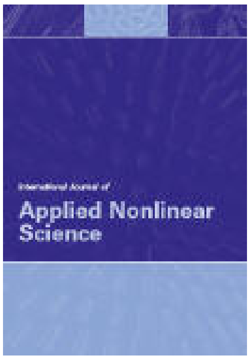

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Efficient algorithm to study the class of Burger's Fisher equation

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Abstract: This paper aims to provide an effective analytical approach to study the family of Fisher's reaction-diffusion equation, namely the reduced differential transform method. These equations are well-known in mathematical biology and have a wide range of applications, including population dynamics, combustion theory, genetic propagation, stochastic processes, and a prototype model for a spreading flame. The proposed method's leverage over other analytical approaches is its capability to handle the nonlinear terms without discretisation, perturbation, or calculation of unneeded terms. The obtained results are more precise and reliable and show a high level of agreement with the exact solution. The convergence criteria and error analysis are also addressed in this paper. The straightforward applicability of the proposed method to convert the complex nonlinear partial differential equation into a simple algebraic system makes it a promising computational method. In this paper, we also provide the algorithm which can be easily implemented in MATLAB.

Keywords: Fisher reaction diffusion equation; reduced differential transform method; RDTM; analytical solution; error analysis; convergence method.

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1 Introduction

The Fisher's reaction diffusion equation is a nonlinear partial differential equation which describes a wide class of physical nonlinear phenomena. It was created in order to trigger the gene distribution within a population. The equation is named after Ronald Aylmer Fisher, and it describes natural population propagation, mass transfer, chemical reaction processes, and heat, as defined by Andrey Nicolaevich Kolmogorov. In this paper, we consider the Fisher's equation

$$\frac{\partial \zeta}{\partial t} = \frac{\partial^2 \zeta}{\partial \xi^2} + \alpha \zeta \left(1 - \zeta\right), \tag{1}$$

$$\frac{\partial \zeta}{\partial t} = \frac{\partial^2 \zeta}{\partial \xi^2} + \alpha \zeta \left(1 - \zeta^{\gamma}\right), \tag{2}$$

and a nonlinear diffusion equation of the Fisher type

$$\frac{\partial \zeta}{\partial t} = \frac{\partial^2 \zeta}{\partial \xi^2} + \alpha \zeta \left(1 - \zeta^{\gamma}\right)\left(\zeta - \rho\right), 0 \le \rho \le 1. \tag{3}$$

where γ, α are the diffusive and reactive constants and $\gamma \ge 0$, $\alpha \ge 0$. Equation (1) was suggested by Fisher as a model for mutant gene propagation, with denoting the density of advantageous mutations. Chemical kinetics (Malfliet, 1992), logistic population growth models (Murray, 1977; Britton, 1986), flame propagation (Frank-Kamenetskii, 2015; Williams, 2018), neurophysiology (Tuckwell, 1988), auto catalytic chemical reactions (Fife and McLeod, 1977; Aronson and Weinberger, 1988), branching Brownian motion processes (Bramson, 1978), gene-culture waves of advance (Aoki, 1987), the spread of early farming in Europe (Ammerman and Cavalli-Sforza, 1971; Ammerman and Avalli-Sforza, 2014), and nuclear reactor theory (Canosa, 1973) all use this equation.

To obtain the analytical and numerical solution of equations (1)–(3) an enormous amount of effort has been made in the last decade. Travelling wave solution was offered by Ablowitz and Zepetella (2014) while Canosa (1973) applied nonlinear eigenvalue problem to obtain the shock-like travelling waves of Fisher's equation. The wavelet-Galerkin method was used by Cattani and Kudreyko (2008) to find the numerical solution of Fisher's equation. A pseudo spectral method was used by Daniel and Shizgal (2006). Gazdag and Canosa (1974) used an accurate space derivative (ASD) approach, while Garey and Shen (1995) used a least-squares FEM. The Sinc collocation approach was used by Khaled (2001). Larson (1978) investigated the solution's transient behavior and examined the nonlinear Fisher's style equations' time-asymptotic convergence. For Fisher's equation, Mickens (1994) created a new class of FDM. Mittal and Jiwari (2009) conducted research by combining the Fisher equation with logistic nonlinearity. Moving mesh technique was used by Qiu and Sloan (1998). Using the model problem FRD equation, Rizwan (2001) investigated the relationship between nodal integral system and FDM. Tan et al. (2007) uses the homotopy analysis approach to obtain a family of travelling waves of the Fisher equation. For the numerical solution of the FRD equation, Sahin and Ozmen (2014) used a B-spline Galerkin method. Tang and Weber (1991) used a Petrov Galerkin FEM to investigate it numerically. Verma et al. (2014) investigated the solution of nonlinear FRD equations using the classical lie symmetry procedure. Nonlinear transformations were introduced by Wang (1988) find the explicit and exact solitary solutions of the generalised FRD equation. Using the Adomian decomposition approach, Wazwaz and Gorguis (2004) were able to obtain exact solutions to FRD type equations. For the FRD equation, Tamsir and Huntul (2021) suggested the extended cubic B-spline differential quadrature approach (EMCB-DQM). For the Fisher equation, Dag et al. (2010) proposed an exponential cubic B-spline algorithm, while Sahin and Ozmen (2014) proposed a quadratic B-spline Galerkin process. Crank-Nicolson (CN) finite difference scheme, ASDs method and discrete singular convolution (DSC) method were proposed by Zhao and Wei (2003). To solve the Fisher's equation,Sahin and Ozmen (2014) used a quintic B-spline collocation method on a uniform mesh, while Mittal and Dahiya (2016) used a quintic B-spline differential quadrature method. For the numerical solution of the nonlinear Fisher equation, Chandraker et al. (2014) suggested a semi-implicit finite difference scheme. Abbas et al. (2014) developed a collocation method based on cubic trigonometric B-spline functions, which they tested using the wave equation. Mickens and Oyedeji (2019) proposed travelling wave solutions to the modified Burgers and diffusionless Fisher's equations, and Agbavon et al. (2019) analysed the Fisher's equation numerically by taking the diffusion term to be smaller than the reaction term. Veeresha et al. (2019) apply q-homotopy analysis transform method to find the solution of nonlinear time-fractional Fisher's equation. Loyinmi and Akinfe (2020) combine two analytical approaches, namely Elzaki transform and homotopy transform perturbation method, to the family of FRD equations in search of an exact solution. Akram et al. (2021) introduce an unconditionally stable and convergent numerical approach to study time-fractional Fisher equation whose result shows an excellent agreement with the exact solution. Many researcher has proposed different numerical approach to study integer and fractional order nonlinear PDE (Adiguzel et al., 2021; Gulbahar et al., 2015; Pankov et al., 2021; Shokri et al., 2018, 2022; Shorki, 2012; Kumar and Verma, 2021; Mandal and Bira, 2021; Yokus and Yavuz, 2021; Yokus et al., 2015; Tamsir et al., 2018a, 2018b; Spiteri and Ruuth, 2002; Sahin et al., 2008; Soori, 2018; Rohila and Mittal, 2018; Onyejekwe, 2018; Mittal and Arora, 2010; Mittal and Jain, 2013; Dag and Ersoy, 2016; Shukla and Tamsir, 2016).

Due to the nonlinearity of PDEs, there is no single best approach for an equation. On some models/problems, some approaches converge quicker to an exact solution, while others do not. That is why nonlinear models require a wide range of analytical, semi-analytical, and computational methods to enhance the integration of the solutions obtained using these methods. This became the motivation behind this investigation.The paper presents the reduced differential transform method (RDTM) to obtain the solution of a one-dimensional nonlinear FRD type equation.

The paper is organised as follows: in Section 2, a description of RDTM is given, and it also addresses convergence and error analysis of RDTM. The procedure for implementing the RDTM is discussed in Section 3. Section 4 discusses the algorithm for MATLAB coding of Burger's Fisher reaction diffusion equation (BFRDE). In Section 5, numerical results are presented for family of a one-dimension nonlinear Fisher's reaction – diffusion equation with graphical and tabular illustrations. The conclusion of the proposed method is given at the end of Section 6.

2 Preliminaries of RDTM

For better understanding, some fundamental concept of RDTM are discussed in this section. Consider a function $\theta(\xi, \Upsilon)$ of two variables and assume that it can be expressed $\theta_1(\xi).\theta_2(\Upsilon)$. According to the concept of a one-dimensional differential transform, $\theta(\xi, \Upsilon)$ can be written as follows:

$$\begin{aligned}\theta(\xi, \Upsilon) &= \left(\sum_{\iota=0}^{\infty} \Theta_1(\iota)\, \xi^{\iota}\right)\left(\sum_{l=0}^{\infty} \Theta_2(\iota)\, \Upsilon^{\iota}\right), \\ &= \sum_{\kappa=0}^{\infty} \Theta_\kappa(\xi)\, \Upsilon^{\kappa}, \end{aligned} \tag{4}$$

where $\Theta_\kappa(\xi)$ is known as Υ-dimensional spectrum function of $\theta(\xi, \Upsilon)$. The definition of RDTM mention as follow (Moosavi Noori and Taghizadeh, 2021):

Definition 1: If the function $\theta(\xi, \Upsilon)$ is analytic and continuously differentiable w.r.t space and time in the domain of interest, then let

$$\Theta_\kappa(\xi) = \frac{1}{\kappa!}\left[\frac{\partial^\kappa}{\partial \Upsilon}\theta(\xi, \Upsilon)\right]_{\Upsilon=\Upsilon_0}, \tag{5}$$

where $\Theta_\kappa(\xi)$ the Υ – spectrum function is the transform function. $\theta(\xi, \Upsilon)$ and $\Theta_\kappa(\xi)$ stands for the original and transform function respectively in this paper.

Definition 2: The differential inverse transform $\Theta_\kappa(\xi)$ is defined as (Moosavi Noori and Taghizadeh, 2021):

$$\theta(\xi, \Upsilon) = \sum_{\kappa=0}^{\infty} \Theta_\kappa(\xi)\,(\Upsilon - \Upsilon_0)^\kappa. \tag{6}$$

Combining equations (5) and (6),we get

$$\theta(\xi, \Upsilon) = \sum_{\kappa=0}^{\infty} \frac{1}{\kappa!}\left[\frac{\partial^\kappa}{\partial \Upsilon}\theta(\xi, \Upsilon)\right]_{\Upsilon=0}(\Upsilon - \Upsilon_0)^\kappa. \tag{7}$$

From equation (7), it is clear that the concept of RDTM is derived from two-dimensional differential transform method. To demonstrate the concept of RDTM, consider NLPDE in terms of operator

$$\Im\left[\theta\left(\xi,\Upsilon\right)\right]+\Re\left[\theta\left(\xi,\Upsilon\right)\right]+N\left[\theta\left(\xi,\Upsilon\right)\right]=\phi\left(\xi,\Upsilon\right), \tag{8}$$

with initial condition

$$\theta\left(\xi,0\right)=\varphi\left(\xi\right),$$

where $\phi\left(\xi,\Upsilon\right)$ is inhomogeneous term, N represent nonlinear operator, $\Im=\frac{\partial}{\partial\Upsilon}$ $\Re$ is a linear operator which has partial derivative. According to RDTM, we construct following recursive formula:

$$\left(\kappa+1\right)\Theta_{\left(\kappa+1\right)}\left(\xi\right)=\Phi_{\kappa}\left(\xi\right)-\Re\left[\Theta_{\kappa}\left(\xi\right)\right]-N\left[\Theta_{\kappa}\left(\xi\right)\right], \tag{9}$$

where $\left(\kappa+1\right)\Theta_{\left(\kappa+1\right)}\left(\xi\right)$, $\Phi_{\kappa}\left(\xi\right)$, $\Re\left[\Theta_{\kappa}\left(\xi\right)\right]$, $N\left[\theta\left(\xi,\Upsilon\right)\right]$ are reduced transform form of $L\left[\theta\left(\xi,y\right)\right]$, $\phi\left(\xi,\Upsilon\right)$, $\Re\left[\theta\left(\xi,\Upsilon\right)\right]$, $N\left[\theta\left(\xi,\Upsilon\right)\right]$ respectively. Applying reduced differential transform to initial condition, we get

$$\Theta_0\left(\xi\right)=\varphi\left(\xi\right). \tag{10}$$

Substituting equation (10) into equation (9) and by straight forward recursive computation, we get the values of $\Theta_{\kappa}\left(x\right)$ for different values of κ. The n^{th} term approximated solution is given by inverse transformation for the set of $\Theta_{\kappa}\left(\xi\right)$ for κ = 1, 2, 3, 4, ... as follows:

$$\theta\left(\xi,\Upsilon\right)=\sum_{\kappa=0}^{n}\Theta_{\kappa}\left(\xi\right)\,\Upsilon^{\kappa}. \tag{11}$$

The exact solution of the stated problem is

$$\theta\left(\xi,\Upsilon\right)=\lim_{n\to\infty}\sum_{\kappa=0}^{n}\Theta_{\kappa}\left(\xi\right)\,\Upsilon^{\kappa}. \tag{12}$$

The fundamental operation performed by RDTM can be easily proved using equations (5) and (6) and are listed in Table 1.

Table 1 Fundamental operation of RDTM

Original function	*Transformed function*
$\theta_1(\xi,\Upsilon)\pm\theta_2(\xi,\Upsilon)$	$\Theta_1\left(\xi\right)\pm\Theta_2\left(\xi\right)$
$\lambda\theta\left(\xi,\Upsilon\right)$	$\lambda\Theta_{\rho}\left(\xi\right)$
$\frac{\partial\theta(\xi,\Upsilon)}{\partial\xi}$	$\frac{\partial\Theta_{\rho}(\xi)}{\partial\xi}$
$\frac{\partial\theta(\xi,\Upsilon)}{\partial t}$	$\left(\vartheta+1\right)\Theta_{\vartheta+1}\left(\xi\right)$
$\frac{\partial\theta(\xi,\Upsilon)}{\partial\xi\,\partial\Upsilon}$	$\left(\vartheta+1\right)\frac{\partial\Theta_{\vartheta+1}(\xi)}{\partial\xi}$
$\theta_1\left(\xi,\Upsilon\right)\theta_2\left(\xi,\Upsilon\right)$	$\sum_{\eta=0}^{\ell}\Theta_{1,\eta}\left(\xi\right)\Theta_{2,\ell-\eta}\left(\xi\right)$
$\xi^{\mathrm{A}}\Upsilon^{B}$	$\xi^{\mathrm{A}}\delta\left(h-B\right)$ where $\delta\left(h-B\right)=\begin{cases}1, h=B\\0, h\neq B\end{cases}$

2.1 Error and convergence analysis

The convergence and error estimation has been addressed in detail by Moosavi Noori and Taghizadeh (2021). The idea of Moosavi Noori and Taghizadeh (2021) was to yield the sufficient condition for convergence of the method for NLPDE. We recall the theorems which guarantees the convergence of RDTM for the general operator equation $\Im\left[\theta\left(\xi, y\right)\right]+\Re\left[\theta\left(\xi, y\right)\right]+N\left[\theta\left(\xi, y\right)\right]=\phi\left(\xi, y\right)$.

Theorem 1: If $\zeta_k\left(\omega, \tau\right)=V_l\left(\omega\right)\left(\tau-\tau_0\right)^l$, then the series solution $\sum_{i=0}^{n}\zeta_l\left(\omega, \tau\right)$ for equation (5) $\forall\, l\in N\cup\{0\}$.

1. is convergent, if there exist $0<\eta<1$ such that $\left\|\zeta_{l+1}\right\|\leq\eta\left\|\zeta_l\right\|$
2. is divergent, if there exist $\eta>1$ such that $\left\|\zeta_{l+1}\right\|\geq\eta\left\|\zeta_l\right\|$.

The truncation error of the series equation (9), which is a specific case of Banach's fixed point theorem (BFPT), is investigated in Theorem 1 (BFPT).

Proof: See Moosavi Noori and Taghizadeh (2021) for the proof.

Theorem 2: Suppose $\sum_{i=0}^{n}\zeta_i\left(\omega, \tau\right)$is required series solution, where $\zeta_l\left(\omega, \tau\right)=V_k\left(\omega\right)\left(\tau-\tau_0\right)^l$, converges to $\varepsilon\left(\omega, t\right)$. If $\sum_{i=0}^{n}\zeta_i\left(\omega, \tau\right)$ is the truncated series used to approximate the solution and then estimated maximum absolute truncated error is as $\left\|\varepsilon\left(\omega, t\right)-\sum_{i=0}^{m}\zeta_i\left(\omega, \tau\right)\right\|\leq\frac{1}{\eta-1}\eta^{j+1}\left\|\zeta_0\right\|$ where $\eta=\max\left\{\eta_i, i=0,1,...,j\right\}$.

Proof: See Moosavi Noori and Taghizadeh (2021) for the proof.

From Theorems 1 and 2, it is concluded that series solution obtained using RDTM for nonlinear equation $\Im\left[\theta\left(\xi, y\right)\right]+\Re\left[\theta\left(\xi, y\right)\right]+N\left[\theta\left(\xi, y\right)\right]=\phi\left(\xi, y\right)$, converges to an exact solution when there exist $0<\eta<1$ such that $\left\|\zeta_{l+1}\right\|\leq\eta\left\|\zeta_l\right\|$, for $\forall\, l\in N\cup\{0\}$. In addition $\left\|\varepsilon\left(\omega, t\right)-\sum_{i=0}^{m}\zeta_i\left(\omega, \tau\right)\right\|\leq\frac{1}{1-\eta}\eta^{m+1}\left\|\zeta_0\right\|$ represents maximum estimated absolute truncated error.

3 Implementation of RDTM to Fisher's reaction diffusion equation

3.1 Case 1

Considering reactive and diffusive constant value, i.e., the parameter values α = 1, γ = 1 equation (2) reduces to

$$\frac{\partial\varsigma}{\partial t}=\frac{\partial\varsigma}{\partial\xi^2}+\varsigma(1-\varsigma), \tag{13}$$

with initial condition

$$\varsigma\left(\xi, 0\right)=\gamma. \tag{14}$$

For the exact solution of Case 1 see Loyinmi and Akinfe (2020). Applying RDTM to equations (13) and (14), the transformed recursive formula and initial condition is

$$(\kappa+1)\,\zeta_{(\kappa+1)}(\xi)=\frac{d^2\zeta_\kappa(\xi)}{d\xi^2}+\zeta_\kappa(\xi)-\sum_{i=0}^{\kappa}\zeta_i(\xi)\zeta_{\kappa-i}(\xi)\,, \tag{15}$$

$$\zeta_0(\xi)=\gamma. \tag{16}$$

Using the transformed condition equation (16) in equation (15), the coefficient of series solution can be obtained for κ = 0, 1, 2, 3, 4, 5, ...

For κ = 0, equation (15) reduces to

$$\zeta_1(\xi)=\frac{d^2\zeta_0(\xi)}{d\xi^2}+\zeta_0(\xi)-{\zeta_0}^2(\xi)\,. \tag{17}$$

Substituting equation (16) in equation (17), we get

$$\zeta_1(\xi)=\gamma-\gamma^2. \tag{18}$$

For κ = 1, equation (15) reduces to

$$2\zeta_2(\xi)=\frac{1}{2}\left(\zeta_1(\xi)-\left(\zeta_0(\xi)\,\zeta_1(\xi)+\zeta_1(\xi)\,\zeta_0(\xi)\right)\right). \tag{19}$$

Substituting equations (16) and (18) in equation (19), we get

$$\zeta_2(\xi)=\frac{1}{4}\left(\gamma-\gamma^2\right)(1-2\gamma)\,. \tag{20}$$

For κ = 2, equation (15) reduces to

$$\zeta_3(\xi)=\frac{1}{3}\left(\zeta_2(\xi)-\left(2\zeta_0(\xi)\,\zeta_2(\xi)+\zeta_1^2(\xi)\,\zeta_1(\xi)\right)\right). \tag{21}$$

Substituting equations (16), (19) and (18) in equation (21), we get

$$\zeta_3(\xi)=\left(\frac{1}{12}\left(\gamma-\gamma^2\right)(1-2\gamma)-\left(\frac{1}{6}\left(\gamma^2-\gamma^3\right)(1-2\gamma)+\left(\gamma-\gamma^2\right)^2\right)\right). \tag{22}$$

For κ = 3, equation (15) reduces to

$$4\zeta_4(\xi)=\frac{d^2\zeta_3(\xi)}{d\xi^2}+\zeta_3(\xi)-\sum_{i=0}^{3}\zeta_i(\xi)\zeta_{3-i}(\xi)\,. \tag{23}$$

Substituting equations (16), (18), (19) and (22) in equation (23), we get

$$\begin{aligned}\zeta_4(\xi)=\frac{1}{4}\Bigg\{&\frac{1}{12}\left(\gamma-\gamma^2\right)(1-2\gamma)-\Bigg(\frac{1}{6}\left(\gamma^2-\gamma^3\right)(1-2\gamma)\\&+\left(\gamma-\gamma^2\right)^2\Bigg)-\Bigg(\Bigg(\frac{1}{6}\left(\gamma^2-\gamma^3\right)(1-2\gamma)-\left(\left(\gamma^3-\gamma^4\right)(1-2\gamma)\right.\\&+\left.\left(\gamma-\gamma^2\right)^2\right)\Bigg)+\frac{1}{2}\left(\gamma-\gamma^2\right)\left(\left(\gamma-\gamma^2\right)(1-2\gamma)\right)\Bigg)\Bigg\}.\end{aligned} \tag{24}$$

In the same way, the remaining coefficient of the series solution can be obtained using the MATLAB software package. The analytical approximate solution in series form up to fourth approximation using equation (12) is given by:

$$\begin{aligned}\zeta(\xi,t) &= \gamma + \left(\gamma-\gamma^2\right)t + \frac{1}{4}\left(\gamma-\gamma^2\right)(1-2\gamma)\,t^2 \\ &+ \left(\frac{1}{12}\left(\gamma-\gamma^2\right)(1-2\gamma) - \left(\frac{1}{6}\left(\gamma^2-\gamma^3\right)(1-2\gamma) + \left(\gamma-\gamma^2\right)^2\right)\right)t^3 \\ &+ \frac{1}{4}\left\{\frac{1}{12}\left(\gamma-\gamma^2\right)(1-2\gamma) - \left(\frac{1}{6}\left(\gamma^2-\gamma^3\right)(1-2\gamma) + \left(\gamma-\gamma^2\right)^2\right)\right. \\ &- \left(\left(\frac{1}{6}\left(\gamma^2-\gamma^3\right)(1-2\gamma) - \left(\left(\gamma^3-\gamma^4\right)(1-2\gamma) + \left(\gamma-\gamma^2\right)^2\right)\right)\right. \\ &+ \left.\left.\frac{1}{2}\left(\gamma-\gamma^2\right)\left(\left(\gamma-\gamma^2\right)(1-2\gamma)\right)\right)\right\}t^4 + \ldots \end{aligned} \tag{25}$$

Case 2

By considering the parameter values $\alpha = 6$, $\gamma = 1$ equation (2) reduces to

$$\frac{\partial\zeta}{\partial t} = \frac{\partial\zeta}{\partial\xi^2} + 6\zeta(1-\zeta), \tag{26}$$

with initial condition

$$\theta(\xi,0) = \frac{1}{\left(1+e^{\xi}\right)^2}. \tag{27}$$

For the exact solution of Case 2 see Loyinmi and Akinfe (2020). Applying RDTM to equations (26) and (27), the transformed recursive formula and initial condition is

$$(\kappa+1)\,\zeta_{(\kappa+1)}(\xi) = \frac{d^2\zeta_\kappa(\xi)}{dx^2} + 6\zeta_\kappa(\xi) - 6\sum_{i=0}^{\kappa}\zeta_i(\xi)\zeta_{\kappa-i}(\xi), \tag{28}$$

$$\zeta_0(\xi) = \frac{1}{\left(1+e^{\xi}\right)^2}. \tag{29}$$

Using transformed condition equation (29) in equation (28), the coefficient of series solution can be obtained for $\kappa = 0, 1, 2, 3, 4, 5, \ldots$

For $\kappa = 0$, equation (28) reduces to

$$\zeta_1(\xi) = \frac{d^2\zeta_0(\xi)}{d\xi^2} + 6\zeta_0(\xi) - 6\zeta_0^2(\xi). \tag{30}$$

Substituting equation (29) in equation (30), we get

$$\zeta_1(\xi) = \frac{10\,e^{\xi}}{\left(1+e^{\xi}\right)^3}. \tag{31}$$

For $\kappa = 1$, equation (28) reduces to

$$2\zeta_2(\xi) = \frac{d^2\zeta_1(\xi)}{d\xi^2} + 6\zeta_1(\xi) - 6\sum_{i=0}^{1}\zeta_i(\xi)\zeta_{1-i}(\xi)\,. \tag{32}$$

Substituting values from equations (29) and (31) in equation (32), we get

$$\begin{aligned}2\zeta_2(\xi) = {} & \frac{10\,e^{\xi}\left(4e^{2\xi} - 7e^{\xi} + 1\right)}{\left(1+e^{\xi}\right)^5} + 6\left(\frac{10\,e^{\xi}}{\left(1+e^{\xi}\right)^3}\right)\\ & - 12\left(\frac{1}{\left(1+e^{\xi}\right)^2}\right)\left(\frac{10\,e^{\xi}}{\left(1+e^{\xi}\right)^3}\right),\end{aligned} \tag{33}$$

$$\zeta_2(\xi) = \frac{25\,e^{\xi}\left(2e^{\xi} - 1\right)}{\left(1+e^{\xi}\right)^4}. \tag{34}$$

For $\kappa = 2$, equation (28) reduces to

$$3\zeta_3(\xi) = \frac{d^2\zeta_2(\xi)}{d\xi^2} + 6\zeta_2(\xi) - 12\zeta_0^2(\xi) - 6\zeta_1^2(\xi)\,. \tag{35}$$

Substituting values from equations (29), (31) and (34) in equation (35), we get

$$\begin{aligned}3\zeta_3(\xi) = {} & -\frac{50\,e^{\xi}\left(33e^{2\xi} - 8e^{3\xi} - 18e^{\xi} + 1\right)}{\left(1+e^{\xi}\right)^6} + \frac{150\,e^{\xi}\left(2e^{\xi} - 1\right)}{\left(1+e^{\xi}\right)^4}\\ & - \left(\frac{300\,e^{\xi}\left(2e^{\xi} - 1\right)}{\left(1+e^{\xi}\right)^6}\right) - 6\left(\frac{10\,e^{\xi}}{\left(1+e^{\xi}\right)^3}\right)^2,\end{aligned} \tag{36}$$

$$\zeta_3(\xi) = -\frac{50\,e^{\xi}\left(15e^{2\xi} - 20e^{3\xi} + 18e^{\xi} - 5\right)}{3\left(1+e^{\xi}\right)^6}. \tag{37}$$

For $\kappa = 3$, equation (28) reduces to

$$4\zeta_4(\xi) = \frac{d^2\zeta_3(\xi)}{d\xi^2} + 6\zeta_3(\xi) - 12\zeta_0(\xi)\,\zeta_3(\xi) - 12\zeta_1(\xi)\,\zeta_2(\xi)\,. \tag{38}$$

Substituting values from equations (29), (31), (34) and (37) in equation (38), we get

$$\begin{aligned}\zeta_4(\xi) = {} & \frac{25\,e^{\xi}\left(386e^{2\xi} + 392e^{3\xi} - 575e^{4\xi} + 80e^{5\xi} - 152e^{\xi} + 5\right)}{2\left(1+e^{x}\right)^8}\\ & - \frac{75\,e^{\xi}\left(15e^{2\xi} - 20e^{3\xi} + 18e^{\xi} - 5\right)}{3\left(1+e^{\xi}\right)^6}\\ & + \left(\frac{50\,e^{\xi}\left(15e^{2\xi} - 20e^{3\xi} + 18e^{\xi} - 5\right)}{\left(1+e^{\xi}\right)^8}\right) - \left(\frac{1,500e^{2\xi}\left(2e^{\xi} - 1\right)}{\left(1+e^{\xi}\right)^7}\right),\end{aligned} \tag{39}$$

$$\zeta_4(\xi) = \frac{50e^{\xi}\left(506\,e^{2\xi} + 32e^{3\xi} - 575e^{4\xi} + 80e^{5\xi} - 12e^{\xi} - 55\right)}{4\left(1+e^{\xi}\right)^8}. \tag{40}$$

In the same way, using the MATLAB software package the remaining coefficient of the series solution can be calculate.The analytical approximate solution of equation (26) with initial condition equation (27) is

$$\theta(\xi,t) = \frac{1}{\left(1+e^{\xi}\right)^{2}} + \left(\frac{10\,e^{\xi}}{\left(1+e^{\xi}\right)^{3}}\right)t + \frac{1}{4}\left(\frac{25\,e^{\xi}\left(2e^{\xi}-1\right)}{\left(1+e^{\xi}\right)^{4}}\right)t^{2}$$
$$+\left(\frac{50\,e^{\xi}\left(15e^{2\xi}-20e^{3\xi}+18e^{\xi}-5\right)}{3\left(1+e^{\xi}\right)^{6}}\right)t^{3}$$
$$+\frac{50e^{\xi}\left(506\,e^{2\xi}+32e^{3\xi}-575e^{4\xi}+80e^{5\xi}-12e^{\xi}-55\right)}{4\left(1+e^{\xi}\right)^{8}}t^{4}+\ldots \tag{41}$$

Case 3

Considering the parameter values $\alpha = 1$, $\gamma = 6$ in equation (2), it reduces to

$$\frac{\partial\zeta}{\partial t} = \frac{\partial\zeta}{\partial\xi^{2}} + \zeta\left(1-\zeta^{6}\right), \tag{42}$$

with initial condition

$$\zeta(\xi,0) = \frac{1}{\sqrt[3]{\left(1+e^{\frac{3}{2}\xi}\right)}}. \tag{43}$$

For the exact solution of Case 3 see Loyinmi and Akinfe (2020). Applying RDTM to equations (42) and (43), the transformed recursive formula and initial condition is

$$(\kappa+1)\,\zeta_{(\kappa+1)}(\xi) = \frac{d^{2}\Theta_{\kappa}(\xi)}{d\xi^{2}} + \Theta_{\kappa}(\xi)$$
$$-\sum_{i=0}^{\kappa}\sum_{\iota=0}^{i}\sum_{\nu=0}^{\iota}\sum_{\omega=0}^{\nu}\sum_{\rho=0}^{\omega}\sum_{\ell=0}^{\rho}\Theta_{\ell}(\xi)\Theta_{\rho-\ell}(\xi)\,\Theta_{\omega-\rho}(\xi)$$
$$\Theta_{\nu-\omega}(\xi)\,\Theta_{\iota-\nu}(\xi)\,\Theta_{i-\iota}(\xi)\,\Theta_{\kappa-i}(\xi), \tag{44}$$

$$\zeta_{0}(\xi) = \frac{1}{\sqrt[3]{\left(1+e^{\frac{3}{2}\xi}\right)}}. \tag{45}$$

Using transformed initial condition equation (43) in equation (44), the coefficient of the series solution can be calculated.

For $\kappa = 0$, equation (44) reduces to

$$\zeta_{1}(\xi) = \frac{d^{2}\zeta_{0}(\xi)}{dx^{2}} + \zeta_{0}(\xi) - \left(\zeta_{0}(\xi)\right)^{7}. \tag{46}$$

Substituting equation (45) in equation (46), we get

$$\zeta_{1}(\xi) = \frac{e^{\frac{3}{2}\xi}\left(e^{\frac{3}{2}\xi}-3\right)}{4\left(1+e^{\frac{3}{2}\xi}\right)^{\frac{7}{3}}} + \frac{1}{\left(1+e^{\frac{3}{2}\xi}\right)^{\frac{1}{3}}} - \frac{1}{\left(1+e^{\frac{3}{2}\xi}\right)^{\frac{7}{3}}}. \tag{47}$$

For $\kappa = 1$, equation (44) reduces to

$$
\begin{aligned}
2\zeta_2(\xi) = {} & \frac{d^2\zeta_1(\xi)}{d\xi^2} + \zeta_1(\xi) \\
& - \sum_{i=0}^{1}\sum_{\iota=0}^{i}\sum_{\nu=0}^{\iota}\sum_{\omega=0}^{\nu}\sum_{\rho=0}^{\omega}\sum_{\ell=0}^{\rho} \zeta_\ell(\xi)\zeta_{\rho-\ell}(\xi)\,\zeta_{\omega-\rho}(\xi)\,\zeta_{\nu-\omega}(\xi) \\
& \zeta_{\iota-\nu}(\xi)\,\zeta_{i-\iota}(\xi)\,\zeta_{\kappa-i}(\xi)\,.
\end{aligned} \tag{48}
$$

Substituting equations (45), (47) in equation (48), we get

$$
\zeta_2(\xi) = \frac{1}{2}\left(\frac{\begin{array}{c} 227e^{3\xi} + 13e^{\frac{3}{2}\xi} + 25e^{6\xi} - 25e^{\frac{9}{2}\xi} - 656\,e^{3\xi}\left(1+e^{\frac{3}{2}\xi}\right)^2 \\ -344\,e^{\frac{3}{2}\xi}\left(1+e^{\frac{3}{2}\xi}\right)^2 - 352e^{6\xi}\left(1+e^{\frac{3}{2}\xi}\right)^2 \\ +672e^{3\xi}\left(1+e^{\frac{3}{2}\xi}\right)^4 + 448e^{\frac{3}{2}\xi}\left(1+e^{\frac{3}{2}\xi}\right)^4 \\ -792e^{\frac{9}{2}\xi}\left(1+e^{\frac{3}{2}\xi}\right)^2 + 112e^{6\xi}\left(1+e^{\frac{3}{2}\xi}\right)^4 \\ +448e^{\frac{9}{2}\xi}\left(1+e^{\frac{3}{2}\xi}\right)^4 - 128\left(1+e^{\frac{3}{2}\xi}\right)^2 \\ +112\left(1+e^{\frac{3}{2}\xi}\right)^4 + 16 \end{array}}{\left(1+e^{\frac{3}{2}\xi}\right)^{\frac{7}{3}}}\right). \tag{49}
$$

$\zeta_3(\xi)$, $\zeta_4(\xi)$ are too long to mention,so they are represented graphical. The analytical approximate solution of equation (42) with initial condition (43) is

$$
\zeta(\xi,t) = \frac{1}{\left(1+e^{\frac{3}{2}\xi}\right)^{\frac{1}{3}}} + \left(\frac{e^{\frac{3}{2}\xi}\left(e^{\frac{3}{2}\xi}-3\right)}{4\left(1+e^{\frac{3}{2}\xi}\right)^{\frac{7}{3}}} + \frac{1}{\left(1+e^{\frac{3}{2}\xi}\right)^{\frac{1}{3}}} - \frac{1}{\left(1+e^{\frac{3}{2}\xi}\right)^{\frac{7}{3}}}\right)t
$$

$$+\frac{1}{2}\left(\frac{\begin{array}{c}227e^{3\xi}+13e^{\frac{3}{2}\xi}+25e^{6\xi}-25e^{\frac{9}{2}\xi}\\ -656\,e^{3\xi}\left(1+e^{\frac{3}{2}\xi}\right)^{2}-344\,e^{\frac{3}{2}\xi}\left(1+e^{\frac{3}{2}\xi}\right)^{2}\\ -352e^{6\xi}\left(1+e^{\frac{3}{2}\xi}\right)^{2}+672e^{3\xi}\left(1+e^{\frac{3}{2}\xi}\right)^{4}\\ +448e^{\frac{3}{2}\xi}\left(1+e^{\frac{3}{2}\xi}\right)^{4}-792e^{\frac{9}{2}\xi}\left(1+e^{\frac{3}{2}\xi}\right)^{2}\\ +112e^{6\xi}\left(1+e^{\frac{3}{2}\xi}\right)^{4}+448e^{\frac{9}{2}\xi}\left(1+e^{\frac{3}{2}\xi}\right)^{4}\\ -128\left(1+e^{\frac{3}{2}\xi}\right)^{2}+112\left(1+e^{\frac{3}{2}\xi}\right)^{4}+16\end{array}}{\left(1+e^{\frac{3}{2}\xi}\right)^{\frac{7}{3}}}\right)t^{2}+\ldots \tag{50}$$

Case 4

Consider the parameter values $\alpha = 1$, $\gamma = 1$, $0 \le \rho \le 1$, equation (3) reduces to

$$\frac{\partial\zeta}{\partial t}=\frac{\partial^2\zeta}{\partial\xi^2}+\zeta\left(1-\zeta\right)\left(\zeta-\rho\right). \tag{51}$$

with initial condition

$$\zeta\left(\xi,0\right)=\frac{1}{\left(1+e^{-\frac{1}{\sqrt{2}}\xi}\right)}. \tag{52}$$

or the exact solution of Case 4 see Loyinmi and Akinfe (2020). Applying RDTM to equations (51) and (52), the transformed recursive formula and initial condition is

$$\begin{aligned}\left(\kappa+1\right)\zeta_{(\kappa+1)}\left(\xi\right)&=\frac{d^2\zeta_\kappa\left(\xi\right)}{d\xi^2}+\left(\rho+1\right)\sum_{i=0}^{\kappa}\zeta_i\left(\xi\right)\zeta_{\kappa-i}\left(\xi\right)-\rho\,\zeta_\kappa\left(\xi\right)\\&\quad-\sum_{i=0}^{\kappa}\sum_{j=0}^{i}\zeta_j\left(\xi\right)\zeta_{i-j}\left(\xi\right)\zeta_{\kappa-i}\left(\xi\right),\end{aligned} \tag{53}$$

$$\zeta_0\left(\xi\right)=\frac{1}{\left(1+e^{-\frac{1}{\sqrt{2}}\xi}\right)}. \tag{54}$$

Using transformed initial condition equation (54) in recursive formula (53) the coefficient of the series solution can be calculated.

For $\kappa = 0$, equation (53) reduces to

$$\zeta_1(\xi) = \frac{d^2\zeta_0(\xi)}{d\xi^2} + (\rho+1)(\zeta_0(\xi))^2 - \rho\,\zeta_0(\xi) - (\zeta_0(\xi))^3. \tag{55}$$

Substituting equation (54) in equation (55), we get

$$\zeta_1(\xi) = -\frac{e^{\frac{1}{\sqrt{2}}\xi}(2\rho-1)}{2\left(1+e^{\frac{1}{\sqrt{2}}\xi}\right)}. \tag{56}$$

For $\kappa = 1$, equation (53) reduces to

$$2\zeta_2(\xi) = \frac{d^2\zeta_1(\xi)}{dx^2} + (\rho+1)\sum_{i=0}^{1}\zeta_i(\xi)\zeta_{\kappa-i}(\xi) - \rho\,\zeta_1(\xi) - \sum_{i=0}^{1}\sum_{j=0}^{i}\zeta_j(\xi)\zeta_{i-j}(\xi)\zeta_{\kappa-i}(\xi), \tag{57}$$

$$2\zeta_2(\xi) = \frac{d^2\zeta_1(\xi)}{d\xi^2} + (2\rho-1)\,\zeta_0(\xi)\,\zeta_1(\xi) - \rho\,\zeta_1(\xi). \tag{58}$$

Substituting value from equations (54) and (56) in equation (58), we get

$$\zeta_2(\xi) = \frac{e^{\frac{1}{\sqrt{2}}\xi}(2\rho-1)\left(2\rho + e^{\sqrt{2}\xi} + 6e^{\frac{1}{\sqrt{2}}\xi} - 2\rho e^{\sqrt{2}\xi} - 1\right)}{8\left(1+e^{\frac{1}{\sqrt{2}}\xi}\right)^4}. \tag{59}$$

For $\kappa = 2$, equation (53) reduces to

$$3\zeta_3(\xi) = \frac{d^2\zeta_3(\xi)}{d\xi^2} + (\rho+1)\sum_{i=0}^{2}\zeta_i(\xi)\zeta_{\kappa-i}(\xi) - \rho\,\zeta_2(\xi) - \sum_{i=0}^{2}\sum_{j=0}^{i}\zeta_j(\xi)\zeta_{i-j}(\xi)\zeta_{\kappa-i}(\xi). \tag{60}$$

Substituting values from equations (54), (56), and (59) in equation (60), we get

$$\zeta_3(\xi) = \frac{e^{\frac{1}{\sqrt{2}}\xi}(2\rho-1)\begin{pmatrix} 4\rho - e^{\sqrt{2}\xi}\left(20+2\rho-12\rho^2\right) \\ +e^{2\sqrt{2}\xi}(17-10\rho) \\ +e^{\frac{1}{\sqrt{2}}\xi}(40-20\rho) \\ +e^{\frac{3}{\sqrt{2}}\xi}\left(76+12\rho+8\rho^2\right) \\ -4\rho^2-1 \end{pmatrix}}{24\left(1+e^{\frac{1}{\sqrt{2}}\xi}\right)^6}. \tag{61}$$

For $\kappa = 3$, equation (53) reduces to

$$4\zeta_4(\xi) = \frac{d^2\zeta_3(\xi)}{d\xi^2} + (\rho+1)\sum_{i=0}^{3}\zeta_i(\xi)\zeta_{\kappa-i}(\xi) - \rho\,\zeta_3(\xi)$$
$$-\sum_{i=0}^{3}\sum_{j=0}^{i}\zeta_j(\xi)\zeta_{i-j}(\xi)\zeta_{\kappa-i}(\xi)\,. \tag{62}$$

Substituting values from equations (54), (56) and (59) in equation (60), we get

$$\zeta_4(\xi) = -\frac{(2\rho-1)\,e^{\frac{1}{\sqrt{2}}\xi}\begin{pmatrix} 897e^{\sqrt{2}\xi} - 6\rho + 1137e^{2\sqrt{2}\xi} - 168e^{\frac{1}{\sqrt{2}}\xi} \\ +17e^{3\sqrt{2}\xi} - 2,160e^{\frac{3}{\sqrt{2}}\xi} + 60e^{\frac{5}{\sqrt{2}}\xi} \\ -432\rho e^{\sqrt{2}\xi} + 402\rho e^{2\sqrt{2}\xi} + 196\rho e^{\frac{1}{\sqrt{2}}\xi} \\ -44\rho e^{3\sqrt{2}x} + 56\rho e^{\frac{3}{\sqrt{2}}\xi} - 332\rho e^{\frac{5}{\sqrt{2}}\xi} \\ -20\rho^2 e^{\sqrt{2}\xi} + 64\rho^3 e^{\sqrt{2}\xi} + 148\rho^2 e^{2\sqrt{2}\xi} \\ -88\rho^2 e^{\frac{1}{\sqrt{2}}\xi} + 20\rho^2 e^{3\sqrt{2}\xi} - 56\rho^3 e^{2\sqrt{2}\xi} \\ +16\rho^3 e^{\frac{1}{\sqrt{2}}\xi} + 240\rho^2 e^{\frac{3}{\sqrt{2}}\xi} + 16\rho^3 e^{\frac{3}{\sqrt{2}}\xi} \\ +8\rho^2 e^{\frac{5}{\sqrt{2}}\xi} - 32\rho^3 e^{\frac{5}{\sqrt{2}}\xi} + 12\rho^2 - 8\rho^3 + 1 \end{pmatrix}}{64\left(1+e^{\frac{1}{\sqrt{2}}\xi}\right)^8}. \tag{63}$$

In the same way, the remaining coefficient can be evaluate using the MATLAB package. The analytical approximate solution of equation (51) with initial condition (52) is

$$\zeta(\xi,t) = \frac{1}{\left(1+e^{-\frac{1}{\sqrt{2}}\xi}\right)} - \left(\frac{e^{\frac{1}{\sqrt{2}}\xi}(2\rho-1)}{2\left(1+e^{\frac{1}{\sqrt{2}}\xi}\right)}\right)t$$
$$+\left(\frac{e^{\frac{1}{\sqrt{2}}\xi}(2\rho-1)\left(2\rho+e^{\sqrt{2}\xi}+6e^{\frac{1}{\sqrt{2}}\xi}-2\rho e^{\sqrt{2}\xi}-1\right)}{8\left(1+e^{\frac{1}{\sqrt{2}}\xi}\right)^4}\right)t^2$$
$$+\frac{e^{\frac{1}{\sqrt{2}}\xi}(2\rho-1)\begin{pmatrix} 4a - e^{\sqrt{2}\xi}\left(20+2\rho-12\rho^2\right) + e^{2\sqrt{2}\xi}(17-10\rho) \\ +e^{\frac{1}{\sqrt{2}}\xi}(40-20\rho) + e^{\frac{3}{\sqrt{2}}\xi}\left(76+12\rho+8a^2\right) \\ -4\rho^2-1 \end{pmatrix}}{24\left(1+e^{\frac{1}{\sqrt{2}}\xi}\right)^6}t^3$$
$$-\frac{(2\rho-1)\,e^{\frac{1}{\sqrt{2}}\xi}\begin{pmatrix} 897e^{\sqrt{2}\xi} - 6\rho + 1,137e^{2\sqrt{2}x} - 168e^{\frac{1}{\sqrt{2}}\xi} \\ +17e^{3\sqrt{2}\xi} - 2,160e^{\frac{3}{\sqrt{2}}\xi} + 60e^{\frac{5}{\sqrt{2}}\xi} \\ -432\rho e^{\sqrt{2}\xi} + 402\rho e^{2\sqrt{2}\xi} + 196\rho e^{\frac{1}{\sqrt{2}}\xi} \\ -44ae^{3\sqrt{2}\xi} + 56ae^{\frac{3}{\sqrt{2}}\xi} - 332ae^{\frac{5}{\sqrt{2}}\xi} \\ -20\rho^2 e^{\sqrt{2}\xi} + 64a^3 e^{\sqrt{2}\xi} + 148a^2 e^{2\sqrt{2}\xi} \\ -88a^2 e^{\frac{1}{\sqrt{2}}\xi}20\rho^2 e^{3\sqrt{2}\xi} - 56\rho^3 e^{2\sqrt{2}\xi} \\ +16\rho^3 e^{\frac{1}{\sqrt{2}}\xi} + 240\rho^2 e^{\frac{3}{\sqrt{2}}\xi} + 16\rho^3 e^{\frac{3}{\sqrt{2}}\xi} \\ +8\rho^2 e^{\frac{5}{\sqrt{2}}\xi} - 32\rho^3 e^{\frac{5}{\sqrt{2}}\xi} + 12\rho^2 - 8\rho^3 + 1 \end{pmatrix}}{64\left(1+e^{\frac{1}{\sqrt{2}}\xi}\right)^8}t^4 + \ldots \tag{64}$$

4 Algorithm for RDTM

Algorithm 1 Algorithm to calculate the series solution of BFRDE

begin
Input: Parameters values, initial condition
Output: Series solution of the (BFRDE)
Step 1 Insert Input parameters.
Step 2 Compute the coefficient of series solution using the recursive formula associated with the given partial differential equation
Create loop for k from 1 to n
Initialise A = 0, B = 0, C = 0
Note: Create the number of loop required depending on power of ζ.
Create loop for i from 1 to k
Create loop for j from 1 to i
$A = A + \zeta(1:\kappa) * \zeta(1:\kappa - i) + \zeta(1:i) * \zeta(1:i-j) * \zeta(1:\kappa - i)$
end
end
$\zeta(\kappa+1) = (diff(\zeta(i), \xi, 2) + A)/(\kappa * (\kappa+1))$
end
Step 3 Compute the series solution unto to n using the coefficient values.
Create loop for i from 1 to n
$Series(t) = \zeta(1:n) * power(t, \kappa - 1)$
end
Step 4 Display the series solution.
end

5 Result and discussion

In this section, we present the result for BFRDE for Cases 1, 2, 3 and 4 obtained by RDTM and compare these result via table, convergence plot with the exact solution as well as the with other well known analytical solution available in the literature (Loyinmi and Akinfe, 2020).

The absolute errors at various times (t) for different values of ξ are obtained and compared with the exact solutions in Tables 1, 2, 3 and 4 for the Cases 1, 2, 3 and 4. Figures 1, 2, 3 and 4 shows 3D plot for Cases 1, 2, 3 and 4 respectively for different values of parameters. To study the accuracy of RDTM solution for Case 1, the absolute error up to four term approximation is listed in Table 2. The comparison of results with other analytical method for Case 1 from Table 2 shows that RDTM is more accurate as compare to EHTPM and HPM. From Figure 5 and Table 1, it is clear that the approximate solution converges rapidly to exact solution. From Figure 5, it clear that series solution with less number of approximation shows an excellent agreement with exact solution. The computational analysis of Case 2 is done in the domain ξ = 1, 2, 3, ..., $0.1 \leq t \leq 0.5$. Table 3 shows the comparison of the present result with the result obtained by Loyinmi and Akinfe (2020). It is found that the present solution is more accurate than Loyinmi and Akinfe (2020) and shows an excellent agreement with the exact solution. From Figure 5, it is clear that the series solution with ten approximation terms shows an excellent agreement with an exact solution in compared to solution with four and seven approximations, respectively. Table 3 shows comparison of present solution with EHTPM and HPM at different value of ξ and t for Case 3 for α = 1, γ = 6 and ρ = 0. We found that the present solution shows an excellent agreement with the exact solution and is more accurate than EHTPM and HPM, which is clear from Figure 6. Table 4 shows the comparison of present solution with EHTPM and HPM at different value of ξ and t for Case 4 for α = 0, β = 6 and $0 \leq a \leq 1$. We found that the present solution shows an excellent agreement with the exact solution and is more accurate than EHTPM and HPM, which is clear from Figure 6. The effect of initial conditions for Cases 1, 2, 3 and 5 can be seen in Tables 2, 3, 4 and 5 respectively.

Figure 1 Analytical approximate solution of Case 1 for parameter values α = 1, γ = 1 (see online version for colours)

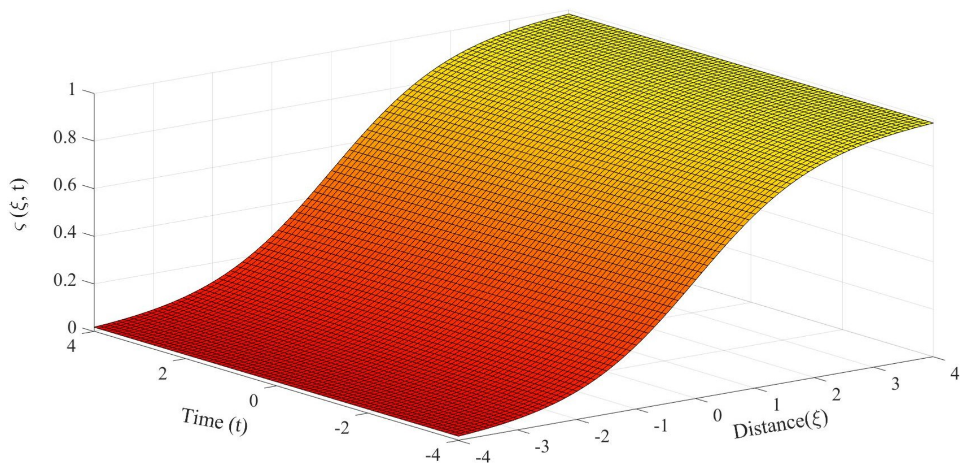


Figure 2 Analytical approximate solution of Case 2 for parameter values α = 6, γ = 1 (see online version for colours)

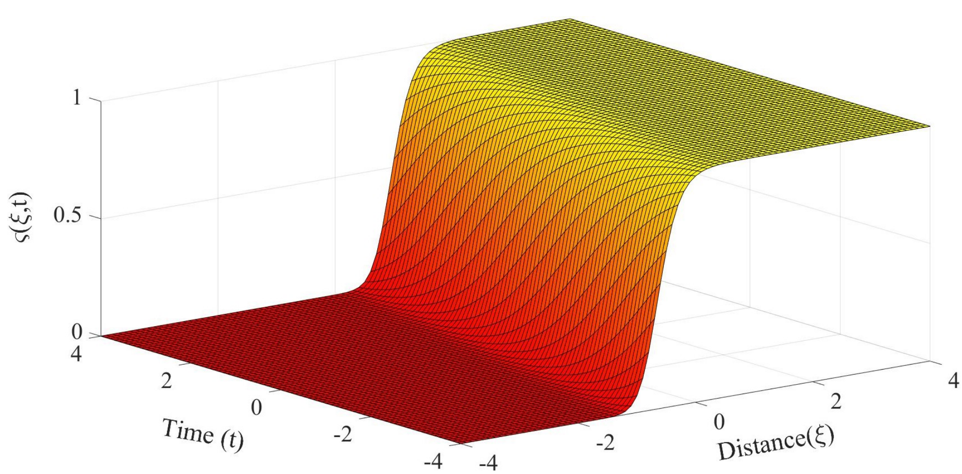


Figure 3 Analytical approximate solution of Case 3 α = 1, γ = 6 (see online version for colours)

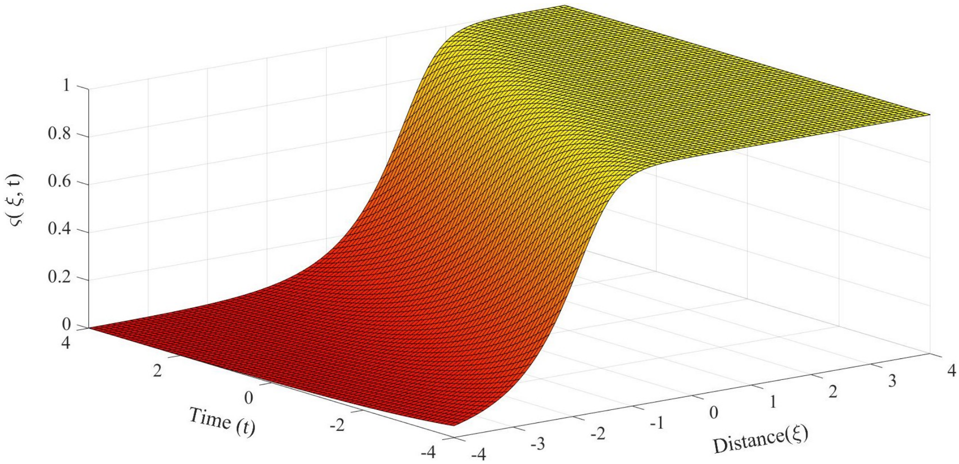


Figure 4 Analytical approximate solution of Case 4 with parameter α = 1, γ = 6, ρ = 0.3 (see online version for colours)

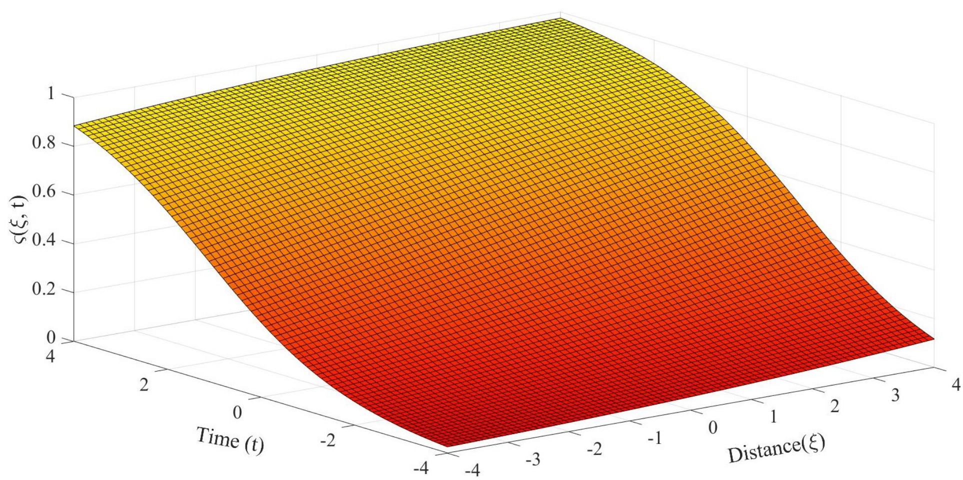


Figure 5 Convergence plot of Cases 1 and 2 respectively (see online version for colours)

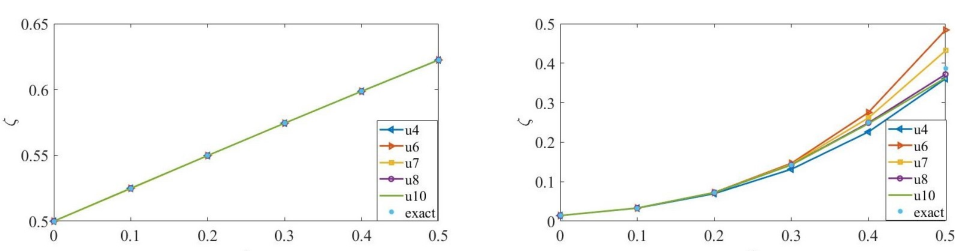


Table 2 Comparison of analytic approximate solution of Case 1 with parameters $\alpha = 1$, $\gamma = 1$

t	*EXACT*	*EHTPM*	*HPM*	*RDTM*	*Absolute error for EHTPM*	*Absolute error for HPM*	*Absolute error for RDTM*
0	0.5	0.5	0.5	0.5	0	0	0
0.1	0.524979188	0.524979187	0.549979188	0.524979187478940	1.0×10^{-10}	0.025	2.106×10^{-11}
0.2	0.549833997	0.549833997	0.599834	0.549833997312522	1.0×10^{-10}	0.050000003	1.25522×10^{-11}
0.3	0.574442517	0.574442517	0.649442563	0.574442516815458	1.0×10^{-10}	0.075000046	8.8542×10^{-11}
0.4	0.59868766	0.598687667	0.698688	0.598687660201764	7.1×10^{-9}	0.10000034	1.76403×10^{-12}
0.5	0.622459331	0.622459605	0.795662	0.622459332232332	2.741×10^{-7}	0.173202669	1.23233×10^{-09}

Table 3 Comparison of analytic approximate solution of Case 2 with parameters $\alpha = 6$, $\gamma = 1$

x	*t*	*Exact*	*EHTPM*	*HPM*	*RDTM*	*Absolute error for EHTPM*	*Absolute error for HPM*	*Absolute error for RDTM*
1	0	0.7937005259841	0.7937005259841	0.7937005259841	0.7937005259841	0	0	0
	0.1	0.1425369566	0.142536957700	0.142536957700	0.142536956596664	1.1×10^{-9}	1.1×10^{-9}	3.336×10^{-12}
	0.2	0.2500	0.25000000880	0.25000000880	0.250000000000000	8.8×10^{-9}	8.8×10^{-9}	0
	0.3	0.387455618900	0.387455610900	0.387455610900	0.387455619000260	8.0×10^{-9}	8.0×10^{-9}	1.0026×10^{-10}
	0.4	0.53444664550	0.53373262850	0.53373262850	0.534446645389474	7.14017×10^{-9}	7.1417×10^{-9}	1.10526×10^{-10}
	0.5	0.668428024300	0.668428157000	0.668428157000	0.668428568541688	1.327×10^{-8}	1.327×10^{-8}	5.44242×10^{-7}
2	0	0.1420933661861	0.1420933661861	0.1420933661861	0.1420933661861	0	0	0
	0.1	0.03327907174	0.033279071749	0.033279071749	0.033279071736024	9.0×10^{-12}	9.0×10^{-12}	3.9765×10^{-12}
	0.2	0.0723294881500	0.0723294884300	0.0723294884300	0.072329488128513	2.8×10^{-10}	2.8×10^{-10}	2.14867×10^{-11}
	0.3	0.14253695660	0.14253695800	0.14253695800	0.142536956596551	1.4×10^{-9}	1.4×10^{-9}	3.44899×10^{-12}
	0.4	0.25000000000	0.25000000260	0.25000000260	0.250000000000000	2.6×10^{-9}	2.6×10^{-9}	0
	0.5	0.3874556189000	0.3874556163000	0.3874556163000	0.387455618598437	2.6×10^{-9}	2.6×10^{-9}	3.01563×10^{-10}
3	0	0.00224921344665465	0.00224921344665465	0.00224921344665465	0.00224921344665465	0	0	0
	0.1	0.00575446348	0.00575446349	0.00575446349	0.005754463476135	1.0×10^{-11}	1.0×10^{-11}	3.86461×10^{-12}
	0.2	0.01420933662	0.01420933638	0.01420933638	0.014209336618610	2.4×10^{-10}	2.4×10^{-10}	1.39×10^{-12}
	0.3	0.03327907174	0.03327907194	0.03327907194	0.033279071736024	2.0×10^{-10}	2.0×10^{-10}	3.9765×10^{-10}
	0.4	0.07232948815	0.07232948819	0.07232948819	0.072329488128513	4.0×10^{-11}	4.0×10^{-11}	2.14867×10^{-11}
	0.5	0.14253695660	0.14253695720	0.14253695720	0.142536956596522	6.0×10^{-10}	6.0×10^{-10}	3.478×10^{-12}

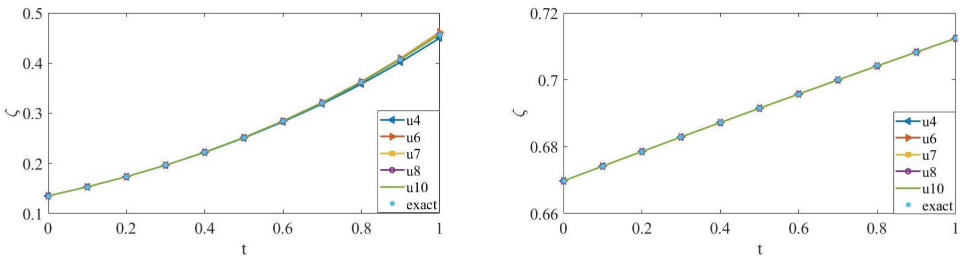
Table 4 Comparison of analytic approximate solution of Case 3 with parameters $\alpha = 1$, $\gamma = 6$

x	t	*Exact*	*EHTPM*	*HPM*	*RDTM*	*Absolute error for EHTPM*	*Absolute error for HPM*	*Absolute error for RDTM*
1	0	0.567146427334617	0.567146427334617	0.567146427334617	0.567146427334617	0	0	0
	0.1	0.6258048411	0.6258048411	0.6258048411	0.625804841005771	0	0	9.4229×10^{-11}
	0.2	0.6845750433	0.6845750439	0.6845750439	0.684575053564968	6.0×10^{-10}	6.0×10^{-10}	1.0265×10^{-08}
	0.3	0.7412818057	0.7412818056	0.7412818056	0.741282017163646	1.0×10^{-10}	1.0×10^{-10}	2.11464×10^{-07}
	0.4	0.793700526	0.79370052640	0.7937005264	0.793701438427354	4.0×10^{-10}	4.0×10^{-10}	9.12427×10^{-07}
	0.5	0.8399823314	0.83998233120	0.8399823312	0.83996392012000	2.0×10^{-10}	2.0×10^{-10}	1.84113×10^{-05}
2	0	0.361969333853025	0.361969333853025	0.361969333853025	0.361969333853025	0	0	0
	0.1	0.4072565019	0.40725650250	0.4072565025	0.407256501777420	6.0×10^{-10}	6.0×10^{-10}	1.2258×10^{-10}
	0.2	0.4568490786	0.45684907950	0.4568490795	0.456849077483068	9.0×10^{-10}	9.0×10^{-10}	1.11693×10^{-09}
	0.3	0.51040112000	0.51040112020	0.51040112020	0.510401061152851	2.0×10^{-10}	2.0×10^{-10}	5.88471×10^{-08}
	0.4	0.56714642730	0.56714642710	0.56714642710	0.567145572933122	2.0×10^{-10}	2.0×10^{-10}	8.54367×10^{-07}
	0.5	0.6258048411	0.6258116411	0.62581164110	0.6257991819957520	6.8×10^{-6}	6.8×10^{-6}	5.6591×10^{-06}
3	0	0.222309976246631	0.222309976246631	0.222309976246631	0.222309976246631	0	0	0
	0.1	0.25149183570	0.2514918356	0.25149183560	0.251491835921418	1.0×10^{-10}	1.0×10^{-10}	2.21418×10^{-10}
	0.2	0.28429339640	0.2842933962	0.28429339620	0.284293396280718	2.0×10^{-10}	2.0×10^{-10}	1.19282×10^{-10}
	0.3	0.32103175570	0.3210317529	0.32103175290	0.321031747034672	2.8×10^{-9}	2.8×10^{-9}	8.66533×10^{-09}
	0.4	0.36196933390	0.3619692659	0.36196926590	0.361969159462556	6.8×10^{-8}	6.8×10^{-8}	1.74437×10^{-07}
	0.5	0.40725650190	0.4072565013	0.40725650130	0.40725466224225	6.0×10^{-10}	6.0×10^{-10}	1.83966×10^{-06}

Table 5 Comparison of analytic approximate solution of Case 4 with parameters $\alpha = 1$, $\gamma = 1$, $\rho = 0.3$

x	t	*Exact*	*EHTPM*	*HPM*	*RDTM*	*Absolute error for EHTPM*	*Absolute error for HPM*	*Absolute error for RDTM*
1	0	0.66976154326657	0.66976154326657	0.66976154326657	0.66976154326657	0	0	0
	0.1	0.6741700549	0.6741700556	0.6741700556	0.6741700548104350	7.0×10^{-10}	7.0×10^{-10}	8.9565×10^{-11}
	0.2	0.6785479545	0.6785479553	0.6785479553	0.6785479547401430	8.0×10^{-10}	8.0×10^{-10}	2.40143×10^{-10}
	0.3	0.6828947004	0.6828947010	0.6828947010	0.6828947003559610	6.0×10^{-10}	6.0×10^{-10}	4.4039×10^{-11}
	0.4	0.6872097630	0.6872097639	0.6872097639	0.6872097630511580	9.0×10^{-10}	9.0×10^{-10}	5.1158×10^{-11}
	0.5	0.6914926343	0.6914926349	0.6914926349	0.6914926344752060	6.0×10^{-10}	6.0×10^{-10}	1.75206×10^{-10}
2	0	0.804429682506957	0.804429682506957	0.804429682506957	0	0	0	0
	0.1	0.8075569884	0.8075569884	0.8075569884	0.8075569887354400	0	0	3.3544×10^{-10}
	0.2	0.8106460584	0.8106460605	0.8106460605	0.8106460584412020	2.1×10^{-9}	2.1×10^{-9}	4.12019×10^{-11}
	0.3	0.8136969821	0.8136969842	0.8136969842	0.8136969821235920	2.1×10^{-9}	2.1×10^{-9}	2.3592×10^{-11}
	0.4	0.8167098629	0.8167098651	0.8167098651	0.8167098631294450	2.2×10^{-9}	2.2×10^{-9}	2.29445×10^{-10}
	0.5	0.8196848174	0.8196848192	0.8196848192	0.8196848172705220	1.8×10^{-9}	1.8×10^{-9}	1.29478×10^{-10}
3	0	0.89295819853483	0.89295819853483	0.89295819853483	0.892958198534830	0	0	0
	0.1	0.8948549056	0.8948549074	0.8948549074	0.8948549058618710	1.8×10^{-9}	1.8×10^{-9}	2.61871×10^{-10}
	0.2	0.8967218917	0.8967218936	0.8967218936	0.8967218919236400	1.9×10^{-9}	1.9×10^{-9}	2.2364×10^{-10}
	0.3	0.8985594848	0.8985594869	0.8985594869	0.8985594852106020	2.1×10^{-9}	2.1×10^{-9}	4.10602×10^{-10}
	0.4	0.9003680153	0.9003680169	0.9003680169	0.9003680155234240	1.6×10^{-9}	1.6×10^{-9}	2.23242×10^{-10}
	0.5	0.9021478138	0.9021478150	0.9021478150	0.9021478137544130	1.2×10^{-9}	1.2×10^{-9}	4.5587×10^{-11}

Figure 6 Convergence plot of Cases 3 and 4 respectively (see online version for colours)



6 Conclusions

In this research, we represent an application of the RDTM by handling the class of reaction-diffusion equations, namely the family of fisher's differential equation. Several conclusions can be drawn from this research:

- It easily handles the nonlinear terms and complex initial conditions and overcomes the disadvantage of computing the unwanted terms and complex calculations.
- Highly nonlinear PDEs can easily handle by the RDTM, and its straightforward applicability makes it easier for users to code in any software language.
- It does not require discretisation, perturbation parameter, or linearisation; instead, it converts the complex differential equation into an algebraic formula that is easy to handle.
- The capability of the method to convert the complex differential equation and initial condition to a simple recursive formula is a massive advantage to researchers as it helps in programming the approach.
- Using MATLAB code, the computation time is reduced if anyone requires a solution by considering more than 30 terms.

We strongly recommend using the proposed RDTM to solve models in various fields, including fluid flow and fluid mechanics, engineering, nonlinear dynamics, acoustics, convection-diffusion, and advection-diffusion models. Furthermore, this approach is versatile enough to be used in the classroom to provide theoretical solutions to equations like the Fisher, Burgers-Huxley, and other nonlinear partial differential equations.

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