Mathematical modelling of breakdowns with soft failures and explicit analytical expressions of an *M/M/*1 queue's transient state probabilities

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Abstract: In the last decade, the increasing usage of digital electronics and software has given rise to a new form of failure occurrence known as soft failure. Soft failure is not the same as catastrophic failure. Catastrophic failure removes all waiting customers, including those receiving service, and the system enters the repair state, whereas soft failure does not require the removal of customers and instead requires the customer to wait for the server reactivation because repair can be accomplished by simply rebooting the system. A new approach for categorising and analysing these events is necessary to characterise and distinguish soft failures from hardware catastrophic failures. As a result, this paper considers a single server queueing model with soft failure. The system is prone to random breakdowns while providing service to the customer. When a system malfunctions, repair work begins right away. During the repair period, new customers are not permitted to join the system. For the first time, the transient state probabilities of an *M*/*M*/1 queue with soft failures are computed explicitly. Some performance metrics, such as availability and reliability, are derived. Numerical examples are also provided to demonstrate the effect of parameters.

Keywords: random breakdown; soft failure; hard failure; time dependent probabilities; Laplace transform; generating function.

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1 Introduction

As the world becomes smaller, the service sectors, as well as the production and manufacturing of goods and processes, will become more important. Increased competition encourages businesses to prioritise efficiency, production, and quality. Even when the services are available without interruption, the quality (however, it is defined because there is no single dimension to its description) is influenced by a number of factors, including server experience, service rate, and variation in the underlying variables. Ideally, the services should be available without interruption. This is not the case in reality.

Many studies in the queueing literature assume that servers are reliable; however, servers are vulnerable to breakdowns and repairs in practise. Throughout the operation of a traditional queueing model, the service facility is always ready to assist clients at regular or slower rates. The service facility may not be continuously available due to:

1. the server taking a vacation when the system becomes empty
2. a server (or machine) failure or an emergency break that requires immediate treatment, after which the service can be resumed.

The server failure is caused by two types of failures. The first type of failure is catastrophic failure, while the second type of failure is soft failure. Catastrophic failure (hard failure) drains the system, necessitates the presence of a repairman, and takes a long time to repair. Soft failure, on the other hand, does not completely deplete the system. Also, it takes less time to repair because the system can be restarted and returned to normal operation.

1.1 Literature survey related to catastrophic failure (hard failure)

Towsley and Tripathi (1991) are the first researchers to investigate a single server queueing model with catastrophic failures. Artalejo (2000) provides an overview of the main results and methods of the queueing model with catastrophes theory. Yang et al. (2002) presented the system size and sojourn time distributions of an $M/G/1$ queueing system with catastrophes. Di Crescenzo et al. (2012) investigated queueing systems with disasters and calculated the transient and steady-state probabilities. Kalidass et al. (2012) investigated an $M/M/1/N$ queue with catastrophes and developed transient state probabilities as well as performance measures. Ammar (2014) recently derived the transient probabilities of a two-processor heterogeneous system with catastrophes. Jiang et al. (2015) calculated the queue length and sojourn time distributions of an $M/G/1$ queue in a multi-phase random environment with disasters. Mytalas and Zazanis (2015) calculated the queue length and sojourn time distribution of a batch arrival queue with disasters under a multiple-adapted vacation policy. Yechiali (2007) calculated expected queue length and customer sojourn time in an $M/M/c$ queue with catastrophes by considering independent abandonments during a repair time. Economou and Kapodistria (2010) conduct similar research for the same model with binomial abandonments. To learn more about queueing systems with catastrophes, readers can read the papers by Kumar et al. (2007), Kumar and Madheswari (2005), Wang et al. (2008), Tarabia (2011), Vijayashree and Janani (2017), Rajadurai (2018) and Rasmi and Jacob (2021).

1.2 Literature survey related to soft failure

With an unstable service station, Wang (1990) developed the *M*/*M*/1 model and discovered steady-state analytic closed-form solutions. He also presents steady-state analytic closed-form solutions for the *M*/*Ek*/1 model with a removable and unreliable server in the year (Wang, 1997); Tang (1997) expanded it to the *M*/*G*/1 model, and Wang et al. (1999) extended it to the *M*/*H2*/1 model. Ke (2006) looked into the control policies for an *M*/*G*/1 queueing system with a start-up server that was unstable. Choudhury and Deka (2008) looked at an *M*/*G*/1 retrial queueing system with two stages of service failure and repair. Wang and Zhang (2009) looked at a discrete time single-server retrial queue with geometrical positive and negative consumer arrivals, as well as server breakdowns and repairs. Choudhury and Tadj (2009) extended the server breakdown concept by include a delay-repair period. A novel sort of queueing model with functioning breakdowns was investigated by Kalidass and Kasturi (2012). They employed probability generating functions to calculate the solutions and probabilities of various states.

Kim and Lee (2014) looked into an *M*/*G*/1 queueing system that had gone through disasters and had operational issues. The reliability of a warm standby repairable system with functioning malfunctions was investigated by Yen et al. (2016). Li and Zhang (2017) explored an *M*/*G*/1 retrial queue with negative arrivals and working breakdowns using the additional variable technique. They also tackled the issue of cost minimisation by developing a cost function and using the parabolic approach to calculate the best service rate during a working breakdown period. Yang and Wu (2017) used a finite-capacity *M*/*M*/1 queue with working breakdowns to conduct transient and steady-state assessments, accounting for reneging customers' retention. In a recent work that adapted Kalidass and Kasturi's (2012) model to the Markovian arrival process, Ye and Liu (2018) employed the matrix-geometric method to evaluate the steady-state probabilities of system size. *M*/*M*/1 queues with working breakdowns and delayed repair were explored by Jiang and Xin (2019), in which the system is either repaired immediately (with probability p) or continues to offer service to customers at a slower rate (with probability $1 - p$) when a breakdown occurs. Liu and Song (2014), Liou (2015), Chen et al. (2016) and Yang and Cho (2019) provide more details on queueing models with working breakdowns. Many scholars, including Avi-Itzhak and Naor (1963), Vinod (1985), Wang and Ke (2002) and Choudhury and Ke (2012), have given queueing models with server breakdown. Krishnamoorthy et al. (2014) investigate queueing systems with server failure in detail.

1.3 Importance of the considered model

According to the literature, the majority of authors in works on server failures concentrated on catastrophic failures. Catastrophic server failures, on the other hand, result in the system being cleared. Consider an ATM that is not working properly. If an ATM malfunctions, customers who are waiting for it are diverted. Aside from natural disasters, ordinary system failures (also known as soft failures) can occur at any time, leaving customers waiting for the server to be restored. Consider a machine that attaches a bottle cap. The equipment may stop working at any time due to overheating. However, after rebooting the machine, it can be restored to working order. Overheating of the computer is another example of ordinary system failure. Dust or debris obstructing air vents, exhaust ports, or a clogged fan might cause our computer to overheat owing to insufficient air movement. Overheating can cause a computer to restart, stop down, or have performance issues.

Figure 1 A device that fixes a bottle cap (see online version for colours)

Various analytical frameworks for unreliable queueing models with steady-state conditions have been investigated to date. Unlike steady-state models, analysing the transient characteristics of queueing systems is difficult; as a result, gaining insights into those systems is difficult. From an application standpoint, steady state models are typically better suited for analysing system behaviours over a finite time period. Transient models, on the other hand, are used to determine the efficiencies of a system over a long period of time. The transient state probabilities of a queueing model with soft failures are provided in this paper for the first time. The rest of this paper is structured as follows:

- Section 2 – model description
- Section 3 – time variant probabilities
- Section 4 – system reliability and availability
- Section 5 – numerical examples
- Section 6 – concluding remarks and future work.

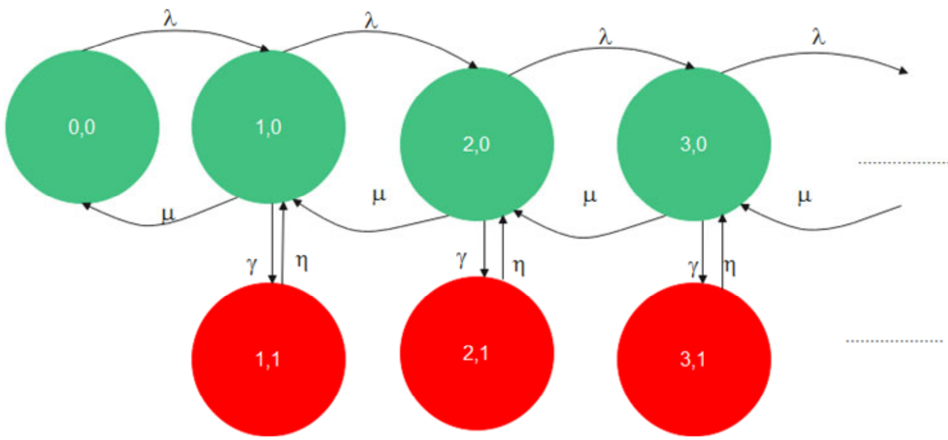
2 Model description

Consider an *M*/*M*/1 queueing system in which customers can join the queue based on the Poisson process with parameter λ. Customers are served at a rate μ, and service time is distributed exponentially. The server is unreliable, and it fails at random while providing service to the customer. Failures occur at a rate γ that is distributed as a Poisson distribution. If a failure occurs, the server immediately enters the repair state and the repair process begins. With parameter η, the repair time is also exponentially distributed.

Furthermore, new customers are not permitted to join the queue during the repair period, and existing customers must wait their turn.

The system state at time t is represented by $\{N(t), J(t)\}$, where $N(t)$ is the number of customers in the system and $J(t)$ is the system state. If $J(t) = 0$, the system is in a normal state; if $J(t) = 1$, the system is in a failure state. The process $\{N(t), J(t)\}$ is then a continuous time Markov chain, with the following transition diagram.

Figure 2 State diagram of the model (see online version for colours)



The system of governing equations of the model is as follows.

$$P'_{0,0}(t) = -\lambda P_{0,0}(t) + \mu P_{1,0}(t), \tag{2.1}$$

$$P'_{n,0}(t) = -(\lambda + \mu + \gamma)P_{n,0}(t) + \mu P_{n+1,0}(t) + \lambda P_{n-1,0}(t) + \eta P_{n,1}(t);\ n = 1, 2, \ldots \tag{2.2}$$

and

$$P'_{n,1}(t) = -\eta P_{n,1}(t) + \gamma P_{n,0}(t);\ n = 1, 2, \ldots \tag{2.3}$$

with the initial condition $P_{0,0}(0) = 1$.

3 Time variant probabilities

In this section, time variant probabilities of the model described in Section 2 are derived with the help of Laplace transform and generating function techniques.

Taking Laplace transform for the equation (2.1) gives,

$$\hat{P}_{0,0}(s) = \frac{1}{s+\lambda} + \frac{\mu}{s+\lambda}\hat{P}_{1,0}(s). \tag{3.1}$$

Taking inverse Laplace transform for the equation (3.1) gives,

$$P_{0,0}(t) = e^{-\lambda t} + \mu e^{-\lambda t} * P_{1,0}(t). \tag{3.2}$$

Similarly taking Laplace transform for the equation (2.3) gives,

$$\hat{P}_{n,1}(s) = \frac{\gamma}{s+\eta}\hat{P}_{n,0}(s). \tag{3.3}$$

Taking inverse Laplace transform for the equation (3.3) gives,

$$P_{n,1}(t) = \gamma e^{-\eta t} * P_{n,0}(t) \tag{3.4}$$

To determine $\sum_{n=1}^{\infty} P_{n,0}(t)$

Define

$$H(t) = \sum_{n=1}^{\infty} P_{n,0}(t). \tag{3.5}$$

Then

$$H'(t) = \sum_{n=1}^{\infty} P'_{n,0}(t). \tag{3.6}$$

On substituting the equation (2.2) in the equation (3.6) yields,

$$H'(t) = -(\lambda+\mu+\gamma)\sum_{n=1}^{\infty} P_{n,0}(t) + \lambda\sum_{n=1}^{\infty} P_{n-1,0}(t) + \mu\sum_{n=1}^{\infty} P_{n+1,0}(t) + \eta\sum_{n=1}^{\infty} P_{n,1}(t) \tag{3.7}$$

Taking Laplace transform for the equation (3.7) gives,

$$(s+\gamma)\hat{H}(s) = \lambda\hat{P}_{0,0}(s) - \mu\hat{P}_{1,0}(s) + \eta\sum_{n=1}^{\infty} \hat{P}_{n,1}(s) \tag{3.8}$$

On substituting the equation (3.1) and (3.3) in the equation (3.8) gives,

$$\sum_{n=1}^{\infty} \hat{P}_{n,0}(s) = \frac{1}{s+\gamma}\sum_{k=0}^{\infty} \left(F_1(s)\right)^k \left[\frac{\lambda}{s+\lambda} + \frac{\lambda\mu}{s+\lambda}\hat{P}_{1,0}(s) - \mu\hat{P}_{1,0}(s)\right], \tag{3.9}$$

where

$$F_1(s) = \frac{\eta\gamma}{(s+\gamma)(s+\eta)}. \tag{3.10}$$

Taking inverse Laplace transform for the equation (3.9) yields,

$$\sum_{n=1}^{\infty} P_{n,0}(t) = e^{-\gamma t} * \sum_{k=0}^{\infty} \left(F_1(t)\right)^{*k} * \left[\lambda e^{-\lambda t} + \lambda\mu e^{-\lambda t} * P_{1,0}(t) - \mu P_{1,0}(t)\right], \tag{3.11}$$

where

$$F_1(t) = \eta\gamma e^{-\gamma t} * e^{-\eta t} \tag{3.12}$$

Taking summation over $n = 1$ to ∞ in the equation (3.3) gives,

$$\sum_{n=1}^{\infty}\hat{P}_{n,1}(s)=\frac{\gamma}{s+\eta}\sum_{n=1}^{\infty}\hat{P}_{n,0}(s). \tag{3.13}$$

Substituting the equation (3.9) in (3.13) gives,

$$\sum_{n=1}^{\infty}\hat{P}_{n,1}(s)=\left(\frac{\gamma}{s+\eta}\right)\left(\frac{1}{s+\gamma}\right)\sum_{k=0}^{\infty}\left(F_1(s)\right)^k\left[\frac{\lambda}{s+\lambda}+\frac{\lambda\mu}{s+\lambda}\hat{P}_{1,0}(s)-\mu\hat{P}_{1,0}(s)\right] \tag{3.14}$$

Taking inverse Laplace transform for the equation (3.14) yields,

$$\sum_{n=1}^{\infty}P_{n,1}(t)=\gamma e^{-\eta t}*e^{-\gamma t}*\sum_{k=0}^{\infty}\left(F_1(t)\right)^{*k}*\left[\lambda e^{-\lambda t}+\lambda\mu e^{-\lambda t}*P_{1,0}(t)-\mu P_{1,0}(t)\right] \tag{3.15}$$

Hence $P_{0,0}(t)$, $\sum_{n=1}^{\infty}P_{n,1}(t)$ and $\sum_{n=1}^{\infty}P_{n,0}(t)$ are expressed in terms of $P_{1,0}$ (t). Therefore, it remains to determine $P_{1,0}$ (t).

To find $P_{1,0}(t)$

By normalisation condition,

$$P_{0,0}(t)+\sum_{n=1}^{\infty}P_{n,0}(t)+\sum_{n=1}^{\infty}P_{n,1}(t)=1. \tag{3.16}$$

Taking Laplace transform for the equation (3.16) and using the equations (3.1), (3.9) and (3.14) yields,

$$\begin{aligned}\hat{P}_{1,0}(s)=\sum_{j=0}^{\infty}\left(F_3(s)\right)^j\Bigg[&\frac{1}{s}-\frac{1}{s+\lambda}-\frac{1}{s+\gamma}\frac{\lambda}{s+\lambda}\sum_{k=0}^{\infty}\left(F_1(s)\right)^k\\&-\left(\frac{\gamma}{s+\eta}\right)\left(\frac{1}{s+\gamma}\right)\left(\frac{\lambda}{s+\lambda}\right)\sum_{k=0}^{\infty}\left(F_1(s)\right)^k\Bigg]\end{aligned} \tag{3.17}$$

where

$$F_3(s)=1-\frac{\mu}{s+\lambda}-F_2(s),$$

and

$$F_2(s)=\frac{1}{s+\gamma}\sum_{k=0}^{\infty}\left(F_1(s)\right)^k\left(\frac{\lambda u}{s+\lambda}-\mu\right)\left(1+\frac{\gamma}{s+\eta}\right).$$

By inverting the equation (3.17) gives,

$$P_{1,0}(t) = \sum_{j=0}^{\infty}\left(F_3(s)\right)^{*j} * \left[1 - e^{-\lambda t} - e^{-\gamma t} * \lambda e^{-\lambda t} * \sum_{k=0}^{\infty}\left(F_1(t)\right)^{*k} - \gamma e^{-\eta t} * e^{-\gamma t} * \lambda e^{-\lambda t} * \sum_{k=0}^{\infty}\left(F_1(t)\right)^{*k}\right], \tag{3.18}$$

where

$$F_3(t) = \delta(t) - \mu e^{-\lambda t} - F_2(t),$$

$$F_2(t) = e^{-\gamma t} * \sum_{k=0}^{\infty}\left(F_1(t)\right)^{*k} * \left(\lambda\mu e^{-\lambda t} - \mu\delta(t)\right) * \left(\delta(t) + \gamma e^{-\eta t}\right),$$

and $\delta(t)$represents dirac delta function.

Hence, $P_{1,0}(t)$ is determined explicitly.

3.1 Special case

3.1.1 Validation of the findings using existing literature

Applying final value theorem of Laplace transform to the equation (3.8) yields,

$$\sum_{n=1}^{\infty} P_{n,0} = \frac{\lambda}{\gamma} P_{0,0} - \frac{\mu}{\gamma} P_{1,0} + \frac{\eta}{\gamma}\sum_{n=1}^{\infty} P_{n,1}. \tag{3.19}$$

Similarly, applying final value theorem of Laplace transform to the equation (3.1) yields,

$$P_{0,0} = \frac{\mu}{\lambda} P_{1,0}. \tag{3.20}$$

On substituting the equation (3.20) in the equation (3.19) gives,

$$\sum_{n=1}^{\infty} P_{n,0} = \frac{\eta}{\gamma}\sum_{n=1}^{\infty} P_{n,1}. \tag{3.21}$$

The normality condition yields,

$$\sum_{n=1}^{\infty} P_{n,1} = \frac{\gamma}{\eta + \gamma}, \tag{3.22}$$

and

$$\sum_{n=1}^{\infty} P_{n,0} = \frac{\eta}{\eta + \gamma}. \tag{3.23}$$

The results in equations (3.21), (3.22) and (3.23) are seen to coincide with the results obtained by Ibe (2016).

4 Availability and reliability of the system

The dependability indices for the model under discussion are supplied in this section, including system availability and system reliability.

Let $A(t)$ be the probability that the system is available (in operation) at time t, i.e., the chance that the server is active or idle at time t.

$$A(t) = 1 - \sum_{n=1}^{\infty} P_{n,1}(t) = 1 - \sum_{n=1}^{\infty} \gamma e^{-\eta t} * P_{n,0}(t) \tag{4.1}$$

The average availability of the system in the interval [0, t] is

$$\begin{aligned} \overline{A}(t) &= \frac{1}{t}\int_0^t A(u)du \\ &= 1 - \frac{1}{t}\int_0^t \sum_{n=1}^{\infty} \gamma e^{-\eta u} * P_{n,0}(u)du \end{aligned} \tag{4.2}$$

Further, the total working time of the system in the interval [0, t] is

$$\begin{aligned} E(A(t)) &= \int_0^t A(u)du \\ &= t - \int_0^t \sum_{n=1}^{\infty} \gamma e^{-\eta u} * P_{n,0}(u)du \end{aligned} \tag{4.3}$$

If $\eta = 0$ then

$$\sum_{n=1}^{\infty} P_{n,1}(t) = \sum_{n=1}^{\infty} \gamma P_{n,0}(t)$$

so that the reliability function R(t) of the system is obtained as

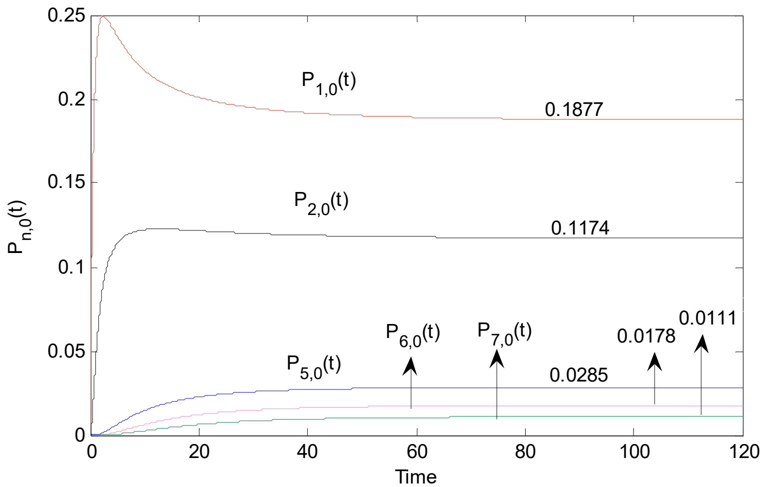
$$R(t) = 1 - \sum_{n=1}^{\infty} P_{n,1}(t) = 1 - \sum_{n=1}^{\infty} \gamma P_{n,0}(t) \tag{4.4}$$

5 Numerical illustrations

In order to support theoretical results, numerical illustrations are depicted below by considering the parameter value as $\lambda = 0.5$, $\mu = 0.8$, $\eta = 0.5$ and $\gamma = 0.2$.

The behaviour of working state probability with respect to time t is depicted in Figure 3. Initially, as time passes, the likelihood of being in a working condition increases and reaches a peak. After then, it progressively declines until it reaches a steady state. It is also noted that as 'n' increases, the likelihood of being in a working condition diminishes. This is due to the machine overheating as it fixes caps to a larger number of bottles in a sequence. This has an impact on the probability of reaching a steady state. It should be emphasised that as 'n' increases, the probability of reaching a steady state diminishes. When 'n' is increased, the machine will obviously stop working due to overheating and will be restored following a reboot. This will undoubtedly have an impact on the steady state probabilities.

Figure 3 $P_{n,0}(t)$ versus t (see online version for colours)



The failure state probability behaviour with regard to time t is shown in Figure 4. Figure 3 and Figure 4 show that the likelihood of being in a working state is higher than the probability of being in a failing condition. Because the failure rate is lower than the repair rate, this is the case. Because the rate of repair is higher than the rate of failure, the likelihood of being in a failure state with a higher value of '*n*' is lower than with a lower value of '*n*'. In practise, if the repair rate is high, a machine that has failed due to overheating will be repaired quickly. As a result, we may argue that the repair rate and the probability of being in a failure state with a higher '*n*' value are linked. The chance of failure with a larger value of '*n*' decreases when the repair rate increases, and vice versa.

Figure 4 $P_{n,1}(t)$ versus t (see online version for colours)

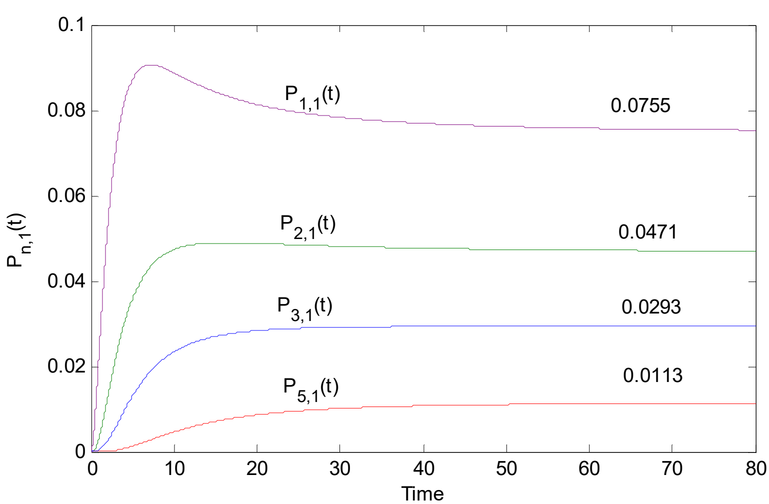


Figure 5 depicts how the probability of the system failing changes over time *t* for various values of η. It has been discovered that as η increases, the chances of being in a failure state diminish. As a result, we can deduce that the probability of failing is inversely related to the rate of η. This indicates that as the repair rate increases, so does the likelihood of being in a failure condition. If a machine stops working due to overheating and is not swiftly rebooted, the chances of it breaking down grow.

Figure 5 Probabilities to be in state one versus time *t* (see online version for colours)

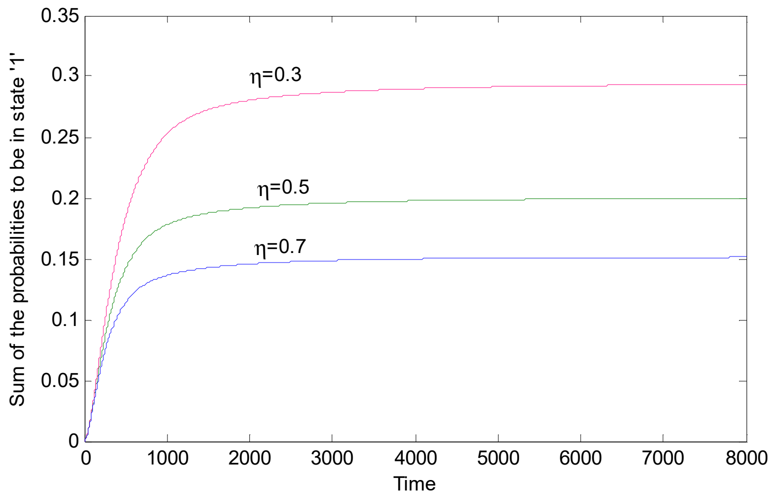


Figure 6 Availability of the server for varying values of η (see online version for colours)

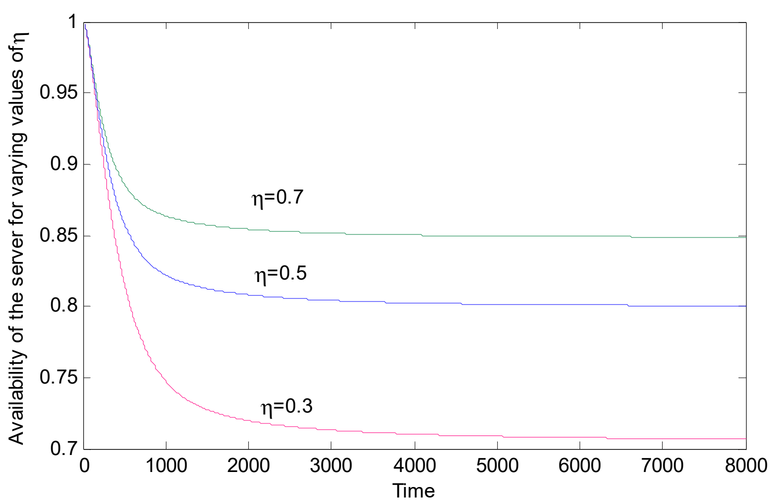
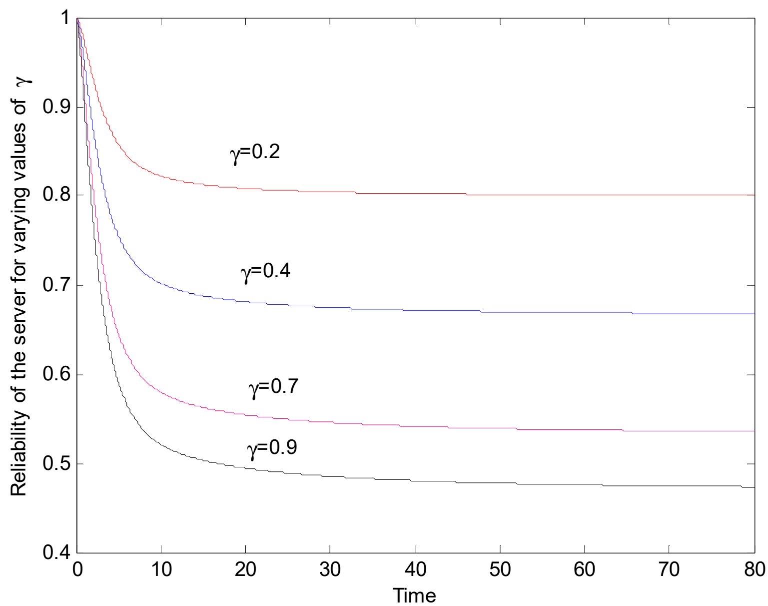


Figure 6 depicts how the system's availability changes over time t for various values of η. It has been discovered that as η increases, the probability of being in an availability state increases. As a result, we can deduce that the probability of being available is proportional to the rate of η. This means that as the rate of repairs increases, so does the possibility of becoming available. If a machine stops working due to overheating and is rapidly rebooted, the server will be available instantly, improving the chances of it being operational.

For various values of γ, Figure 7 shows how the system's reliability changes over time t. It has been discovered that as γ decreases, the reliability increases. As a result, we can conclude that the rate of γ is inversely proportional to the reliability. This means that reliability improves as the rate of failure reduces. The reliability of a machine degrades if it stops working frequently due to overheating.

Figure 7 Reliability of the server for varying values of γ (see online version for colours)



6 Concluding remarks and future work

This study takes a look at a single server queueing model with soft failure. While providing service to the consumer, the system is prone to periodic breakdowns. When a system fails, repair work begins immediately. New clients are not permitted to join the system during the maintenance period. The transient state probabilities of an $M/M/1$ queue with soft failures are explicitly estimated for the first time. Some performance measurements are derived, such as availability and reliability. There are additional numerical examples offered to highlight the effect of parameters. This work can be extended even further by allowing customers to join the queue during the repair period. In addition, the model can be constrained to a finite capacity, along with many other things.

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References

Ammar, S. (2014) 'Transient behavior of a two-processor heterogeneous system with catastrophes, server failures and repairs', *Applied Mathematical Modelling*, Vol. 38, Nos. 7–8, pp.2224–2234.

Artalejo, J. (2000) 'G-Networks: a versatile approach for work removal in queueing networks', *European Journal of Operational Research*, Vol. 126, No. 2, pp.233–249.

Avi-Itzhak, B. and Naor, P. (1963) 'Some queuing problems with the service station subject to breakdown', *Operations Research*, Vol. 11, No. 3, pp.303–320.

Chen, J.Y., Yen, T.C. and Wang, K.H. (2016) 'Cost optimization of a single-server queue with working breakdowns under the N policy', *Journal of Testing and Evaluation*, Vol. 44, No. 5, pp.2059–2067.

Choudhury, G. and Deka, K. (2008) 'An M/G/1 retrial queueing system with two phases of service subject to the server breakdown and repair', *Performance Evaluation*, Vol. 65, No. 10, pp.714–724.

Choudhury, G. and Ke, J.C. (2012) 'A batch arrival retrial queue with general retrial times under Bernoulli vacation schedule for unreliable server and delaying repair', *Applied Mathematical Modelling*, Vol. 36, No. 1, pp.255–269.

Choudhury, G. and Tadj, L. (2009) 'An M/G/1 queue with two phases of service subject to the server breakdown and delayed repair', *Applied Mathematical Modelling*, Vol. 33, No. 6, pp.2699–2709.

Di Crescenzo, A., Giorno, V., Kumar, B. and Nobile, A. (2012) 'A double-ended queue with catastrophes and repairs and a jump-diffusion approximation', *Methodology and Computing in Applied Probability*, Vol. 14, No. 4, pp937–954.

Economou, A. and Kapodistria, S. (2010) 'Synchronized abandonments in a single server unreliable queue', *European Journal of Operational Research*, Vol. 203, No. 1, pp.143–155.

Ibe, O.C. (2016) 'Vacation queueing models of service systems subject to failure and repair', in *Principles of Performance and Reliability Modeling and Evaluation*, pp.481–496, Springer, Cham.

Jiang, T. and Xin, B. (2019) 'Computational analysis of the queue with working breakdowns and delaying repair under a Bernoulli-schedule-controlled policy', *Communications in Statistics-Theory and Methods*, Vol. 48, No. 4, pp.926–941.

Jiang, T., Liu, L. and Li, J. (2015) 'Analysis of the M/G/1 queue in multi-phase random environment with disasters', *Journal of Mathematical Analysis and Applications*, Vol. 430, No. 2, pp.857–873.

Kalidass, K. and Kasturi, R. (2012) 'A queue with working breakdowns', *Computers and Industrial Engineering*, Vol. 63, No. 4, pp.779–783.

Kalidass, K., Gopinath, S., Gnanaraj, J. and Ramanath, K. (2012) 'Time dependent analysis of an *M/M/*1*/N* queue with catastrophes and a repairable server', *Opsearch*, Vol. 49, No. 3, pp.39–61.

Ke, J.C. (2006) 'Optimal NT policies for *M/G/*1 system with a startup and unreliable server', *Computers and Industrial Engineering*, Vol. 50, No. 3, pp.248–262.

Kim, B.K. and Lee, D.H. (2014) 'The *M/G/*1 queue with disasters and working breakdowns', *Applied Mathematical Modelling*, Vol. 38, Nos. 5–6, pp.1788–1798.

Krishnamoorthy, A., Pramod, P.K. and Chakravarthy, S.R. (2014) 'Queues with interruptions: a survey', *Top*, Vol. 22, No. 1, pp.290–320.

Kumar, B. and Madheswari, S. (2005) 'Transient analysis of an *M/M/*1 queue subject to catastrophes and server failures', *Stochastic Analysis and Applications*, Vol. 23, No. 2, pp.329–340.

Kumar, B., Krishnamoorthy, A., Madheswari, S. and Basha, S. (2007) 'Transient analysis of a single server queue with catastrophes, failures and repairs', *Queueing Systems*, Vol. 56, Nos. 3–4, pp.133–141.

Li, T. and Zhang, L. (2017) 'An *M/G/*1 retrial G-queue with general retrial times and working breakdowns', *Mathematical and Computational Applications*, Vol. 22, No. 1, p.15.

Liou, C.D. (2015) 'Markovian queue optimisation analysis with an unreliable server subject to working breakdowns and impatient customers', *International Journal of Systems Science*, Vol. 46, No. 12, pp.2165–2182.

Liu, Z. and Song, Y. (2014) 'The $M^X/M/1$ queue with working breakdown', *RAIRO-Operations Research*, Vol. 48, No. 3, pp.399–413.

Mytalas, G. and Zazanis, M. (2015) 'An $M^X/G/1$ queueing system with disasters and repairs under a multiple adapted vacation policy', *Naval Research Logistics*, Vol. 62, No. 3, pp.171–189.

Rajadurai, P. (2018) 'A study on *M/G/*1 retrial queueing system with three different types of customers under working vacation policy', *International Journal of Mathematical Modelling and Numerical Optimisation*, Vol. 8, No. 4, pp.393–417.

Rasmi, K. and Jacob, M.J. (2021) 'Analysis of a multiserver queueing inventory model with self-service', *International Journal of Mathematical Modelling and Numerical Optimisation*, Vol. 11, No. 3, pp.275–291.

Tang, Y.H. (1997) 'A single-server *M/G/*1 queueing system subject to breakdowns – some reliability and queueing problems', *Microelectronics Reliability*, Vol. 37, No. 2, pp.315–321.

Tarabia, A. (2011) 'Transient and steady state analysis of an *M/M/*1 queue with balking, catastrophes, server failures and repairs', *Journal of Industrial and Management Optimization*, Vol. 7, No. 4, pp.811–823.

Towsley, D. and Tripathi, S.K. (1991) 'A single server priority queue with server failures and queue flushing', *Operations Research Letters*, Vol. 10, No. 6, pp.353–362.

Vijayashree, K.V. and Janani, B. (2017) 'Stationary and transient analysis of *M/M/*1*/N* queue subject to disasters and subsequent repair', *International Journal of Operational Research*, Vol. 29, No. 4, pp.433–459.

Vinod, B. (1985) 'Unreliable queueing systems', *Computers and Operations Research*, Vol. 12, No. 3, pp.323–340.

Wang, J. and Zhang, P. (2009) 'A discrete-time retrial queue with negative customers and unreliable server', *Computers and Industrial Engineering*, Vol. 56, No. 4, pp.1216–1222.

Wang, J., Liu, B. and Li, J. (2008) 'Transient analysis of an *M/G/*1 retrial queue subject to disasters and server failures', *European Journal of Operational Research*, Vol. 189, No. 3, pp.1118–1132.

Wang, K.H. (1990) 'Infinite source *M/M/*1 queue with breakdown', *Journal of the Chinese Institute of Industrial Engineers*, Vol. 7, No. 7, pp.47–55.

Wang, K.H. (1995) 'Optimal operation of a Markovian queueing system with a removable and non-reliable server', *Microelectronics Reliability*, Vol. 35, No. 8, pp.1131–1136.

Wang, K.H. (1997) 'Optimal control of an $M/E^K/1$ queueing system with removable service station subject to breakdowns', *Journal of the Operational Research Society*, Vol. 48, No. 9, pp.936–942.

Wang, K.H. and Ke, J.C. (2002) 'Control policies of an *M/G/*1 queueing system with a removable and non-reliable server', *International Transactions in Operational Research*, Vol. 9, No. 2, pp.195–212.

Wang, K.H., Chang, K.W. and Sivazlian, B.D. (1999) 'Optimal control of a removable and non-reliable server in an infinite and a finite *M/H2/*1 queueing system', *Applied Mathematical Modelling*, Vol. 23, No. 8, pp.651–666.

Yang, D.Y. and Cho, Y.C. (2019) 'Analysis of the N-policy *GI/M/*1*/K* queueing systems with working breakdowns and repairs', *The Computer Journal*, Vol. 62, No. 1, pp.130–143.

Yang, D.Y. and Wu, Y.Y. (2017) 'Analysis of a finite-capacity system with working breakdowns and retention of impatient customers', *Journal of Manufacturing Systems*, Vol. 44, No. 1, pp.207–216.

Yang, W.S., Kim, J.D. and Chae, K.C. (2002) 'Analysis of *M/G/*1 stochastic clearing systems', *Stochastic Analysis and Applications*, Vol. 20, No. 5, pp.1083–1100.

Ye, Q. and Liu, L. (2018) 'Analysis of *MAP/M/*1 queue with working breakdowns', *Communications in Statistics-Theory and Methods*, Vol. 47, No. 13, pp.3073–3084.

Yechiali, U. (2007) 'Queues with system disasters and impatient customers when system is down', *Queueing Systems*, Vol. 56, No. 3, pp.195–202.

Yen, T.C., Chen, W.L. and Chen, J.Y. (2016) 'Reliability and sensitivity analysis of the controllable repair system with warm standbys and working breakdown, *Computers and Industrial Engineering*, Vol. 97, pp.84–92.