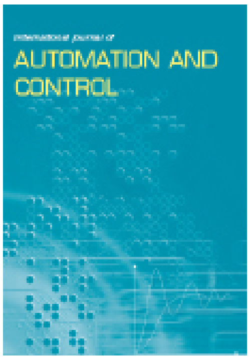


International Journal of Automation and Control

ISSN online: 1740-7524 - ISSN print: 1740-7516
https://www.inderscience.com/ijaac

H_∞ stabilisation of uncertain discrete time-delayed system with actuator saturation by using Wirtinger inequality

Komal Agrawal, Richa Negi, Vipin Chandra Pal, Vishwajeet Patel

DOI: 10.1504/IJAAC.2022.10044198

Article History:

Received:	30 May 2021
Accepted:	07 August 2021
Published online:	30 November 2022

H_∞ stabilisation of uncertain discrete time-delayed system with actuator saturation by using Wirtinger inequality

Komal Agrawal* and Richa Negi

Department of Electrical Engineering,
Motilal Nehru National Institute of Technology,
Prayagraj, Uttar Pradesh, 211002, India
Email: komal@mnnit.ac.in
Email: richa@mnnit.ac.in
*Corresponding author

Vipin Chandra Pal

Department of Electronics and Instrumentation Engineering,
National Institute of Technology Silchar,
Assam, 788010, India
Email: vipin.vchandra@gmail.com
Email: vipin@ei.nits.ac.in

Vishwajeet Patel

Department of Electrical Engineering,
Motilal Nehru National Institute of Technology,
Prayagraj, Uttar Pradesh, 211002, India
Email: vjp251@gmail.com

Abstract: This paper contemplates the H_∞ control of discrete-time delayed systems together with actuator saturation, parametric uncertainties and disturbances. H_∞-based state feedback controller is conceived to stabilise the closed loop system. Lyapunov Krasovskii functional (LKF), discrete Wirtinger-based summation inequality and convex hull approach are combined to obtain novel regional stability conditions. The estimated attraction domain is maximised using an optimisation method along with linear matrix inequality (LMI). A comparative study is shown between the obtained and existing findings. The results are found to be less conservative than the prior ones. Finally, instances signify efficacy of presented approaches.

Keywords: time-delay; discrete system; actuator saturation; linear matrix inequality; LMI; H_∞ control; Wirtinger inequality.

Reference to this paper should be made as follows: Agrawal, K., Negi, R., Pal, V.C. and Patel, V. (2023) 'H_∞ stabilisation of uncertain discrete time-delayed system with actuator saturation by using Wirtinger inequality', *Int. J. Automation and Control*, Vol. 17, No. 1, pp.43–72.

Biographical notes: Komal Agrawal is currently pursuing her research in Control Systems and Applications from the MNNIT, Prayagraj. She received her BTech in Electrical and Electronics from the United College of Engineering and Research, Allahabad (then affiliated to V.B.S. Purvanchal University, Jaunpur) in 2002, and MTech in PE and ASIC Design from MNNIT, Prayagraj in 2014. Her research area includes stability of discrete time-delayed systems, linear and nonlinear systems and robust control.

Richa Negi is a Professor in the Electrical Engineering Department, MNNIT, Prayagraj. She received her BTech in Electrical Engineering and MTech in Power System from the Kurukshetra University, in 1992 and 1994, respectively. She received her PhD in Discrete Time Linear Control System from the MNNIT, Prayagraj, in 2013. Her current interest covers time delay systems, robust and adaptive control, Lyapunov-based design, modelling of 1D and 2D dynamical systems, stabilisation of linear and nonlinear multi-dimensional systems and control of renewable energy.

Vipin Chandra Pal is an Assistant Professor in the Department of Electronics and Instrumentation Engineering, National Institute of Technology Silchar, Assam, India. He received his BTech in Electronics Instrumentation and Control from the Dr. A.P.J. Abdul Kalam Technical University (AKTU) (formerly UPTU), Lucknow, in 2002 and MTech in Control and Instrumentation from MNNIT, Prayagraj, in 2012. He received a gold medal also. He obtained his PhD in Stability of Time Delayed Control Systems from the MNNIT, Prayagraj, in 2017. His current interest covers time delay systems, robust and adaptive control, Lyapunov stability, fractional order systems, modelling of dynamical systems, linear and nonlinear multi-dimensional systems, biological control system, and control of renewable energy.

Vishwajeet Patel is currently pursuing his research in control systems at the NIT, Kurukshetra. He received his BTech in Electrical Engineering from the Rajkiya Engineering College, Ambedkar Nagar, in 2016 and MTech in Control and Instrumentation from MNNIT, Prayagraj, in 2019. His area of interest includes stability analysis of time-delayed systems.

1 Introduction

Actuator saturation is the most common inherent nonlinearity of real-time practical systems. As a sequel of saturation, a closed loop control system becomes an open loop. Thus, the implementation of the system degrades and also causes instability (Jishi et al., 2013; Ma et al., 2018; Shen et al., 2019). It is not possible in a real-time system to provide a very high magnitude/rate of control signal due to safety constraints. Some examples of these limitations are flow volume limitation, rate in hydraulic actuators, voltage limits in electrical actuators and deflection limits in aircraft actuators (Bezzaoucha et al., 2012). Stability persual of the control system coupled with input saturation has become vital and till now, it is a very prominent area of research (Flores et al., 2013; Jishi et al., 2013; Ma and Chen, 2015; Mahjoub et al., 2014; Negi et al., 2012; Pal and Negi, 2012, 2018; Pal et al., 2019; Qian et al., 2015; Sun et al., 2019; Xu et al., 2012; Yuana et al., 2019). From stabilisation point of view, mainly two approaches are used:

1 anti-windup compensator, i.e., two step approach- at first, the nominal controller is designed and in the next step, the effect of saturation is considered (Negi et al., 2012; Pal and Negi, 2012)

2 direct controller design in which the constraint in input is considered initially (Flores et al., 2009, 2012; Sbarbaro et al., 2009).

In these methods, mostly saturation is represented by a convex hull or sector bounded approach. A lot of research is found in literature giving methods to mitigate the effect of saturation in process control systems (Pal and Negi, 2012).

Virtually, the time delay is a ubiquitous phenomenon in almost all real-time systems in industries such as information and technology, nuclear reactor, ship stabilisation, aircraft stabilisation, hydraulic processes, chemical systems and temperature processes, etc. (Ma et al., 2018; Song and Wang, 2013). The most common reasons for a time delay are communication between data processing in computer aided automated industries or limited speed of information processing/bandwidth, transmission delays, or synchronisation delays in a system (Gyurkovics et al., 2017; Han et al., 2016; Jiang and Yang, 2016; Kandanvli and Kar, 2008; Lee and Park, 2015; Ma et al., 2018; Negi et al., 2012; Pal and Negi, 2018; Pal et al., 2019; Park et al., 2011; Qian et al., 2015; Ramakrishnan and Ray, 2012; Seuret and Gouaisbaut, 2012a; Stojanovic et al., 2018; Sun et al., 2018, 2019; Tadepalli et al., 2017; Xian and Cheng, 2015; Zhang et al., 2018; Zhao and Sun, 2016). The delay in the system may deteriorate the execution of the system. Therefore, a significant amount of effort has been done to stabilise both linear and nonlinear time delayed systems (Chen et al., 2015; Jiang and Yang, 2016; Ramakrishnan and Ray, 2012; Zhang et al., 2016; Zhao and Sun, 2016). To stabilise the delayed system, two approaches are used, i.e., delay-dependent and delay-independent. The delay-dependent approach utilises the information of delay and gives less conservative results for a limited range of delay as compared to the delay-independent method.

The delay in the control system plays a very important and significant role in stabilisation. Therefore, the well-known problem delay has been adequately referred to in Bensaker et al. (2020) and Gu (2000).

All systems are liable to uncertainties due to numerous factors like parameter variations, component aging, modelling errors. These have an adverse effect on system performance. The design of robust control is very effective for stabilising and improving the performance of time delayed systems. Now it has become an integral part of the control system and popular among researchers. Therefore, a robust controller for delayed systems is needed for technology advancement. Zhu and Wang (2018) introduced K-filters with time-varying low-gain in the design of output feedback control by integrating backstepping framework for a class of stochastic feedforward systems with unknown output function and unknown control coefficients. Again Zhu (2019) established the input-to-state practically exponential mean-square stability of the continuous-time stochastic nonlinear delay system with event triggered feedback control. Wang and Zhu (2020) calculated global stability by employing an adaptive control scheme, adding-a-power-integrator technique and novel state feedback control law to a stochastic nonlinear systems with time-varying delay, parametric uncertainties, stochastic inverse dynamics and unknown powers.

From reviewing the stabilisation of delayed systems, it has been noticed that more and more focus is given to obtain less conservative results. In this direction, mainly two-fold actions are appearing, i.e.:

a modification in Lyapunov Krasovskii functional (LKF)

b different inequalities such as free matrix-based integral inequality (Chen et al., 2016), Park's inequality (Fridman and Shaked, 2002), convex combination method (Park et al., 2011; Ramakrishnan and Ray, 2012; Tadepalli et al., 2017), Jensen inequality (Chen et al., 2016; Park et al., 2011; Seuret and Gouaisbaut, 2012b), Wirtinger inequality (Lee and Park, 2015; Seuret and Gouaisbaut, 2012a, 2012b; Seuret et al., 2015; Tadepalli and Kandanvli, 2017), for bounding the cross-product in solving the difference of LKF to obtain linear matrix inequality (LMI)-based stability conditions.

Jensen's inequality was introduced in Gu (2000) to bound quadratic integral terms of state. It was reconsidered in Shao (2008) to bound tightly quadratic integral terms of first order state derivative and obtained lesser conservative sequel. A new sequence of single summation inequalities was proposed by Chen et al. (2016) which introduces few free matrices including free-matrix-based, Jensen's and Wirtinger summation inequalities. Further, this concept has been extended to solve multiple-summation-inequality (Gyurkovics, 2017; Nam et al., 2015). Thus, in one framework almost all the summation inequalities are embraced. This technique can be widely applied to systems like sampled data and networked control systems. Presently, the comparison between Jensen's inequality and Wirtinger inequality is increasing to reduce the conservatism in their findings (Kammler, 2007; Kandanvli and Kar, 2008; Seuret et al., 2015; Shen et al., 2019; Zhang et al., 2017). A new version of summation Wirtinger inequality is derived for the stability of the discrete delayed system in Seuret et al. (2015) by LMI optimising setup. In Zhang et al. (2018), an augmented LKF is considered in which all the information regarding delay like upper and lower bounds, derivative of upper bound, marginal and exact delayed states, current states are taken into account. The extended reciprocal convex and Wirtinger-based inequality are utilised to solve the delay-dependent discrete-time neural networks (Meng et al., 2010; Stojanovic et al., 2018; Sun et al., 2018). Further, different approaches are developing along with Wirtinger's inequality for obtaining less conservative results (Lee and Park, 2015; Seuret and Gouaisbaut, 2012a, 2012b; Xian and Cheng, 2015). Anti-windup compensator is designed for a delta operated system making use of small gain theorem, input output approach, Wirtinger inequality and actuator saturation to obtain stability of the system by Rachid et al. (2019). Solgi et al. (2020) proposed a non-monotonic LKF to deal with stability of linear discrete time delay system. Bouazizi (2021) designed a linear parameter varying observer combined with LKF and LMI dialation techniques with Wirtinger inequality.

The environmental noise and gradual changing parameters (Kwon et al., 2011), make it difficult to get an accurate mathematical model. Therefore uncertainties are unavoidable and they will affect the system performance and stability. To mitigate their effect on the system, the various robust controller has been designed and studied (Bejaoui et al., 2019; Bensaker et al., 2020; Flores et al., 2009; Kwon et al., 2011; Lee and Park, 2015; Li et al., 2019; Liu, 2019; Ma and Chen, 2015; Ma et al., 2018; Pal et al., 2017; Ramakrishnan and Ray, 2012; Zhao et al., 2019). The problem can be analysed either in the time domain using Lyapunov stability theory or the frequency domain using the decomposition of matrices. The problem of robust stability in delayed systems has been discussed by many researchers; see Kandanvli and Kar (2008), Kwon et al. (2011), Lee and Park (2015), Ma et al. (2018), Ma and Chen (2015), Ramakrishnan and Ray (2012), Sun et al. (2018), Tadepalli et al. (2017), Xu et al. (2012), and some of the work also included with actuator saturation.

Dong et al. (2019) analysed robust stability and synthesised H_∞ controller for state feedback uncertain switched systems with nonlinear disturbances. Haghighi and Tavassoli (2020) discussed network control systems with random induced delays in forward as well as feedback channels and modelled as Markov chains. Robust control for output feedback has been developed. Viegas et al. (2020) discussed the design and optimisation of a distributed controller for discrete-time systems. Dong and He (2019) designed the robust H_∞ controller with time delays and uncertainties for conic-type discrete-time nonlinear systems. Lefebvre et al. (2020) investigated optimal robust problems subjected to dynamical uncertainty using polynomial approximation theory. Venkatesh et al. (2021) designed a controller for stabilisation state-delayed systems using reciprocal convex and Wirtinger's inequality. Wu et al. (2020) discussed robust stability for uncertain systems using delay partitioning, reciprocal convex and quadruple-integral terms. Chang et al. (2021) employed a delay partitioning approach to analyse fractional disturbances employing H_∞ application. Sun et al. (2021) considered Takagi-Sugeno fuzzy systems comprising of disturbance, uncertainty and faults. They realised a robust reliable H infinity control to attain stability of the system. Pratap and Sharma (2021) designed a robust controller for linearised twin rotor control system (TRCS) with parametric uncertainty using quantitative feedback theory (QFT) to mitigate the effect of various other nonlinearities. Ding et al. (2021) discussed intermittent estimator-based mixed passive and H_∞ control for high speed train with multiple noise using semi-Markov switching mode to accelerate convergence time. Ding and Zhu (2021a) addressed mean square exponential stability of T-S fuzzy flexible spacecraft with input saturation, random occurrence parameter uncertainties and stochastic faults using fuzzy intermittent control for switching system stability and again in 2021 also designed an observer for extended dissipative anti-disturbance control delayed switched singular semi-Markovian jump systems with multiple disturbance.

Seuret et al. (2015) have developed Wirtinger inequality for stabilisation of discrete time-varying delayed systems. It is less conservative than Jensen's inequality. They have combined it with reciprocal convex to obtain less conservative and reduced ciphering inequalities than the regular LKF approach with Jensen inequality. But they have not included any nonlinearity and also considered the autonomous system. Pal et al. (2017) have designed a H_∞-based output feedback controller with saturation and disturbance but they have not taken into account uncertainty and Wirtinger inequality. Pal and Negi (2018) have solved the problem using triple LKF but not used Wirtinger inequality. Thus, the region of stabilisation is less. Pal et al. (2019) have employed Wirtinger inequality but have not considered any nonlinearity. There already exist results that employed Wirtinger-based inequality and the convex hull embedding saturation in Chen et al. (2019) but without external disturbance, instead used polytopic approach. Moreover, delay bound is lesser than the proposed work.

In this paper a novel H_∞ controller has been established using state feedback controller, $u(\vartheta) = Kx(\vartheta)$, convex hull, Wirtinger inequality and reciprocal convexity to stabilise a discrete time-varying delay system including external disturbance as well as uncertainty subjected to actuator saturation. It is followed by realising the attraction domain of the plant. The benefactions of this manuscript are summed up as follows:

1 The discrete time varying delayed systems are considered with parametric norm bounded uncertainty, bounded disturbance and saturation nonlinearity to calculate asymptotic stability of the system.

2 This work is an extension of the results of Seuret et al. (2015) and Pal et al. (2019), of a discretised delayed system with uncertainties, extraneous interference and saturation.

3 To upgrade the robustness of the system a novel H_∞-based state feedback controller $u(\vartheta) = Kx(\vartheta)$ is derived by using an augmented LKF, Wirtinger inequality and reciprocal convex inequality besides convex hull method. To the finest of our knowingness, no work has been done using a combination of these techniques together with the aforesaid nonlinearities thus deriving the improved and new less conservative stability conditions.

4 The superiority of this work lies in the fact that the closed loop system with H_∞ controller results in higher delay bound, the disturbance attenuation level has reduced to a large extent, LMI-based stability conditions are proposed which are less conservative than the existing ones. The state control trajectories and control effort plots show faster stabilisation of the system. Comparative study in Table 1 proves that this work is better than the existing ones.

5 The difference between this paper and other existing papers are that various authors have considered nonlinearities like parametric uncertainties, disturbance, actuator saturation with different techniques such as Wirtinger inequality, triple LKF, polytopic approach, delay partitioning in 1D and 2D. But none has considered the combination of all these nonlinearities with Wirtinger inequality. This novel work, for the first time has taken into account all these nonlinearities with Wirtinger inequality.

6 To maximise the attraction domain, an optimisation procedure is proposed.

7 Numerical examples prove the effectiveness of the derived conditions and the criteria used. Three different types of numerical examples are solved. First has a simple plant, second is an unstable system and third is a practical missile control one. The state control trajectories and control effort plots show faster stabilisation of the system. Comparative study in Table 1 proves that this work is better than the existing ones.

The rest of the paper is structured as follows. The system considered is specified in Section 2. In Section 3, the extraction of delay-dependent asymptotically stable terms has been done utilising the LMI approach. Numerical instances are illustrated in Section 4 to prove the potency and superiority of the obtained results. Conclusions are given in Section 5.

Notations: $\Re^{q\times n}$ is the set of $q \times n$ real matrices, $\Re^q$ denotes set of $q \times 1$ real matrices, $P > 0$ (≥ 0) denotes that P is real symmetric and positive definite (positive semidefinite) matrix, 0 is a null matrix or null vector, I is an identity matrix with appropriate dimension, $\lambda_{\max}(\bar{A})$ denotes maximum eigenvalue of any given matrix $\bar{A}$, symbol '*' represents symmetric terms in a symmetric matrix, $co\{.\}$ denotes convex hull, $\|.\|$

represents the norm of a vector, $He(A) = A + A^T$, $diag(Y, Z)$ symbolises block diagonal matrix $\begin{bmatrix} Y & 0 \\ 0 & Z \end{bmatrix}$, $\|q\|_2 = \sqrt{\sum_{i=0}^{\infty} \|q(i)\|^2}$, is l_2 norm of signal $q(i) \in l_2\{(0, \infty)\}$, if $\|q\|_2 < \infty$.

2 System description

Consider a discrete-time system consisting of a time-varying delay, actuator saturation, disturbance and uncertainty:

$$x(\vartheta+1) = (A_p + \Delta A_p)x(\vartheta) + (A_{dp} + \Delta A_{dp})x(\vartheta - g(\vartheta)) + B_w w(\vartheta) + B_p sat(u(\vartheta)) \quad (1a)$$

$$y(\vartheta) = C_p x(\vartheta) \quad (1b)$$

$$z(\vartheta) = C_z x(\vartheta) + D_z w(\vartheta) \quad (1c)$$

$$\boldsymbol{x}(\vartheta) = \Upsilon(\vartheta),\ \vartheta = -g_h, -g_h + 1, \ldots, 0 \quad (1d)$$

The $x(\vartheta) \in \Re^t$, $u(\vartheta) \in \Re^q$, $y(\vartheta) \in \Re^p$, and $z(\vartheta) \in \Re^m$, are the state, the input, measured output and controlled output vectors, respectively. The external interference is represented by $w(\vartheta) \in \Re^n$. $A_p \in \Re^{t\times t}$, $A_{dp} \in \Re^{t\times t}$, $B_w \in \Re^{t\times n}$, $B_p \in \Re^{t\times q}$, $C_p \in \Re^{p\times t}$, $D_z \in \Re^{m\times t}$, are the known constant matrices of proper dimensions. The parametric uncertainties in the plant are denoted by $\Delta A_p \in \Re^{t\times t}$ and $\Delta A_{dp} \in \Re^{t\times t}$, initial condition and time-varying delay are $\Upsilon(\vartheta)$ and $g(\vartheta)$, respectively.

The time-varying delay $g(\vartheta)$ satisfies the following relation:

$$g_l \le g(\vartheta) \le g_h, \quad (1e)$$

where g_l is lower and g_h is upper delay bound.

The saturation function for $a = 1, 2, \ldots, 2^q$ is defined as follows:

$$sat(u_{(a)}) = \begin{cases} 1 & if \quad u_{(a)} > 1 \\ u_{0(a)} & if \quad -1 \le u_{(a)} \le 1 \\ -1 & if \quad u_{(a)} < -1 \end{cases} \quad (1f)$$

The state feedback controller is designed as below:

$$u(\vartheta) = Kx(\vartheta) \quad (1g)$$

where K is the $q \times t$ controller gain.

The uncertainties in the system can be expressed as

$$\Delta A_p = \Gamma_0 F_0 L_0, \quad (2a)$$

$$\Delta A_{dp} = \Gamma_1 F_1 L_1, \quad (2b)$$

where $\Gamma_\vartheta \in \Re^{t\times b_\vartheta}$ and $L_\vartheta \in \Re^{e_\vartheta \times t}$, $\vartheta = 0, 1$ are constant known matrices. The unknown matrix $F_\vartheta = \Re^{b_\vartheta \times e_\vartheta}$ $(\vartheta = 0,1)$ denotes parametric uncertainty which satisfies

$$F_\vartheta^T F_\vartheta \le I,\ \ \vartheta = 0,1. \quad (3)$$

A feedback controller is given by $u(\vartheta) = Kx(\vartheta)$. The stabilised system (1) occurs as follows:

$$x(\vartheta+1) = \left(A_p + \Delta A_p\right)x(\vartheta) + \left(A_{dp} + \Delta A_{dp}\right)x\left(\vartheta - g(\vartheta)\right) + B_w w(\vartheta) + B_p sat\left(Kx(\vartheta)\right) \tag{4a}$$

$$y(\vartheta) = C_p x(\vartheta) \tag{4b}$$

$$z(\vartheta) = C_z x(\vartheta) + D_z w(\vartheta). \tag{4c}$$

Consider K and $H \in \Re^{m\times n}$ are the gain matrices. Suppose a set of diagonal matrices D of the order of $q\times q$. The number of elements in each matrix is 2^q. Its diagonal elements are 0 or 1. All elements of D are written being D_n, $n = 1, 2, \ldots 2^q$ and if $D_n \in D$, then $D_n^- \in D$.

$\Phi(P, 1)$, an ellipsoid, is represented as follows for:

$$0 < P < \Re^{t\times t} \tag{5}$$

$\rho(H)$, a polyhedral set, is defined as follows:

$$\rho(H) = \left\{x(\vartheta) \in \Re^t : \left|H_b x(\vartheta)\right| \le \tau, b = 1, 2 \ldots q\right\} \tag{6}$$

H_b is the b^{th} row of the matrix H and τ denotes the saturation level.

When $x(\vartheta) \in \rho(H)$, using well known Lemma 6 from Pal and Negi (2018) and Hu et al. (2002)

$$sat\left(Kx(\vartheta)\right) = \sum_{n=1}^{\hat{N}} \Theta_n \left(\left(D_n K + D_n^- H\right)x(\vartheta)\right). \tag{7}$$

$\sum_{n=1}^{\hat{N}} \Theta_n = 1$ where $\hat{N} = 2^q$ with $\Theta_1 \ge 0, \ldots, \Theta_n \ge 0$.

Lemma 1 (Seuret et al., 2015): In a given symmetric positive definite matrix $U = \Re^{n\times n}$, the sequence of a discrete-time variable $x(\vartheta)$ in $[-g, 0] \cap \mathbb{Z} \to \Re^n$, where $g \ge 1$, the inequality is as follows:

$$\sum_{p=-g+1}^{0} \Omega^T(p) U \Omega(p) \ge \frac{1}{g}\begin{bmatrix}\Xi_0 \\ \Xi_1\end{bmatrix}^T \begin{bmatrix} U & 0 \\ 0 & 3\left(\dfrac{g+1}{g-1}\right)U\end{bmatrix}\begin{bmatrix}\Xi_0 \\ \Xi_1\end{bmatrix}, \tag{8}$$

where

$$\Omega(p) = x(p) - x(p-1),$$
$$\Xi_0 = x(0) - x(-g),$$
$$\Xi_1 = x(0) + x(-g) - \frac{2}{g+1}\sum_{p=-g}^{0} x(p).$$

In some practical systems having time-varying delay, the factor $\left(\frac{g+1}{g-1}\right)$ is difficult to tackle. Hence, it is removed using Lemma 2.

Lemma 2 (Seuret et al., 2015): In a given symmetric positive definite matrix $U = \Re^{n\times n}$, the sequence of a discrete-time variable $x(k)$ in $[-g, 0]\cap\mathbb{Z}\rightarrow\Re^n$, where $g \geq 1$, the following inequality holds:

$$\sum_{p=-g+1}^{0} \Omega^T(p)U\Omega(p) \geq \frac{1}{g}\begin{bmatrix}\Xi_0\\ \Xi_1\end{bmatrix}^T \begin{bmatrix}U & 0\\ 0 & 3U\end{bmatrix}\begin{bmatrix}\Xi_0\\ \Xi_1\end{bmatrix} \tag{9}$$

where

$$\Omega(p) = x(p) - x(p-1),$$

$$\Xi_0 = \boldsymbol{x}(0) - \boldsymbol{x}(-g), \qquad \text{''}$$

$$\Xi_1 = \boldsymbol{x}(0) + \boldsymbol{x}(-g) - \frac{2}{g+1}\sum_{p=-g}^{0} \boldsymbol{x}(p).$$

The initial condition for equation (4) is given below:

$$\boldsymbol{x}(\vartheta) = \Upsilon(\vartheta),\ \vartheta = -g_h, -g_h+1, \ldots, 0. \tag{10a}$$

The attraction domain of origin of equation (4) is as follows:

$$\hat{\boldsymbol{\eta}} \triangleq \left\{\Upsilon(\vartheta), \quad \vartheta = -g_h, -g_h+1, \ldots, 0 : \lim_{k\rightarrow\infty} \Upsilon\left(\vartheta, x_0\right) = 0\right\} \tag{10b}$$

Succeeding the nudge of Negi et al. (2012) and Pal and Negi (2018), an estimated domain of attraction is given via

$$E_\Theta \subset \hat{\eta},$$

where

$$E_\Theta \triangleq \left\{\Upsilon_x(\vartheta), -g_h \leq \vartheta \leq 0 : \max\left\|\Upsilon_x(\vartheta)\right\| \leq \Theta\right\} \tag{11}$$

Employing well known Lemma 6 from Pal and Negi (2018) and Hu et al. (2002), and substituting the value of equation (7) in equation (4), it can be written as follows:

$$\begin{aligned} x(\vartheta+1) &= \sum_{n=1}^{\hat{N}} \Theta_n \tilde{A}_n x(\vartheta) + \Delta A_p x(\vartheta) + \left(A_{dp} + \Delta A_{dp}\right) x\left(\vartheta - g(\vartheta)\right) + B_w w(\vartheta) \\ &= \left(\tilde{A} + \Delta A_p\right) x(\vartheta) + \left(A_{dp} + \Delta A_{dp}\right) x\left(\vartheta - g(\vartheta)\right) + B_w w(\vartheta) \end{aligned} \tag{12a}$$

$$y(\vartheta) = C_p x(\vartheta) \tag{12b}$$

$$z(\vartheta) = C_z x(\vartheta) + D_z w(\vartheta), \tag{12c}$$

where

$$\tilde{A} = \sum_{n=1}^{\hat{N}} \Theta_n \tilde{A}_n ; \tilde{A}_n = \begin{bmatrix} \tilde{A}_{11}^n & \tilde{A}_{12}^n \\ \tilde{A}_{21}^n & \tilde{A}_{22}^n \end{bmatrix};$$

$$\tilde{A}_{ij}^n = A_{ij} + B_i D_n K_j + B_i D_n^- H_j .$$

D and H are given from Lemma 6 Pal and Negi (2018) and Hu et al. (2002).

Assume that the disturbance satisfies $\|w(\vartheta)\|_2 \le \Lambda^2$ where $\Lambda > 0$ is a constant.

3 Main results

The findings of the manuscript are expressed successively.

3.1 Discrete-time systems in the ubeity of parametric uncertainty

Theorem 1: Consider system (12) devoiding external disturbance (i.e., $w(\vartheta) = 0$), for given integers g_l, g_h, j_0 and j_1 satisfying $0 < g_l < g_h, j_0 > 0, j_1 > 0$, if there exists symmetric matrices $0 < G = \text{diag}(G_1, G_2, G_3) \in \Re^{3t\times 3t}$, $0 < E_\vartheta(\vartheta = 1,2) \in \Re^{t\times t}$, $0 < T_\vartheta(\vartheta = 1,2) \in \Re^{t\times t}$, $0 < R_\vartheta$ $(\vartheta = 1, \ldots, 3) \in \Re^{t\times t}$, controller gain matrix $K = \Re^{q\times t}$, matrix $H = \Re^{q\times t}$, matrices Y_ϑ $(\vartheta = 1, \ldots, 4)$ with suitable dimensions fulfilling inequalities (13)–(15)

$$\varphi = \begin{bmatrix} \tilde{T}_1 & 0 & 0 \\ * & \tilde{T}_2 & Y \\ * & * & \tilde{T}_2 \end{bmatrix} > 0 \tag{13}$$

$$\begin{bmatrix} G_1 & H_b^T \\ * & \tau^2 \end{bmatrix} > 0, b = 1, 2, \ldots q \tag{14}$$

with

$$\begin{bmatrix}
\chi_{11} + j_0 L_0^T L_0 & -2T_1 & 0 & 0 & 6T_1 & 0 & 0 & \tilde{A}^T & g_l(\tilde{A}-I)^T & g_{lh}(\tilde{A}-I)^T & 0 & 0 \\
* & \chi_{22} & \chi_{23} & \chi_{24} & 6T_1 & 6T_2 & 2Y_2 + 2Y_4 & 0 & 0 & 0 & 0 & 0 \\
* & * & \chi_{33} + j_1 L_1^T L_1^T & \chi_{34} & 0 & \chi_{36} & \chi_{37} & A_{dp}{}^T & g_l A_{dp}{}^T & g_{lh} A_{dp}{}^T & 0 & 0 \\
* & * & * & \chi_{44} & 0 & -2Y_3^T + 2Y_4^T & 6T_2 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -12T_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -12T_2 & -4Y_4 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -12T_2 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -2R_1 + R_1 G_1 R_1 & 0 & 0 & \Gamma_0^T & \Gamma_1^T \\
* & * & * & * & * & * & * & * & -2R_2 + R_2 T_1 R_2 & 0 & g_l \Gamma_0^T & g_l \Gamma_1^T \\
* & * & * & * & * & * & * & * & * & -2R_3 + R_3 T_2 R_3 & g_{lh} \Gamma_0^T & g_{lh} \Gamma_1^T \\
* & * & * & * & * & * & * & * & * & * & -j_0 I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -j_1 I
\end{bmatrix} < 0, \tag{15}$$

where

$$\chi_{11} = -G_1 + G_2 + E_1 - 4T_1 \tag{16}$$

$$\chi_{22} = -G_2 + G_3 - E_1 + E_2 - 4T_1 - 4T_2 \tag{17a}$$

$$\chi_{23} = -2T_2 - Y_1 - Y_2 - Y_3 - Y_4 \tag{17b}$$

$$\chi_{24} = Y_1 - Y_2 + Y_3 - Y_4 \tag{17c}$$

$$\chi_{33} = -8T_2 + Y_1^T + Y_1 + Y_2^T + Y_2 - Y_3^T - Y_3 - Y_4^T - Y_4 \tag{18a}$$

$$\chi_{34} = -2T_2 - Y_1 + Y_2 + Y_3 - Y_4 \tag{18b}$$

$$\chi_{36} = 6T_2 + 2Y_3^T + 2Y_4^T \tag{18c}$$

$$\chi_{37} = 6T_2 - 2Y_2 + 2Y_4 \tag{18d}$$

$$\chi_{44} = -4T_2 - G_3 - E_2 \tag{19}$$

$$\tilde{T}_1 = diag(T_1, 3T_1), \tilde{T}_2 = diag(T_2, 3T_2), g_{lh} = g_h - g_l, Y = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix} \in \Re^{2t \times 2t},$$

then, equation (12) is asymptotically stable operating controller of gain K.

Proof: Define

$$\eta(p) = x(p) - x(p-1). \tag{20}$$

Following Lyapunov Krasvoskii functional has been considered for stabilisation

$$V(x(\vartheta)) = \sum_{n=1}^{3} V_n(x(\vartheta)), \tag{21}$$

where

$$V_1(x(\vartheta)) = \begin{bmatrix} x^T(\vartheta) & \sum_{p=\vartheta-g_l}^{\vartheta-1} x^T(p) & \sum_{p=\vartheta-g_h}^{\vartheta-g_l-1} x^T(p) \end{bmatrix} G \begin{bmatrix} x(\vartheta) \\ \sum_{p=\vartheta-g_l}^{\vartheta-1} x(p) \\ \sum_{p=\vartheta-g_h}^{\vartheta-g_l-1} x(p) \end{bmatrix}. \tag{22}$$

$$V_2(x(\vartheta)) = \sum_{p=\vartheta-g_l}^{\vartheta-1} x^T(p) E_1 x(p) + \sum_{p=\vartheta-g_h}^{\vartheta-g_l-1} x^T(p) E_2 x(p). \tag{23}$$

$$V_3(\boldsymbol{x}(\vartheta)) = g_l \sum_{p=-g_l+1}^{0} \sum_{j=\vartheta+i}^{\vartheta} \boldsymbol{\eta}^T(j) \boldsymbol{T}_1 \boldsymbol{\eta}(j) + g_{lh} \sum_{p=-g_h+1}^{-g_l} \sum_{j=\vartheta+i}^{\vartheta} \boldsymbol{\eta}^T(j) \boldsymbol{T}_2 \boldsymbol{\eta}(j). \tag{24}$$

$\xi(\vartheta)$, an augmented vector, is delineated as

$$\xi(\vartheta) = \begin{bmatrix} x^T(\vartheta) & x^T(\vartheta - g_l) & x^T(\vartheta - g(\vartheta)) & x^T(\vartheta - g_h) & h_1^T(\vartheta) & h_2^T(\vartheta) & h_3^T(\vartheta) \end{bmatrix}^T, \tag{25}$$

where

$$h_1(\vartheta) = \frac{1}{g_l + 1} \sum_{p=\vartheta-g_l}^{\vartheta} x(p). \tag{26}$$

$$h_2(\vartheta) = \frac{1}{g(\vartheta) - g_l + 1} \sum_{p=\vartheta-g(\vartheta)}^{\vartheta-g_l} x(p). \tag{27}$$

$$h_3(\vartheta) = \frac{1}{g_h - g(\vartheta) + 1} \sum_{p=\vartheta-g_h}^{\vartheta-g(\vartheta)} x(p). \tag{28}$$

For complete proof kindly refer Appendix A.

Remark 1: In equation (15) of Theorem 1, the values of R_ϑ ($\vartheta = 1, \ldots, 3$) are found using, the method adopted in Chen and Fong (2010). The values can be computed iteratively.

Remark 2: Choice of appropriate LKF is the main difficulty for achieving stability of delayed system. Tremendous effort has been put by the researchers to produce less conservative results in terms of delay. Moreover, another complexity can be found in the process of finding the feasibility in the stabilisation problems analysed by the LKF approach. It can be observed that the complication arises in two aspects:

1 model transformation

2 bound of some cross-terms to calculate the difference of function above.

Moreover, reciprocal convexity helps to make calculations simpler though elaborated. Hence, it is a trade-off between big size LMI and simplicity.

The approximate domain of attraction for equation (12) is given by $\Psi_\Theta \le 1$

$$\Psi_\Theta = \Theta^2 \begin{bmatrix} \lambda_{\max}(G_1) + g_l \lambda_{\max}(G_2) + (g_h - g_l)\lambda_{\max}(G_3) + g_l \lambda_{\max}(E_1) \\ +(g_h - g_l)\lambda_{\max}(E_2) + 2g_l(g_l+1)\lambda_{\max}(T_1) \\ +2(g_h - g_l)(g_h + g_l + 1)\lambda_{\max}(T_2) \end{bmatrix}. \tag{29}$$

Now, presenting a criterion that includes both an external disturbance and parametric uncertainty.

3.2 Discrete-time system including both external disturbance as well as uncertainties

Corollary 1 is given when an external disturbance is added with uncertainties in system (12).

Corollary 1: Given positive integers g_l, g_h, β, λ, j_0 and j_1 satisfying $0 < g_l < g_h$, $\beta > 0$, $\lambda > 0$, $j_0 > 0$, $j_1 > 0$, if there exists symmetric matrices $0 < G = \text{diag}(G_1, G_2, G_3) \in \Re^{3t\times 3t}$, $0 < E_\vartheta$ ($\vartheta = 1, 2$) $\in \Re^{t\times t}$, $0 < T_\vartheta$ ($\vartheta = 1,2$) $\in \Re^{t\times t}$, $\mathbf{0} < \boldsymbol{R}_\vartheta$ ($\vartheta = 1, \ldots, 3$) $\in \Re^{t\times t}$, controller gain matrix $K = \Re^{q\times t}$, matrix $H = \Re^{q\times t}$, matrices Y_ϑ($\vartheta = 1, \ldots, 4$) with suitable dimensions complying with equations (30)–(32).

$$\varphi = \begin{bmatrix} \tilde{T}_1 & 0 & 0 \\ * & \tilde{T}_2 & Y \\ * & * & \tilde{T}_2 \end{bmatrix} > 0 \tag{30}$$

$$\begin{bmatrix} G_1 & H_b^T \\ * & \tau^2 \end{bmatrix} > 0, b = 1, 2, \dots.q \tag{31}$$

with

$$\overline{\chi_m} = \begin{bmatrix}
\chi_{11} + j_0 L_0^T L_0 & -2T_1 & 0 & 0 & 6T_1 & 0 & 0 & 0 & \tilde{A}^T & g_l\left(\tilde{A} - I\right)^T & g_{lh}\left(\tilde{A} - I\right)^T & C_z^T & 0 & 0 \\
* & \chi_{22} & \chi_{23} & \chi_{24} & 6T_1 & 6T_2 & 2Y_2 + 2Y_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \chi_{33} + j_1 L_0^T L_0 & \chi_{34} & 0 & \chi_{36} & \chi_{37} & 0 & A_{dp}^T & g_l A_{dp}^T & g_{lh} A_{dp}^T & 0 & 0 & 0 \\
* & * & * & \chi_{44} & 0 & -2Y_3^T + 2Y_4^T & 6T_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -12T_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -12T_2 & -4Y_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -12T_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\lambda^2 I & B_w^T & g_l B_w^T & g_{lh} B_w^T & D_z^T & 0 & 0 \\
* & * & * & * & * & * & * & * & -G_1^{-1} & 0 & 0 & 0 & \Gamma_0^T & \Gamma_1^T \\
* & * & * & * & * & * & * & * & * & -T_1^{-1} & 0 & 0 & g_l\Gamma_0^T & g_l\Gamma_1^T \\
* & * & * & * & * & * & * & * & * & * & -T_2^{-1} & 0 & g_{lh}\Gamma_0^T & g_{lh}\Gamma_1^T \\
* & * & * & * & * & * & * & * & * & * & * & -I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & -j_0 I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & 0 & -j_1 I
\end{bmatrix} \tag{32}$$

then via controller, $u(\vartheta) = Kx(\vartheta)$, system (12) possess a H_∞ disturbance attenuation level λ for entire initial conditions fulfilling $\Psi_\Theta \leq 1$. The asymptotic stability region is specified by an ellipsoid.

$$\Phi\left(P, 1 + \lambda^2\beta^2\right) = \left\{x \in \Re^t; x^T Px \leq 1 + \lambda^2\beta^2\right\}. \tag{33}$$

The attraction domain is estimated as

$$\Psi_\Theta = \Theta^2 \begin{bmatrix} \lambda_{\max}\left(G_1\right) + g_l\lambda_{\max}\left(G_2\right) + \left(g_h - g_l\right)\lambda_{\max}\left(G_3\right) + g_l\lambda_{\max}\left(E_1\right) \\ +\left(g_h - g_l\right)\lambda_{\max}\left(E_2\right) + 2g_l\left(g_l + 1\right)\lambda_{\max}\left(T_1\right) \\ +2\left(g_h - g_l\right)\left(g_h + g_l + 1\right)\lambda_{\max}\left(T_2\right) \end{bmatrix}, \tag{34}$$

where

$$\tau = \frac{1}{1 + \lambda^2\beta^2}.$$

$$\hat{\Gamma}_0^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_0^T & g_l\Gamma_0^T & g_{lh}\Gamma_0^T & 0 & 0 & 0 \end{bmatrix} \tag{35a}$$

$$\hat{\Gamma}_1^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_1^T & g_l\Gamma_1^T & g_{lh}\Gamma_1^T & 0 & 0 & 0 \end{bmatrix} \tag{35b}$$

$$\hat{L}_0 = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{36a}$$

$$\hat{L}_1 = \begin{bmatrix} 0 & 0 & L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{36b}$$

and

$$0 < G = diag\left(G_1, G_2, G_3\right) \in \Re^{3t \times 3t}.$$

For matrices $0 < R_\vartheta$ ($\vartheta = 1, \ldots, 3$), we have

$$-G_1^{-1} \leq -2R_1 + R_1 G_1 R_1, -T_1^{-1} \leq -2R_2 + R_2 T_1 R_2, -T_2^{-1} \leq -2R_3 + R_3 T_2 R_3. \tag{37}$$

Proof: When disturbance is present in a system (12), disturbance attenuation can be intended in sense of H_∞ by considering the following equation (38):

$$\alpha(\vartheta) = \Delta v\left(x(\vartheta)\right) + z^T(\vartheta) z(\vartheta) - \lambda^2 w^T(\vartheta) w(\vartheta). \tag{38}$$

For complete proof, please see Appendix B.

3.3 Maximisation of attraction basin

The optimisation approach to find the maximum value of the attraction basin is as follows.

Theorem 2: For the closed loop system (12) and initial condition (10), the capitalised attraction basin can be evaluated with subsequent convex optimisation problem.

Diminish r, with

$$\begin{aligned} r = {} & r_1 + g_l r_2 + \left(g_h - g_l\right) r_3 + g_l r_4 + \left(g_h - g_l\right) r_5 + 2g_l\left(g_l + 1\right) r_6 \\ & + 2\left(g_h - g_l\right)\left(g_h + g_l + 1\right) r_7 \end{aligned} \tag{39}$$

subjected to equations (13)–(15) and

$$\begin{aligned} & r_1 I - G_1 \geq 0, r_2 I - G_2 \geq 0, r_3 I - G_3 \geq 0, r_4 I - E_1 \geq 0, \\ & r_5 I - E_2 \geq 0, r_6 I - T_1 \geq 0, r_7 I - T_2 \geq 0 \end{aligned} \tag{40}$$

owns a feasible decipher for the weighting parameters $r_i > 0$, $p = 1,2,\ldots 7$, positive definite symmetric matrices $G_1 \in \Re^{t \times t}$, $G_2 \in \Re^{t \times t}$, $G_3 \in \Re^{t \times t}$, $E_1 \in \Re^{t \times t}$, $E_2 \in \Re^{t \times t}$, $T_1 \in \Re^{t \times t}$, $T_2 \in \Re^{t \times t}$, $\mathrm{K} = \Re^{q \times t}$, $\boldsymbol{R}_\vartheta$ ($\vartheta = 1, 2, 3$) $\in \Re^{t \times t}$, $\boldsymbol{H} \in \Re^{q \times t}$, $\boldsymbol{Y} \in \Re^{2t \times 2t}$.

In this condition, gain matrix K furnishes a capitalised evaluation of attraction basin specified by

$$\Theta_{\max} = \frac{1}{\sqrt{\varsigma}},$$

where

$$\begin{aligned} \varsigma = {} & \lambda_{\max}\left(G_1\right) + g_l \lambda_{\max}\left(G_2\right) + \left(g_h - g_l\right)\lambda_{\max}\left(G_3\right) + g_l \lambda_{\max}\left(E_1\right) \\ & + \left(g_h - g_l\right)\lambda_{\max}\left(E_2\right) + 2g_l\left(g_l + 1\right)\lambda_{\max}\left(T_1\right) \\ & + 2\left(g_h - g_l\right)\left(g_h + g_l + 1\right)\lambda_{\max}\left(T_2\right). \end{aligned} \tag{41}$$

Proof: If the constraints in equation (42) hold good, then

$$\begin{aligned} & r_1 I \geq \lambda_{\max}\left(G_1\right), r_2 I \geq \lambda_{\max}\left(G_2\right), r_3 I \geq \lambda_{\max}\left(G_3\right), r_4 I \geq \lambda_{\max}\left(E_1\right), \\ & r_5 I \geq \lambda_{\max}\left(E_2\right), r_6 I \geq \lambda_{\max}(T_1), r_7 I \geq \lambda_{\max}\left(T_2\right). \end{aligned}$$

From equation (32), we get $\Theta = \frac{\Psi_\Theta}{\sqrt{\varsigma}}$. The solution of equation (39) along with equations (13)–(15) proves a maximised domain of attraction given by Θ.

4 Examples

To demonstrate the main result three instances are given in this segment.

Example 1: Let us consider the systems denoted by equation (12).

The parameters are:

$$A_p = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.7 \end{bmatrix}, A_{dp} = \begin{bmatrix} -0.015 & -0.01 \\ 0 & -0.013 \end{bmatrix},$$

$$B_p = \begin{bmatrix} 0.65 \\ -0.4 \end{bmatrix}, C_p = \begin{bmatrix} 1 & 0 \end{bmatrix}, B_w = \begin{bmatrix} 0.008 & 0.005 \\ 0.008091 & 0.008 \end{bmatrix},$$

$$C_z = \begin{bmatrix} -0.01 & 0.3 \\ 0 & 0.01 \end{bmatrix}, D_z = \begin{bmatrix} 0.09 & 0 \\ 0 & 0.01 \end{bmatrix}, \Gamma_0 = \begin{bmatrix} -0.01 \\ 0.1 \end{bmatrix},$$

$$L_0 = \begin{bmatrix} -0.01 & -0.081 \end{bmatrix}, \Gamma_1 = \begin{bmatrix} 0.01 \\ 0.001 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -0.01 & 0.05 \end{bmatrix}, F_0 = F_1 = \sin(\vartheta), \alpha = 0.01, w(\vartheta) = 0.05e^{-0.05\vartheta}.$$

Application of LMI toolkit (Gahinet et al., 1995), the LMIs [equations (27)–(32)] referred to in Corollary 1 are discovered to be feasible in favour of delay span $1 \le g(\vartheta) \le 20$ and the capitalised attraction basin estimated is 0.1606.

The unknown parameters and gain K that stabilises the unstable system given above are obtained as follows:

$$\lambda = 0.7; 1 \le g(\vartheta) \le 20$$

$$K = \begin{bmatrix} 0.1030 & -0.0945 \end{bmatrix}, G_1 = \begin{bmatrix} 11.9307 & 2.3488 \\ 2.3488 & 19.1249 \end{bmatrix}, G_2 = \begin{bmatrix} 0.0595 & 0.0510 \\ 0.0510 & 0.1221 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} 0.0220 & 0.0254 \\ 0.0254 & 0.0627 \end{bmatrix}, E_1 = \begin{bmatrix} 0.0595 & 0.0510 \\ 0.0510 & 0.1221 \end{bmatrix}, E_2 = \begin{bmatrix} 0.0220 & 0.0254 \\ 0.0254 & 0.0627 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 1.3776 & 0.2585 \\ 0.2585 & 1.7889 \end{bmatrix}, T_2 = \begin{bmatrix} 0.0053 & 0.0029 \\ 0.0029 & 0.0091 \end{bmatrix}, Y_1 = \begin{bmatrix} -0.0416 & -0.0416 \\ -0.0434 & -0.1204 \end{bmatrix},$$

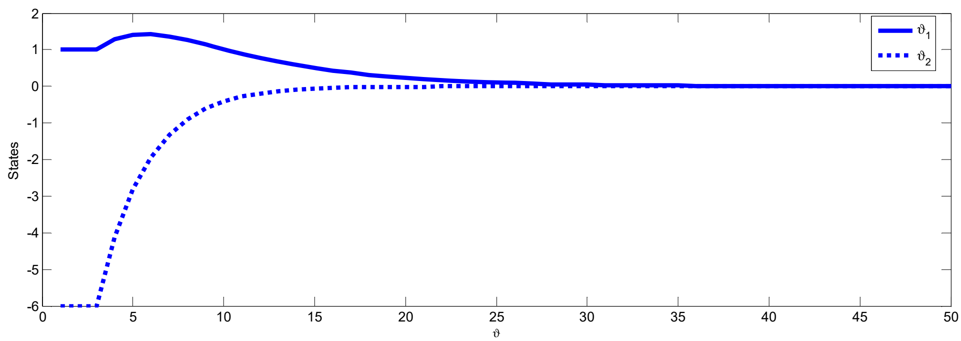
$$Y_2 = 10^{-6} \times \begin{bmatrix} 0.8081 & -0.4075 \\ -0.3212 & 0.1564 \end{bmatrix}, Y_3 = 10^{-7} \times \begin{bmatrix} 0.1714 & 0.5707 \\ 0.8639 & -0.0547 \end{bmatrix},$$

$$Y_4 = 10^{-8} \times \begin{bmatrix} -0.1594 & 0.1104 \\ -0.1721 & 0.1137 \end{bmatrix}, H = \begin{bmatrix} 0.1023 & -0.0966 \end{bmatrix}$$

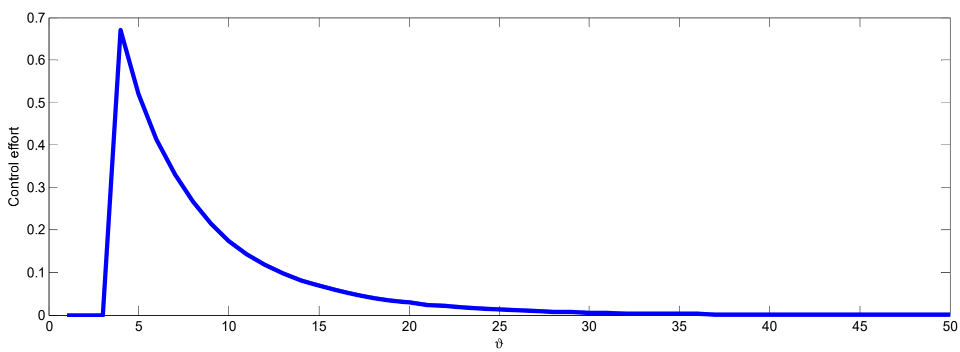
It is inferred that the delay limits in Negi et al. (2012), is $1 \le g(\vartheta) \le 5$ whereas the delay limits in the suggested techniques appear to be $1 \le g(\vartheta) \le 20$.

Figure 1(a) reflects the state trajectories of equation (12) described for initial state $x_0 = [1\ -0.6]^T$. The control effort is represented in Figure 1(b). From Figure 1, it can be inferred, the unstable system is asymptotically stable influenced by the controller $u(\vartheta) = Kx(\vartheta)$.

Figure 1 (a) State trajectories (b) Control effort (see online version for colours)



(a)



(b)

Notes: Through Table 1, it can be seen that the upper limit on the delay bound has risen relative to other earlier outcomes (Negi et al., 2012; Qian et al., 2015; Xu et al., 2012). In contrast to other earlier works, the disturbance has also attenuated further. By implementing the state feedback law $u(\vartheta) = Kx(\vartheta)$, the H_∞ performance enhancement is shown to minimise the amount of disturbance attenuation level λ.

The comparison of the outcomes is depicted in Table 1.

Remarkably, the range of delay has been increased by using the proposed approach in comparison with the previous results as well as the domain of attraction has also been calculated. Therefore, the region of initial condition for which states are starting from the initial condition and after time tending to infinity, the states are reaching the equilibrium states, i.e., zero. Exponentially decreasing term of disturbance has been considered in the example by $w(\vartheta)$ and considering ±10% uncertainty in the system; all the states converge to origin for the given initial conditions.

Although the states in the previous results are also reaching zero and the magnitude of control effort is in saturation limit, the main findings of this work can be seen in terms of increasing the delay range also and getting a less conservative result. Next, the domain of attraction has also been increased.

Table 1 Delay bounds

Method	*Delay range* ($g_l \le g(\vartheta) \le g_h$)	*Nonlinearities*
Theorem 2 (Negi et al., 2012)	$1 \le g(\vartheta) \le 5$	With saturation and time varying delay.
Theorem 1 (Xu et al., 2012)	$1.4 \le g(\vartheta) \le 3.8$	With saturation, time varying delay and interference
Theorem 1 (Qian et al., 2015)	$1 \le g(\vartheta) \le 4$	With saturation, time varying delay and interference with H_∞ level $\lambda = 1.3424$
Theorem 2 (Qian et al., 2015)	$1 \le g(\vartheta) \le 3$	With saturation, time varying delay and interference with H_∞ attenuation level $\lambda = 1.4683$
Corollary 2 (Pal et al., 2018)	$1 \le g(\vartheta) \le 9$	With saturation, time varying delay, external interference and uncertainties with H_∞ level $\lambda = 1$
Theorem 1 (Chen et al., 2018)	$1 \le g(\vartheta) \le 5$	With saturation, time varying delay and uncertainty
Theorem 1 (Chen et al., 2018)	$1 \le g(\vartheta) \le 8$	With saturation, time varying delay and uncertainty (special case)
Corollary 1 (De Souza et al., 2018)	$1 \le g(\vartheta) \le 11$	With saturation, time varying delay
Corollary 1 (proposed work)	$1 \le g(\vartheta) \le 20$	With saturation, time varying delay, external interference and uncertainties with H_∞ level $\lambda = 0.7$

Example 2: In view of unstable discrete delay systems with guidelines as follows:

$$A_p = \begin{bmatrix} -1.27 & 0 \\ 0 & -0.07 \end{bmatrix}, A_{dp} = \begin{bmatrix} -0.015 & -0.01 \\ 0 & -0.013 \end{bmatrix}, B_p = \begin{bmatrix} 0.65 \\ -0.4 \end{bmatrix}, C_p = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$B_w = \begin{bmatrix} 0.008 & 0.005 \\ 0.008091 & 0.008 \end{bmatrix}, C_z = \begin{bmatrix} -0.01 & 0.3 \\ 0 & 0.01 \end{bmatrix}, D_z = \begin{bmatrix} 0.09 & 0 \\ 0 & 0.01 \end{bmatrix},$$

$$\Gamma_0 = \begin{bmatrix} -0.01 \\ 0.1 \end{bmatrix}, L_0 = \begin{bmatrix} -0.01 & -0.081 \end{bmatrix}, \Gamma_1 = \begin{bmatrix} 0.01 \\ 0.001 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -0.01 & 0.05 \end{bmatrix}, F_0 = F_1 = \sin(\vartheta), \alpha = 0.01, w(\vartheta) = 0.05e^{-0.05\vartheta}.$$

These parameters can be used to find the system denoted by equation (12). It is noted system matrix A has eigenvalues [–1.27, –0.07]. The aforementioned unstable system is affected by time-varying delay, actuator saturation, uncertainties together with disturbance. Employing Corollary 1, the system becomes stable using the feedback controller referred to in equation (4) expressed by $u(\vartheta) = Kx(\vartheta)$.

The application of the workbox (Gahinet et al., 1995), to the LMIs [equations (27)–(32)] shown in Corollary 1 is considered feasible for the parameters described below:

$$K = [1.9499 \quad -0.0708],\ G_1 = \begin{bmatrix} 32.8760 & -1.7734 \\ -1.7734 & 7.6445 \end{bmatrix},\ G_2 = \begin{bmatrix} 0.1588 & 0.1183 \\ 0.1183 & 0.1305 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} 0.0822 & 0.0489 \\ 0.0489 & 0.0670 \end{bmatrix},\ E_1 = \begin{bmatrix} 0.1321 & 0.0969 \\ 0.0969 & 0.1091 \end{bmatrix},\ E_2 = \begin{bmatrix} 0.0660 & 0.0409 \\ 0.0409 & 0.0569 \end{bmatrix},$$

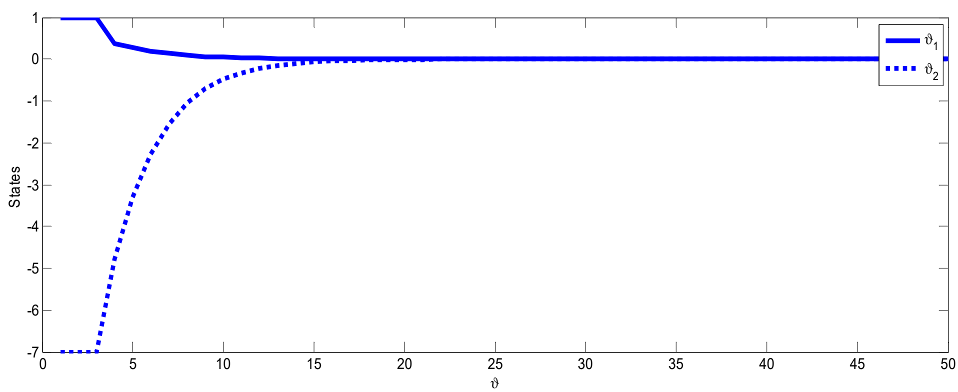
$$T_1 = \begin{bmatrix} 0.1170 & 0.0144 \\ 0.0144 & 0.1162 \end{bmatrix},\ T_2 = \begin{bmatrix} 0.0467 & 0.0164 \\ 0.0164 & 0.0484 \end{bmatrix},\ Y_1 = \begin{bmatrix} -0.0631 & -0.0261 \\ -0.0363 & -0.0560 \end{bmatrix},$$

$$Y_2 = 10^{-6} \times \begin{bmatrix} -0.5097 & -0.5219 \\ 0.3025 & 0.1169 \end{bmatrix},\ Y_3 = 10^{-5} \times \begin{bmatrix} 0.1107 & -0.0676 \\ 0.0830 & -0.0522 \end{bmatrix},$$

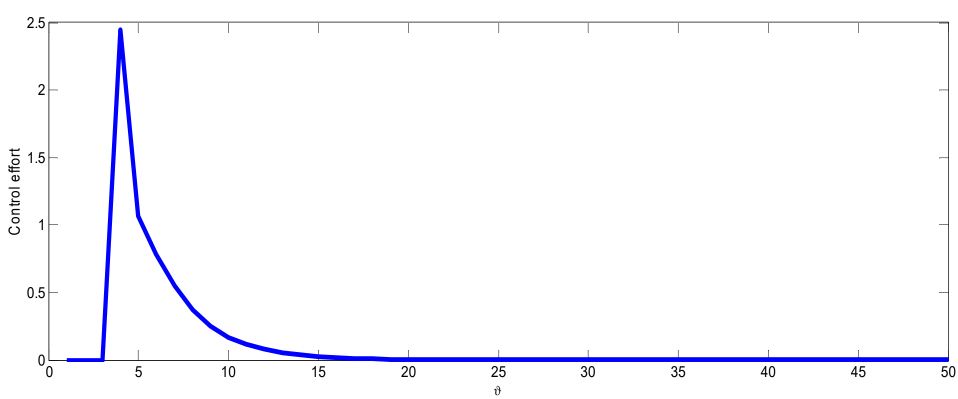
$$Y_4 = 10^{-8} \times \begin{bmatrix} -0.1664 & -0.0811 \\ -0.1402 & -0.0633 \end{bmatrix},\ H = [0.9809 \quad 0.1941],\ \lambda = 0.2, 1 \le g(\vartheta) \le 3.$$

As stated in Example 2, the system's state trajectories are revealed in Figure 2(a) for initial state $x_0 = [1 - 7]^T$. Figure 2(b) depicts the control effort. The controller gain K here is stabilising the unstable system and is depicted in Table 2 for various delay ranges.

Figure 2 (a) State trajectories (b) Control effort (see online version for colours)



(a)



(b)

Table 2 Computational result

Delay range $g_l \le g(\vartheta) \le g_h$	*Controller gain (K)*
$1 \le g(\vartheta) \le 2$	$K = [1.9628 \quad -0.0661]$
$1 \le g(\vartheta) \le 3$	$K = [1.9499 \quad -0.0708]$
$1 \le g(\vartheta) \le 4$	*Infeasible*

To stabilise the unstable system, Example 2 is capable of showing the fruitfulness of the suggested controller. The gauge attraction basin is 0.1796. The upper delay limit improvement, in the instance, reveals that in LKF, the introduction of Wirtinger inequality lowers the conservatism.

4.1 Implementation of missile control system

Example 3: (Pal and Negi, 2018; Nise, 2010). The missile control system can be marked by equation (15) with the guidelines as follows:

$$A_p = \begin{bmatrix} -0.2428 & -0.6238 \\ 1 & 0 \end{bmatrix}, A_{dp} = \begin{bmatrix} -0.016 & -0.012 \\ 0 & -0.013 \end{bmatrix}, B_p = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$C_p = [1.31 \quad 0.5566], B_w = \begin{bmatrix} 0.009 & 0.006 \\ 0.0079 & 0.008 \end{bmatrix}, C_z = \begin{bmatrix} -0.014 & 0.253 \\ 0 & 0.01 \end{bmatrix},$$

$$D_z = \begin{bmatrix} 0.109 & 0 \\ 0 & 0.012 \end{bmatrix}, \Gamma_0 = \begin{bmatrix} -0.02 \\ 0.99 \end{bmatrix}, L_0 = [-0.01 \quad -0.05], \Gamma_1 = \begin{bmatrix} 0.012 \\ 0.002 \end{bmatrix},$$

$$L_1 = [-0.03 \quad 0.06], \boldsymbol{K} = [0.6417 \quad 0.6586], \boldsymbol{F}_0 = \boldsymbol{F}_1 = \sin(\vartheta), \alpha = 0.02,$$

$$w(\vartheta) = 0.04e^{-0.03\vartheta}.$$

The control feedback $u(\vartheta) = Kx(\vartheta)$ is conceived to stabilise the aforementioned missile control scheme when using LMI Toolkit, see Gahinet et al. (1995). It is noted that the LMI constraints (33)–(35) in Corollary 1 are feasible for the parameters that are unknown as below:

$$G_3 = \begin{bmatrix} 0.0158 & -0.0003 \\ -0.0003 & 0.0360 \end{bmatrix}, E_1 = \begin{bmatrix} 0.0159 & -0.0085 \\ -0.0085 & 0.0644 \end{bmatrix}, E_2 = \begin{bmatrix} 0.0158 & -0.0003 \\ -0.0003 & 0.0360 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 0.1173 & 0.0001 \\ 0.0001 & 0.1278 \end{bmatrix}, T_2 = \begin{bmatrix} 0.0077 & 0.0001 \\ 0.0001 & 0.0086 \end{bmatrix}, Y_1 = \begin{bmatrix} -0.0200 & -0.0026 \\ 0.0025 & -0.0329 \end{bmatrix},$$

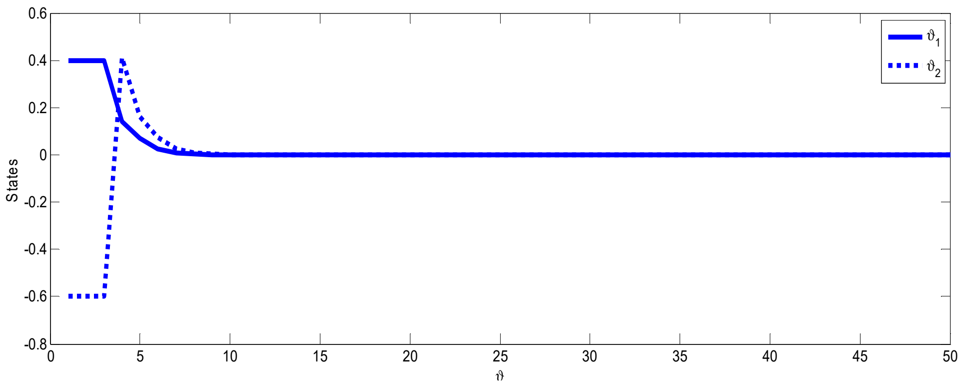
$$Y_2 = 10^{-7} \times \begin{bmatrix} -0.0644 & -0.1249 \\ -0.1482 & -0.2748 \end{bmatrix}, Y_3 = 10^{-5} \times \begin{bmatrix} 0.0274 & -0.2233 \\ 0.0018 & -0.0131 \end{bmatrix},$$

$$Y_4 = 10^{-9} \times \begin{bmatrix} 0.4468 & 0.8672 \\ 0.0272 & 0.0540 \end{bmatrix}, \boldsymbol{K} = [0.6417 \quad 0.6586], H = [0.6423 \quad 0.6582],$$

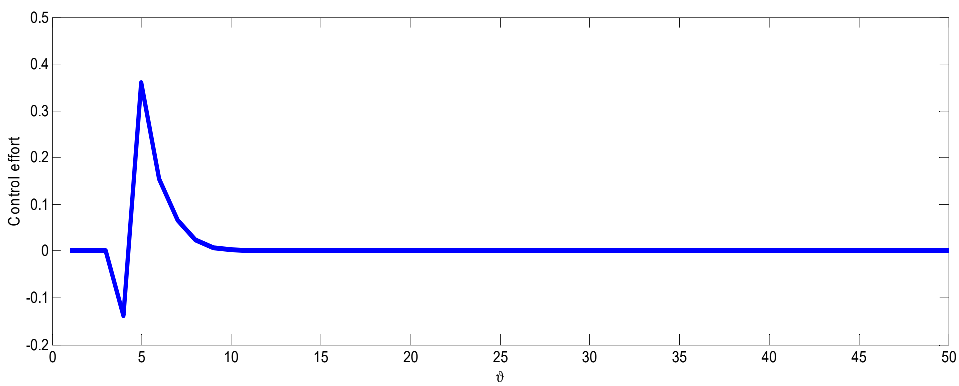
$$\lambda = 0.2, 1 \le g(\vartheta) \le 6.$$

Figure 3(a) shows state trajectories of the missile control system for the initial state $x_0 = [0.4 \quad -0.6]^T$ and Figure 3(b) shows the control effort. It can be inferred that in the presence of the aforesaid nonlinearities, all the states converge to origin from any arbitrary initial conditions and the conceived controller gain K makes the system stable. The upper limit of the delay range has increased to $1 \leq g(\vartheta) \leq 6$ in comparison to Pal and Negi (2018) $1 \leq g(\vartheta) \leq 5$.

Figure 3 (a) State trajectories (b) Control effort (see online version for colours)



(a)



(b)

The same problem can be targeted for two-dimensional (2D) discrete time delayed systems coupled with of nonlinearities like saturation, etc.

5 Conclusions

In this work, a H_∞ state feedback controller is designed to obtain a robust stability criterion that stabilises and optimises a discrete delayed system together with actuator saturation, uncertainty and external disturbance using discrete Wirtinger-based inequality. The enhanced delay span is the principal attainment of the proffered technique for the system considered with nonlinearities. The presented criterion helps to obtain a larger stability region than the previous criterion. Numerical instances reveal the skill of the suggested method. The computed results are less conservative as compared to previous methods and show an increase in the upper bound of the stable region. The disadvantage of the proposed method is that robust controllers are insensitive to minor system changes. Robustness is the sensitivity to the unexplained upshots in the analysis plus design. Model transformation and bound of some cross-terms for finding the difference poses some complications. It can be seen that by applying these methods, the complexity will increase in establishing the stability conditions based on the LMI technique. Several assumptions have been taken. Certain matrices, for example matrix G, is considered diagonal in Theorem 1. On removal of these assumptions, results will be better.

The same problem can be done using:

1 delay partitioning, both uniform and non-uniform (Feng et al., 2015) with simple and multiple time varying delays

2 finite word length (Singh et al., 2021),

3 polytopic approach (Chen et al., 2019)

4 control for switching systems, stochastic system and chaotic system

5 non-monotonic LKF can be used in place of conventional monotonic ones

6 2D systems.

References

Bejaoui, I., Saidi, I., Xibilia, M.G. and Soudani, D. (2019) 'Internal model control of discrete non-minimum phase over-actuated systems with multiple time delays and uncertain parameters', *Journal of Engineering Science and Technology Review*, Vol. 12, No. 2, pp.111–118.

Bensaker, N., Kherfane, H. and Bensaker, B. (2020) 'New robust stability criteria for uncertain neutral time-delay systems with discrete and distributed delays', *Iranian Journal of Electrical and Electronic Engineering (IJEEE)*, 24pp, Iran University of Science and Technology, ISSN: 1735-2827.

Bezzaoucha, S., Marx, B., Maquin, D. and Ragot, J. (2012) 'Linear feedback control input under actuator saturation: a Takagi-Sugeno approach', in *2nd International Conference on Systems and Control (ICSC'12)*, Marrakech, Morocco.

Bouazizi, M.H. (2021) 'An observer-based H_∞ linear parameter varying controller for time delayed linear parameter varying systems using dilated linear matrix inequalities and Wirtinger inequality', *Transactions of the Institute of Measurement and Control*, Vol. 43, No. 9, pp.1915–1923.

Boyd, S., EI-Ghaousi, L., Feron E. and Balakrishnan, V. (1994) *Linear Matrix Inequalities in System and Control Theory*, Society for Industrial and Applied Mathematics (SIAM), Philadelphia.

Chang, H.C., Lien, C.H. and Yu, K.W. (2021) 'H_∞ performance analysis and switching control design for uncertain discrete switched time-delay systems', *International Journal of Automation and Control*, Vol. 15, No. 2, p.170.

Chen, F., Hu, Y., Wang, W. and Lei, J. (2015) 'Research on delay-independent stability for discrete-time interval systems with time-varying delay', *Proceedings of 2015 IEEE International Conference on Mechatronics and Automation* (ICMA), Beijing, pp.145–150.

Chen, J., Xu, S., Jia, X. and Zhang, B. (2016) 'Novel summation inequalities and their applications to stability analysis for systems with time-varying delay', *IEEE Transactions on Automatic Control*, Vol. 62, No. 5, pp.2470–2475.

Chen, K.F. and Fong, I.K. (2010) 'Stability analysis and output-feedback stabilization of discrete time systems with an interval time-varying state delay', *IET Control Theory and Application*, Vol. 4, No. 4, pp.563–572.

Chen, Y., Wang, Z., Fei, S. and Han, Q.L. (2019) 'Regional stablization for discrete time-delay systems with actuator saturations via a delay-dependent polytopic approach', *IEEE Transactions on Automatic Control*, Vol. 64, No. 3, pp.1257–1264.

De Souza, C., Leite, V.J.S., Silva, L.F.P. and Castelan, E.B. (2019) 'ISS robust stabilization of state-delayed discrete-time systems with bounded delay variation and saturating actuators', *IEEE Transactions on Automatic Control*, Vol. 64, No. 9, pp.3913–3919.

Ding, K. and Zhu, Q. (2021a) 'Extended dissipative anti-disturbance control for delayed switched singular semi-Markovian jump systems with multi-disturbance via disturbance observer', *Automatica*, Vol. 128, pp.1–13, Article 109556 [online] doi.org/10.1016/j.automatica.2021. 109556.

Ding, K. and Zhu, Q. (2021b) 'Reliable intermittent extended dissipative control for uncertain fuzzy flexible spacecraft systems with Bernoulli stochastic distribution', *IET Control Theory and Applications*, Vol. 15, No. 5, pp.911–925.

Ding, K., Zhu, Q. and Yang, X. (2021) 'Intermittent estimator-based mixed passive and H∞ control for high-speed train with actuator stochastic fault', *IEEE Transactions on Cybernetics*, DOI: 10.1109/TCYB.2021.3079437.

Dong, X. and He, S. (2019) 'Controlling the discrete-time conic-type nonlinear system by a time-delayed robust H_∞ controller', *Optimal Control Applications and Methods*, Vol. 41, No. 2, pp.369–380.

Dong, Y., Liang, S. and Wang, H. (2019) 'Robust stability and H_∞ control for nonlinear discrete-time switched systems with interval time-varying delay', *Mathematical Methods in the Applied Sciences*, Vol. 22, No. 6, pp.1999–2015.

Feng, Z., Lam, J. and Yang, G-H. (2015) 'Optimal partitioning method for stability analysis of continuous/discrete delay systems', *International Journal of Robust and Nonlinear Control*, Vol. 25, No. 4, p.5592574.

Flores, J.V., Da Silva Jr., G.J.M. and Sartori, R. (2013) 'Tracking and rejection of periodic signals for discrete-time linear systems subject to control saturation', *IET Control Theory and Applications*, Vol. 7, No. 3, pp.363–371.

Flores, J.V., Gomes da Silva, J.M. and Sbarbaro, D. (2009) 'Robust periodic reference tracking for uncertain linear systems subject to control saturations', *Proceedings of the 48h IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference*, Shanghai, pp.7960–7965.

Flores, J.V., Gomes da Silva, J.M., Pereira, L.F.A. and Sbarbaro, D.G. (2012) 'Repetitive control design for MIMO systems with saturating actuators', *IEEE Transactions on Automatic Control*, Vol. 57, No. 1, pp.192–198.

Fridman, E. and Shaked, U. (2002) 'A descriptor system approach to H_∞ control of linear time-delay systems', *IEEE Transactions on Automatic Control*, Vol. 47, No. 2, pp.253–270.

Gahinet, P., Nemirovski, A., Laub, A.J. and Chilali, M. (1995) *LMI Control Toolbox: For Use with MATLAB*, Math Works Inc., 3 Apple Hill Drive Natick, MA.

Gu, K. (2000) 'An integral inequality in the stability problem of time-delay systems', *Proceedings of the 39th IEEE Conference on Decision and Control*, Sydney, NSW, Vol. 3, pp.2805–2810, Cat. No. 00CH37187.

Gyurkovics, E., Kiss, K., Nagy, I. and Takács, T. (2017) 'Multiple summation inequalities and their application to stability analysis of discrete-time delay systems', *Journal of the Franklin Institute*, Vol. 354, No. 1, pp.123–144.

Haghighi, P. and Tavassoli, B. (2020) 'Robust H_∞ output feedback design for networked control systems with partially known delay probabilities', *Optimal Control Applications and Methods*, Vol. 41, No. 4, pp.1052–1067.

Han, L., Dong, X., Li, Q. and Zhang, R. (2016) 'Formation tracking control for time-delayed multi-agent systems with second-order dynamics', *Chinese Journal of Aeronautics*, Vol. 30, No. 1, pp.348–357, 14pp.

Hu, T., Lin, Z. and Chen, B.M. (2002) 'Analysis and design for discrete-time linear systems subject to actuator saturation', *Systems & Control Letters*, Vol. 45, No. 2, pp.97–112.

Jiang, X. and Yang, S. (2016) 'A delay-dependent stability criterion for a class of systems with time-varying delay', *2016 Chinese Control and Decision (CCDC)*, Yinchuan, pp.2296–2299.

Jishi, Z., Yan, W., Yanwu, W. and Zhonghu, G. (2013) 'Stability analysis and controller design of positive linear systems subject to actuator saturation', *32nd Chinese Control Conference*, Xian China, pp.33–38.

Kammler, D.W. (2007) *A First Course in Fourier Analysis*, 2007 – Mathematics, 798pp, Cambridge University Press.

Kandanvli, V.K.R. and Kar, H. (2008) 'Robust stability of discrete-time state-delayed systems employing generalized overflow nonlinearities', *Nonlinear Analysis: Theory, Methods & Applications*, Vol. 69, No. 9, pp.2780–2787.

Kwon, O.M., Lee, S.M. and Park, J.H. (2011) 'On the reachable set bounding of uncertain dynamic systems with time-varying delays and disturbances', *Information Sciences*, Vol. 181, No. 17, pp.3735–3748.

Lee, T.H. and Park, J.H. (2015) 'Further results on robust stability of neutral time-delay systems using Wirtinger-based inequality', *27th Chinese Control and Decision Conference (2015CCDC)*, Qingdao, pp.6384–6388.

Lefebvre, T., Frederik, D.B. and Crevecoeur, G.(2020) 'A framework for robust quadratic optimal control with parametric dynamic model uncertainty using polynomial chaos', *Optimal Control Applications and Methods*, Vol. 41, No. 3, pp.1–16.

Lefebvre, T., Frederik, D.B. and Crevecoeur, G.(2020) 'A framework for robust quadratic optimal control with parametric dynamic model uncertainty using polynomial chaos', *Optimal Control Applications and Methods*, Vol. 41, No. 3, pp.1–16, DOI: 10.1002/oca.2575.

Li, H., Wang, X. and Zhang, X. (2019) 'Lyapunov matrices approach to robust stability analysis for linear discrete-time systems with multiple delays', *Proceedings of the 38th Chinese Control Conference*, Guangzhou, China, pp.1312–1316.

Li, T., Song, A. and Fei, S. (2009) 'Novel stability criteria on discrete-time neural networks with time varying and distributed delays', *International Journal of Neural Systems*, Vol. 19, No. 4, pp.269–283.

Liu, L. (2019) 'QCs characterization of robust stability with simultaneous uncertainties in plant and controller', *Systems & Control Letters*, Vol.133, ISSN 104550, 8pp.

Ma, Y., Jia, X. and Zhang, Q. (2018) 'Robust observer-based finite-time control for discrete-time singular Markovian jumping system with time delay and actuator saturation', *Nonlinear Analysis: Hybrid Systems*, May, Vol. 28, pp.1–22 [online] https://doi.org/10.1016/j.nahs.2017.10.008.

Ma, Y.C. and Chen, M.H. (2015) 'Memory feedback H_∞ control of uncertain singular T-S fuzzy time-delay system under actuator saturation', *Computational and Applied Mathematics*, 2018, pp.1–19, DOI: 10.1007/s40314-015-0240-5.

Mahjoub, A., Giri, F. and Derbel, N. (2014) 'Delayed system control in presence of actuator saturation', *Alexandria Engineering Journal*, Vol. 53, No. 3, pp.553–561.

Meng, X., Lam, J., Du, B. and Gao, H. (2010) 'A delay-partitioning approach to the stability analysis of discrete-time systems', *Automatica*, Vol. 46, No. 3, pp.610–614, DOI: 10.1016/j.automatica.2009.12.004.

Nam, P.T., Trinh, H. and Pathirana, P.N. (2015) 'Discrete inequalities based on multiple auxiliary functions and their applications to stability analysis of time-delay systems', *Journal of the Franklin Institute*, Vol. 352, No. 12, pp.5810–5831.

Negi, R., Purwar, S. and Kar, H. (2012) 'Delay-dependent stability analysis of discrete time delay systems with actuator saturation', *Intelligent Control and Automation*, Vol. 3, No. 1, pp.34–43.

Nise, N.S. (2010) *Control System Engineering*, 6th ed., 7 December, John Wiley & Sons, Inc., USA.

Pal, V.C. and Negi, R. (2012) 'Performance analysis of discrete time systems with input saturation: a LMI-based approach', *Students Conference on Engineering and Systems (SCES)*, MNNIT Allahabad, pp.1–6.

Pal, V.C. and Negi, R. (2018) 'Delay-dependent stability criterion for uncertain discrete time systems in presence of actuator saturation', *Transactions of the Institute of Measurement and Control*, Vol. 40, No. 6, pp.1873–1891.

Pal, V.C., Kumar, J., Negi, R. and Pati, A. (2017) 'H_∞ control of discrete time delayed systems in presence of actuator saturation', *Indian Control Conference (ICC)*, Indian Institute of Technology, Guwahati, India, pp.295–302.

Pal, V.C., Negi, R. and Zhu, Q. (2019) 'Stabilization of discrete-time delayed systems in presence of actuator saturation based on Wirtinger inequality', *Mathematical Problems in Engineering*, Vol. 2019, Article ID 5954642, 14pp., Research Article, Hindawi [online] http://doi.org/10.1155/2019/5954642.

Park, P., Ko, J.W. and Jeong, C. (2011) 'Reciprocally convex approach to stability of systems with time-varying delays', *Automatica*, Vol. 47, No. 1, pp.235–238.

Pratap, B. and Sharma, J. (2020) 'Robust controller design for nonlinear twin rotor control system using quantitative feedback theory', *International Journal of Automation and Control*, Vol. 14, No. 3, p.355.

Qian, Y., Xiang, Z. and Karimi, H.R. (2015) 'Disturbance tolerance and rejection of discrete switched systems with time-varying delay and saturating actuator', *Nonlinear Analysis: Hybrid Systems*, Vol. 16, pp.81–92 [online] http://doi.org/10.1016/j.nahs.2014.11.001.

Rachid, H., Lamrabet, O. and Tissir, E.H. (2019) 'Stabilization of delta operator systems with actuator saturation via an anti-windup compensator', *Symmetry*, Vol. 11, No. 9, p.1084.

Ramakrishnan, K. and Ray, G. (2012) 'Reciprocal convex approach to delay-dependent stability of uncertain discrete-time systems with time-varying delay', *American Control Conference (ACC)*, Montreal, QC, pp.5450–5453.

Sbarbaro, D., Tomizuka, M. and de la Barra, B.L. (2009) 'Repetitive control system under actuator saturation and windup prevention', *ASME, Journal of Dynamic Systems. Measurement, and. Control*, Vol. 131, No. 4, 8pp, ISSN: 044505.

Seuret, A. and Gouaisbaut, F. (2012a) 'On the use of the Wirtinger's inequalities for time-delay systems', *Proc. of the10th IFAC Workshop on Time Delay Systems (IFAC TDS'12*), Boston, MA, USA, Vol. 45, No. 14, pp.260–265.

Seuret, A. and Gouaisbaut, F. (2012b) 'Reducing the gap of the Jensen's inequality by using the Wirtinger's inequality', *Automatica*, 16pp, HAL Id: hal-00765870 [online] https://hal.archives-ouvertes.fr/hal-00765870 (accessed 17 December 2012).

Seuret, A., Gouaisbaut, F. and Fridman, E. (2015) 'Stability of discrete time systems with time-varying delays via a novel summation inequality', *IEEE Transactions on Automatic Control*, Vol. 60, No. 10, pp.2740–2745.

Shao, H.Y. (2008) 'Improved delay-dependent stability criteria for systems with a delay varying in a range', *Automatica*, Vol. 44, No. 12, pp.3215–3218.

Shen, Z., Li, C., Li, H. and Cao, Z. (2019) 'Estimation of the domain of attraction for discrete-time linear impulsive control systems with input saturation', *Applied Mathematics and Computation*, Vol. 362, No. 6, p.124502, DOI: 10.1016/j.amc.2019.06.016.

Singh, K., Kandanvli, V.K.R. and Kar, H. (2021) 'Delay partitioning approach to the robust stability of discrete-time systems with finite wordlength nonlinearities and time-varying delays', *Transactions of the Institute of Measurement and Control*, Vol. 43, No. 4, pp.958–974, DOI: 10.1177/0142331220947566.

Solgi, Y., Fatehi, A. and Shariati, A. (2020) 'Non-monotonic Lyapunov-Krasovskii functional approach to stability analysis and stabilization of discrete time-delay systems', *IEEE/CAA Journal of Automatica Sinica*, Vol. 7, No. 3, pp.752–763.

Song, G.F. and Wang, Z. (2013) 'A delay partitioning approach to output feedback control for uncertain discrete time-delay systems with actuator saturation', *Nonlinear Dynamics*, Vol. 74, Nos. 1–2, pp.189–202, DOI: 10.1007/s11071-013-0957-x.

Stojanovic, S., Stojanovic, M. and Stevanovic, M. (2018) 'Novel delay-dependent stability criteria for discrete-time neural networks with time-varying delay', *Mathematical Problems in Engineering*, p.5397870, 15pp, Article ID 5397870 [online] https://doi.org/10.1155/2018/5397870.

Sun, S., Zhang, H., Su, H. and Liang, Y. (2021) 'Robust reliable H∞ optimization control for uncertain discrete-time Takagi-Sugeno fuzzy systems with time-varying delay', *Optimal Control Appl. Method*, Vol. 42, No. 3, pp.848–876.

Sun, W., Wang, L. and Xie, X. (2019) 'An improved stability analysis method for Hamiltonian systems with input saturation and delay', *12th Asian Control Conference (ASCC)*, Kitakyushu-shi, Japan, pp.1030–1035.

Sun, Y., Li, N., Shen, M., Wei, Z. and Sun, G. (2018) 'Robust control of uncertain linear system with interval time-varying delays by using Wirtinger inequality', *Applied Mathematics and Computation*, Vol. 335, pp.1–11 [online] http://doi.org/10.1016/j.amc.2018.04.027.

Tadepalli, S.K. and Kandanvli, V.K.R. (2017) 'Delay-dependent stability of discrete-time systems with multiple delays and nonlinearities', *International Journal of Innovative Computing, Information and Control*, Vol. 13, No. 3, pp.891–904, ISSN: 1349-4198.

Tadepalli, S.K., Kandanvli, V.K.R. and Vishwakarma, A. (2017) 'Criteria for stability of uncertain discrete-time systems with time varying delays and finite wordlength nonlinearities', *Transactions of the Institute of Measurement and Control*, Vol. 40, No. 9, pp.2868–2880.

Tadepalli, S.K., Kandanvli, V.K.R. and Vishwakarma, A. (2018) 'Criteria for stability of uncertain discrete-time systems with time varying delays and finite wordlength nonlinearities', *Transactions of the Institute of Measurement and Control*, Vol. 40, No. 9, pp.2868–2880.

Venkatesh, M., Patra, S. and Ray, G. (2021) 'Observer-based controller design for linear time-varying delay systems using a new Lyapunov-Krasovskii functional', *International Journal of Automation and Control*, January, Vol. 15, No. 1, pp.99–123.

Viegas, D., Batista, P., Oliveira, P. and Silvestre, C. (2020) 'Distributed controller design and performance optimization for discrete-time linear systems', *Optimal Control Applications and Methods*, Vol. 42, No. 1, pp.126–143.

Wang, H. and Zhu, Q. (2020) 'Global stabilization of a class of stochastic nonlinear time-delay systems with SISS inverse dynamics', *IEEE Transactions on Automatic Control*, Vol. 65, No. 10, pp.4448–4455.

Wu, Y., Zhang, H., Li, G., Sun, D. and Li, Y. (2020) 'Novel robust stability condition for uncertain systems with interval time-varying delay and nonlinear perturbations', *International Journal of Automation and Control*, January, Vol. 14, No. 1, pp.98–114.

Xian, Z. and Cheng, G. (2015) 'Further improvement of Wirtinger-based integral inequality for systems with time-varying delay', *34th Chinese Control Conference (CCC)*, Hangzhou, pp.1545–1549.

Xu, S., Feng, G., Zou, Y. and Huang, J. (2012) 'Robust controller design of uncertain discrete time-delay systems with input saturation and disturbances', *IEEE Transactions on Automatic Control*, Vol. 57, No. 10, pp.2604–2609.

Yuana, Y., Wang, Z., Yu, Y., Guo, L. and Yang, H. (2019) 'Active disturbance rejection control for a pneumatic motion platform subject to actuator saturation: an extended state observer approach', *Automatica*, Vol. 107, No. 12, pp.353–361.

Zhang, C., He, Y., Jiang, L., Wu, M. and Zeng, H. (2017) 'Summation inequalities to bounded real lemmas of discrete-time systems with time-varying delay', *IEEE Transactions on Automatic Control*, Vol. 62, No. 5, pp.2582–2588.

Zhang, C.K., He, Y., Jiang, L., Wu, M. and Zeng, H.B. (2016) 'Delay-variation-dependent stability of delayed discrete-time systems', *IEEE Transactions on Automatic Control*, Vol. 61, No. 9, pp.2663–2669.

Zhang, L., He, L. and Song, Y. (2018) 'New results on stability analysis of delayed systems derived from extended Wirtinger's integral inequality', *Neurocomputing*, Vol. 283, pp.98–106, doi:10.1016/j.neucom.2017.12.044.

Zhao, M.M. and Sun, Y.G. (2016) 'Stability analysis for time-varying positive linear systems with delays', *School of Mathematical Sciences, 16th International Conference on Control, Automation and Systems (ICCAS 2016)*, pp.782–785.

Zhao, T., Huang, M. and Dian, S. (2019) 'Robust stability and stabilization conditions for nonlinear networked control systems with network-induced delay via T-S fuzzy model', *IEEE Transactions on Fuzzy Systems*, Vol. 29, No. 3, pp.486–499.

Zhu, Q and Wang, H. (2018) 'Output feedback stabilization of stochastic feedforward systems with unknown control coefficients and unknown output function', *Automatica*, Vol. 87, pp.166–175 [online] http://doi.org/10.1016/j.automatica.2017.10.004.

Zhu, Q. (2019) 'Stabilization of stochastic nonlinear delay systems with exogenous disturbances and the event-triggered feedback control', *IEEE Transactions on Automatic Control*, Vol. 64, No. 9, pp.3764–3771.

Appendix A

Proof of Theorem 1

The forward difference of the Lyapunov functional of the system (12) is given as

$$\begin{aligned}\Delta V\left(x(\vartheta)\right) &= \sum_{a=1}^{3}\left(V_a\left(x(\vartheta+1)\right) - V_a\left(x(\vartheta)\right)\right) \\ &= \sum_{a=1}^{3}\left(\Delta V_a\left(x(\vartheta)\right),\right.\end{aligned} \tag{A1}$$

Now,

$$\begin{aligned}\Delta V_1\left(x(\vartheta)\right) &= \xi^T(\vartheta)\begin{bmatrix}\left(\Omega_1 + \alpha\left(g(\vartheta)\right)^T G(\Omega_1 + \alpha\left(g(\vartheta)\right)\right) \\ -(\Omega_2 + \alpha\left(g(\vartheta)\right)^T G(\Omega_2 + \alpha\left(g(\vartheta)\right)\right)\end{bmatrix}\xi(\vartheta) \\ &= \xi^T(\vartheta)\left[\left(\Omega_1^T G\Omega_1 - \Omega_2^T G\Omega_2\right) + He\left(\alpha^T\left(g(\vartheta)G\left(\Omega_1 - \Omega_2\right)\right]\xi(\vartheta),\right.\right.\end{aligned} \tag{A2}$$

where

$$\Omega_1 = \begin{bmatrix} A+\Delta A_p - I & 0 & A_{dp}+\Delta A_{dp} & 0 & 0 & 0 & 0 \\ 0 & -I & 0 & 0 & I & 0 & 0 \\ 0 & 0 & -I & -I & 0 & I & I \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -I & 0 & 0 & 0 & I & 0 & 0 \\ 0 & -I & -I & 0 & 0 & I & I \end{bmatrix}, \tag{A3}$$

$$\alpha(g(\vartheta)) = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_l I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (g(\vartheta)-g_l)I & (g_h - g(\vartheta))I \end{bmatrix}.$$

$$\begin{aligned} \Delta V_2(x(\vartheta)) &= \sum_{p=\vartheta-g_l+1}^{\vartheta} x^T(p)E_1x(p) + \sum_{p=\vartheta-g_h+1}^{\vartheta-g_l} x^T(p)E_2x(p) \\ &\quad - \sum_{p=\vartheta-g_l}^{\vartheta-1} x^T(p)E_1x(p) + \sum_{p=\vartheta-g_h}^{\vartheta-g_l-1} x^T(p)E_2x(p) \\ &= x^T(\vartheta)E_1x(\vartheta) - x^T(\vartheta-g_l)(E_1-E_2)x(\vartheta-g_l) \\ &\quad - x^T(\vartheta-g_h)E_2x(\vartheta-g_h) \\ &= \xi^T(\vartheta)\hat{E}\xi(\vartheta), \end{aligned} \tag{A4}$$

where

$$\hat{E} = diag(E_1, -E_1+E_2, 0, -E_2, 0, 0, 0).$$

$$\begin{aligned} \Delta V_3(\boldsymbol{x}(\vartheta)) &= g_l \sum_{p=g_l+1}^{0} \sum_{j=\vartheta+p+1}^{\vartheta+1} \boldsymbol{\eta}^T(j)\boldsymbol{T}_1\boldsymbol{\eta}(j) + g_{lh} \sum_{p=-g_h+1}^{-g_l} \sum_{j=\vartheta+p+1}^{\vartheta+1} \boldsymbol{\eta}^T(j)\boldsymbol{T}_2\boldsymbol{\eta}(j) \\ &\quad - g_l \sum_{p=g_l+1}^{0} \sum_{j=\vartheta+p}^{\vartheta} \eta^T(j)T_1\eta(j) - g_{lh} \sum_{p=-g_h+1}^{-g_l} \sum_{j=\vartheta+p}^{\vartheta} \eta^T(j)T_2\eta(j) \end{aligned}$$

where

$$\tilde{T}_1 = diag(T_1, 3T_1), \tilde{T}_2 = diag(T_2, 3T_2),$$

$$T_3 = \begin{bmatrix} \tilde{T}_1 & 0 & 0 \\ 0 & \frac{g_{lh}}{g(\vartheta)-g_l}\tilde{T}_2 & 0 \\ 0 & 0 & \frac{g_{lh}}{g_h - g(\vartheta)}\tilde{T}_2 \end{bmatrix}. \tag{A5}$$

If there exists a matrix $Y = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix} \in \Re^{2t\times 2t}$ such that equation (13) holds true then, applying from Lemma 2 of Pal and Negi (2018) and Park et al. (2011), the following relation is obtained:

$$\Delta V_3(\vartheta) \le \xi^T(\vartheta)\Omega_1^T \tilde{T}_{12}\Omega_1\xi(\vartheta) - \xi^T(\vartheta)\Xi^T(\vartheta)\varphi\Xi(\vartheta)\xi(\vartheta), \tag{A6}$$

where

$$\tilde{T}_{12} = \begin{bmatrix} g_l^2 T_1 + g_{lh}^2 T_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \varphi = \begin{bmatrix} \tilde{T}_1 & 0 & 0 \\ 0 & \tilde{T}_2 & Y \\ 0 & Y^T & \tilde{T}_2 \end{bmatrix},$$

$$\Xi(\vartheta) = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & -2I & 0 & 0 \\ 0 & I & -I & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 & -2I & 0 \\ 0 & 0 & I & -I & 0 & 0 & 0 \\ 0 & 0 & I & I & 0 & 0 & -2I \end{bmatrix}. \tag{A7}$$

Using equations (A2)–(A6), $\Delta V(x(\vartheta))$ [equation (A1)] can be written as:

$$\Delta V\left(x(\vartheta)\right) \le \xi^T(\vartheta)\hat{E}\xi(\vartheta) + \xi^T(\vartheta)\begin{bmatrix} \left(\Omega_1^T G\Omega_1 - \Omega_2^T G\Omega_2\right) \\ +He\left(\alpha^T\left(g(\vartheta)\right)G\left(\Omega_1 - \Omega_2\right)\right) \end{bmatrix}\xi(\vartheta) \tag{A8}$$
$$+\xi^T(\vartheta)\Omega_1^T\tilde{T}_{12}\Omega_1\xi(\vartheta) - \xi^T(\vartheta)\Xi^T(\vartheta)\varphi\,\Xi(\vartheta)\xi(\vartheta).$$

$$\Delta V\left(x(\vartheta)\right) \le \xi^T(\vartheta)\chi\xi(\vartheta), \tag{A9}$$

$$\chi = \begin{bmatrix} \chi_{11} + l_{11} & -2T_1 & \chi_{13} & 0 & 6T_1 & 0 & 0 \\ * & \chi_{22} & \chi_{23} & \chi_{24} & 6T_1 & 6T_2 & 2Y_2 + 2Y_4 \\ * & * & \chi_{33} + l_{33} & \chi_{34} & 0 & \chi_{36} & \chi_{37} \\ * & * & * & \chi_{44} & 0 & -2Y_3^T + 2Y_4^T & 6T_2 \\ * & * & * & * & -12T_1 & 0 & 0 \\ * & * & * & * & * & -12T_2 & -4Y_4 \\ * & * & * & * & * & * & -12T_2 \end{bmatrix}, \tag{A10}$$

$$l_{11} = \left(\tilde{A} + \Delta A_p - I\right)^T g_l^2 T_1\left(\tilde{A} + \Delta A_p - I\right) + \left(\tilde{A} + \Delta A_p - I\right)^T g_{lh}^2 T_2\left(\tilde{A} + \Delta A_p - I\right)$$
$$+\left(\tilde{A} + \Delta A_p\right)^T G_1\left(\tilde{A} + \Delta A_p\right) \tag{A11a}$$

$$\chi_{13} = \left(\tilde{A} + \Delta A_p\right)^T G_1\left(\tilde{A}_{dp} + \Delta A_{dp}\right)$$
$$+\left(\tilde{A} + \Delta A_p - I\right)^T\left(g_l^2 T_1 + g_{lh}^2 T_2\right)\left(A_{dp} + \Delta A_{dp}\right) \tag{A11b}$$

$$l_{33} = \left(A_{dp} + \Delta A_{dp}\right)^T G_1\left(\tilde{A} + \Delta A_p\right) + \left(A_{dp} + \Delta A_{dp}\right)^T g_l^2 T_1\left(A_{dp} + \Delta A_{dp}\right)$$
$$+\left(A_{dp} + \Delta A_{dp}\right)^T g_{lh}^2 T_2\left(A_{dp} + \Delta A_{dp}\right) \tag{A12}$$

Employing Schur's complement on equation (A10), we get

$$\chi_1 = \begin{bmatrix} \chi_{11} & -2T_1 & 0 & 0 & 6T_1 & 0 & 0 & (\tilde{A}+\Delta A_p)^T & g_l(\tilde{A}+\Delta A_p - I)^T & g_{lh}(\tilde{A}+\Delta A_p - I)^T \\ * & \chi_{22} & \chi_{23} & \chi_{24} & 6T_1 & 6T_2 & 2Y_2+2Y_4 & 0 & 0 & 0 \\ * & * & \chi_{33} & \chi_{34} & 0 & \chi_{36} & \chi_{37} & (A_{dp}+\Delta A_{dp})^T & g_l(A_{dp}+\Delta A_{dp})^T & g_{lh}(A_{dp}+\Delta A_{dp})^T \\ * & * & * & \chi_{44} & 0 & -2Y_3^T+2Y_4^T & 6T_2 & 0 & 0 & 0 \\ * & * & * & * & -12T_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -12T_2 & -4Y_4 & 0 & 0 & 0 \\ * & * & * & * & * & * & -12T_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -G_1^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & * & -T_1^{-1} & 0 \\ * & * & * & * & * & * & * & * & * & -T_2^{-1} \end{bmatrix}. \quad \text{(A13)}$$

For matrices $0 < R_\vartheta$ ($\vartheta = 1, \ldots, 3$), we have

$$-G_1^{-1} \le -2R_1 + R_1G_1R_1, -T_1^{-1} \le -2R_2 + R_2T_1R_2, -T_2^{-1} \le -2R_3 + R_3T_2R_3. \quad \text{(A14)}$$

It can be spotted that $\Delta V(x(\vartheta)) < 0$ for $\xi(\vartheta) \neq 0$ together with equations (13)–(14) are constraints for asymptotic stability of equation (12). Here, $\Delta V(x(\vartheta)) < 0$ is denoted as $\chi_1 < 0$.

The compliance of condition mentioned in equation (14) represents that $\Phi(P_1) = \{x \in \Re^t; x^TP_1x \le 1\}$ is contained in the polyhedral set $\rho(H)$ as stated in equation (6). It can be demonstrated that $\Phi(P_1) = \{x \in \Re^t; x^TP_1x \le 1\}$ is comparable to Boyd et al. (1994).

$$G_1H_{(b)}^TH_{(b)}\tau^{-2} > 0, b = 1, 2, \ldots, q. \quad \text{(A15)}$$

Pre and post multiplying equation (A15) by x^T and x sequentially, conforms that $x \in \rho(H)$ for all $x \in \Phi^*(G_1)$. LMI [equation (14)] established by practising Schur's complement on equation (A15).

Taking into account the parametric uncertainties (2a) in system (12), the resultant matrix (A13) is written as follows:

$$\chi_1 + \overline{\Gamma}_0\overline{F}_0\overline{L}_0 + \overline{L}_0^T\overline{F}_0^T\overline{\Gamma}_0^T < 0. \quad \text{(A16a)}$$

On applying Lemma 3 of Pal and Negi (2018), Boyd et al. (1994) and Tadepalli et al. (2018), equation (A16a) is similar to

$$\chi_1 + j_0^{-1}\overline{\Gamma}_0\overline{\Gamma}_0^T + j_0\overline{L}_0^T\overline{L}_0 < 0, \quad \text{(A16b)}$$

where

$$\overline{\Gamma}_0^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_0^T & g_l\Gamma_0^T & g_{lh}\Gamma_0^T \end{bmatrix}, \quad \text{(A17)}$$

$$\overline{L}_0 = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad \text{(A18)}$$

Employing Schur's complement, equation (A16b) can be expressed as equation (15).

Following the steps similar to equations (A16)–(A18), leads to equation (15) by considering equation (2b). This verifies Theorem 1.

Appendix B

Verification of Corollary 1

Equation (A11) will be modified as:

$$\alpha(\vartheta) \le \xi_m^T(\vartheta)\chi_m\xi_m(\vartheta), \tag{B1}$$

where

$$\chi_m = \begin{bmatrix} \chi_{m11}+l_{11} & -2T_1 & \chi_{13} & 0 & 6T_1 & 0 & 0 & \chi_{m18} \\ * & \chi_{22} & \chi_{23} & \chi_{24} & 6T_1 & 6T_2 & 2Y_2+2Y_4 & 0 \\ * & * & \chi_{33} & \chi_{34} & 0 & \chi_{36} & \chi_{37} & \chi_{m38} \\ * & * & * & \chi_{44} & 0 & -2Y_3^T+2Y_4^T & 6T_2 & 0 \\ * & * & * & * & -12T_1 & 0 & 0 & 0 \\ * & * & * & * & * & -12T_2 & -4Y_4 & 0 \\ * & * & * & * & * & * & -12T_2 & 0 \\ * & * & * & * & * & * & * & \chi_{m88} \end{bmatrix}. \tag{B2}$$

$$\xi_m^T(\vartheta) = \left[x^T(\vartheta) \quad x^T(\vartheta - g_l) \quad x^T(\vartheta - g(\vartheta)) \quad x^T(\vartheta - g_h) \quad h_1^T(\vartheta) \quad h_2^T(\vartheta) \quad h_3^T(\vartheta) \quad w^T(\vartheta)\right] \tag{B3}$$

$$\chi_{m11} = -G_1 + G_2 + E_1 - 4T_1 + C_z^T C_z \tag{B4a}$$

$$\chi_{m18} = B_w^T G_1 \tilde{A} + B_w^T\left(g_l^2 T_1 + g_{lh}^2 T_2\right)\left(\tilde{A} - I\right) + C_z^T D_z \tag{B4b}$$

$$\chi_{m38} = \tilde{A} G_1 B_w^T + B_w^T\left(g_l^2 T_1 + g_{lh}^2 T_2\right) A_{dp} \tag{B5}$$

$$\chi_{m88} = D_z^T D_z + B_w G_1 B_w^T + B_w^T\left(g_l^2 T_1 + g_{lh}^2 T_2\right) B_w - \lambda^2 I. \tag{B6}$$

Employing Schur's complement on equation (B2) and following the steps (A16)–(A18), the term χ_m, associated with the difference of Lyapunov function (B1), can be represented as equation (32).

The fulfilment of equation (31) for each initial state satisfying $\Psi_\Theta < 1$, conform to $x^T(\vartheta)Px(\vartheta) < 1 + \lambda^2\beta^2$. Thus, the trajectories of the system beginning from $\Psi_\Theta < 1$ will persist inside the ellipsoid given by $\Phi(P, 1 + \lambda^2\beta^2)$.

The LMI conditions (30)–(32) is sufficient conditions for equation (12) to be asymptotically stable by virtue of controller $u(\vartheta) = Kx(\vartheta)$ for every initial condition gratifying $\Psi_\Theta < 1$ along with an established H_∞ level λ.