t) \in \Omega(v_i, 1)$ *will remain inside* $\bigcup_i^J \Omega(v_i, 1)$*, where* $N_0 = 0$.

Proof: By equation (8), the system (2) with state saturation can be rewritten as:

$$\dot{x}(t) = A_i x(t) + B_i \text{sat}(L_i(F_i + E_i H_i)\hat{x}(t)).$$

By Lemma 4, we have

$$\begin{aligned}\dot{x}(t) = \textstyle\sum_{g=1}^{2^r} \ell_{ig}[&A_i x(t) + B_i G_{ig} L_i (F_i + E_i H_i) x(t) \\ &+ B_i G_{ig} L_i (F_i + E_i H_i) x_e(t) \\ &+ B_i G_{ig}^- M_i)(x(t) + x_e(t))].\end{aligned} \tag{23}$$

Noting the condition $G_{ig} \succeq 0$, we obtain $\mathbf{1}_r^T K_{di} G_{ig}^T B_i^T v_i \geq 0$. Then, for initial state condition $x(t_0) \succeq 0$, one can obtain from equations (6), (14) and (15) that

$$\begin{aligned}\dot{x}(t_0) \succeq \sum_{g=1}^{2^r} \ell_{ig}[&A_i + \frac{1}{\gamma} \frac{B_i L_{di} \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{+T}}{\mathbf{1}_r^T L_{di}^T G_{ig}^T B_i^T v_i} \Psi \\ &+ \gamma \frac{B_i L_{di} \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di}^T G_{ig}^T B_i^T v_i} \Upsilon \\ &+ \frac{1}{\gamma} \frac{B_i L_{di} E_i \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T}}{\mathbf{1}_r^T L_{di}^T G_{ig} B_i^T v_i} \Psi \\ &+ \gamma \frac{B_i L_{di} G_{ig}^T E_i \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di}^T G_{ig}^T B_i^T v_i} \Upsilon \\ &+ B_i G_{ig}^- M_i \Upsilon] x(t_0).\end{aligned} \tag{24}$$

By equation (21a), we have

$$\begin{aligned}A_i &+ \frac{\Im_{is1} \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{+T} \Psi}{\aleph_{is}} + \frac{\Im_{is2} \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{-T} \Upsilon}{\aleph_{is}} \\ &+ \frac{\Im_{is3} \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} \Psi}{\aleph_{is}} + \frac{\Im_{is4} \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} \Upsilon}{\aleph_{is}} \\ &+ \Im_{is5} \Upsilon + \frac{\delta_i}{\aleph_{is}} I \succeq 0.\end{aligned}$$

Thus,

$$\begin{aligned}A_i &+ B_i G_{ig} L_i F_i^+ \Psi + B_i G_{ig} L_i F_i^- \Upsilon + B_i G_{ig} L_i E_i H_i^+ \Psi \\ &+ B_i G_{ig} L_i E_i H_i^- \Upsilon + B_i G_{ig}^- M_i \Upsilon + \frac{\delta_i}{\mathbf{1}_r^T L_{di}^T G_{ig}^T B_i^T v_i} I \\ &\succeq 0.\end{aligned}$$

By Lemma 3, $A_i + B_i G_{ig} L_i F_i^+ \Psi + B_i G_{ig} L_i F_i^- \Upsilon + B_i G_{ig} L_i E_i H_i^+ \Psi + B_i G_{ig} L_i E_i H_i^- \Upsilon + B_i G_{ig}^- M_i \Upsilon$ is a Metzler matrix. Hence, $\sum_{g=1}^{2^r} \ell_{ig}[A_i +$

$B_iG_{ig}L_iF_i^+\Psi + B_iG_{ig}L_iF_i^-\Upsilon + B_iG_{ig}L_iE_iH_i^+\Psi + B_iG_{ig}L_iE_iH_i^-\Upsilon + B_iG_{ig}^-M_i\Upsilon]$ is Metzler. Similar to the method used in Theorem 1, we can obtain the rest of the proof.

Choose a linear co-positive Lyapunov function as (16), then

$$\begin{aligned}\dot{V}(x(t)) = \sum_{g=1}^{2^r} \ell_{ig}[&(B_iG_{ig}L_i(F_i + E_iH_i)x(t)\\ &+ (B_iG_{ig}L_iF_i + B_iG_{ig}L_iE_iH_i)x_e(t)\\ &+ B_iG_{ig}^-M_i(x(t) + x_e(t)) + A_ix(t)]^T v_i.\end{aligned}$$

By equation (15), we have

$$\begin{aligned}\dot{V}(x(t)) \le \sum_{g=1}^{2^r} \ell_{ig}x^T(t)[&A_i + B_iG_{ig}L_iF_i^+\Upsilon\\ &+ B_iG_{ig}L_iF_i^-\Psi + B_iG_{ig}L_iE_iH_i^+\Upsilon\\ &+ B_iG_{ig}L_iE_iH_i^-\Psi + B_iG_{ig}^-M_i\Psi]^T v_i.\end{aligned} \tag{25}$$

Using equations (21f), (21g), and (22) follows that

$$\begin{aligned}&(B_iG_{ig}L_iF_i^+)^T v_i = \frac{\sum_{\iota=1}^r \xi_{i\iota}^+ \mathbf{1}_r^{(\iota)T} L_i^T G_{ig}^T B_i^T v_i}{\mathbf{1}_r^T L_{ui}^T G_{ig}^T B_i^T v_i}\\ &\preceq \frac{\xi_i^+ \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)T} L_i^T G_{ig}^T B_i^T v_i}{\mathbf{1}_r^T L_{ui}^T G_{ig}^T B_i^T v_i} \prec \xi_i^+,\end{aligned} \tag{26a}$$

$$\begin{aligned}&(B_iG_{ig}L_iF_i^-)^T v_i = \frac{\sum_{\iota=1}^r \xi_{i\iota}^- \mathbf{1}_r^{(\iota)T} L_i^T G_{ig}^T B_i^T v_i}{\mathbf{1}_r^T L_{di}^T G_{ig}^T B_i^T v_i}\\ &\preceq \frac{\xi_i^- \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)T} L_i^T G_{ig}^T B_i^T v_i}{\mathbf{1}_r^T L_{di}^T G_{ig}^T B_i^T v_i} \prec \xi_i^-,\end{aligned} \tag{26b}$$

$$\begin{aligned}&(B_iG_{ig}L_iE_iH_i^+)^T v_i\\ &\preceq \frac{\zeta_i^+ \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)T} E_i^T L_i^T G_{ig}^T B_i^T v_i}{\mathbf{1}_r^T L_{ui}^T G_{ig}^T B_i^T v_i} \prec \theta_2\zeta_i^+,\end{aligned} \tag{26c}$$

$$\begin{aligned}&(B_iG_{ig}L_iE_iH_i^-)^T v_i\\ &\preceq \frac{\zeta_i^- \sum_{\iota=1}^r \mathbf{1}_r^{(\iota)T} E_i^T L_i^T G_{ig}^T B_i^T v_i}{\mathbf{1}_r^T L_{di}^T G_{ig}^T B_i^T v_i} \prec \theta_1\zeta_i^-.\end{aligned} \tag{26d}$$

Case 1: When $G_{ig} \neq I$ and $G_{ig} \neq 0$, we obtain from equations (25) and (26) that

$$\begin{aligned}\dot{V}(x(t)) \le x^T(t)(&A_i^T v_i + \Upsilon\xi_i^+ + \Psi\xi_i^- + \theta_2\Upsilon\zeta_i^+\\ &+ \theta_1\Psi\zeta_i^- + \Psi M_i^T G_{ig}^{-T} B_i^T v_i).\end{aligned} \tag{27}$$

Case 2: When $G_{ig} = I$,

$$\begin{aligned}
&(B_i G_{ig} L_i F_i^{+})^T v_i \prec \xi_i^{+}, (B_i G_{ig} L_i F_i^{-})^T v_i \prec \xi_i^{-}, \\
&(B_i G_{ig} L_i E_i H_i^{+})^T v_i \prec \theta_2 \zeta_i^{+}, \\
&(B_i G_{ig} L_i E_i H_i^{-})^T v_i \prec \theta_1 \zeta_i^{-}, B_i G_{ig}^{-} M_i = 0.
\end{aligned} \tag{28}$$

By equations (25) and (28), we have

$$\begin{aligned}
\dot{V}(x(t)) \leq\ & x^T(t)(A_i^T v_i + \Upsilon \xi_i^{+} + \Psi \xi_i^{-} \\
& + \theta_2 \Upsilon \zeta_i^{+} + \theta_1 \Psi \zeta_i^{-}).
\end{aligned} \tag{29}$$

Case 3: When $G_{ig} = 0$,

$$\begin{aligned}
&(B_i G_{ig} K_i E_i^{+})^T v_i = 0, (B_i G_{ig} K_i E_i^{-})^T v_i = 0, \\
&(B_i G_{ig} K_i F_i H_i^{+})^T v_i = 0, (B_i G_{ig} K_i F_i H_i^{-})^T v_i = 0,
\end{aligned} \tag{30}$$

Connecting equations (25) and (30) gives

$$\dot{V}(x(t)) \leq x^T(t)(A_i^T v_i + \Psi M_i^T G_{ig}^{-T} B_i^T v_i). \tag{31}$$

Using equations (21b)–(21d), (27), (29), and (31) yields

$$\dot{V}(x(t)) \leq -\mu x^T(t) v_i. \tag{32}$$

The rest of the proof can be obtained by using the similar method in Theorem 1.

Finally, we will discuss the invariance of the system (2). Driving from (32), we can get that $x^T(t)v_i \leq e^{-N_0 \ln \lambda} x^T(t_0) v_i \leq 1$, where $N_0 = 0$. That means $x(t) \in \bigcup_i^J \Omega(v_i, 1)$ for any $x(t_0) \in \bigcup_i^J \Omega(v_i, 1)$. From (21h), it implies that $\Omega(v_i, 1) \subseteq T(M_i)$. Consequently, the $\bigcup_i^J \Omega(v_i, 1)$ is an estimate of domain of attraction. □

Theorem 1 only considers the problem of actuator fault of positive switched systems. Indeed, actuator saturation is an inevitable phenomenon in actual control systems due to the limited capacity of components and the constraints of variables. Actuator saturation will deteriorate the system performances and even lead to the instability of systems. Thus, actuator saturation is taken into account in Theorem 2. Theorem 2 designs a non-fragile event-triggered controller. Under the designed controller, the considered systems are positive and stable even if the systems are subject to faults. This implies that the systems can resist the risk from the faults of the actuator.

Remark 3: In Wang and Zhao (2016), Zhang et al. (2019a), Park et al (2018), Qi et al. (2017), Zhang et al. (2014) and Zhang and Raïssi (2019), the saturation problem of positive systems was explored. However, these literature ignored the practicability of the design and the fragility of the controller owing to the parameter fluctuation. The obtained controller may fail when the systems are subject to uncertain fluctuation. Considering these problems in the literature, Theorem 2 proposes the non-fragile event-triggered control design. The designed controller in Theorem 2 can resist the risk from the faults of the actuator and the parameter fluctuation of the controller gain.

Remark 4: In Zuo et al. (2010), a saturation avoidance feedback approach was proposed to deal with the saturation issue. In Selvaraj et al. (2017, 2018), the saturation term was represented in terms of convex hull. The conditions in the literature were determined by using linear matrix inequalities. Linear matrix inequalities are more complex than linear programming. In particular, it is not easy to guarantee the positivity of the systems under the framework of linear matrix inequalities. What's more, these existing results are all based on the time-triggered control strategy. Considering these points, Theorem 2 designs a non-fragile event-triggered controller for positive switched systems in terms of linear programming.

Remark 5: Theorem 1 investigates the non-fragile event-triggered control of positive switched systems with actuator faults and Theorem 2 further considers the actuator saturation issue. Indeed, the systems may be subject to constraints in practice (Mayne et al., 2000; Zhang et al., 2016, 2019b). In Zhang et al. (2019b), a model predictive control controller was designed for positive systems with constrained state and input by virtue of linear programming. Motivated by these results, it is interesting and possible to consider the non-fragile event-triggered control of constrained positive switched systems in the future work.

4 Illustrative example

In Yang et al. (2019), a communication network with three nodes was established via positive switched systems in Figure 1. The communication network has two modes, namely, busy time and idle time, and switches between the two modes. In practice, it is clear that the communication is always subject to faults and saturation owing to the frequent use and too many users. It implies that the model in (2) is more suitable for describing the communication network. To avoid the network crash, it is necessary to restrict the speed of some communication channel at some time. This is a typical event-triggered problem. Considering these points, we will continue to study the communication network under the control strategy in this paper.

Consider the system (2) with:

$$A_1 = \begin{pmatrix} -2.1 & 1.6 & 1.5 \\ 2 & -1.8 & 2 \\ 1.3 & 1.3 & -2.5 \end{pmatrix}, B_1 = \begin{pmatrix} 0.1 & 0.25 \\ 0.25 & 0.1 \\ 0.15 & 0.25 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -2.1 & 1.8 & 0.7 \\ 2.5 & -2.1 & 1.5 \\ 1.3 & 1.4 & -2.3 \end{pmatrix}, B_2 = \begin{pmatrix} 0.2 & 0.3 \\ 0.4 & 0.2 \\ 0.15 & 0.2 \end{pmatrix}.$$

Given

$$E_1 = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.25 \end{pmatrix}, L_{d1} = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.2 \end{pmatrix}, L_{u1} = \begin{pmatrix} 0.33 & 0 \\ 0 & 0.26 \end{pmatrix},$$

$$E_2 = \begin{pmatrix} 0.65 & 0 \\ 0 & 0.65 \end{pmatrix}, L_{d2} = \begin{pmatrix} 0.42 & 0 \\ 0 & 0.3 \end{pmatrix}, L_{u2} = \begin{pmatrix} 0.42 & 0 \\ 0 & 0.35 \end{pmatrix},$$

$$M_1 = \begin{pmatrix} -0.03 & -0.04 & -0.03 \\ -0.05 & -0.05 & -0.04 \end{pmatrix}, M_2 = \begin{pmatrix} -0.05 & -0.04 & -0.03 \\ -0.04 & -0.06 & -0.04 \end{pmatrix},$$

then $\theta_1 = 0.25$ and $\theta_2 = 0.65$. Choose $\mu = 0.2,\ \lambda = 1.1,\ \gamma = 1.1,$ and $\alpha = 0.15$. By Theorem 2, we get

$$v_1 = \begin{pmatrix} 157.4397 \\ 145.6831 \\ 148.4657 \end{pmatrix}, \xi_1^- = \begin{pmatrix} -95.6526 \\ -99.9504 \\ -97.7878 \end{pmatrix}, \xi_1^+ = \begin{pmatrix} 149.7315 \\ 148.5994 \\ 149.7920 \end{pmatrix},$$

$$\zeta_1^+ = \begin{pmatrix} 156.2888 \\ 154.4419 \\ 157.4469 \end{pmatrix}, \zeta_1^- = \begin{pmatrix} -71.5978 \\ -72.2095 \\ -71.8075 \end{pmatrix}, v_2 = \begin{pmatrix} 158.0387 \\ 147.8074 \\ 133.5772 \end{pmatrix},$$

$$\xi_2^+ = \begin{pmatrix} 143.6352 \\ 145.4154 \\ 149.2430 \end{pmatrix}, \xi_2^- = \begin{pmatrix} -113.1081 \\ -98.4270 \\ -54.8880 \end{pmatrix}, \zeta_2^+ = \begin{pmatrix} 152.2234 \\ 153.8178 \\ 157.8917 \end{pmatrix},$$

$$\zeta_2^- = \begin{pmatrix} -66.5949 \\ -63.2587 \\ -43.9530 \end{pmatrix}, \delta_1 = 199.4658, \delta_2 = 277.5274,$$

$$\tau^* \geq 0.4766.$$

Then,

$$F_1^+ = \begin{pmatrix} 1.7121 & 1.5851 & 1.6959 \\ 1.6726 & 1.6746 & 1.6727 \end{pmatrix},$$

$$H_1^+ = \begin{pmatrix} 1.9970 & 1.8931 & 2.0357 \\ 1.9851 & 1.9225 & 2.0286 \end{pmatrix},$$

$$F_1^- = \begin{pmatrix} -8.4347 & -9.6408 & -8.6827 \\ -8.5210 & -8.3283 & -8.6947 \end{pmatrix},$$

$$H_1^- = \begin{pmatrix} -6.1276 & -6.2604 & -6.1527 \\ -6.1398 & -6.1247 & -6.1512 \end{pmatrix},$$

$$F_2^+ = \begin{pmatrix} 1.0729 & 1.1858 & 1.1201 \\ 1.0280 & 1.1110 & 1.4166 \end{pmatrix},$$

$$H_2^+ = \begin{pmatrix} 1.3800 & 1.4754 & 1.4185 \\ 1.3406 & 1.3960 & 1.7269 \end{pmatrix},$$

$$F_2^- = \begin{pmatrix} -8.2851 & -6.8274 & -5.3568 \\ -7.6407 & -6.7329 & -2.6148 \end{pmatrix},$$

$$H_2^- = \begin{pmatrix} -4.4992 & -4.1893 & -3.3239 \\ -4.3220 & -4.1613 & -2.5157 \end{pmatrix}.$$

Choose $L_1 = \begin{pmatrix} 0.32 & 0 \\ 0 & 0.25 \end{pmatrix}$, $L_2 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.33 \end{pmatrix}$ and denote

$$N_1 = A_1 + \sum_{g=1}^{2^r} \ell_g(B_1G_{1g}L_1F_1^+ + B_1G_{1g}L_1F_1^- + B_1G_{1g}L_1E_1H_1^+ + B_1G_{1g}L_1E_1H_1^- + B_1G_{1g}^-M_1),$$

$$N_2 = A_2 + \sum_{g=1}^{2^r} \ell_g(B_2G_{2g}L_2F_2^+ + B_2G_{2g}L_2F_2^- + B_2G_{2g}L_2E_2H_2^+ + B_2G_{2g}L_2E_2H_2^- + B_2G_{2g}^-M_2).$$

Then, under the event-triggered control law (8), we have

$$N_1 = \begin{pmatrix} -2.4783 & 1.2046 & 1.1136 \\ 1.5850 & -2.2697 & 1.5729 \\ 0.8589 & 0.8305 & -2.9513 \end{pmatrix},$$

$$N_2 = \begin{pmatrix} -2.8038 & 1.1235 & 0.3873 \\ 1.4647 & -2.9513 & 0.9955 \\ 0.7329 & 0.9240 & -2.5270 \end{pmatrix}.$$

The simulation of system states is showed in Figures 2 with the initial condition $x(t_0) = [0.5\ 0.2\ 0.3]^T$, and the event-triggered signal can be found in Figure 3, and the domain of attraction can be seen in Figure 4.

Figure 1 Data communication network consisting of three nodes

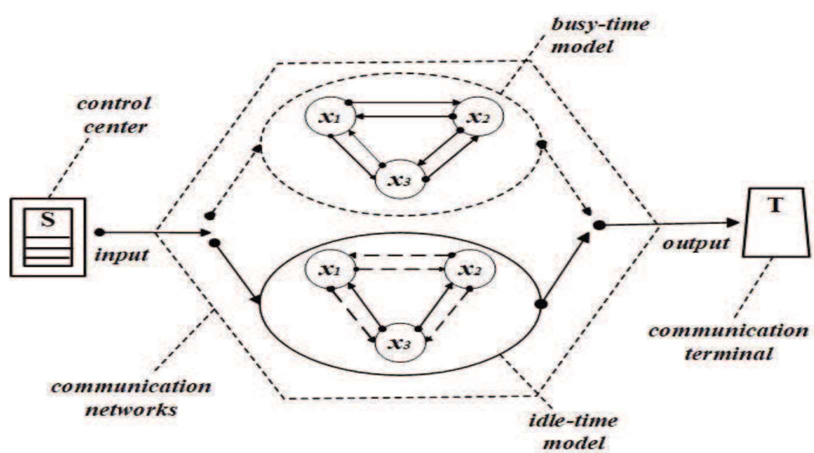


Figure 2 Simulation results of the system state $x(t)$ under ADT switching (see online version for colours)

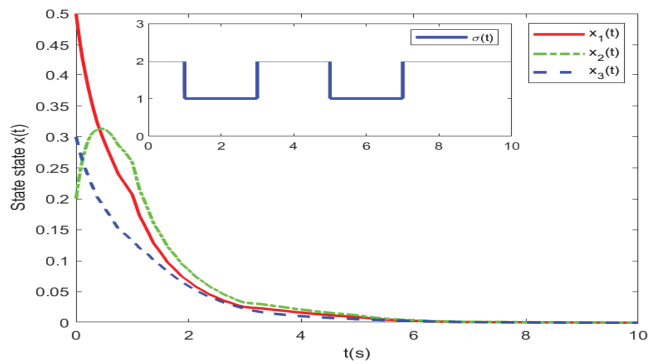


Figure 3 The event-triggered signal (see online version for colours)

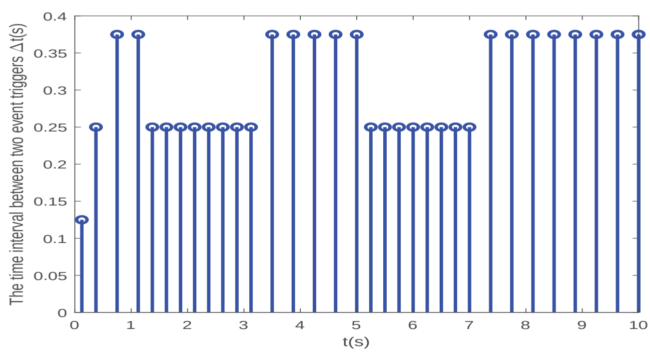
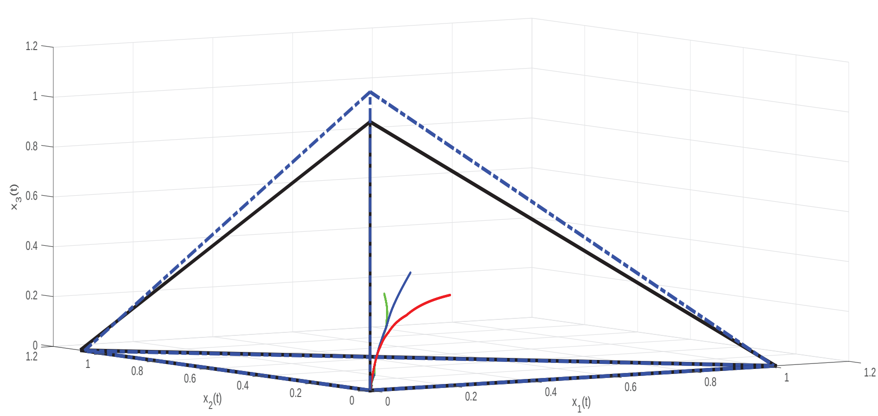


Figure 4 Domain of attraction (see online version for colours)



5 Conclusions

This paper proposed a non-fragile control approach for positive switched system without/with actuator saturation based on event triggering. First, an event-triggering mechanism is established in the form of 1-norm. On this basis, the controller is designed considering non-fragile control integrated with event-triggering mechanism. A new type matrix decomposition technique was employed to divided the gain matrix into a normal gain matrix and a gain perturbation matrix. Then, actuator saturation is taken into account. All the conditions given in this paper are obtained by linear programming. In the future work, the presented non-fragile event-triggered control framework can be extended to other hybrid positive systems in the presence of actuator saturation.

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