A queuing network and Markov chain approach for balancing assembly line: a case study

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Abstract: Having a balanced assembly line is a strength point for any manufacturing company and results in reducing costs as well as decreasing processing time and increasing productivity. Allocating activities to workstations regarding least number of workstations and idle time of each station with respect to the prerequisite relationships of assembly line activities are the main purposes in balancing assembly line problems. Accordingly, in this study, a mathematical model has been developed and considering the assembly line as a queuing network, a Markov chain has been formed and balance rate equations added into the basic model. In addition, obtained results from a real case study in Iran, Esfahan, approved the applicability and efficiency of the proposed approach and indicated an improvement in the levelling of the idle time of resources at workstations.

Keywords: queuing network; Markov chain; balancing assembly line; idle time.

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1 Introduction

Some companies have customers who do not accept the slightest tardiness, and as soon as they are faced with a delay, they will dismiss their order. In these situations, it is vital for either industries or service centres to meet delivery times. In addition, meeting deadlines could decrease the costs. However, in order to minimising costs, manufacturing companies have been relying on assembly lines for the mass production of commodity goods (Pereira et al., 2018).

Assembly line is a production process that is used in the mass production of standard products. In each assembly line, there are a number of consecutive workstations which are connected to each other by a material handling system, such as a conveyor belt. For example, in the automobile production process, the assembly line arranged in a chain of several stations and linked to a material handling system. Each station has the particular assigned time, known as the cycle time, to complete one or more tasks per product (Vila and Pereira, 2014).

One of the most important subjects in designing an assembly line is determining the number of workstations. Accordingly, there are different approaches towards number of workstations. In some research such as Becker and Scholl (2006), Battaïa and Dolgui (2013) and Boysen and Fliedner (2008), the number of work stations are constant while in other research including Wu et al. (2008) and Hu et al. (2010) number of stations are variable.

Nowadays, balancing assembly lines is a key step in planning production systems. This is especially important in mass production systems since a balanced assembly line leads to taking full advantage of the capacity and equipment while an imbalanced assembly line could not work efficiently. The main purpose of balancing assembly line is to allocate the activities to workstations considering minimisation of number of workstations and idle time of each station with respect to the prerequisite relationships of assembly line activities.

Due to the nature of the problem of balancing assembly line and parts waiting in each workstation, the formation of a queue is expectable. Consequently, a queuing theory approach could be used in modelling the balancing assembly line problem. Therefore, in this paper, we have developed a model proposed by Zhang et al. (2013) for balancing assembly line of a real case study applying Markov chain approach.

The remaining parts of the present research are as follows.

First, a comprehensive literature review is presented. Next, the research methodology is elaborated. After that, the case study will be introduced briefly. Following this, by using the basic model and forming the Markov chain, the developed model is formulated. In the next stage, the results of solving the model for the case study are presented and analysed. Finally, in the conclusions section, the most important results are discussed and some directions for future research are suggested.

2 Literature review

There are different researches within the literature which combine different approaches with mathematical modelling for balancing assembly lines. For example, taking into account the random processing time for activities along with the manpower factor, Celik et al. (2014) studied the balancing assembly line problem by using the ant-colony algorithm. Tarimoradi et al. (2015) formulated a multi-objective model for balancing assembly line problem by considering the processing time of activities and using fuzzy logic and simulation method. Alavidoost et al. (2016) investigated the balancing assembly line problem by using an interactive fuzzy programming method and proposed a bi-objective fuzzy linear programming model. In Bautista et al. (2016), given that ergonomic hazards in multi-product assembly lines can affect both workers and line productivity, a mathematical model has been formulated to balance these lines regarding features such as time, place and ergonomic factors. Chica et al. (2016) applied a multi-objective model along with evolutionary algorithms for balancing assembly lines by taking into account the time of activities and the available space in the assembly lines with respect to uncertain demands. Li et al. (2020) developed an ant colony optimisation algorithm with ten heuristics in an agent-based environment for solving mixed-model parallel two-sided assembly line balancing problem. Dong et al. (2018) addressed a stochastic assembly line balancing problem with flexible task times and zoning constraints. To do so, a bi-objective chance-constrained mixed integer programming model has been developed to simultaneously minimise the cycle time and the equipment cost. Bukchin and Raviv (2018) applied the constraint programming for the simple assembly line balancing problem as well as some of its generalisations. Janardhanan et al. (2019) studied robotic assembly line balancing problem with the aim of minimising cycle time by considering sequence-dependent setup times. Zhang et al. (2020) formulated a U-shaped assembly worker assignment and balancing problem to simultaneously minimise cycle times and ergonomic risks. Finco et al. (2020) proposed a bi-objective manual assembly line design model, aiming to avoid excessive daily vibration exposures. The developed model allows to minimise both total equipment costs and vibration levels by respecting the threshold values defined in the ISO 5349-1. Schmid et al. (2020) formulated a mathematical programming model to evaluate different policies for feeding, i.e., the provision of material to assembly stations. For this, it is not only decided which feeding policy is used for a part but also where exactly it is stored at the assembly stations. Furthermore, they proposed a mechanism to decide on the available space per station, allowing cheaper but more space consuming policies at some stations. Analysing state dependent breakdown in bulk arrival and batch service queuing system with vacation was described in Niranjan (2020), scheduling problem in assembly job shop systems with machine breakdowns in Paul et al. (2018), introducing an integrated disassembly line balancing and routing problem (Kenger et al., 2020), integrated disassembly line balancing and routing problem with mobile (Kenger et al., 2021), a new branch, bound and remember (BBR) algorithm to minimise the number of mated-stations in two-sided assembly lines (Li et al., 2020), U-shaped assembly lines and the increase of labour costs and subsequent utilisation of robots (Zhang et al., 2019a), a disassembly line balancing problem (DLBP) in remanufacturing that aims to allocate a set of tasks to workstations to disassemble a product (Zhou et al., 2020), an energy-oriented balancing and sequencing problem of mixed-model assembly line (Zhang et al., 2020) and U-shaped assembly lines to replace operators (Zhang et al., 2019b) are noteworthy additional works on assembly line in literature.

In addition, application of queuing networks in the service and production systems could be seen in the literature of industries and balancing assembly line problems. For instance, Regattieri et al. (2009) proposed a methodology based on the M/M/m queuing model for parametric analyses of system performance according to the different possible ranges of input parameters. Cruz et al. (2010) investigated buffer and throughput trade-offs in M/G/1/K queuing networks. Avşar and Zijm (2014) presented a queuing analysis for capacitated multi-stage inventory systems. Rashid et al. (2015) presented a mathematical model for an inventory control system in which customers' demands and suppliers' service time are considered as stochastic parameters. The proposed problem is solved through queuing theory for a single item. Zavanella et al. (2015) proposed an analytical approach, based on the application of queuing theory, to model the power request and the consequent energy use in a production system. Chen et al. (2017) used wafer lots transfer probability to capture flow rate of wafer lots and constructed an open queuing network model constructed for material handling systems. Hanukov et al. (2019) proposed an approach, based on a combined queuing and inventory model for the typical fast food service system.

However, some studies have used queuing theory particularly in assembly lines. In this category, Azaron et al. (2006) developed an open queuing network for optimal design of multi-stage assemblies in which each service station represents a manufacturing or assembly operation. Perkgoz et al. (2007) modelled a multi-stage assembly system as an open queuing network and multi-objective problem and an improved genetic algorithm was applied to solve it. Manitz (2008) studied the production process on multi-stage assembly lines. These production systems comprise simple processing as well as assembly stations. The two-station subsystems are analysed by using G/G/1/N stopped-arrival queuing models. Li et al. (2010) considered a semiconductor assembly and test factory which was a three-segment-constant work-in-process system with overlapping machines. In the proposed model, the system modelled as a three-loop closed queue network and a genetic algorithm designed to obtain near optimal solutions. De Boeck and Vandaele (2011) using queuing concepts modelled a generic first-come first-serve assembly system, consisting of two generally distributed component input streams. Park and Lee (2013) studied a multi-product assembly production system in which individual components are made to meet various order types as a closed queuing network. Yaghoubi et al. (2013) for modelling multi-class multi-stage assembly systems, considered every class separately and converted the queuing network of each class into an appropriate stochastic network. Then by using the concept of continuous-time Markov processes, a system of differential equations created to obtain the distribution function of manufacturing lead time for any type of product, which is actually the time between receiving the order and the delivery of finished product. Furthermore, they developed a multi-objective model with three conflicting objectives to optimally control the service rates, and used goal attainment method to solve a discrete time approximation of the original multi-objective continuous-time problem. Molavi et al. (2014) developed a robust multi-objective model for lead time optimal control problem in a multi-stage assembly system using the open queuing network. Kim and Lim (2016) investigated overtaking in an M/M/c queuing network in flexible assembly systems and telecommunication systems. Two distributions were considered to describe the amount of overtaking: the number of customers that an arbitrary (tagged) customer overtakes and the number of customers who overtake the tagged customer. Evdokimova et al. (2018) considered a Markovian queuing system with multiple coupled queues and customer impatience for assembly processes.

Table 1 compares some of the aforementioned papers with the applied approaches in the current paper.

Despite the fact that queuing theory can be applied for balancing assembly line and there are some articles that have used this concept for smoothing production process, we did not observe any particular study on balancing assembly line using Markov chain. Given that having a balanced assembly line is a strength for any organisation and can reduce costs and processing time and increase profits and productivity, and since there is a research gap in the literature of balancing assembly line problem, in this research, considering manpower as one of the main pillars of assembly lines and by using Markov chain, the assembly line of a real factory has been studied as a queuing system and by developing a mathematical model the assembly line is balanced while cycle times as well as costs, including salaries are as well.

Table 1 Comparison of selected related works with current paper

Author(s)	*Assembly line*				*Decision variable(s)*						*Objective function(s)*							
	Multi-product	*Two-sided*	*Straight*	*U-shaped*	*Time of activities*	*Parts consumption rate, production rate, available space and machine speed*	*Cycle time*	*Number and type of manpower, training, productivity and ergonomic factors*	*Number of stations*	*Demand level*	*Repetitive activities and ergonomic hazards (min.)*	*Costs and production rate deviations (min.)*	*Number of stations (min.)*	*Number of manpower (min.)*	*Blocking time (min.)*	*Cycle time (min.)*	*Production rates and profits (max.)*	*Employee productivity and effectiveness (max.)*
Celik et al. (2014)				✓	✓			✓						✓		✓		
Tarimoradi et al. (2015)	✓				✓											✓		
Bautista et al. (2016)	✓				✓			✓			✓							✓
Chica et al. (2016)	✓				✓	✓				✓							✓	
Li et al. (2020)		✓			✓				✓				✓			✓		
Alavidoost et al. (2016)			✓	✓			✓									✓		
Present research			✓					✓						✓	✓	✓		

3 Methodology

The steps of this research are as follows:

1 *Defining basic model:* A model proposed by Zhang et al. (2013) is presented as the basic model for this research.

2 *Constructing related Markov chain:* Considering state variables and probability distribution of stations time a Markov chain has been constructed.

3 *Formulating developed model:* By adding balance rate equations into the basic model, developed model has been reached.

4 *Solving the model for the case study:* The model is solved by using real data of the case study.

4 Basic model

Indices

$i = 1, 2, \ldots, m$	index of station
$j, k = 1, 2, \ldots, n$	indices of task
$w = 1, 2, \ldots, w$	index of worker.

Decision variables

x_{ij}	if task j is assigned to station i and otherwise 0
y_{iw}	if the worker w is assigned to station i and otherwise 0.

Parameters

$m = 1, \ldots, 4$	number of stations
$n = 1, \ldots, 22$	number of tasks
t_{jw}	the processing time of j^{th} task by the worker w
$t(s_i)$	processing time at each station i
$Pre(j)$	predictive set of activities j
$w = 1, \ldots, 10$	workers.

The objective function of the model is minimising the assembly line's cycle time.

$$\text{Min } CT = \underset{1 \le i \le m}{\text{Max}} \sum_{j}^{n} \sum_{w}^{m} t_{jw} x_{ij} y_{iw} \tag{1}$$

Constraints for assigning tasks and workers to the stations, so that, each activity and workforce is assigned to a station are as follows:

$$\sum_{i=1}^{m} x_{ij} \leq \sum_{i=1}^{m} x_{ik}, \quad \forall j, k \in pre(j) \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = 1, \quad \forall j \tag{3}$$

$$\sum_{w=1}^{m} y_{iw} = 1, \quad \forall i \tag{4}$$

$$\sum_{i=1}^{m} y_{iw} = 1, \quad \forall w \tag{5}$$

$x_{ij}, y_{iw} \in \{0, 1\}$.

5 Markov chain

5.1 Defining state variables

In a series system, all customers receive services from all stations, and the order of services is also the same for everyone. Therefore, there are four Markov chain state stations. The status of each station is as follows:

0 empty

1 fill

b block.

For example, the mode (1, 1, 0, 1) represents the state of the process that there is part in the first, second, and fourth stations, and the third station is vacant. Also, blocked status refers to a situation where the activity is completed at the station, but due to the fact that the next station is busy, the part waits at this station. In this situation, the station is blocked and cannot serve. After reviewing all possible scenarios and eliminating unreasonable states, we arrive at the 21 states in the system which shown in Table 2.

Table 2 Possible states

Status code	*Status description*	*Status code*	*Status description*	*Status code*	*Status description*
1	(1, 0, 0, 0)	8	(1, 1, 1, 0)	15	(1, *b*, *b*, 1)
2	(1, 0, 0, 1)	9	(1, 1, 1, 1)	16	(*b*, 1, 1, 0)
3	(1, 0, 1, 0)	10	(1, 1, *b*, 1)	17	(*b*, 1, 1, 1)
4	(1, 0, 1, 1)	11	(*b*, 1, 0, 0)	18	(*b*, 1, *b*, 1)
5	(1, 0, *b*, 1)	12	(*b*, 1, 0, 1)	19	(*b*, *b*, 1, 0)
6	(1, 1, 0, 0)	13	(1, *b*, 1, 0)	20	(*b*, *b*, 1, 1)
7	(1, 1, 0, 1)	14	(1, *b*, 1, 1)	21	(*b*, *b*, *b*, 1)

5.2 Probability distribution of stations time

The time of the factory assembly line activities was estimated by using the Kolmogorov-Smirnov test within SPSS software. Results of this test showed that station times follow an exponential distribution.

5.3 State transition diagram

In the Markov chain, stop times of system in each mode is a random variable with exponential distribution.

Moreover, the continuous Markov chains are usually characterised by its state transitions diagram, and then calculate the steady-state probabilities.

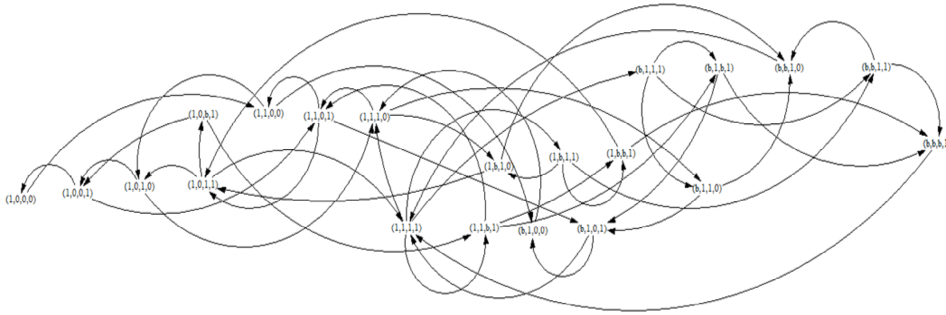
The state transition diagram of assembly line modes for our case study problem has been presented in Figure 1.

Due to the fact that the servers of each station do not operate in parallel, each station will be an M/M/1 queuing system.

Table 3 Balance rate equations of states

State	*Rate balance equation*
1	$\pi_1\mu_1 = \pi_2\mu_4$
2	$\pi_2\mu_1 + \pi_2\mu_4 = \pi_3\mu_3 + \pi_5\mu_4$
3	$\pi_3\mu_1 + \pi_3\mu_3 = \pi_4\mu_4 + \pi_6\mu_2$
4	$\pi_4\mu_1 + \pi_4\mu_3 + \pi_4\mu_4 = \pi_7\mu_2 + \pi_{13}\mu_3 + \pi_{15}\mu_4$
5	$\pi_5\mu_1 + \pi_5\mu_4 = \pi_4\mu_3$
6	$\pi_6\mu_1 + \pi_6\mu_2 = \pi_1\mu_1 + \pi_7\mu_4$
7	$\pi_7\mu_1 + \pi_7\mu_4 + \pi_7\mu_2 = \pi_2\mu_1 + \pi_8\mu_3 + \pi_{10}\mu_4$
8	$\pi_8\mu_1 + \pi_8\mu_2 + \pi_8\mu_3 = \pi_3\mu_1 + \pi_{11}\mu_2 + \pi_9\mu_4$
9	$\pi_9\mu_1 + \pi_9\mu_2 + \pi_9\mu_3 + \pi_9\mu_4 = \pi_4\mu_1 + \pi_{21}\mu_4 + \pi_{19}\mu_3 + \pi_{12}\mu_2$
10	$\pi_9\mu_1 + \pi_9\mu_2 + \pi_9\mu_3 + \pi_9\mu_4 = \pi_4\mu_1 + \pi_{21}\mu_4 + \pi_{19}\mu_3 + \pi_{12}\mu_2$
11	$\pi_{10}\mu_1 + \pi_{10}\mu_2 + \pi_{10}\mu_4 = \pi_9\mu_3 + \pi_5\mu_1$
12	$\pi_{11}\mu_2 = \pi_{12}\mu_4 + \pi_6\mu_1$
13	$\pi_{12}\mu_2 + \pi_{12}\mu_4 = \pi_7\mu_1 + \pi_{16}\mu_3 + \pi_{18}\mu_4$
14	$\pi_{13}\mu_1 + \pi_{13}\mu_3 = \pi_{14}\mu_4 + \pi_8\mu_2$
15	$\pi_{14}\mu_1 + \pi_{14}\mu_3 + \pi_{14}\mu_4 = \pi_9\mu_2$
16	$\pi_{15}\mu_1 + \pi_{15}\mu_4 = \pi_{14}\mu_3 + \pi_{10}\mu_2$
17	$\pi_{16}\mu_2 + \pi_{16}\mu_3 = \pi_{17}\mu_4 + \pi_8\mu_1$
18	$\pi_{17}\mu_2 + \pi_{17}\mu_3 + \pi_{17}\mu_4 = \pi_9\mu_1$
19	$\pi_{18}\mu_2 + \pi_{18}\mu_4 = \pi_{17}\mu_3 + \pi_{10}\mu_1$
20	$\pi_{19}\mu_3 = \pi_{20}\mu_4 + \pi_{16}\mu_2 + \pi_{13}\mu_1$
21	$\pi_{20}\mu_4 + \pi_{20}\mu_3 = \pi_{17}\mu_2 + \pi_{14}\mu_1$

Figure 1 State transition diagram of assembly line modes for the case study



5.4 Balance rate equations

According to Table 2, the balance rate equations which formed from the Markov chain has been presented in Table 3.

6 Developed model

In this section, by entering the balance rate equations into the initial mathematical model and adding the objective function of levelling the idle time and blocking of the stations, a developed model is formulated. Assumptions of the developed model are as follows:

- The number of workers in each station is specific.
- The processing time of activities depends on the number of workers.
- Up to four workers can work at each station.
- The number of possible states is numbered 1 to 21.
- The prerequisites for activities are specific and constant.
- When a worker is assigned to a station, it only deals with the processor and receives the station.
- A task cannot be divided between two or more stations.
- The time of processing and doing the task of each worker for each specific task.
- Time of displacement, loading and unloading, switching of the tool, etc. can be ignored or taken into account in the amount of processing time.
- The processing time is different among workers, because of differences in the worker's experience.
- A worker has the ability to do all the task, but experience and skill are different.
- Workstations are embedded on a rail link.

Decision variables are similar to the basic model but following additional parameters are needed:

t_j time required for task numbered j

$t_{j\max}$ maximum of time required for task numbered j that obtained with one worker

$t_{j\min}$ minimum of time required for task numbered j that obtained with maximum number of workers

r maximum number of workers that can assigned to tasks.

The objective function is minimising the idle time of workstations. Thus, the idle time of a station is the time wasted due to the lack of work at the station (mode 0) and due to the block being in the station (mode b). In this case, π_1', the idle time of station 1 will be sum of these π's: ($\pi_{11} + \pi_{12} + \pi_{16} + \pi_{17} + \pi_{18} + \pi_{19} + \pi_{20} + \pi_{21}$). Other ($\pi_i$)'s can also be defined in the same way.

$$\min z = \max\left(\pi_1' \cdot \pi_2' \cdot \pi_3' \cdot \pi_4'\right) \tag{6}$$

$$\pi_1' = \left(\pi_{11} + \pi_{12} + \pi_{16} + \pi_{17} + \pi_{18} + \pi_{19} + \pi_{20} + \pi_{21}\right) \tag{7}$$

$$\pi_2' = \left(\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_7 + \pi_{13} + \pi_{14} + \pi_{15} + \pi_{19} + \pi_{20} + \pi_{21}\right) \tag{8}$$

$$\pi_3' = \left(\pi_1 + \pi_2 + \pi_5 + \pi_6 + \pi_{10} + \pi_{11} + \pi_{12} + \pi_{15} + \pi_{18} + \pi_{21}\right) \tag{9}$$

$$\pi_4' = \left(\pi_1 + \pi_3 + \pi_6 + \pi_8 + \pi_{11} + \pi_{13} + \pi_{15} + \pi_{16} + \pi_{19}\right) \tag{10}$$

This function is nonlinear. We have use below equation for linearisation:

$$\min z = \pi_{\max} \tag{11}$$

$$\pi_i' \le \pi_{\max} \quad i = 1, \ldots, 4 \tag{12}$$

All constraints of the basic model must still be met. The other constraints are as follows:

$$\mu_i = \frac{60}{\sum_j \sum_w t_j x_{ij} y_{iw}} \quad i = 1, \ldots, 21 \tag{13}$$

which $\sum_j \sum_w t_j x_{ij} y_{iw}$ is the average time of i^{th} station and $\frac{60}{\sum_j \sum_w t_j x_{ij} y_{iw}}$ service rate of this station.

$$t_j = t_{j\max} + \left(\frac{t_{j\min} - t_{j\max}}{r-1}\right)\left(\left(\sum_w y_{iw} * x_{ij}\right) - 1\right) \tag{14}$$

The above equation calculates the time of each activity which is done according to the number of workers assigned to the workstation.

The set of equations of balance rate equations is also added to the model constraints.

Given the fact that the objective function of minimising cycle time is eliminated from the objectives of the model, the optimal cycle time can be set in the constraints. In this way, by solving the model with the objective of minimising cycle time, we obtain a value of 15.720 hours for the cycle time. In the current state of this assembly line, the cycle time is 48 hours. These numbers can be constrained as two bounds for the cycle time.

$$15.720 \leq \underset{1 \leq i \leq m}{\text{Max}} \sum_{j}^{n} \sum_{w}^{m} t_j x_{ij} y_{iw} \leq 48. \quad (15)$$

7 Computational results

To examine the applicability of the developed model, a real case study in Esfahan, Iran has been selected. The case study is active in the field of manufacturing trucks. This company has now been able to supplying not only the need of internal market for truck parts but also exporting a wide range of its products to different countries. The current study carried out in the body assembly line of this factory where there are four stations

The input parameters of the model collected from factory body assembly line have been given in Appendix 1. Also, the allocation of workers and activities to the stations has been listed in Appendix 2. To solve the model, we have used CPLEX solver of GAMS software on a laptop with Intel Core i5 CPU, 2.5 GHz and 6 GB of RAM.

By allocating workers and activities to the four stations, the service rate at each station (μ_i) is determined and based on them, balance rate equations will be solved. Accordingly, the cycle time is equal to 48 and the maximum idle time is 0.381.

Moreover, by solving the model, the maximum idle time of the stations is 0.283.

By comparing the results obtained from the solution of the model manually and software, it can be concluded that the output of the model is justified and can be considered as a good scale in the allocation of human resources. The first objective function, which is the cycle time, decreased from 48 hours to 15.720 and the second objective function, which is blocking time has been decreased from 0.745 to 0.120. Results of the calculation are mentioned in Table 4.

Table 4 Results obtained from model solving with GAMS

Station number	*Number of tasks assigned*	*Number of workers*
1	7	4
2	7	2
3	3	2
4	5	2
Cycle time	15.720	
Blocking time	0.120	

Table 5 shows the effect of changes in the cycle time on blocking time. As it can be seen in Table 4, there is a direct relation between them.

Table 5 Sensitivity analysis of cycle time

Increase in cycle time			*Decrease in cycle time*		
ID	*Cycle time*	*Time being blocked*	*ID*	*Cycle time*	*Time being blocked*
1	4.210	0.298	1	5.930	0.342
2	5.148	0.345	2	5.640	0.327
3	6.204	0.400	3	4.680	0.282
4	6.520	0.418	4	4.210	0.243

8 Managerial insights

The managerial values of this research are two folds. In general, results and method of this research can be considered as an effective procedure in balancing assembly lines and increasing productivity in production units. In addition, the findings of this research help managers and engineers to achieve a proper flow of materials within their assembly lines as well as decreasing costs. It also helps professionals to have a proper state of resources, work shifts, and performance indicators for production units.

In particular and for the case study, the approach of this study led to increasing the efficiency and productivity of workers in the assembly line as well as optimal allocation of works to workstations.

9 Conclusions

Failure to achieve a balanced production system means lack of full usage of the system's capacity. Hereby, balancing production lines due to high production costs has been one of the most important purposes of researchers and engineers in research centres and industrial units. In this regard, balancing assembly/production lines lead to reducing total cost and more effective usage of both human and operational resources in the production lines.

The main purpose of this paper is to provide a framework for analysing the balancing assembly line problem so that the benchmarking for the levelling of unemployment periods at stations can be clearly and explicitly examined. To do so, the assembly line has been considered as a queuing network and then a Markov chain has been formed and balance rate equations have been added into the basic model.

To examine the applicability of the proposed approach, the body assembly line at a Truck manufacturing company is modelled with the purpose of manpower allocation to the work stations and results have compared with current procedure in the case study. Results indicated an improvement in the levelling of the idle time of resources at workstations, which is considered as the most important purpose of this study.

In future research, the problem could be developed and solved for in larger scales. Also, considering other assumptions along with developing either heuristic or metaheuristic techniques would be interesting directions for future research.

References

Alavidoost, M.H., Babazadeh, H. and Sayyari, S.T. (2016) 'An interactive fuzzy programming approach for bi-objective straight and U-shaped assembly line balancing problem', *Applied Soft Computing*, Vol. 40, No. C, pp.221–235.

Avşar, Z.M. and Zijm, W.H. (2014) 'Approximate queueing models for capacitated multi-stage inventory systems under base-stock control', *European Journal of Operational Research*, Vol. 236, No. 1, pp.135–146.

Azaron, A., Katagiri, H., Kato, K. and Sakawa, M. (2006) 'Modelling complex assemblies as a queueing network for lead time control', *European Journal of Operational Research*, Vol. 174, No. 1, pp.150–168.

Battaïa, O. and Dolgui, A. (2013) 'A taxonomy of line balancing problems and their solution approaches', *International Journal of Production Economics*, Vol. 142, No. 2, pp.259–277.

Bautista, J., Batalla-García, C. and Alfaro-Pozo, R. (2016) 'Models for assembly line balancing by temporal, spatial and ergonomic risk attributes', *European Journal of Operational Research*, Vol. 251, No. 3, pp.814–829.

Becker, C. and Scholl, A. (2006) 'A survey on problems and methods in generalized assembly line balancing', *European Journal of Operational Research*, Vol. 168, No. 3, pp.694–715.

Boysen, N. and Fliedner, M. (2008) 'A versatile algorithm for assembly line balancing', *European Journal of Operational Research*, Vol. 184, No. 1, pp.39–56.

Bukchin, Y. and Raviv, T. (2018) 'Constraint programming for solving various assembly line balancing problems', *Omega*, Vol. 78, No. c, pp.57–68.

Celik, E., Kara, Y. and Atasagun, Y. (2014) 'A new approach for rebalancing of U-lines with stochastic task times using ant colony optimisation algorithm', *International Journal of Production Research*, Vol. 52, No. 24, pp.7262–7275.

Chen, W., Wang, Z. and Chan, F.T. (2017) 'Robust production capacity planning under uncertain wafer lots transfer probabilities for semiconductor automated material handling systems', *European Journal of Operational Research*, Vol. 261, No. 3, pp.929–940.

Chica, M., Bautista, J., Cordón, Ó. and Damas, S. (2016) 'A multiobjective model and evolutionary algorithms for robust time and space assembly line balancing under uncertain demand', *Omega*, January, Vol. 58, pp.55–68.

Cruz, F.R.B., Van Woensel, T. and Smith, J.M. (2010) 'Buffer and throughput trade-offs in M/G/1/K queueing networks: a bi-criteria approach', *International Journal of Production Economics*, Vol. 125, No. 2, pp.224–234.

De Boeck, L. and Vandaele, N. (2011) 'Analytical analysis of a generic assembly system', *International Journal of Production Economics*, Vol. 131, No. 1, pp.107–114.

Dong, J., Zhang, L. and Xiao, T. (2018) 'A hybrid PSO/SA algorithm for bi-criteria stochastic line balancing with flexible task times and zoning constraints', *Journal of Intelligent Manufacturing*, Vol. 29, No. 4, pp.737–751.

Evdokimova, E., De Turck, K. and Fiems, D. (2018) 'Coupled queues with customer impatience', *Performance Evaluation*, Vol. 118, pp.33–47.

Finco, S., Abdous, M.A., Calzavara, M., Battini, D. and Delorme, X. (2020) 'A bi-objective model to include workers' vibration exposure in assembly line design', *International Journal of Production Research*, Vol. 59, No. 13, pp.4017–4032.

Hanukov, G., Avinadav, T., Chernonog, T. and Yechiali, U. (2019) 'Performance improvement of a service system via stocking perishable preliminary services', *European Journal of Operational Research*, Vol. 274, No. 3, pp.1000–1011.

Hu, X., Wu, E., Bao, J. and Jin, Y. (2010) 'A branch-and-bound algorithm to minimize the line length of a two-sided assembly line', *European Journal of Operational Research*, Vol. 206, No. 3, pp.703–707.

Janardhanan, M.N., Li, Z., Bocewicz, G., Banaszak, Z. and Nielsen, P. (2019) 'Metaheuristic algorithms for balancing robotic assembly lines with sequence-dependent robot setup times', *Applied Mathematical Modelling*, Vol. 65, pp.256–270.

Kenger, Z.D., Koç, Ç. and Özceylan, E. (2020) 'Integrated disassembly line balancing and routing problem', *International Journal of Production Research*, Vol. 58, No. 23, pp.7250–7268.

Kenger, Z.D., Koç, Ç. and Özceylan, E. (2021) 'Integrated disassembly line balancing and routing problem with mobile additive manufacturing', *International Journal of Production Economics*, Vol. 235, No. c, p.108088.

Kim, W.S. and Lim, D.E. (2016) 'Analysis of overtaking in M/M/c queues', *Computers and Industrial Engineering*, Vol. 101, No. c, pp.177–183.

Li, N., Yao, S., Liu, G. and Zhuang, C. (2010) 'Optimization of a multi-constant work-in-process semiconductor assembly and test factory based on performance evaluation', *Computers and Industrial Engineering*, Vol. 59, No. 2, pp.314–322.

Li, Z., Kucukkoc, I. and Zhang, Z. (2020) 'Branch, bound and remember algorithm for two-sided assembly line balancing problem', *European Journal of Operational Research*, Vol. 284, No. 3, pp.896–905.

Manitz, M. (2008) 'Queueing-model based analysis of assembly lines with finite buffers and general service times', *Computers and Operations Research*, Vol. 35, No. 8, pp.2520–2536.

Molavi, M., Hamzeli, H. and Dalfard, V.M. (2014) 'Multi-objective lead-time control problem with stochastic constraints', *Journal of Applied Research and Technology*, Vol. 12, No. 5, pp.927–938.

Niranjan, S.P. (2020) 'Managerial decision analysis of bulk arrival queuing system with state dependent breakdown and vacation', *International Journal of Advanced Operations Management*, Vol. 12, No. 4, pp.351–376.

Park, C.W. and Lee, H.S. (2013) 'Performance evaluation of a multi-product CONWIP assembly system with correlated external demands', *International Journal of Production Economics*, Vol. 144, No. 1, pp.334–344.

Paul, M., Ramanan, T.R. and Sridharan, R. (2018) 'Simulation modelling and analysis of dispatching rules in an assembly job shop production system with machine breakdowns', *International Journal of Advanced Operations Management*, Vol. 10, No. 3, pp.234–251.

Pereira, J., Ritt, M. and Vásquez, Ó.C. (2018) 'A memetic algorithm for the cost-oriented robotic assembly line balancing problem', *Computers and Operations Research*, Vol. 99, pp.249–261.

Perkgoz, C., Azaron, A., Katagiri, H., Kato, K. and Sakawa, M. (2007) 'A multi-objective lead time control problem in multi-stage assembly systems using genetic algorithms', *European Journal of Operational Research*, Vol. 180, No. 1, pp.292–308.

Rashid, R., Hoseini, S.F., Gholamian, M.R. and Feizabadi, M. (2015) 'Application of queuing theory in production-inventory optimization', *Journal of Industrial Engineering International*, Vol. 11, No. 4, pp.485–494.

Regattieri, A., Gamberini, R., Lolli, F. and Manzini, R. (2009) 'Designing production and service systems using queuing theory: principles and application to an airport passenger security screening system', *International Journal of Services and Operations Management*, Vol. 6, No. 2, pp.206–225.

Schmid, N.A., Limère, V. and Raa, B. (2020) 'Mixed model assembly line feeding with discrete location assignments and variable station space', *Omega*, Vol. 102, No. 15, p.102286.

Tarimoradi, M., Alavidoost, M.H. and Zarandi, M.F. (2015) 'Comparative corrigendum note on papers "Fuzzy adaptive GA for multi-objective assembly line balancing" continued "Modified GA for different types of assembly line balancing with fuzzy processing times": differences and similarities [*Appl. Soft Comput.* 34 (September 2015) 655–677]', *Applied Soft Computing*, Vol. 35, pp.786–788.

Vila, M. and Pereira, J. (2014) 'A branch-and-bound algorithm for assembly line worker assignment and balancing problems', *Computers and Operations Research*, Vol. 44, pp.105–114.

Wu, E.F., Jin, Y., Bao, J.S. and Hu, X.F. (2008) 'A branch-and-bound algorithm for two-sided assembly line balancing', *The International Journal of Advanced Manufacturing Technology*, Vol. 39, Nos. 9–10, pp.1009–1015.

Yaghoubi, S., Noori, S. and Azaron, A. (2013) 'Lead time control in multi-class multi-stage assembly systems with finite capacity', *Computers and Industrial Engineering*, Vol. 66, No. 4, pp.808–817.

Zavanella, L., Zanoni, S., Ferretti, I. and Mazzoldi, L. (2015) 'Energy demand in production systems: a queuing theory perspective', *International Journal of Production Economics*, Vol. 170, pp.393–400.

Zhang, B., Xu, L. and Zhang, J. (2020) 'A multi-objective cellular genetic algorithm for energy-oriented balancing and sequencing problem of mixed-model assembly line', *Journal of Cleaner Production*, Vol. 244, p.118845.

Zhang, W., Xu, W. and Gen, M. (2013) 'Multi-objective evolutionary algorithm with strong convergence of multi-area for assembly line balancing problem with worker capability', *Procedia Computer Science*, Vol. 20, pp.83–89.

Zhang, Z., Tang, Q. and Zhang, L. (2019a) 'Mathematical model and grey wolf optimization for low-carbon and low-noise U-shaped robotic assembly line balancing problem', *Journal of Cleaner Production*, Vol. 215, No. 1, pp.744–756.

Zhang, Z., Tang, Q., Li, Z. and Zhang, L. (2019b) 'Modelling and optimisation of energy-efficient U-shaped robotic assembly line balancing problems', *International Journal of Production Research*, Vol. 57, No. 17, pp.5520–5537.

Zhou, Y., Guo, X. and Li, D. (2020) 'A dynamic programming approach to a multi-objective disassembly line balancing problem', *Annals of Operations Research*, https://doi.org/10.1007/s10479-020-03829-9.

Appendix 1

Table A1 The input parameters of the model on the assembly line of factory under study

ID	*Station number*	*Task description*	t_{max} *(people-hours)*	t_{min} *(people-hours)*	*Prerequisite*	*Number of workers*	*Task's time*	*Station time*	*Rate of service*
1	1	Three-piece assembly and foot tray assembly	2.5	2.3	-	3	2.433	13.317	4.506
2		Assembly tray under the dashboard	2.2	1.9	1		2.1		
3		Assembly of a glass pane and a chunk of the front column	2.3	2	2		2.2		
4		Complete chassis assembly of the floor	2	1.5	3		1.833		
5		Rear assembly and assembly	3	2.75	4		2.917		
6		Assembly three pieces of foot tray	2	1.5	5		1.833		
Total time			14	11.95					
7	2	Assembly behind the room	2.5	2	6	2	2.167	12.35	4.858
8		Assemble the door lock on the chassis	1.5	1.5	7		1.5		
9		Assemble double vent valve	2	2	8		2		
10		Complete three-line assembly	3	2.75	9		2.833		
11		Steering column assembly	1	0.5	10		0.667		
12		Assembling the seat belt to the tray floor and ceiling	2.45	2.3	11		2.35		
13		Assembly of the closet under the ceiling	1	0.75	12		0.833		
Total time			134.5	11.8					
14		Assembling a fuse holder	1	1	13	3	1	8.1	7.407
15		Assembly of the tray inside the engine and submachamber	2.4	2.2	14		2.333		
16		Assembly of a double-sided valve and a rack around the ceiling and working point	1.75	1.6	15		1.7		
17		Assembling the driver's seat and driver's side chassis and the dashboard	2.2	1.8	16		2.067		
18		Column valve assembly	1	1	17		1		
			8.35	7.6					
19		Assembling doors and hood	4.5	4.2	18	2	4.3	14.233	
20		Window assembly	1	0.75	19		0.833		
21		Stone making, polishing and tin work	6.2	5.8	20		5.933		
22		Completion and final confirmation	3.5	3	21		3.167		
Total time			15.2	13.75					
Cycle time					48				

Appendix 2

Table A2 Blocking and idle time in the current situation

ID	*Station number*	*Task description*	*Idle time notation (percent)*	*Idle time value*
1	1	Three-piece assembly and foot tray assembly	π_1	0.038
2		Assembly tray under the dashboard	π_2	0.03
3		Assembly of a glass pane and a chunk of the front column	π_3	0.028
4		Complete chassis assembly of the floor	π_4	0.034
5		Rear assembly and assembly	π_5	0.029
6		Assembly three pieces of foot tray	π_6	0.039
Total time				0.198
7	2	Assembly behind the room	π_7	0.047
8		Assemble the door lock on the chassis	π_8	0.04
9		Assemble double vent valve	π_9	0.045
10		Complete three-line assembly	π_{10}	0.037
11		Steering column assembly	π_{11}	0.057
12		Assembling the seat belt to the tray floor and ceiling	π_{12}	0.056
13		Assembly of the closet under the ceiling	π_{13}	0.023
Total time				0.305
14	3	Assembling a fuse holder	π_{14}	0.019
15		Assembly of the tray inside the engine and submachamber	π_{15}	0.036
16		Assembly of a double-sided valve and a rack around the ceiling and working point	π_{16}	0.021
17		Assembling the driver's seat and driver's side chassis and the dashboard	π_{17}	0.018
18		Column valve assembly	π_{18}	0.033
Total time				0.127
19	4	Assembling doors and hood	π_{19}	0.036
20		Window assembly	π_{20}	0.015
21		Stone making, polishing and tin work	π_{21}	0.028
22		Completion and final confirmation	π_{22}	0.036
Total time				0.115
Total idle time in all stations				0.745