Six Sigma-based RS, MDSS and MDSRS control charts

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Abstract: In this paper, well-defined Six Sigma-based repetitive sampling control charts (RSCCs) and multiple dependent state sampling control charts (MDSSCCs) are proposed. The Six Sigma-based multiple dependent state repetitive sampling control charts (MDSRSCCs) are also studied. The average run length (ARL) performance of the proposed charts are numerically evaluated and compared with the existing RSCCs, MDSSCCs and MDSRSCCs. Since the proposed charts are based on the Six Sigma methodology, these charts ensure the Six Sigma goal of 3.4 defects per million opportunities. It is observed that the proposed charts perform better than the existing ones by means of better average run lengths and also suggest minimum process disturbance for smaller shifts in the process mean. The charts are easier to apply by the quality practitioners. A numerical example is given to illustrate the better performance of the proposed control charts.

Keywords: average run length; multiple dependent state sampling; repetitive sampling; Six Sigma; zone control charts.

Reference to this paper should be made as follows: Zacharia, A. and Ravichandran, J. (2021) 'Six Sigma-based RS, MDSS and MDSRS control charts', *Int. J. Six Sigma and Competitive Advantage*, Vol. 13, No. 4, pp.435–455.

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1 Introduction

In statistical quality control (SQC)/statistical process control (SPC), control charts developed by Shewhart (1931) serve as a key technique mainly to monitor a manufacturing process and also to prevent products from going out of the given specification limits (Aslam et al., 2014a). In fact, given a manufacturing set up with product specification limits – lower specification limit (LSL), upper specification limit (USL) and target (T), the manufacturer always wishes to produce products of high quality meaning that the products produced so will meet the set specification requirements or the ones provided by the customers in the form of specification limits (Ravichandran, 2019a). Control charts consisting of lower control limit (LCL), upper control limit (UCL) and process average, are treated as an important tool for detecting the causes that significantly influence the process variation as early as possible. More details about understanding the relationship between control limits and specification limits can be found in Ravichandran (2019b).

As discussed by Aslam et al. (2015a, 2015b) and Aldosari et al. (2017), a process of interest is always expected to produce products in tune with the set target and the related specification limits. However, there is every possibility that a process may experience a shift from the target due to internal disturbances (influenced by chance causes) and/or external disturbances (influenced by assignable causes). For example, refer to Grant and Leavenworth (1996) and Montgomery (2009) for more details about these concepts. While traditional control charts introduced by Shewhart (1931) are found to be effective in detecting large shifts from the target, the control charts such as cumulative sum (CUSUM) charts and exponentially weighted moving average (EWMA) charts are very effective in detecting small shifts quickly. Readers are referred to Gadre and Rattihalli (2004), Reynolds and Lou (2010), Saleh et al. (2015) and Ravichandran (2017) for more on the implications of small and large shifts and their detections.

It could be seen from literature that researchers have developed many advanced control charts (either attribute-type or variable-type) depending on the problem under study. While Luo and Wu (2002) developed variable sample size and variable sampling interval control charts for fraction defective using the principle of optimisation methods, Wu and Jiao (2008) proposed a new control chart for monitoring the process mean based on attribute inspection. Similarly, useful studies on variable-type control charts are also made by many authors. Schoonhoven et al. (2009, 2011) proposed various design schemes by using different estimators for the underlying parameters of variable-type control charts. Lee (2011) proposed adaptive range charts with variable parameters. Recently, Ravichandran (2017) proposed Six Sigma-based variable-type control chart for high quality processes.

In addition, there are several studies on run rule-based control charts (Champ and Woodall, 1987), zone control charts (Jaehn, 1987a, 1987b, 1987c), modified zone control charts (Davis et al., 1990), Six Sigma-based zone control charts (Ravichandran, 2019a), self-starting charts (Hawkins, 1987; Quesenberry, 1991, 1995; Sullivan and Jones, 2002; Keefe et al., 2015), and repetitive sampling (RS) and multiple dependent state sampling (MDSS) control charts (Aslam et al., 2014a, 2014b; Aldosari et al., 2017). Also, Aldosari et al. (2018) studied the performance of multiple dependent state repetitive sampling (MDSRS) control charts. Ravichandran (2019c) proposed Six Sigma-based self-starting control charts.

It is observed that, the zone control charts have gained momentum that resulted in the development of RS, MDSS and MDSRS control charts. Originally, RS, MDSS and MDSRS methods have been introduced for studies related to lot acceptance (e.g. Sherman, 1965; Balamurali and Jun, 2006). Because of their better performance, these sampling methods have been used for the construction of RS control charts (RSCCs) (Aslam et al., 2014a), MDSS control charts (MDSSCCs) (Aslam et al., 2015a) and MDSRS control charts (MDSRSCCs) (Aldosari et al., 2018). In these charts, double control limits are proposed using a pre-set average run length (ARL) value. Because of the usage of double control limits, both repetitive and multiple dependent control charts, in fact, split the standard control chart limits into three zones either side from the target: control zone, doubtful zone and out-of-control zone. In the doubtful zone, sampling is repeated to further confirm the state of the process. Ravichandran (2019a) developed Six Sigma-based zone control charts. In this paper, we modify the zones appropriately to propose more meaningful Six Sigma-based ready to use RSCCs, MDSSCCs and MDSRSCCs. Since the proposed charts are based on the Six Sigma methodology, these charts ensure the Six Sigma goal of 3.4 defects per million opportunities (DPMO). It is observed that the proposed Six Sigma-based charts perform better than the existing counterparts by means of better average run lengths and also suggest minimum process disturbance for smaller shifts in the process mean. The charts are easier to apply by the quality practitioners.

Remainder of the paper is organised as follows: In Section 2, we discuss elaborately:

1 various traditional control charts
2 Six Sigma-based control charts
3 RS, MDSS and MDSRS control charts to know how developments in this area of research have evolved over years.

The importance of zone control charts is also discussed in this section. Well-defined procedures of the proposed Six Sigma-based RS, MDSS and MDSRS control charts are given in Sections 3, 4 and 5 respectively. Section 6 is dedicated to numerical results and performance evaluation of the proposed Six Sigma-based RSC, MDSSC and MDSRS control charts. In this section various observations made from the numerical results with regard to these control charts are consolidated. This section presents the findings based on comparison with the existing RS, MDSS and MDSRS control charts. While Section 7 presents an illustrative numerical example, in Section 8 the results are discussed and concluding remarks are given.

2 Traditional, zone, Six Sigma-based, RS, MDSS and MDSRS control charts

Looking at the disadvantages of basic and advanced quality control charts proposed by various authors in getting estimators of unknown population parameters such as mean and standard deviation, a number of methods have been proposed for the estimation of such parameters (e.g., Schoonhoven et al., 2009, 2011). According to Hill (1956) and Carr (1989), a well-controlled process is expected to satisfy the respective specification requirements. This aspect prompted them to develop modified control charts. Also, Aslam et al. (2014a, 2014b) observed that control charts are key techniques in SQC to monitor a manufacturing process and to prevent products from going out of the given specification limits. Ravichandran (2017) focused on this to estimate the population parameters from the given specification limits that can satisfy Six Sigma quality requirements (the goal of 3.4 DPMO). Accordingly, Ravichandran (2017) developed a Six Sigma-based control chart keeping in mind the expectations from high quality processes. Further, Six Sigma-based zone control charts and Six Sigma-based self-starting control charts have also been studied by and Ravichandran (2019a, 2019c). Later, while presenting a review of specification limits and control limits from the perspective of Six Sigma quality processes, Ravichandran (2019b) argued that both control and specification limits should work like hand-in-glove to keep satisfying customer needs.

A typical Six Sigma control chart (SSCC) proposed by Ravichandran (2017) for mean has the limits as given below:

$$\begin{aligned} &\text{Upper control limit (LCL)} = \bar{\bar{x}} + 4.5\sigma_{ss}\big/\sqrt{n} \\ &\text{Central line (CL)} = \hat{\mu} = \bar{\bar{x}} \\ &\text{Lower control limit (UCL)} = \bar{\bar{x}} - 4.5\sigma_{ss}\big/\sqrt{n} \end{aligned} \tag{1}$$

where

$$P[-4.5 \le Z \le 4.5)] = 1 - 6.8 \times 10^{-6} \tag{2}$$

In equation (2), Z is the standard normal variate. The control limits given in (1) will result in DPMO of 3.4 either below LCL or above UCL. Note that in equation (1), $\hat{\mu} = \bar{\bar{x}} = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_i$ is the grand mean with $\bar{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij}$, $i = 1, 2, \ldots, m$, where $x_{1j}, x_{2j}, \ldots, x_{ij}, \ldots, x_{mj}$ is the collection of m samples drawn from the population, each of size n (i.e., $j = 1, 2, \ldots, n$). Further, we assume that the measurable quality characteristic, say X, with a unique specification (LSL, T and USL), follows normal distribution with mean T and variance σ_{ss}^2, that is $X \sim N(T, \sigma_{ss}^2)$, with $P(T - K\sigma_{ss} \le X \le T + K\sigma_{ss}) = 1 - \alpha$, where α is a prespecified probability value such that $\alpha = P(X < T - K\sigma_{ss}) + P(X > T + K\sigma_{ss})$. Then, from $T \pm K\sigma_{ss}$, under the normality assumption we get $\sigma_{ss} = (USL - T)/6$ or $\sigma_{ss} = (T - LSL)/6$ as half of the Six Sigma process spread with $K = 6$.

Referring to the works of Jaehn (1989), Davis et al. (1990, 1994) and Zhang et al. (2016) on zone control charts (ZCCs) and their performances, Ravichandran (2019a) proposed Six Sigma zone control charts (SSZCCs) and studied the transition probabilities and ARL performance of such SSZCCs. A typical SSZCC has ten zones (five each on either side of the chart) as shown in Table 1.

According to Six Sigma quality program introduced by Motorola, the Six Sigma quality process tolerates a shift of ±1.5 times of standard deviation from the target in the long process run, since it still results in a defect rate of just 3.4 DPMO (Harry, 1998; Lucas, 2002). Hence, SSZCC considers the zones 5 and 6 given in Table 1 as safe zones which covers the region $T - 1.5\sigma_{ss}\big/\sqrt{n} \le \bar{\bar{x}} \le T + 1.5\sigma_{ss}\big/\sqrt{n}$ and is called the *target range* (Ravichandran, 2006).

As discussed earlier in the introduction section, since the repetitive sampling approach proposed by Sherman (1965) and Balamurali and Jun (2006) for acceptance sampling plans is found to be more efficient, Aslam et al. (2014a, 2014b, 2015a, 2015b) studied the RSCCs for both attribute and variable-type cases. Unlike the traditional control chart, the RSCC uses two pairs of control limits. Accordingly, the process of interest is declared to be in control if a plotted statistic falls within the inner control limits, while it is declared out-of-control if the statistic is plotted beyond the outer control limits. If the statistic is located between the inner and outer control limits, a new sample should be gathered and inspected. Clearly, in RSCCs there are six zones, there are six zones, three zones in either side from the target. These zones can be classified as control zone (or safe zone), doubtful zone and out-of-control zone. In the doubtful zone, the sampling is repeated to further confirm the state of the process. The SSZCC proposed by Ravichandran (2019a) has ten zones and in this paper we modify the zones appropriately as safe, doubtful and out-of-control zones to propose a more meaningful Six Sigma-based ready-to-use RSCCs.

Table 1 The ten zones of SSZCC

Zone	*Region in the chart*
10	Above $\hat{\mu}+4.5\sigma_{ss}/\sqrt{n}$
9	Between $\hat{\mu}+3.5\sigma_{ss}/\sqrt{n}$ and $\hat{\mu}+4.5\sigma_{ss}/\sqrt{n}$
8	Between $\hat{\mu}+2.5\sigma_{ss}/\sqrt{n}$ and $\hat{\mu}+3.5\sigma_{ss}/\sqrt{n}$
7	Between $\hat{\mu}+1.5\sigma_{ss}/\sqrt{n}$ and $\hat{\mu}+2.5\sigma_{ss}/\sqrt{n}$
6	Between $\hat{\mu}=\bar{\bar{x}}$ and $\hat{\mu}+1.5\sigma_{ss}/\sqrt{n}$
5	Between $\hat{\mu}-1.5\sigma_{ss}/\sqrt{n}$ and $\hat{\mu}=\bar{\bar{x}}$
4	Between $\hat{\mu}-2.5\sigma_{ss}/\sqrt{n}$ and $\hat{\mu}-1.5\sigma_{ss}/\sqrt{n}$
3	Between $\hat{\mu}-3.5\sigma_{ss}/\sqrt{n}$ and $\hat{\mu}-2.5\sigma_{ss}/\sqrt{n}$
2	Between $\hat{\mu}-4.5\sigma_{ss}/\sqrt{n}$ and $\hat{\mu}-3.5\sigma_{ss}/\sqrt{n}$
1	Below $\hat{\mu}-4.5\sigma_{ss}/\sqrt{n}$

3 Six Sigma-based repetitive sampling control chart

A typical RSCC has six zones as shown in Table 2 since the RSCCs proposed by various authors (e.g., Aslam et al., 2014a, 2014b) consist of two pairs of control limits (two LCLs and two UCLs) each on either side of the control chart instead of the traditional Shewhart-type control charts that consist of only one LCL and one UCL.

The control limits of RSCC are given as:

$$\begin{aligned} UCL_1 &= \bar{\bar{x}} + k_1\sigma/\sqrt{n} \\ UCL_2 &= \bar{\bar{x}} + k_2\sigma/\sqrt{n} \\ CL &= \bar{\bar{x}} \\ LCL_2 &= \bar{\bar{x}} - k_2\sigma/\sqrt{n} \\ LCL_1 &= \bar{\bar{x}} - k_1\sigma/\sqrt{n} \end{aligned} \tag{3}$$

Table 2 The six zones of RSCC

Zone	*Region in the chart*
6	Above $\hat{\mu} + k_1\sigma/\sqrt{n}$
5	Between $\hat{\mu} + k_2\sigma/\sqrt{n}$ and $\hat{\mu} + k_1\sigma/\sqrt{n}$
4	Between $\hat{\mu} = \bar{\bar{x}}$ and $\hat{\mu} + k_2\sigma/\sqrt{n}$
3	Between $\hat{\mu} - k_2\sigma/\sqrt{n}$ and $\hat{\mu} = \bar{\bar{x}}$
2	Between $\hat{\mu} - k_1\sigma/\sqrt{n}$ and $\hat{\mu} - k_2\sigma/\sqrt{n}$
1	below $\hat{\mu} - k_1\sigma/\sqrt{n}$

The standard deviation σ given in Table 2 and in equation (3) is to be estimated from the process data if the population standard deviation is unknown. Also, the unknown constants k_1 and k_2 are to be determined by fixing the target in-control ARL. The values of k_1 and k_2 are also dependent on the sub sample size n.

It may be noted that most of the time, quality practitioners/process operators prefer ready-to-use control charts rather than the one that has to be determined every time. Also, ever since the concept of Six Sigma came into being, these quality practitioners/process operators aim for achieving the Six Sigma goal of 3.4 DPMO. Therefore, given the disadvantages/complexities of RSCC and also the overwhelming interest shown by quality practitioners/process operators on the use of Six Sigma practices, in this paper we propose to develop a Six Sigma-based RSCC which has control limits as follows:

$$\begin{aligned} UCL(SS)_1 &= \bar{\bar{x}} + 4.5\sigma_{ss}/\sqrt{n} \\ UCL(SS)_2 &= \bar{\bar{x}} + 1.5\sigma_{ss}/\sqrt{n} \\ CL(SS) &= \bar{\bar{x}} \\ LCL(SS)_2 &= \bar{\bar{x}} - 1.5\sigma_{ss}/\sqrt{n} \\ LCL(SS)_1 &= \bar{\bar{x}} - 4.5\sigma_{ss}/\sqrt{n} \end{aligned} \tag{4}$$

In equation (4), the abbreviation *SS* given in parentheses represents Six Sigma. After these control limits are established, a two-step procedure given below may be followed to make decision based on a fresh sample.

A two-step procedure for Six Sigma-based RSCC

Step 1 Set i, ($i = 1, 2, \ldots, m$), take a fresh subsample of size n. Obtain the sample mean $\bar{X}_i$.

Step 2 Take one of the following decisions:

1. the process is declared to be out-of-control if either $\bar{X}_i > UCL(SS)_1$ or $\bar{X}_i < LCL(SS)_1$
2. the process is declared to be in-control if $LCL(SS)_2 \le \bar{X}_i \le UCL(SS)_2$
3. if either $LCL(SS)_1 \le \bar{X}_i \le LCL(SS)_2$ or $UCL(SS)_2 \le \bar{X}_i \le UCL(SS)_1$, set $i = i + 2$, go to Step 1 to repeat sampling.

Let us define the probabilities

$$\left.\begin{aligned} P_1 &= P\left(LCL(SS)_2 \le \bar{X} \le UCL(SS)_2\right) \\ P_2 &= P\left(LCL(SS)_1 \le \bar{X} \le LCL(SS)_2\right) \\ P_3 &= P\left(UCL(SS)_2 \le \bar{X} \le UCL(SS)_1\right) \\ P_4 &= P\left(\bar{X} < LCL(SS)_1\right) \\ P_5 &= P\left(\bar{X} > UCL(SS)_1\right) \end{aligned}\right\} \tag{5}$$

In equation (5), P_1 is the probability that the process is in control, P_2 and P_3 are the probabilities that the process requires resampling and P_4 and P_5 are the probabilities that the process is out-of-control. According to Balamurali and Jun (2006) and Aslam et al. (2014a), the probability that the process is in-control under RS for the proposed Six Sigma-based RSCC can be given as

$$P_{rs}(SS)_{in} = \frac{P_1}{P_1 + (P_4 + P_5)} = \frac{P_1}{1 - (P_2 + P_3)} = \frac{P_1}{1 - P_{rs}(SS)} \tag{6}$$

where $P_{rs}(SS)$ in equation (6) is the probability of repetition of sampling and can be expressed as

$$P_{rs}(SS) = 2\{\phi(4.5) - \phi(1.5)\} \tag{7}$$

In equation (7), $\phi(\cdot)$ represents the cumulative probability of standard normal distribution. Accordingly, equation (6) can be expressed as

$$P_{rs}(SS)_{in} = \frac{2\phi(1.5) - 1}{1 - 2\{\phi(4.5) - \phi(1.5)\}} \tag{8}$$

It may be noted that if $\bar{\bar{x}} \ne T$, then the process is not centred and the shift can be measured as $\bar{\bar{x}} = T \pm K\sigma_{ss}$ where K represents the level of shift. Hence, given the shift, the in-control probability under RS, denoted by $P_{rs}^{*}(SS)_{in}$ can be given as

$$P_{rs}^{*}(SS)_{in} = \frac{\phi\left(1.5 - K\sqrt{n}\right) + \phi\left(1.5 + K\sqrt{n}\right) - 1}{\phi\left(1.5 + K\sqrt{n}\right) - \phi\left(4.5 + K\sqrt{n}\right) - \phi\left(4.5 - K\sqrt{n}\right) + \phi\left(1.5 - K\sqrt{n}\right) + 1} \tag{9}$$

Therefore, the in-control ARL for the proposed Six Sigma-based RSCC with shifted mean can be obtained as follows:

$$ARL_{rs}^{*}(SS)_{in} = \frac{1}{1 - P_{rs}^{*}(SS)_{in}} \tag{10}$$

If the process is centred at the target, then we have $K = 0$ and hence equation (9) will reduce to equation (8). Therefore, with $K = 0$, equation (10) will give in-control ARL for the process with centred mean. Accordingly, for a given subsample size n, the in-control ARL given in equation (10) can be computed for different values of $0 \le k < 1.5$. We shall consider the values of $1.5 \le K \le 3$ as well for comparison purpose with other existing charts in Section 6.

Remark 3.1: For an initial sample under Six Sigma-based RSCC, probability that the process is out-of-control can be given as

$$P_{rs}(SS)_{out} = P_4 + P_5 = 2[1 - \phi(4.5)]$$

and hence the in-control ARL, with $K = 0$, can be given as

$$ARL_{rs}(SS)_{in} = \frac{1}{p_4 + p_5} = \frac{1}{2[1 - \phi(4.5)]}$$

4 Six Sigma-based multiple dependent state sampling control chart

As shown in Table 2 for RSCC, the MDSSCC too has six zones with the same form of control limits though the constants are computed differently. Unlike RSCC, in fact, MDSSCC utilises the sample information from previous subgroups in addition to the current subgroup (Aslam et al., 2014b). Based on this idea, in this paper, Six Sigma-based MDSSCC is proposed. Given a set of control limits as shown in equation (4), the Six Sigma-based MDSSCC works as given in the following steps:

A two-step procedure for Six Sigma-based MDSSCC

Step 1 Set i, ($i = 1, 2, \ldots, m$), take a fresh subsample of size n. Obtain the sample mean $\overline{X}_i$.

Step 2 Make one of the following decisions:

1 The process is declared to be out-of-control if either $\overline{X}_i > UCL(SS)_1$ or $\overline{X}_i < LCL(SS)_1$.

2 The process is declared to be in-control if $LCL(SS)_2 \le \overline{X}_i \le UCL(SS)_2$.

3 If either $LCL(SS)_1 \le \overline{X}_i \le LCL(SS)_2$ or $UCL(SS)_2 \le \overline{X}_i \le UCL(SS)_1$, then decide as follows:

a the process is in control if the preceding k, $1 \le k \le i - 1$, subsample means are in control.

b Otherwise the process is out-of control.

It may be noted that the proposed Six Sigma-based MDSSCC chart involves the parameter k, where k is the number of preceding subgroups to be decided by the quality practitioner. It is also interesting to note that initially for $i = 1$ we have $k = 0$ and then the chart behaves like the Six Sigma-based RSCC and essentially if the subsample mean $\bar{X}_1$ falls in the doubtful region given in Step 2.3, then one has to decide if the process is either in-control or out-of-control. However, for the next subsample $i = i + 1 = 2$ we have $\bar{X}_2$ and if $\bar{X}_2$ falls in the region given in Step 2.3, then we have only one preceding mean, i.e., $\bar{X}_1$, and therefore when i^{th} mean is plotted then we can have $1 \le k \le i - 1$, with $i \ge 2$.

Given the probabilities in equation (5) for various regions, in case of Six Sigma-based MDSSCC, while P_1 is the probability that the process is in control, P_4 and P_5 are the probabilities that the process is out-of-control. However, P_2 and P_3 are the probabilities that no immediate decision is possible unless there are k preceding subsample means that are in control. Now, following Aslam et al. (2014b), the probability, denoted by $P_{mdss}(SS)_{in}$ that the process is in-control under MDSS for the proposed Six Sigma-based MDSSCC when the process is centred at target can be given as

$$P_{mdss}(SS)_{in} = P_1 + (P_2 + P_3)(P_1)^k \tag{11}$$

By using the cumulative standard normal probabilities, equation (11) can be expressed as

$$P_{mdss}(SS)_{in} = \{2\phi(1.5) - 1\} + 2\{\phi(4.5) - \phi(1.5)\}\{2\phi(1.5) - 1\}^k \tag{12}$$

For a shifted process, the probability that the process is in control, denoted by $P^*_{mdss}(SS)_{in}$ can be obtained as

$$\begin{aligned} P^*_{mdss}(SS)_{in} = &\left\{\phi\left(1.5 + K\sqrt{n}\right) + \phi\left(1.5 - K\sqrt{n}\right) - 1\right\} \\ &+ \left[\left\{\phi\left(-1.5 - K\sqrt{n}\right) - \phi\left(-4.5 - K\sqrt{n}\right)\right\}\right. \\ &+ \left.\left\{\phi\left(4.5 - K\sqrt{n}\right) - \phi\left(1.5 - K\sqrt{n}\right)\right\}\right] \\ &\left\{\phi\left(1.5 + K\sqrt{n}\right) + \phi\left(1.5 - K\sqrt{n}\right) - 1\right\}^k \end{aligned} \tag{13}$$

Therefore, the in-control ARL for the proposed Six Sigma-based MDSSCC can be obtained as follows:

$$ARL^*_{mdss}(SS)_{in} = \frac{1}{1 - P^*_{mdss}(SS)_{in}} \tag{14}$$

If the process is centred at the target, then $K = 0$ and hence equation (13) will reduce to equation (12) and hence equation (14) will result in the in-control ARL for the proposed Six Sigma-based MDSSCC with centred mean. Now, for a given subsample size n, ARL given in equation (14) can be computed for different values of $0 \le k < 1.5$ and $1 \le k < i$. We shall consider the values of $1.5 \le K \le 3$ as well for comparison purpose with other existing charts in Section 6.

5 Six Sigma-based multiple dependent state repetitive sampling control chart

Similar to that of RSCC and MDSCC, the multiple dependent state repetitive sampling control chart (MDSRSCC) too has six zones with the same form of control limits though the constants are computed differently (refer to Table 2 for zones). It may be noted that unlike MDSSCC, in case of MDSRSCC, samples are repeated if at any point of time the subsample mean falls in the doubtful region and the preceding *k* subsample means are not in control (Aldosari et al., 2017, 2018). In this paper, the Six Sigma-based MDSRSCC is proposed. Given a set of control limits as shown in equation (4), the Six Sigma-based MDSRSCC works as given in the following steps:

A two-step procedure for Six Sigma-based MDSRSCC

Step 1 Set i, ($i = 1, 2, \ldots, m$), take a fresh subsample of size n. Obtain the sample mean $\bar{X}_i$.

Step 2 Make one of the following decisions:

1. The process is declared to be out-of-control if either $\bar{X}_i > UCL(SS)_1$ or $\bar{X}_i < LCL(SS)_1$.
2. The process is declared to be in-control if $LCL(SS)_2 \le \bar{X}_i \le UCL(SS)_2$.
3. If either $LCL(SS)_1 \le \bar{X}_i \le LCL(SS)_2$ or $UCL(SS)_2 \le \bar{X}_i \le UCL(SS)_1$, then decide as follows:
   a. the process is in control if the preceding k, $1 \le k \le i - 1$, subsample means are in control.
   b. otherwise set $i = i + 1$, go to Step 1 to repeat sampling.

It may be noted that similar to that of MDSSCC, the proposed Six Sigma-based MDSRSCC also involves the parameter k, where k is the number of preceding subgroups to be decided by the quality practitioner. Here also, it may be note that initially for $i = 1$, we have $k = 0$ and the MDSRSCC behaves like the Six Sigma-based RSCC/MDSSCC and essentially if the subsample mean $\bar{X}_1$ falls in the region given in Step 2.3, then one has to move on to repeat sampling. For the next subsample $i = i + 1 = 2$, we have $\bar{X}_2$ and if $\bar{X}_2$ falls in the doubtful region given in Step 2.3, then we have only one preceding mean, i.e., $\bar{X}_1$, and therefore when i^{th} mean is plotted then we can have $1 \le k \le i - 1$, with $i \ge 2$.

With the various probabilities given in equation (5), in case of MDSRSCC, P_2 and P_3 are the probabilities that the process requires resampling whereas the purpose of other probabilities remains the same. Then, according to Aldosari et al. (2018), the probability, denoted by $P_{mdsrs}(SS)_{in}$, that the process is in-control for the proposed Six Sigma-based MDSRSCC with centred mean can be obtained as

$$P_{mdsrs}(SS)_{in} = \frac{P_1 + (P_2 + P_3)(P_1)^k}{1-(P_2+P_3)\left\{1-(P_1)^k\right\}} = \frac{P_1 + (P_2 + P_3)(P_1)^k}{1 - P_{rs(mdsrs)}(SS)} \quad (15)$$

where $P_{rs(mdsrs)}(SS)$ in equation (15) is the probability of repetition of sampling under MDSRS and is given as

$$P_{rs(mdsrs)}(SS) = (P_2 + P_3)\left\{1-(P_1)^k\right\} \quad (16)$$

Using the cumulative probability of standard normal distribution, equation (15) can be expressed as

$$P_{mdsrs}(SS)_{in} = \frac{\{2\phi(1.5)-1\} + 2\{\phi(4.5)-\phi(1.5)\}\{2\phi(1.5)-1\}^k}{1-2\{\phi(4.5)-\phi(1.5)\}\left\{1-\{2\phi(1.5)-1\}^k\right\}} \quad (17)$$

It may be recalled that if $\bar{\bar{x}} \neq T,$ then the process is not centred and the shift can be measured as $\bar{\bar{x}} = T \pm K\sigma_{ss}$ where K represents the level of shift. Given the shift, the in-control probability under the proposed Six Sigma-based MDSRSCC, denoted by $P^*_{mdsrs}(SS)_{in}$ can be expressed as

$$P^*_{mdsrs}(SS)_{in} = \frac{\left\{\phi(1.5-K\sqrt{n}) + \phi(1.5+K\sqrt{n}) - 1\right\} + 2\left\{\phi(4.5+K\sqrt{n}) - \phi(1.5+K\sqrt{n})\right\}\left\{\phi(1.5-K\sqrt{n}) + \phi(1.5+K\sqrt{n}) - 1\right\}^k}{1-2\left\{\phi(4.5+K\sqrt{n}) - \phi(1.5+K\sqrt{n})\right\}\left\{1-\left\{\phi(1.5-K\sqrt{n}) + \phi(1.5+K\sqrt{n}) - 1\right\}^k\right\}} \quad (18)$$

Therefore, the in-control ARL for the proposed Six Sigma-based MDSRSCC can be obtained as follows:

$$ARL^*_{mdsrs}(SS)_{in} = \frac{1}{1-P^*_{mdsrs}(SS)_{in}} \quad (19)$$

It may be noted that if the process is centred at the target, then $K = 0$ and hence equation (18) will reduce to equation (17) and hence equation (19) will result in the in-control ARL for the proposed Six Sigma-based MDSRSCC with centred mean. Now, for a given subsample size n, ARL given in equation (19) can be computed for different values of $0 \leq K < 1.5$. We shall consider the values of $1.5 \leq K \leq 3$ as well for comparison purpose with other existing charts in Section 6.

6 Numerical results and performance evaluation

In order to numerically evaluate the performance of the proposed Six Sigma-based RSCC, MDSSCC and MDSRSCC, the ARL values are computed for different parameter settings for the respective charts as shown in Table 3. This table also briefs how these charts work and what happens once initial plot of mean is done. The ARL values for all the three proposed control charts are now computed based on the parameter settings and given in Tables 4, 5 and 6 respectively for Six Sigma-based RSCC, MDSSCC and MDSRSCC. The observations drawn from these tables are consolidated and given below:

1 When initial sample is plotted, the in-control ARL values are same for all the three types of proposed control charts. This is due to the reason that when the process is centred (or in-control), i.e., $K = 0$, and when there is no preceding subsample, i.e., $k = 0$, the MDSRSCC will reduce to MDSSCC and both the charts will reduce to the RSCC with the initial sample (refer to Remark 3.1). This can be seen from the ARL values given in the shaded rows of Tables 4 and 5. In fact, the ARL values shown in the shaded row of Table 4 represent that the process is in the initial stage and it is given here to compare with those of Six Sigma-based MDSSCC and MDSRSCC for initial case.

2 For all the charts, as the process shift increases beyond $K = 0$, the ARL values keep decreasing steadily, meaning that when the shift increases the charts are able to detect the out-of-control situation quickly. Therefore, the proposed control charts outperform the existing charts because of higher ARL with regard to false alarm true alarm rates (Ravichandran, 2019a)

3 In all the three charts, for a given shift level K, when the subsample size n increases, the ARL values are decreasing meaning that the estimator obtained becomes more robust as the subsample size increases and hence detection of out-of-control situation is accomplished at much higher rate.

4 In cases of MDSSCC and MDSRSCC, as the number of preceding subsamples, k, increases, it is observed that the ARL values keep decreasing and this means that higher the preceding subsamples, quicker the detection of the process that is out-of control.

5 While the inner control limits of the proposed Six Sigma-based RSCC is within ±1.5 of the standard deviation, the existing RSCCs have the inner control limits ±1.0 or even less in most of the situations (Aslam et al., 2014a). This implies that even when a sample mean is closer to the process mean (or target), the existing RSCCs suggest for a resample which may not be warranted.

6 In case of the proposed Six Sigma-based MDSSCC and MDSRSCC, the difference between the inner control limits and the outer control limits is within ±3 of the standard deviation, where as existing MDSSCC and MDSRSCC have the inner control limits that are too close to the outer control limits with a difference of ±1.0

approximately (Aslam et al., 2014b; Aldosari et al., 2018). Due to this fact, it may not be significant to look for multiple decisions since the process has already moved close to the outer control limit. Also, there is a likelihood that a subsample mean that is closer to the outer control limit may result in deciding the process as in-control process.

7 It is clear that the proposed Six Sigma-based RSCC/MDSSCC/MDSRSCCs suggest for a resample only if the sample mean exceeds an allowable shift of ±1.5 times of the standard deviation. This avoids the frequent resampling and process disturbances. This has prompted the researchers (e.g., Carr, 1989; Davis et al., 1990) to develop modified ZCCs that suggest that as long as the shift in the process mean is around the target, no much or frequent adjustments are warranted. This leads to the conclusion that the proposed Six Sigma-based RSCCs, MDSSCCs and MDSRSCCs perform far better than their existing counterparts and ensure the Six Sigma goal of 3.4 DPMO.

Table 3 Parameter settings considered for numerical evaluations

Parameter	*Six Sigma-based chart type*			*Description*
	RSCC	*MDSSCC*	*MDSRSCC*	
Shift (K)	K = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 1.0, 1.5, 2.0, 3.0			When K = 0 the resulting values are in-control ARLs
Subsample size (n)	n = 10, 20, 30, 40, 50			Not applicable
Preceding subsample (k)	Not applicable	k = 0, 1, 2, 3, 4	k = 0, 1, 2, 3, 4 subsamples	When k = 0, the resulting ARL values are for initial stage before repetition is effected. Hence all the three charts will have the same in-control ARL values for K = 0 and k = 0
How the chart works	The chart starts with the initial plot of mean. After each plot of mean, the chart repeats sampling if in-control or out-of-control decision is not reached.	The chart starts with initial plot of mean. For each plot of mean, in-control or out-of-control decision is reached. It uses the nature and counts of pre-defined in-control preceding subsamples if required for a decision.	The chart starts with initial plot of mean. After each plot of mean, the chart repeats sampling if in-control or out-of-control decision is not reached even after checking the nature and counts of pre-defined in-control preceding subsamples.	

Table 4 ARL values for Six Sigma-based RSCCs

Shift (K)	*Subsample size (n)*				
	10	*20*	*30*	*40*	*50*
0.0 (initial)	147,160.00	147,160.00	147,160.00	147,160.00	147,160.00
0.0	142,857.14	142,857.14	142,857.14	142,857.14	142,857.14
0.1	55,555.55	32,258.06	20,790.33	14,345.31	10,362.91
0.2	10,638.29	4,629.63	1,970.09	971.53	525.52
0.3	3,663.00	707.71	207.68	75.62	31.74
0.4	971.82	116.06	24.37	7.21	2.94
0.5	266.52	20.57	3.76	1.53	1.12
1.0	1.53	1.00	1.00	1.00	1.00
1.5	1.00	1.00	1.00	1.00	1.00
2.0	1.00	1.00	1.00	1.00	1.00
3.0	1.00	1.00	1.00	1.00	1.00

Table 5 ARL values for Six Sigma-based MDSSCCs

Shift (K)	*Preceding sample (k)*	*Subsample size (n)*				
		10	*20*	*30*	*40*	*50*
0.0	0	147,160	147,160	147,160	147,160	147,160
	1	55.995	55.995	55.995	55.995	55.995
	2	30.007	30.007	30.007	30.007	30.007
	3	21.402	21.402	21.402	21.402	21.402
	4	17.142	17.142	17.142	17.142	17.142
0.1	0	66,377	38,939	25,690	18,141	13,414
	1	42.724	33.789	27.455	22.801	19.288
	2	23.147	18.491	15.182	12.740	10.889
	3	16.673	13.449	11.149	9.445	8.148
	4	13.479	10.972	9.176	7.841	6.822
0.2	0	18,142	6,420	3,018	1,645	985
	1	22.801	12.672	8.261	5.939	4.562
	2	12.740	7.377	5.005	3.740	10.889
	3	9.445	5.672	3.981	3.069	2.518
	4	7.841	4.863	3.512	2.776	2.325
0.3	0	5,217	1,260	467	216	115
	1	11.253	5.169	3.205	2.324	1.853
	2	6.617	3.317	2.227	1.732	1.468
	3	5.132	2.762	1.962	1.592	1.391
	4	4.434	2.525	1.864	1.549	1.371

Table 5 ARL values for Six Sigma-based MDSSCCs (continued)

Shift (K)	*Preceding sample (k)*	*Subsample size (n)*				
		10	*20*	*30*	*40*	*50*
0.4	0	1,645	298	96.000	41.000	21.000
	1	5.939	2.642	1.745	1.382	1.207
	2	3.740	1.911	1.407	1.205	1.110
	3	3.069	1.727	1.343	1.183	1.102
	4	2.776	1.665	1.329	1.179	1.101
0.5	0	569	84.000	25.585	11.050	5.974
	1	3.505	1.682	1.249	1.099	1.039
	2	2.395	1.372	1.133	1.053	1.022
	3	2.086	1.316	1.122	1.051	1.021
	4	1.968	1.304	1.121	1.051	1.021
1.0	0	11.050	2.046	1.197	1.035	1.005
	1	1.099	1.002	1.000	1.000	1.000
	2	1.053	1.002	1.000	1.000	1.000
	3	1.051	1.001	1.000	1.000	1.000
	4	1.051	1.001	1.000	1.000	1.000
≥1.5	0	1.677	1.014	1.000	1.000	1.000
	1	1.000	1.000	1.000	1.000	1.000
	2	1.000	1.000	1.000	1.000	1.000
	3	1.000	1.000	1.000	1.000	1.000
	4	1.000	1.000	1.000	1.000	1.000

Table 6 ARL values for Six Sigma-based MDSRSCCs

Shift (K)	*Preceding sample (k)*	*Subsample size (n)*				
		10	*20*	*30*	*40*	*50*
0.0	0	147,160	147,160	147,160	147,160	147,160
	1	144,533	144,533	144,533	144,533	144,533
	2	142,256	142,256	142,256	142,256	142,256
	3	140,284	140,284	140,284	140,284	140,284
	4	138,576	138,576	138,576	138,576	138,576
0.1	0	11.965	8.301	6.659	5.670	4.993
	1	11.838	8.228	6.607	5.631	4.962
	2	11.731	8.167	6.565	5.600	4.938
	3	11.639	8.117	6.532	5.576	4.920
	4	11.562	8.075	6.504	5.557	4.906

Table 6 ARL values for Six Sigma-based MDSRSCCs (continued)

Shift (K)	*Preceding sample (k)*	*Subsample size (n)*				
		10	*20*	*30*	*40*	*50*
0.2	0	5.670	3.787	2.957	2.474	2.155
	1	5.631	3.769	2.947	2.468	2.151
	2	5.600	3.756	2.941	2.465	2.149
	3	5.576	3.747	2.937	2.463	2.148
	4	5.557	3.741	2.934	2.462	2.148
0.3	0	3.527	2.299	1.788	1.529	1.365
	1	3.512	2.295	1.797	1.528	1.365
	2	3.501	2.293	1.796	1.528	1.365
	3	3.494	2.291	1.795	1.528	1.365
	4	3.480	2.290	1.795	1.528	1.365
0.4	0	2.474	1.631	1.121	1.051	1.101
	1	2.462	1.630	1.324	1.179	1.101
	2	2.460	1.629	1.324	1.179	1.101
	3	2.459	1.629	1.324	1.178	1.101
	4	2.459	1.629	1.324	1.178	1.101
0.5	0	1.882	1.300	1.121	1.051	1.021
	1	1.880	1.300	1.121	1.051	1.021
	2	1.879	1.300	1.121	1.051	1.021
	3	1.878	1.300	1.121	1.051	1.021
	4	1.878	1.300	1.121	1.051	1.021
1.0	0	1.051	1.001	1.000	1.000	1.000
	1	1.051	1.001	1.000	1.000	1.000
	2	1.051	1.001	1.000	1.000	1.000
	3	1.051	1.001	1.000	1.000	1.000
	4	1.051	1.001	1.000	1.000	1.000
≥1.5	0	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	1.000	1.000	1.000
	2	1.000	1.000	1.000	1.000	1.000
	3	1.000	1.000	1.000	1.000	1.000
	4	1.000	1.000	1.000	1.000	1.000

7 Numerical example

In this section we consider a numerical example to illustrate the application of the proposed Six Sigma-based RSCC/MDSSCC/MDSRSCCs with the existing RSCC/MDSSCC/MDSRSCC. We consider the example used by Ravichandran (2016) to study the Six Sigma-based X-bar Control chart. Twenty samples (i.e., m = 20) each with five subsamples (i.e., n = 5) are taken from a process where the quality characteristic represents the thickness of a transparent film with specification limit given as 180 ± 7. The population mean (or the target *T*) and the standard deviation $\sigma_{ss} = (USL - T)/6$ are obtained as 180 and 1.166667 respectively. The means of the 20 subsamples are given as 184.0, 179.6, 184.4, 179.8, 179.2, 181.4, 178.4, 183.8, 180.0, 178.6, 179.6, 182.8, 182.4, 180.8, 178.0, 182.6, 178.6, 181.4, 181.4, 178.6.

The actual process mean $\hat{\mu} = \bar{\bar{x}}$ and the process standard deviation σ are computed as 180.6 and 2.04 respectively. Now, Six Sigma-based control chart is constructed using equation (4) and is shown in Figure 1. Similarly, we have constructed the existing RSCC (Figure 2) and MDSRSCC (Figure 3) with the parameters given in Aslam et al. (2014a, 2014b) for RSCC and in Aldosari et al. (2018) for MDSRSCC. That is, in Figure 2, the parameters used are $n = 5$, $k_1 = 3.052$, $k_2 = 0.9699$ and in Figure 3 we considered $n = 5$, $k_1 = 2.9996$, $k_2 = 2.7784$ for computing the control limits. In all, it is assumed that the parameter $k = 2$, representing the preceding number of subsample to be checked in the event that no decision is taken.

We have compared Figures 1 and 2 for the performance of Six Sigma-based RSCC and the existing RSCC. It can be seen that while the proposed chart declares subsample number 15 as out-of-control, this subsample suggests for resampling in case of existing RSCC. This means that the proposed chart is capable of detecting the shifts at an early stage itself. Similarly, we compared Figures 1 and 3 for the performance of Six Sigma-based MDSRSCC with the existing MDSRSCC by assuming $k = 2$. In the proposed chart subsample 15 shows that the process is out-of-control whereas the existing MDSSCC in Figure 3 asks for checking the preceding two subsamples and decides the process is in control. Also, in Figure 1, subsample 7 says the process needs resampling whereas in Figure 3, the subsample is well in-control. This is mainly due to the reason that the inner control limits are too close to the outer control limits. These contrasting observations demonstrate that the proposed Six Sigma charts are superior to the existing counterparts in the early detection of shifts.

Figure 1 Six Sigma-based RSCC/MDSSCC/MDSRSCC (see online version for colours)

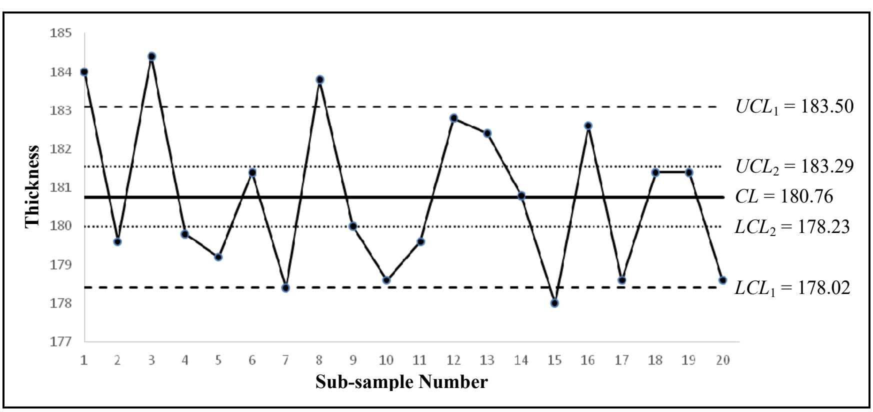


Figure 2 Existing RSCC based on Aslam et al. (2014a)

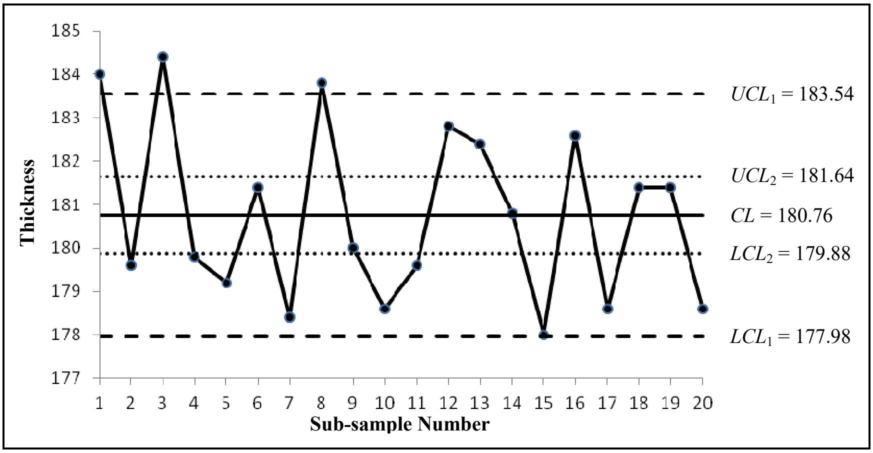
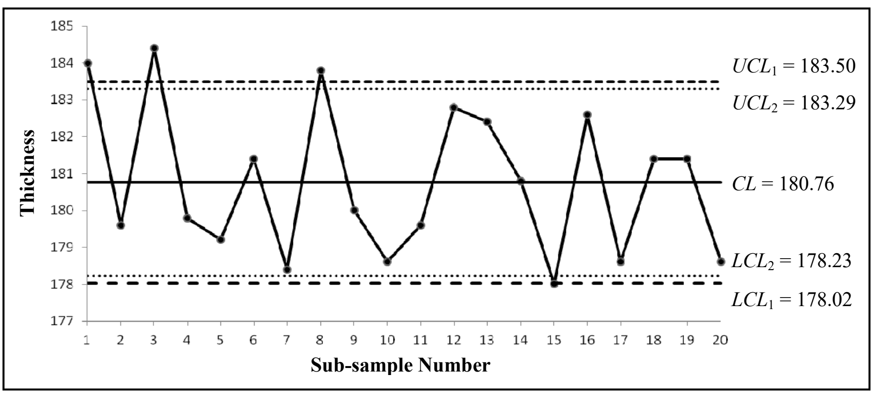


Figure 3 Existing MDSRSCC based on Aldosari et al. (2018)



8 Discussion and Conclusions

In this paper, well-defined Six Sigma-based RS, MDSS and MDSRS control charts are proposed. It is observed that unlike the existing RSCC, MDSSCC and MDSRSCC, the proposed control charts consider a more meaningful and ready-to-use control limits from the perspective of Six Sigma quality practitioners/process operators. The Six Sigma-based RSCC, MDSSCC and MDSRSCC are designed in such a way that the Six Sigma goal of 3.4 DPMO is ensured without much disturbance to the process when the process mean is closer to the target. We have numerically evaluated the performance of the proposed Six Sigma-based RSCC, MDSSCC and MDSRSCC, and the results are compared with the existing RSCC, MDSSCC and MDSRSCC. From the ARL values it is observed that the proposed chart performs better for increased subsample size as the estimator of interest becomes robust and hence the shift is detected at a higher rate. Further, it is observed that the proposed charts perform better than the existing ones by means of better average run lengths for detecting false alarm and true alarm in the process and also suggest for minimum process disturbance for smaller shifts in the process mean as expected by most practitioners/process operators. From the numerical example, we have shown that the proposed Six Sigma-based control charts perform better than the existing counterparts in early detection of the out-of-control process situations.

Like any traditional control charts, the proposed Six Sigma-based charts in this paper have some limitations and provide scope for further study as well. The proposed charts cannot be used as it is to monitor a process for attributes. Applicability of the existing ‘run rules’ to the proposed charts needs further study. Also, it is important to check if the process is used for a long run and hence the allowance of ±1.5 sigma shift is meaningful. Under this case, reasonable large subsamples may be used.

Acknowledgements

Authors would like to thank one of the Associate Editors and three anonymous referees for their constructive comments that helped to improve the paper to a greater extent.

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