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Lyapunov-based MPC for nonlinear process with online triggered linearised model

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Abstract: Most of industrial processes are nonlinear. Model predictive control (MPC) using an explicit nonlinear model can achieve satisfactory performance, however, it will bring a high computational burden. Although linear MPC is widely used in practice, a linear model cannot deal with the highly nonlinear system dynamic that is well overall in a wide operating region. In this study, an error trigger rule evoking a re-modelling algorithm to re-linearise the known nonlinear analytical model has been proposed for closed-loop nonlinear systems with input constraints. The error-triggering can be conducted by an error quantiser that quantifies model error and the re-linearisation program is triggered when the accumulated error exceeds the set threshold. The stability of the process is maintained by using the Lyapunov-based MPC. The effectiveness of the proposed control algorithm is validated by a chemical process simulation.

Keywords: model predictive control; MPC; nonlinear systems; online linearisation; error-trigger; computation time.

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1 Introduction

Model predictive control (MPC) has become a significant control strategy with the highest potential because it can deal with multi-input and multi-output constrained systems (Mayne et al., 2000; Mayne, 2014). At each sampling time step, a constrained optimisation problem utilising an explicit system prediction model is solved online to reckon the optimal control sequence. Most of the practical industrial processes are nonlinear and nonlinear MPC (NMPC) can treat the nonlinear dynamic and constraints directly and explicitly. However, linear MPC is still more widely used in practice than NMPC. This is because that NMPC may meet some difficulties in applications, such as

a computation complexity

b the nonconvex properties of the optimisation problem

c stability and robustness (Rahideh and Shaheed, 2011).

Hence the applications of NMPC techniques are still limited in practical situations (Camacho and Alba, 2013).

A precise explicit system model using for prediction in MPC plays an important role in achieving the desirable performance (Zhang and Shi, 2020). Generally, a single linear model cannot deal with the highly nonlinear system dynamic well overall operating regions, unless the process is limited to work in the neighborhood of the target operating point. Hence, it is not enough for MPC to use only one linear model to obtain satisfying performance in the whole operating region of a nonlinear process. With the development of nonlinear modelling methods, there are increasingly many mature techniques for modelling a nonlinear system. It includes analytical approaches Gustafsson (1984); Fjeld et al. (1974), experimental modelling methods [e.g., Wiener model (Li and Li, 2016), Hammerstein model (Huo et al., 2008), and Volterra model (Doyle et al., 2002)], and artificial intelligence-based technique (Ostafew et al., 2016; Wang et al., 2015; Boulkaibet et al., 2017) (e.g., neural networks and Gaussian process). Owing to the simplicity and maturity of the linear model-based control strategies (Gu and Gupta, 2008), in some case, the global nonlinear model can be approximated by several local linear models (Banerjee et al., 1997), including multiple-model modelling methods (Gu and Gupta, 2008; Wang et al., 2007; Dougherty and Cooper, 2003) and/or adaptive linear modelling (Zhang and Shi, 2020; Fukushima et al., 2007). For multiple-model modelling methods, several subregions are segmented offline from the entire operating region of the nonlinear system. Each subregion is approximated by a local linear model. In light of the current operating point, the subregion and the corresponding local linear model are determined to achieve the prediction. However, how to decide the local model numbers is still required to address. More local models lead to a better nonlinear approximation, but it may increase the computational complexity (Dougherty and Cooper, 2003). In the adaptive linear modelling methods, the updating of the linear model depends on the linearisation of the nonlinear analytical model or the system observations like using recursive formulations to update the linear model parameters at each sample time (Zhang and Shi, 2020; Fukushima et al., 2007). It is noted that the recursive estimation scheme requires the data to contain sufficient (Dougherty and Cooper, 2003). Further, it may lead to expensive computational costs and frequent changes to the control due to updating the linear model at each sampling instance, which

are undesirable in practice. Some studies utilise an offset-free framework to reduce the effect of disturbances and handle modelling errors, especially for linear modelling. In the context of state-space linear systems, one usual approach is augmenting the system with additional disturbance states and a disturbance observer employed to obtain disturbance estimates (Maeder et al., 2009; Pratap and Purwar, 2019; Son et al., 2010, 2021; Huang et al., 2020), the selection of disturbance model and parameters will directly affect the control performance. An alternative approach is based on a velocity form linear model in which the extended state is composed by the state increments and the output error, while the manipulated variable is the control increment (Betti et al., 2013; Pannocchia et al., 2015).

Recently, there have been a number of studies on saving communication and computation resources by introducing triggering mechanisms (Yu et al., 2016), a strategy was proposed for triggering to precompute an input trajectory over a prediction horizon (Ellis and Christofides, 2015), a forecast-trigger MPC with sensor-controller communication constraints was developed (Xue and El-Farra, 2018), a robust event-triggered MPC scheme was designed to reduce the frequency of solving linear matrix inequality problems (Hu et al., 2021). To focus on the model mismatch problem, the triggering mechanisms are introduced to update the machine-learning-based predictive model (Wu et al., 2019). It has been noted that as the prediction error generated by the current linear model is within a certain range, the model still can be used at the next moment until the error exceeds a pre-defined threshold. The error-triggered algorithm designed in this study is to handle the model mismatch caused by the linearisation. A trigger rule will evoke the re-modelling algorithm to re-linearise the known nonlinear analytical model at the current operative point. By using this philosophy, the model updating frequency and computational cost will be reduced.

Lyapunov-based MPC (LMPC) utilises an additional Lyapunov function that can enforce the closed-loop stability (Mayne et al., 2000), in some recent results, a Lyapunov-based adaptive MPC was employed for unconstrained nonlinear systems with parametric uncertainties (Zhu and Xia, 2016), meanwhile, The Lyapunov-based design was adopted to ensure good control performance of some data-driven MPC (Kheradmandi and Mhaskar, 2018; Narasingam and Kwon, 2019). Therefore, an LMPC scheme has been designed in this study due to its ability to explicitly characterising the stability region and reducing complexity optimisation. Note that most existing MPC algorithms have no prior closed-loop stability regions and have higher computational cost (Witsenhausen, 1968; Scokaer and Mayne, 1998; de la Peña and Christofides, 2008) by comparing with LMPC. In this paper, a moving horizon error quantiser is derived initially, it ensures the model accuracy and avoids linearisation at each sample step. By comprehensively considering the computational burden, model accuracy, and control performance, a novel LMPC in light of the mentioned trigger rule-based linearised method have been proposed for a class of strongly nonlinear process.

The LMPC using an online triggered linearised model can reduce the model update frequency compared to the conventional adaptive linear MPC method and maintain model accuracy within a reasonable range. The computation burden is greatly reduced compared with the nonlinear LMPC. Meanwhile, compared with the traditional LMPC using only one linear model, it possesses a better control performance.

The remainder of the paper is organised as follows: a class of nonlinear process with input constraints is considered and the auxiliary control is introduced briefly in Section 2. In Section 3, the trigger rule to update the linear model is proposed for the

novel LMPC which will be described detailly in Section 4. In Section 5, the execution and flowchart of the algorithm are given in details. In Section 6, a strongly nonlinear chemical process simulation is demonstrated to validate the proposed approach. In final Section 7, some conclusions are drawn.

1.1 Notations

The notation $|\cdot|$ denotes Euclidean norm of a vector and $\|\cdot\|$ means the matrix norm. $\|x\|_Q^2 := x^T Q x$ indicates the weighted norm, where Q is a positive definite symmetric matrix and the transpose of the vector x is denoted x^T. The standard Lie derivative defined by $L_f V(x) := \frac{\partial V(x)}{\partial x} f(x)$. A function $\alpha(\cdot) : [0, a) \to [0, \infty)$ is refered to class K if it is continuous, strictly increasing and satisfies $\alpha(0) = 0$. The notation Ω_r denotes the set $\Omega_r := \{x \in R^n : V(x) \leq r\}$ where V is a sufficiently smooth scalar function and $r > 0$, and the notation $S(\Delta)$ refers to the family of piecewise constant functions with period Δ.

2 Problem formulation

The continuous-time nonlinear process with input constraints is given in the following description:

$$\dot{x}(t) = F(x(t), u(t), w(t)) \tag{1}$$

$$u \in U, \tag{2}$$

where $F(x(t),\ u(t),\ w(t)) = f(x(t)) + g(x(t))u + h(x(t))w,\ x(t) \in R^n$ denotes the state vector, $u(t) \in R^m$ denotes the vector of control input, $w(t) \in R^m$ denotes the vector of disturbance, $x(t_0) = x_0$. Control inputs are restricted in a nonempty convex set $U := \{u \in R^m : |u| \leq u^{norm}, u^{norm} > 0\}$. The vector function $F(x(t),\ u(t),\ w(t))$ is locally Lipschitz and the origin is an equilibrium point of the unforced nominal system $F(x(t),\ w(t))$, that is, $F(0, 0) = 0$.

Assumption 1: For a nonlinear stabilisable system (1), there is a Lyapunov-based controller that makes the nominal system ($w(t) \equiv 0$) asymptotically stable in the open neighborhood of the origin.

In light of converse Lyapunov theorems (Christofides and El-Farra, 2005; Lin et al., 1996), Assumption 1 indicates that there exists a continuously differentiable Lyapunov scalar function $V : R^n \to R_+$, for the closed-loop system of equation (1) under $u(t) = u_L(x(t))$ satisfying the following inequalities:

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \tag{3a}$$

$$\frac{\partial V(x)}{\partial x} F(x, u_L(x), 0) \leq -\alpha_3(|x|) \tag{3b}$$

$$\left| \frac{\partial V(x)}{\partial x} \right| \leq \alpha_4(|x|) \tag{3c}$$

where $\alpha_i(\cdot), i = 1, 2, 3, 4$ are functions of class K. $\Omega_\rho \subseteq D$ is the stable region of the closed-loop system under the Lyapunov-based control, D is an open neighborhood of the origin for all $x \in D \subseteq R^n$. The candidate control $u_L(x)$ is given in the following form Lin and Sontag (1991):

$$u_L(x) = \begin{cases} -k(x)(L_gV)^T(x), & \left|(L_gV)^T(x)\right| \neq 0 \\ 0, & \left|(L_gV)^T(x)\right| = 0 \end{cases} \tag{4a}$$

$$k(x) = \frac{L_f^*V(x) + \sqrt{\left(L_fV(x)\right)^2 + \left(u^{norm}|(L_gV)^T(x)|\right)^4}}{|(L_gV)^T(x)|^2\left[1 + \sqrt{1 + \left(u^{norm}|(L_gV)^T(x)|\right)^2}\right]} \tag{4b}$$

where $L_gV(x) = \left[L_{g^1}V, ..., L_{g^m}V\right]$ is a row vector, where g^i is the i^{th} column of $g(x)$, $L_f^*V(x)$=$L_f^*V(x) + \rho^*V(x)$, $\rho^* > 0$. By using a standard Lyapunov argument, a region where the time-derivative of V is rendered negatively under the control $u_L(x)$ that can be characterised as:

$$\Phi_u = \left\{x \in R^n : L_f^*V(x) \leq u^{norm}\left|(L_gV)^T(x)\right|\right\}, \tag{5}$$

According to equation (5), the control satisfies the input constraints. Then the closed-loop stability can be guaranteed for the system (1) by defining a level set of V, i.e.,

$$\Omega_\rho = \{x \in R^n | V(x) \leq \rho\} \tag{6}$$

where $\rho > 0$ is the largest number for which $\Omega_\rho \subseteq \Phi_u$. When sampling period Δ is selected properly and $w(t)$ is bounded, system (1) can be stabilised by the Lyapunov-based control (Mhaskar et al., 2005).

In the control design, a nonlinear model is derived initially which will be used to update a serial of linear models at each operational point. Then the linear models will be used in the following LMPC design.

$$\dot{x} = A_ix(t) + B_iu(t) + d_i \tag{7}$$

$$A_i = \left.\frac{\partial F}{\partial x}\right|_{(x_i,u_i,0)}, B_i = \left.\frac{\partial F}{\partial u}\right|_{(x_i,u_i,0)} \tag{8}$$

$$d_i = F(x_i, u_i, 0) - A_ix_i - B_iu_i \tag{9}$$

where $x_i \in R^n$ and $u_i \in R^m$ denote the i^{th} operation point used to linearise (1), $A_i \in R^{n\times n}$ and $B_i \in R^{n\times m}$ are constant matrices corresponding to the i^{th} linearised model, d_i is a modelling error vector generated by Taylor expansion linearisation, particularly, d_i is zero when $x_i \equiv 0$ and $u_i \equiv 0$ ($i = 1, ..., \tilde{M}$).

Assumption 2: There are a set of controllers $u_{L1}(x)$, $u_{L2}(x)$, ..., $u_{L\tilde{M}}(x)$ designed by using several linearised models and each controller can stabilise system (1) asymptotically.

According to Assumption 2, there is a differentiable Lyapunov function $\hat{V} : R^n \to R_+$ satisfy (Khalil and Grizzle, 2002):

$$\hat{\alpha}_1(|x|) \leq \hat{V}(x) \leq \hat{\alpha}_2(|x|) \tag{10a}$$

$$\frac{\partial \hat{V}(x)}{\partial x} F(x, u_{Li}(x), 0) \leq -\hat{\alpha}_{3i}(|x|), i = 1, ..., \tilde{M} \tag{10b}$$

$$\left| \frac{\partial \hat{V}(x)}{\partial x} \right| \leq \hat{\alpha}_4(|x|) \tag{10c}$$

for all $x \in D_{Li} \subset R^n$, D_{Li} is an open neighborhood of the origin, the functions $\hat{a}_j(\cdot), j = 1, 2, 4$, and $\hat{a}_{3i}(\cdot), i = 1, ..., \tilde{M}$, are belong to class K. Under control $u_{Li}(x)$, system (1) has the stability region $\Omega_{\hat{\rho}i} \subset D_{Li}, i = 1, ..., \tilde{M}$.

The control objective is to design Lyapunov-based MPC in light of the model error trigger rule to make a class of strong nonlinear processes to be stable. The proposed approach will reduce the online calculation burden compared with general nonlinear model predictive and improve control performance compared with general linear predictive control using a single predictive linear model.

3 Error-based triggering rule for evoking the lineariser

An error-triggering rule is proposed in this section to update the model by running the lineariser, which can be used in the design and implementation of the LMPC. It can avoid updating the process model at every sampling period, which may lead to an incessant adjustment of the control law and expensive computation that are undesirable in practice. Further, the proposed mechanism reduces the large model mismatch as the traditional linearisation method which uses a single linear model in the whole operation region.

An error quantiser reckoning the moving horizon error cumulant e_d at time t_k is designed for quantifying the model error:

$$e_d(t_k) = \sum_{i=0}^{M} \frac{|\bar{x}(t_{k-i}) - x(t_{k-i})|}{|x(t_{k-i})|} \exp^{(\beta - \gamma i)} \tag{11}$$

$$M = \begin{cases} k & 0 \leq t_k \leq t_{M_t} \\ M_t & t_k > t_{M_t} \end{cases} \tag{12}$$

where M is the number of sampling periods before t_k that contributes to the prediction error quantification, M_t denotes the maximum calculated number, t_{M_t} denotes the sampling time when calculated number to the maximum from initial time t_0. $x(t_{k-i})$ and $\bar{x}(t_{k-i})$, $i = 0, ..., M$ are the past measurements and prediction state of the system by the proposed model at sampling times in the interval of $[t_{k-M}, t_k]$. Forgetting factor $\exp^{(\beta - \gamma i)}$ is added, $\beta > 0$ and $\gamma > 0$ are parameters which can adjust the size of the forgetting factor. $e_{d,T}$ is the threshold. When the quantiser value surpasses the threshold, i.e., $e_d(t_k) > e_{d,T}$, it triggers the model update process at the current

operating point. The selection of $e_{d,T}$ should comprehensively consider the number of moving horizons, weight coefficient in equation (11), and model accuracy requirements.

Remark 1: At the initial time, it cannot know the control input for the linearisation process to acquire the initial linearised model, so the initial linear model can be acquired by linearising the normal nonlinear system (1) at the origin or the closest equilibrium point to the initial point. It may capture the nonlinear dynamic better and contribute to model-based control design easier.

Remark 2: M_t should be chosen appropriately such that the past error information could be fully utilised, and unnecessary data storage could be avoided. $e_{d,T}$ should be determined based on the value of M_t in such a way that the linearised frequency is reasonable to guarantee the model prediction and reduce the model update frequency.

Remark 3: Because of the update of the predictive model, further computation may be required to get the new control law for some model-based control laws (e.g., on account of the new model, Sontag-type control law which uses in equation (4) will require to be re-calculated), so u_{Li} may be updated when the process model is updated.

Remark 4: In terms of the reliability of the linear model, additional state constraints have been added to forces the state to stay in the neighborhood of the linearised working point. It can ensure predictive linearisation models available in the neighborhood of the optimisation problem (Carvalho et al., 2013). Note that additional constraints may shrink the feasible region and increase the complexity of computation. Since the proposed triggering method ensures that the accuracy of the predictive linear model used around the working point is at a reasonable accuracy, it can avoid adding additional state constraints.

4 LMPC in light of the linear models with the triggering mechanism

The LMPC with the aforementioned model is designed to solve the following issue:

$$J := \min_{u \in S(\Delta)} \int_{t_k}^{t_k+N\Delta} L(\hat{x}(t), u(t))\mathrm{dt} \tag{13a}$$

$$\text{s.t.} \quad \dot{\hat{x}}(t) = A_i\hat{x}(t) + B_i u(t) + d_i \tag{13b}$$

$$\hat{x}(t_k) = x(t_k) \tag{13c}$$

$$u(t) \in U, \forall t \in [t_k, t_{k+N\Delta}) \tag{13d}$$

$$\begin{aligned} &\frac{\partial \hat{V}(x(t_k))}{\partial x}(A_i x(t_k) + B_i u(t_k) + d_i) \\ &\leq \frac{\partial \hat{V}(x(t_k))}{\partial x}(A_i x(t_k) + B_i u_{Li}(x(t_k)) + d_i) \quad \text{if } \hat{V}(x(t_k)) > \delta \end{aligned} \tag{13e}$$

$$\hat{V}(x(t)) \leq \delta \quad \forall t \in [t_k, t_k + N\Delta) \quad \text{if } \hat{V}(x(t_k)) \leq \delta \tag{13f}$$

where $\hat{x}(t) \in R^n$ is the predictive state trajectory with the calculated control input trajectory $u(t) \in R^m$, δ is a positive real number, and N is the prediction horizon. Starting with the initial state constrainted condition (13c) acquired from the process state measurement at t_k, the LMPC utilises the proposed linearised model (13b) with the i^{th} linear model (which is updated by the error-triggered law) to predict the state trajectory. $u_{Li}(x)$ and $\hat{V}(x)$ are designed for the Lyapunov-based constraints (13) with the proposed linearised model. The manipulated inputs constraints are characterised by equation (13d). The stage cost (13a) is formulated as follows:

$$L(\hat{x}(t), u(t)) = \|x(t)\|_Q^2 + \|u(t)\|_R^2 \tag{14}$$

where $Q \in R^{n \times n}$ and $R \in R^{m \times m}$ are positive semi-definite and strictly positive definite, symmetric matrices. Only the first element $u^*(t_k|t_k)$ of optimal control sequence $u^*(t|t_k)$, $t \in [t_k, t_{k+N\Delta})$ is applied to the plant at each sampling time.

Remark 5: In general, $\hat{V}$ (i.e., $\hat{V} = x^T P x$) is designed by the linearised model at the target equilibrium for a nonlinear system and the stability region depends on the level set of the additional Lyapunov function. It is not be required to update $\hat{V}$ when the model of equation (13b) is updated if it is a set point tracking problem.

5 LMPC implementation with triggering mechanism

M_t, β, γ and $e_{d,T}$ are set off-line initially. The implementation strategy of the LMPC with proposed trigger schemes is shown in Figure 1. The implementation strategy of the LMPC with proposed trigger schemes is summarised as Algorithm 1.

Remark 6: Noted that the value of the additional Lyapunov function is reduced under the Lyapunov-based constraint of equation (13e) over the first sampling period. If the values of M_t, β, γ and $e_{d,T}$ are selected suitably to make the prediction based on a reasonably accurate model, the closed loop system will be stable under the stability constraint (Alanqar et al., 2017).

Remark 7: More recently, Koopman operator has received much attention. The basic idea is to transform the nonlinear dynamics into a higher dimensional space where its evolution is approximately bilinear or linear (Narasingam and Kwon, 2019), it can be easily embedded into the range of mature linear mode-based control design techniques (Huang et al., 2018; Ma et al., 2019; Narasingam and Kwon, 2020a,b). However, the key issue is to find an appropriate tradeoff between nonlinearity and higher dimensionality. A finite-dimensional approximation obtained from data-driven numerical techniques leads to plant-model mismatch (Son et al., 2010). Incorporating the proposed model update online triggering mechanism, to get a better approximation of Koopman operator and update the data-driven model by using the most recent process data, might be a scheme for reducing model mismatch.

Algorithm 1 Lyapunov-based MPC with trigger scheme

Step 1 Linearise at the set-point to obtain the first linear model (A_1 and B_1). Given the model, input, and a control Lyapunov function $\hat{V}$, design the Lyapunov-based controller $u_{L_1}(x(t_0))$ of equation (4);
Step 2 Receive a state measurement $x(t_k)$ and compute the moving horizon error metric $e_d(t_k)$ of equation (11) where the value of is determined by equation (12);
Step 3
if $e_d(t_k) > e_{d,T}$ **then**
| Go to Step 4;
else
| Go to Step 5;
end
Step 4 Given the new model, the constraints on the input, the state measurement $x(t_k)$, and the control Lyapunov function $\hat{V}$, design the control $u_{L_i}(x(t_k))$ and update the Lyapunov constraint in the optimisation of equation (13e).
Step 5 The process is operated under the LMPC of equation (13) based on the current linearised model (A_i and B_i) and the state measurement is used in the constraints of equations (13c) and (13e).
Step 6 If meet the simulation termination criteria, the algorithm is stop, otherwise, repeat Steps 2 to 5.

6 Numerical example

To verify the effectiveness of the control technology, the LMPC with the online triggered linearised model is adopted to control a First-order irreversible exothermic reaction of the form $A \rightarrow B$ taking place in the continuous stirred tank reactor (CSTR). The process mechanism model is given as follows:

$$\begin{aligned}\dot{C}_A &= \frac{F}{V}(C_{A0} - C_A) - k_0 \text{exp}^{(-E/RT_R)} C_A \\ \dot{T}_R &= \frac{F}{V}(T_{A0} - T_R) + \frac{(-\Delta H)}{\rho_\sigma c_p} k_0 \text{exp}^{(-E/RT_R)} C_A + \frac{Q_\sigma}{\rho_\sigma c_p V}\end{aligned} \tag{15}$$

where the variables used in the above model can be explained in Table 1, and the corresponding parameter value can be found in Table 2 (Mhaskar et al., 2006).

The control objective is to make the reactor track quickly and accurately to the set point (C_A^s, T_R^s) = (0.57 Kmol/^{3}, 395.3 K), in which the control inputs are Q_σ and change in inlet concentration $\Delta C_{A0} = C_{A0} - C_{A0_s}$ with the constraints $|Q_\sigma| \leq$ 0.0167 KJ/min and $|\Delta C_{A0}| \leq 1$ Kmol/m^3.

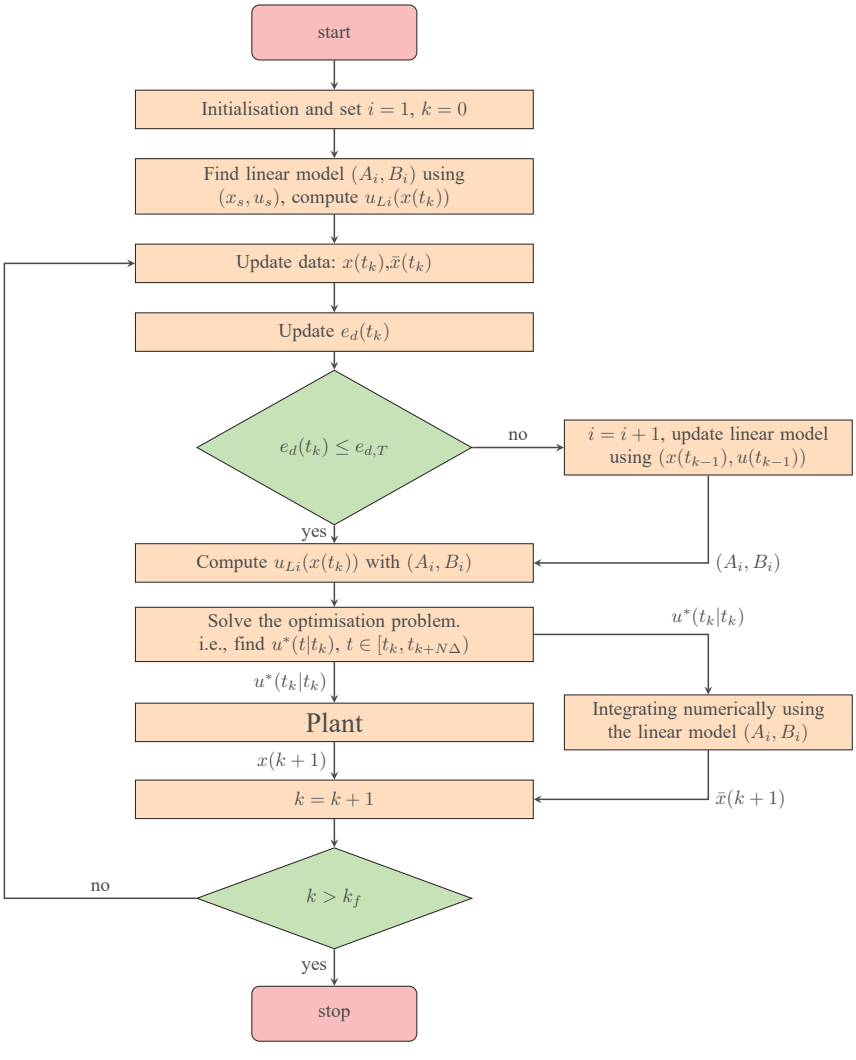
A Lyapunov-based controller of equation (4) with $V(x) = x^T P x$ was constructed, matrix P can be computed by using the linearised system, where

$$x = (C_A - C_A^s, T_R - T_R^s) \quad P = \begin{bmatrix} 9.35 & 0.41 \\ 0.41 & 0.02 \end{bmatrix},$$

The parameters in the objective function of equation (14) are selected as $Q = I$, $R = I$. The predictive horizon N is chosen to be 10. The quantiser reckons the relative

prediction error as stated in equation (11), where the $\beta = 0.8$ and $\gamma = 0.04$ are set in the forgetting factor, M_t in equation (12) is chosen to be 50. In the simulation, the explicit Euler numerical integration method was adopted in which the integration step size is $h = 10^{-4}$ min. The constrained optimisation problem is solved using the MPCTOOLS with CasADi solver (Risbeck and Rawlings, 2016; Andersson et al., 2019).

Figure 1 Flowchart of the proposed ETLMPC approach (see online version for colours)



Case 1 An initial point (C_A, T_R) = (0.44 Kmol/m^3, 397 K) near the equilibrium point

The threshold value of aforementioned error-trigger LMPC (ETLMPC) is chosen as $e_{d,T}$ = 1.0E–4. From the initial state, the closed-loop trajectories under nonlinear LMPC (NLMPC), linear LMPC (LLMPC), and ETLMPC as the predictive model are as shown

in Figure 2. From the closed-loop state trajectory, all three control schemes can achieve stabilisation of the closed-loop system successfully and track to the setpoint well. Compare to NLMPC (red solid lines), the input profiles of the aforementioned ETLMPC (blue dotted lines) are more smooth. The single linear model will be not accurate when the working point is far from the equilibrium point. As shown in Figures 3 and 4, the error quantiser is designed to reduce the linearisation frequency and ensure the accuracy of the prediction model within a certain range, where the moment with amplitude of 1 represents triggered moment while the moment with amplitude of 0 represents an untriggered moment, and the number of total simulation steps is 300 and it is triggered 71 times all the time.

Table 1 The definitions of process variables

C_A	Concentration of the species A
Q_σ	Heat removed from the reactor
k_0	Pre-exponential constant
ΔH	Enthalpy of the reaction
ρ_σ	Fluid density
T_R	Temperature of the reactor
V	Volume of the reactor
E	Activation energy
c_p	Heat capacity

Table 2 Process parameters and steady-state values

V = 0.1 m^3
R = 8.314 KJ/KmolK
C_{A0_s} = 1.0 Kmol/m^3
T_{A0_s} = 310.0 K
Q_s = 0.0 KJ/min
ΔH = –4.78 × 10^4 KJ/Kmol
k_0 = 72 × 10^9 min^{-1}
E = 8.314 × 10^4 KJ/Kmol
c_p = 0.239 KJ/KgK
ρ_σ = 1,000.0 Kg/m^3
F = 100 × 10^{-3} m^3/min
T_{Rs} = 395.33 K
C_{As} = 0.57 Kmol/m^3

Figure 5 shows the computation time required, at each sampling period, to solve the optimisation problem for the NLMPC, LLMPC, and ETLMPC, respectively. The solver for ETLMPC converged in less than 0.03 s (Figure 5). The total computation time required to solve the NLMPC and ETLMPC optimisation problems are 12.06 s and 4.40 s. The total computation time of NLMPC is 174% greater than the ETLMPC. There is no need to update the model for LLMPC, and the predictive model is a single linear model, so the total computation time required to solve LLMPC in the simulation was 2.72 s and it is the minimum computation time of three control scheme. But note that the control performance of the proposed approach is best among these three methods.

Figure 2 Closed-loop state (top) and input (bottom) profiles under the controller of NLMPC (red solid lines), under the controller of LLMPC (black dashed lines) and under the controller of ETLMPC (blue dotted lines) where the initial point is (0.44 Kmol/m^3, 397 K) (see online version for colours)

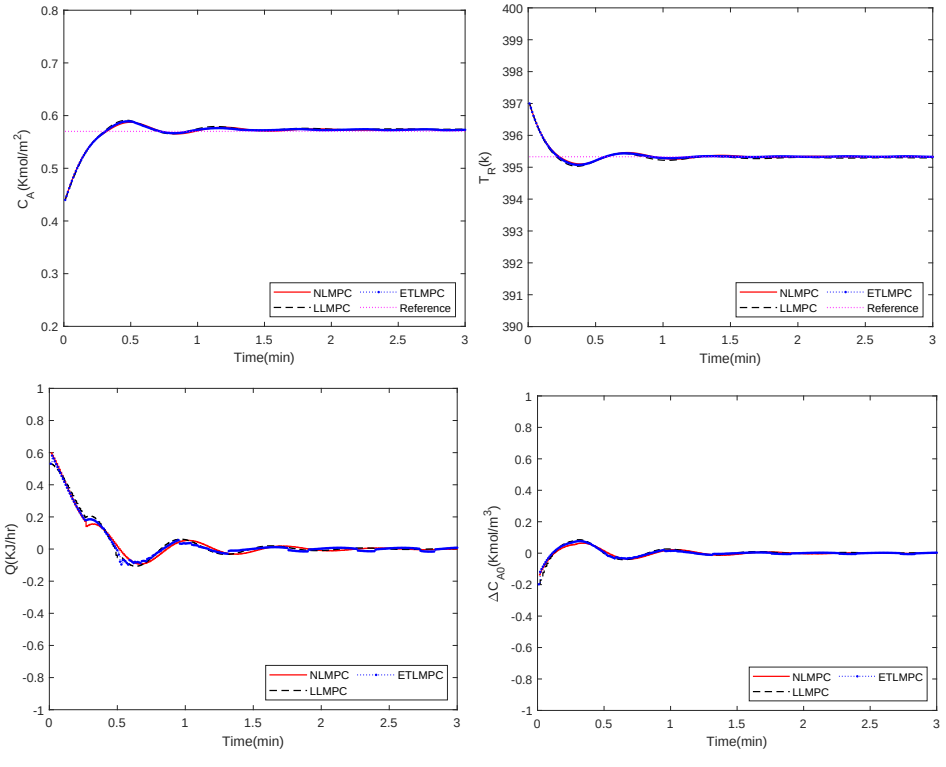


Figure 3 The trigger time of ETLMPC where the initial point is (0.44 Kmol/m^3, 397 K) (see online version for colours)

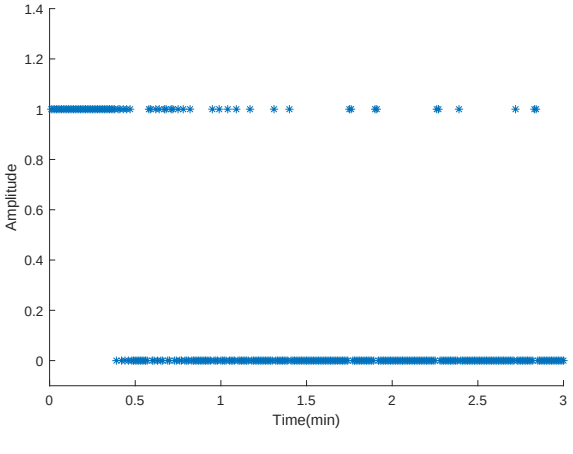


Figure 4 Predictive model errors of LLMPC and NLMPC where the initial point is (0.44 Kmol/m^3, 397 K) (see online version for colours)

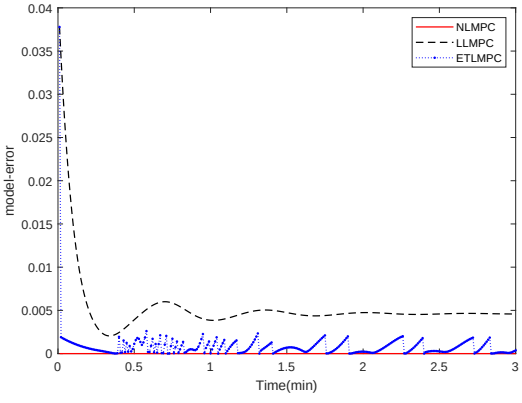
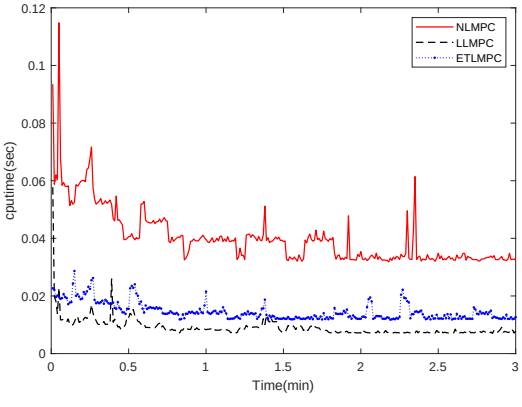


Figure 5 The computation time required to solve the NLMPC (red solid lines), the LLMPC (black dashed lines) and the ETLMPC (blue dotted lines) where the initial point is (0.44 Kmol/m^3, 397 K) (see online version for colours)



Case 2 An initial point (C_A, T_R) = (0.22 Kmol/m^3, 401 K) is far from the equilibrium point

In this case, the increased linearisation error generated by the single linear model may lead to the model mismatch. The threshold value of aforementioned ETLMPC is chosen as $e_{d,T}$ = 1E–4. Figure 6 shows the state and input trajectories of the CSTR under the NLMPC, ETLMPC, and LLMPC throughout the three-min simulation, and starts from the same initial condition. It is noted that when at some simulation time like 0.68 min, 0.71 min, 0.80 min, etc. the optimisation problem of LLMPC is unsolvable which may due to the model mismatch, the control law of the previous moment is adopted at the above simulation time. As shown in Figure 7, even if the working point is far from the setpoint, ETLMPC can achieve stabilisation of the closed-loop state trajectory and track to the setpoint better compared to the LLMPC. The number of total simulation steps is 300 and it is triggered 88 times all the time, the trigger-scheme can reduce the model update frequency effectively

Figure 6 Closed-loop state (top) and input (bottom) profiles under the controller of NLMPC (red solid lines), under the controller of LLMPC (black dashed lines) and under the controller of ETLMPC (blue dotted lines) where the initial point is (0.22 Kmol/m^3, 401 K) (see online version for colours)

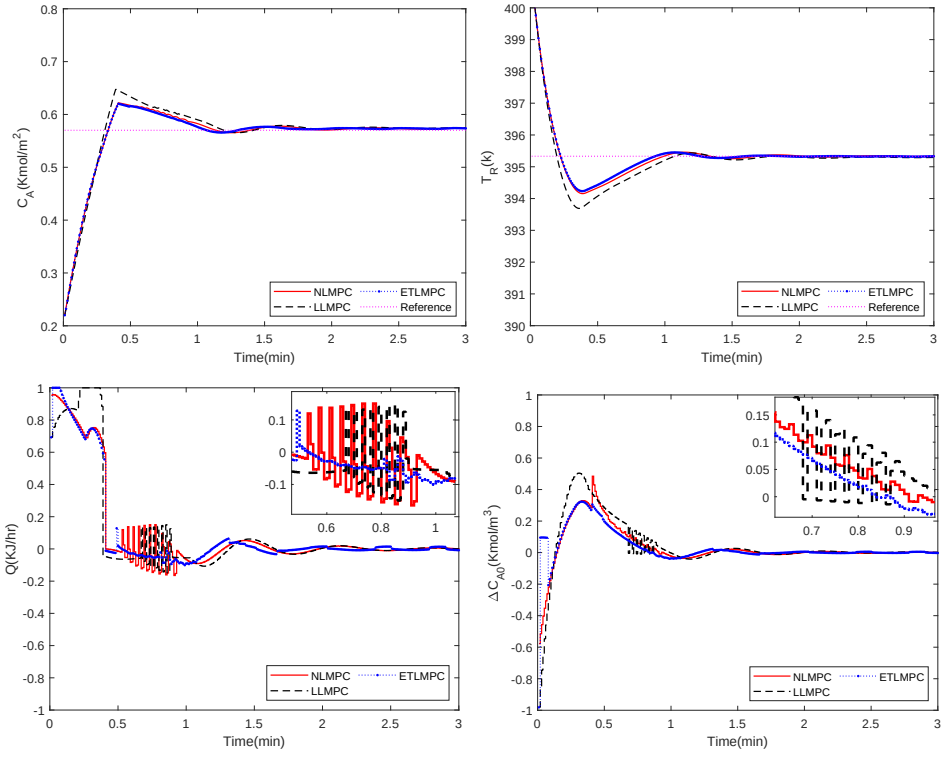
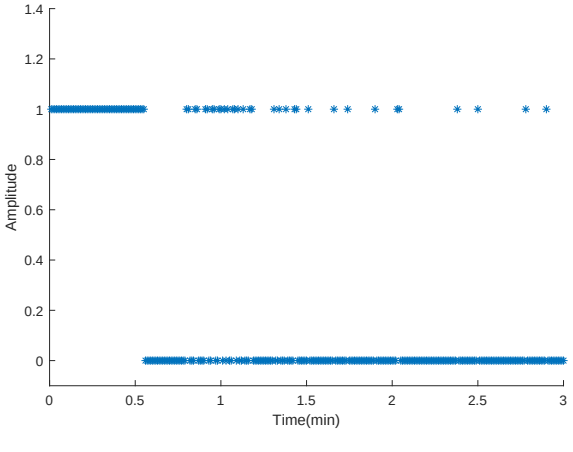


Figure 7 The trigger time of ETLMPC where the initial point is (0.22 Kmol/m^3, 401 K) (see online version for colours)



Tables 3 and 4 show the average computation time needed for solving the optimisation problems in each step and other performance indexes of the NLMPC, LLMPC and ETLMPC systems at the initial condition (0.44 Kmol/m^3, 397 K) and (0.22 Kmol/m^3, 401 K). As can be seen, the average computation time of ETLMPC is a bit longer than LLMPC, however, both of them are significantly shorter than that of NLMPC. In the case of MPC with the linear model, the solution of the optimal control problem can be converted to the solution of a quadratic program which can be solved efficiently, but, MPC with the nonlinear model has to solve a nonlinear program, which is in general computationally expensive (Allgower et al., 2004). For assessing the overall trajectory of the entire transition process, the integral forms of the error including integrated time and absolute error (ITAE) and integral time square error (ITSE) are used to analyse the control performance. J_{total} describes the stage cost (14) accumulated overall simulation. It is shown that ETLMPC has a lower value of the three performance index than other controller schemes.

Table 3 The performance of the three approaches at the initial point (0.44 Kmol/m^3, 397 K)

	Average computation time	*ITAE*	*ITSE*	J_{total}
NLMPC	0.0402 s	124.0147	75.5697	21.5424
LLMPC	0.0091 s	127.2069	76.2715	22.0146
ETLMPC	0.0146 s	117.0217	74.7177	21.3357

Table 4 The performance of the three approaches at the initial point (0.22 Kmol/m^3, 401 K)

	Average computation time	*ITAE*	*ITSE*	J_{total}
NLMPC	0.0491 s	503.5168	1031.8269	256.2012
LLMPC	0.0104 s	592.7555	1,169.1803	292.3252
ETLMPC	0.0165 s	470.4964	991.4939	246.8389

In this section, LMPC control systems using three different models are simulated to compare their performances at two initial conditions, respectively. The closed-loop state trajectory illustrating the convergence of the states from the initial conditions (0.44 Kmol/m^3, 397 K) under the three control schemes are plotted in Figure 2, but when initial point (0.22 Kmol/m^3, 401 K) is far from the setpoint, LMPC may be unsolvable at some working points, the closed-loop state trajectory demonstrated the efficacy of the NLMPC and ETLMPC control are plotted in Figure 6. The error-trigger approach used in ETLMPC reduce the model update frequency and maintain model accuracy within a reasonable range. As reported in Tables 3 and 4, the total computation time, and the transition process performance indexes of the ETLMPC over all the sampling periods are less than NLMPC with same initial conditions. Through above comparisons, the effectiveness of the ETLMPC systems can be demonstrated.

7 Conclusions

A Lyapunov-based MPC is initially developed based on a new model updated rule. The error-triggering rule is evoked by an error quantiser and triggers model re-linearisation when the accumulated predictive error exceeds a threshold. The chemical process

simulation demonstrated that the control strategy can stabilise the system and maintain model accuracy even if the operation point is far from the set point compared to the LMPC using a linear model, it also can greatly reduce computation time compared to the nonlinear LMPC. The experiments will be used to demonstrate the proposed approach in the near future.

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