Robust H_2/H_∞ DPDC dynamic output feedback controller for an uncertain tensor product model of statically unstable missile

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Abstract: In this paper, the model filtering method is used to optimise the missile model. Based on the tensor product (TP) model transformation method, the uncertain TP model of the statically unstable missile is obtained and its robust variable gain mixed H_2/H_∞ performance dynamic parallel distributed compensation (DPDC) dynamic output feedback controller is designed. Firstly, the nonlinear model of statically unstable missile is optimised by model filtering method. The filtered missile model conforms to the actual engineering requirements, and meets the performance design requirements of H_2. Then, the TP model transformation method is used to convert the nonlinear model of the statically unstable missile obtained after filtering into the TP model. Finally, the gain matrix of the robust variable gain mixed H_2/H_∞ performance dynamic output feedback controller is obtained by solving linear matrix inequalities (LMIs). The simulation results show that the designed DPDC dynamic output feedback controller.

Keywords: TP model; dynamic output feedback control; mixed H_2/H_∞; HOSVD; DPDC; LMIs.

Reference to this paper should be made as follows: He, P., Chen, Z-a., Zhou, Z., Liu, Z. and Qian, D. (2023) 'Robust H_2/H_∞ DPDC dynamic output feedback controller for an uncertain tensor product model of statically unstable missile', *Int. J. Advanced Mechatronic Systems*, Vol. 10, No. 1, pp.33–40.

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1 Introduction

The traditional missiles have the characteristic of statically stability, which ensures the missile has good stability in the flight process, but it also makes the missile lose flexibility. Missiles with statically unstable characteristics gain good maneuverability by sacrificing part of the stability characteristics of the missile's flight process, and statically unstable missiles are easily affected by environmental factors and aerodynamic parameters during flight, so the design of high-performance control systems associated with stabilising missiles becomes particularly important. The robust variable gain mixed performance dynamic output feedback controller has a strong realisation value in the practical engineering control of static unstable missiles. In practical engineering, the state feedback controller will be limited because some states cannot be measured, and the dynamic output feedback controller can well avoid these problems, and it also has good stability and tracking effect (Noge et al., 2021; Ueno et al., 2021).

The common method of converting nonlinear model into linear model is using Jacobi linearisation for model conversion. Qin et al (2011) used Jacobi linearisation method to convert aircraft nonlinear model into linear parameter varying (LPV) model, and combined with tensor product (TP) model conversion method to convert LPV system into polytopic model, and then used H∞ robust control to design gain controller. The TP model has a lot of valuable work in the field of converting the nonlinear model to the linear model. Baranyi et al (2015) first proposed the transformation method of TP model and applied it to the nonlinear model. They also proposed the relationship among TP model, T-S model and polytopic model, and applied parallel distributed compensation (PDC) fuzzy controller to TP model. Zhang et al (2014) converted TP model into a polytopic model to design a variable gain robust controller according to the actual measured state variables. In Tao et al (2014), the parameter uncertainty model is converted into a polytope LPV model and solved the state feedback controller of the model combined with LMIs. The controller design of the uncertain nonlinear model of the statically unstable missile is mostly based on state feedback. In practical engineering, some states of the missile cannot be measured by instruments, so it has become a trend to design the control system based on the output feedback controller. Reza et al (2021) studied the problem of robust switched gain scheduling H2 output feedback (RSGSOF) controller with control convexity constraints for polytopic uncertain continuous time LPV systems. Márcia et al (2021) proposed a new comprehensive condition for static output feedback control of gain scheduling for discrete linear systems with time varying parameters, and extended the stability condition to deal with H∞ control problem. The static output feedback has poor ability to suppress noise interference, poor system robustness, and no dynamic characteristics. Its feedback can only feedback local information. The control system using static output feedback is difficult to achieve global stability of the system. Dynamic output feedback has many advantages over static output feedback, such as all poles can be configured to achieve state feedback performance index problem (Muneomi et al., 2019; Szollosi et al., 2018; Kuti et al., 2017; Deng et al., 2022). In addition, the dynamic parallel distributed compensation (DPDC) dynamic output feedback controller sets a compensator and combines the characteristics of the observer to achieve global feedback. The DPDC dynamic output feedback controller based on the T-S model has always been applied in the field of fuzzy control (Lien et al., 2021; Zhao and Wang, 2019; Hui et al., 2010; Li et al., 2021; Liu et al., 2020). However, there is no relevant research on the application of DPDC dynamic output feedback controller to the TP model of static and unstable missiles.

In the rest of this article, Section 2 introduces the TP model of missile uncertain dynamic model. Section 3 designs a robust variable gain mixed H_2/H_∞-DPDC dynamic output feedback controller. Section 4 shows the simulation results of the control system. Section 5 is the conclusions.

2 Uncertainty TP model of statically unstable missile

This section focuses on the design of the uncertain TP model for statically unstable missiles. Firstly, the nonlinear model of the statically unstable missile is given, and the nonlinear model of the missile is optimised by the model filtering method. Then, based on the filtered nonlinear model of the missile, the nonlinear uncertainty model of the statically unstable missile is further designed. Finally, the TP model conversion method is used to convert the uncertain nonlinear model of the missile into an uncertain TP model.

2.1 Statically unstable missile model

The longitudinal nonlinear model of a statically unstable missile can be described by the following state space expression:

$$\begin{bmatrix}\dot{\alpha}(t)\\ \ddot{\vartheta}(t)\end{bmatrix}=\begin{bmatrix}-b_\alpha(v(t),h(t)) & 1\\ -a_\alpha(v(t),h(t)) & -a_\omega(v(t),h(t))\end{bmatrix}\begin{bmatrix}\alpha(t)\\ \dot{\vartheta}(t)\end{bmatrix}+\begin{bmatrix}-b_\delta(v(t),h(t))\\ -a_\delta(v(t),h(t))\end{bmatrix}[\delta_e(t)]$$
$$[a_y(t)]=[v(t)b_\alpha(v(t),h(t)) \quad 0]\begin{bmatrix}\alpha(t)\\ \dot{\vartheta}(t)\end{bmatrix}+[v(t)b_\delta(v(t),h(t))][\delta_e(t)] \tag{1}$$

Table 1 Explanation of coefficient symbols

Symbol	*Explain*
a_α	Pitching moment caused by angle of attack
a_ω	Pitch damping moment caused by pitch rate
a_δ	Pitching moment caused by the elevator
b_α	Pitch force caused by angle of attack
b_δ	Pitch force caused by the elevator

where, variable parameters $v(t)$ is the flight speed and variable parameters $h(t)$ flight altitude of the missile; the state variables are α and $\dot{\vartheta}$, α is the angle of attack and $\dot{\vartheta}$ is the pitch rate; the input δ_e is the pitch rudder angle; the output a_y is the overload of the missile; b_δ change slightly with $v(t)$ and $h(t)$ which can be regarded as a constant. Due to the missile is statically unstable, the pitching moment caused by angle of attack is less than zero (a_α).

The nonlinear equation (1) can be simplified to equation (2)

$$\begin{aligned}\dot{\boldsymbol{x}}(t)&=\boldsymbol{A}(p(t))\boldsymbol{x}(t)+\boldsymbol{B}(p(t))\boldsymbol{u}(t)\\ \boldsymbol{y}(t)&=\boldsymbol{C}(p(t))\boldsymbol{x}(t)+\boldsymbol{D}(p(t))\boldsymbol{u}(t)\end{aligned} \tag{2}$$

where vector $\boldsymbol{x}(t)=\begin{bmatrix}\alpha(t)\\ \dot{\vartheta}(t)\end{bmatrix}$ is the system state: $\boldsymbol{y}(t) = [a_y(t)]$ is the control output; $\boldsymbol{u}(t) = [\delta_e(t)]$ is the control input; variable parameters $v(t)$ and $h(t)$ can be written as $p(t) = (v(t), h(t))$, $p(t)\in\boldsymbol{\Omega}$ is an N-dimensional variable parameter vector, in the closed interval $\boldsymbol{\Omega} = [a, b] \times\ldots\times [a_N, b_N]\subset \mathbf{R}^N$ The variable parameter $p(t)$ can contain some state variables of the dynamic system, and it can be measured online over time.

$$\boldsymbol{A}(p(t))=\begin{bmatrix}-b_\alpha(v(t),h(t)) & 1\\ -a_\alpha(v(t),h(t)) & -a_\omega(v(t),h(t))\end{bmatrix}$$

$$\boldsymbol{B}(p(t))=\begin{bmatrix}-b_\delta(v(t),h(t))\\ -a_\delta(v(t),h(t))\end{bmatrix}$$

$$\boldsymbol{C}(p(t))=[v(t)b_\alpha(v(t),h(t)) \quad 0]$$

$$\boldsymbol{D}(p(t))=[v(t)b_\delta(v(t),h(t))]$$

2.1.1 Model filtering method

The output a_y of equation (1) is not the overload output measured by the accelerometer of statically unstable missiles in actual engineering. In order to make the missile model in this paper more suitable for practical engineering applications, this section proposes to use the model post-filtering method to optimise the missile model. And the equation (2) obtained by post-filtering also meets the basic requirements of the H2 performance controller design for the model, that is, the straight-through term D of the state space is zero, G(s) = (A, B, C, D = 0). Model filtering method has two filtering methods, pre-filtering and post-filtering. Prefiltering needs to define a new control input $\tilde{\boldsymbol{u}}$, postfiltering needs to define a new measurement output $\tilde{\boldsymbol{y}}$ (Apkarian et al., 1995). An ideal model can be obtained by using the model filtering method to meet the design requirements of the controller. Model pre-filtering method is to define a new control output $\tilde{\boldsymbol{u}}$.

$$\begin{aligned}\dot{\boldsymbol{x}}_u&=\boldsymbol{A}_u\boldsymbol{x}_u+\boldsymbol{B}_u\tilde{\boldsymbol{u}}\\ \boldsymbol{u}&=\boldsymbol{C}_u\boldsymbol{x}_u\end{aligned} \tag{3}$$

Model post-filtering is to define a new measurement output $\tilde{\boldsymbol{y}}$.

$$\begin{aligned}\dot{\boldsymbol{x}}_y&=\boldsymbol{A}_y\boldsymbol{x}_y+\boldsymbol{B}_y\boldsymbol{y}\\ \tilde{\boldsymbol{y}}&=\boldsymbol{C}_y\boldsymbol{x}_y\end{aligned} \tag{4}$$

where $\boldsymbol{A}_u$ and $\boldsymbol{A}_y$ are the state matrices of the filtered model, which are obtained by pre-filtering and post-filtering transformation of matrix $\boldsymbol{A}$, respectively. If the statically unstable missile equation (1) is pre-filtered, it is equivalent to adding a limiter to limit the output feedback of the controller in practical engineering. In this paper, the dynamic output feedback controller is mainly designed for statically unstable missiles. Therefore, only the post-model filtering method is needed to optimise the model, so that the model output state space expression meets the design requirements of the subsequent control system and meets the needs of practical engineering applications. The pre-model filtering will not be described too much for the time being. Combined with the equations (2) and (4), the filtered equation (5) can be obtained

$$\begin{bmatrix}\mathbf{x}\\ \dot{\mathbf{x}}_y\end{bmatrix}=\begin{bmatrix}\mathbf{A}(p(t)) & \mathbf{0}\\ \mathbf{B}_y\mathbf{C}(p(t)) & \mathbf{A}_y\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \mathbf{x}_y\end{bmatrix}+\begin{bmatrix}\mathbf{B}(p(t))\\ \mathbf{B}_y\mathbf{D}(p(t))\end{bmatrix}\mathbf{u}$$
$$\tilde{\mathbf{y}}=[\mathbf{0} \quad \mathbf{C}_y]\begin{bmatrix}\mathbf{x}\\ \mathbf{x}_y\end{bmatrix} \tag{5}$$

Let $\bar{\mathbf{x}} = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_y \end{bmatrix}$, $\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A}(p(t)) & \mathbf{0} \\ \mathbf{B}_y\mathbf{C}(p(t)) & \mathbf{A}_y \end{bmatrix}$, $\bar{\mathbf{B}} = \begin{bmatrix} \mathbf{B}(p(t)) \\ \mathbf{B}_y\mathbf{D}(p(t)) \end{bmatrix}$, $\bar{\mathbf{C}} = [0 \quad \mathbf{C}_y]$, equation (5) can be written as.

$$\begin{aligned} \dot{\bar{\mathbf{x}}} &= \bar{\mathbf{A}}(p(t))\bar{\mathbf{x}} + \bar{\mathbf{B}}(p(t))\mathbf{u} \\ \tilde{\mathbf{y}} &= \bar{\mathbf{C}}\bar{\mathbf{x}} \end{aligned} \tag{6}$$

2.2 Statically unstable missile TP uncertainty model

Based on equation (6), considering static instability missile uncertainty model as shown in equation (3).

$$\begin{aligned} \dot{\bar{\boldsymbol{x}}} &= \boldsymbol{A}_1(p(t))\bar{\boldsymbol{x}}(t) + \boldsymbol{B}_1(p(t))\boldsymbol{\omega}(t) + \boldsymbol{B}_2(p(t))\boldsymbol{u}(t) \\ \boldsymbol{z}_1 &= \boldsymbol{C}_1(p(t))\bar{\boldsymbol{x}}(t) + \boldsymbol{D}_{11}(p(t))\boldsymbol{\omega}(t) + \boldsymbol{D}_{12}(p(t))\boldsymbol{u}(t) \\ \boldsymbol{z}_2 &= \boldsymbol{C}_2(p(t))\bar{\boldsymbol{x}}(t) + \boldsymbol{D}_{21}(p(t))\boldsymbol{\omega}(t) + \boldsymbol{D}_{22}(p(t))\boldsymbol{u}(t) \\ \tilde{\boldsymbol{y}} &= \boldsymbol{C}_3(p(t))\bar{\boldsymbol{x}}(t) + \boldsymbol{D}_{31}(p(t))\boldsymbol{\omega}(t) \end{aligned} \tag{7}$$

$\bar{\mathbf{x}}(t) = \begin{bmatrix} \alpha(t) \\ \dot{\vartheta}(t) \\ x_y(t) \end{bmatrix}$ is the state variable; uncertainty input considered $\boldsymbol{\omega} = \begin{bmatrix} \omega_{ay} \\ \omega_{dn} \end{bmatrix}$ is disturbance produced by measuring instrument; $\tilde{\boldsymbol{y}}$ is the measured output; $\mathbf{z}_1$ is the control output under performance index; $\mathbf{z}_2$ is the control output under H_∞ performance index.

where,

$$\mathbf{A} = \bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A}(\mathbf{p}(t)) & \mathbf{0} \\ \mathbf{B}_y\mathbf{C}(\mathbf{p}(t)) & \mathbf{A}_y \end{bmatrix}, \mathbf{B}_1 = [0_{3\times 2}],$$

$$\mathbf{B}_2 = \bar{\mathbf{B}} = \begin{bmatrix} \mathbf{B}(\mathbf{p}(t)) \\ \mathbf{B}_y\mathbf{D}(\mathbf{p}(t)) \end{bmatrix}; \mathbf{C}_1 = -[v(t)b_\alpha(\mathbf{p}(t)) \quad 0 \quad 0],$$

$$\mathbf{D}_{11} = [1 \quad -1], \mathbf{D}_{12} = -[v(t)b_\beta(\mathbf{p}(t))];$$

$$\mathbf{C}_2 = -[v(t)b_\alpha \quad 0 \quad 0], \mathbf{D}_{21} = [1 \quad -1],$$

$$\mathbf{D}_{22} = -[v(t)b_\beta(\mathbf{p}(t))]; \mathbf{C}_3 = -\bar{\mathbf{C}} = -[0 \quad 0 \quad \mathbf{C}_y],$$

$$\mathbf{D}_{31} = [0 \quad -1].$$

2.2.1 Design of TP uncertain model for statically unstable missile

According to equation (7), the system matrix (8) of the uncertainty model for statically unstable missiles is defined.

$$\mathbf{S}(p(t)) = \begin{pmatrix} \mathbf{A}_1(p(t)) & \mathbf{B}_1(p(t)) & \mathbf{B}_2(p(t)) \\ \mathbf{C}_1(p(t)) & \mathbf{D}_{11}(p(t)) & \mathbf{D}_{12}(p(t)) \\ \mathbf{C}_2(p(t)) & \mathbf{D}_{21}(p(t)) & \mathbf{D}_{22}(p(t)) \\ \mathbf{C}_3(p(t)) & \mathbf{D}_{31}(p(t)) & \mathbf{0} \end{pmatrix} \tag{8}$$

The equation (8) can be transformed into equation (9) by using the TP transformation method. The uncertain TP equation (9) of the statically unstable missile is composed of a few vertex systems and the weighted combination of the weight function.

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{z}_1(t) \\ \mathbf{z}_1(t) \\ \tilde{\mathbf{y}}(t) \end{pmatrix} \underset{\varepsilon}{\approx} \bar{\wp}^n \bigotimes_{n=1}^{N} \bar{\mathbf{w}}_{n(p_n(t))} \begin{pmatrix} \bar{\mathbf{x}}(t) \\ \boldsymbol{\omega}(t) \\ \mathbf{u}(t) \end{pmatrix} \tag{9}$$

The TP transformation method can be divided into three steps.

- First: The variation range of the variable parameters $p(t)$ is gridded. Select the desired work area for variable parameters and set it as parameter space $\boldsymbol{\Omega} = [a_1, b_1] \times \ldots \times [a_N, b_N] \subset \mathbf{R}^N$, defining a rectangular sampling grid $P_{g_1,g_2,\ldots,g_N} = [p_{1g_1} \quad p_{2g_2} \quad \cdots \quad p_{Ng_N}]^T$ using the equal division method.
- Second: Sample the system equation (8) at each grid point. Compute the linear time invariant (LTI) model of the system matrix at each grid point $\mathbf{S}(p_{ig_i})$, and store all LTI models in a system tensor of order N + 2 $\wp^s(g_1, g_2, \ldots, g_N, :, :) = \mathbf{S}(P_{g_1,g_2,\ldots,g_N})$.
- Third: Perform a higher order singular value decomposition (HOSVD) of the system tensors $\wp^s$. The purpose is to extract the matrix storing the singular value vectors related to the weight function $\boldsymbol{w}_n(P_n(t)$, and the vertex system matrix $\mathbf{S}_{I_1}, \mathbf{S}_{I_2}, \ldots, \mathbf{S}_{I_n}$ of the system equation (9). Perform HOSVD on the tensor $\wp^s$ to get $\wp^s = \wp^n \bigotimes_{n=1}^{N} [\mathbf{U}_n \quad \mathbf{U}_n^d]$, where $\wp^n$ is the core tensor containing the vertex system matrix of the system matrix. Matrix $\boldsymbol{U}_n$ stores the vectors of singular values greater than zero, and matrix $\mathbf{U}_n^d$ stores the remaining vectors of singular values that can be discarded. The HOSVD result for a tensor $\wp^s$ can also be written approximately as $\wp^s \approx \wp^n \bigotimes_{n=1}^{N} \mathbf{U}_n$ .The sampled value of the weight function $\boldsymbol{w}_n(P_n(t))$ at the grid point is defined as the g_N row of matrix $\boldsymbol{U}_n$.
- Fourth: The results obtained by HOSVD are subjected to convex normalisation. The weight function $\boldsymbol{w}_n(P_n(t))$ must meet the normalisation requirements:

$$\left\{ \begin{aligned} \forall n,\ p_n(t) : \sum_{r=1}^{I_n} w_{n,r}(p_n(t)) = 1, \forall r, n, \\ p_n(t) : w_{n,r}(p_n(t)) \geq 0 \end{aligned} \right\}$$

Baranyi (2014) The convex normalisation of TP model is explained in detail, which is not repeated in this paper. The normalised system matrix can be expressed as $\wp^s \approx \bar{\wp}^n \bigotimes_{n=1}^{N} \bar{\mathbf{w}}_{n(p_n(t))}$, the uncertain TP model of the statically unstable missile obtained by the final conversion is shown in equation (9). Where ε is the approximate error generated by the TP model transformation method $\left\| \mathbf{S}(p(t)) - \bar{\wp}^n \bigotimes_{n=1}^{N} \bar{\mathbf{w}}_n(p_n(t)) \right\| \leq \varepsilon$.

3 Design of H_2/H_∞–DPDC dynamic output feedback controller

The LTI model expression of each vertex of the statically unstable missile TP uncertainty model is shown in equation (10).

$$\begin{aligned} \dot{\bar{\mathbf{x}}} &= \mathbf{A}_{1,i}\bar{\mathbf{x}}(t) + \mathbf{B}_{1,i}\boldsymbol{\omega}(t) + \mathbf{B}_{2,i}\mathbf{u}(t) \\ \mathbf{z}_1 &= \mathbf{C}_{1,i}\bar{\mathbf{x}}(t) + \mathbf{D}_{11,i}\boldsymbol{\omega}(t) + \mathbf{D}_{12,i}\mathbf{u}(t) \\ \mathbf{z}_2 &= \mathbf{C}_{2,i}\bar{\mathbf{x}}(t) + \mathbf{D}_{21,i}\boldsymbol{\omega}(t) + \mathbf{D}_{22,i}\mathbf{u}(t) \\ \tilde{\mathbf{y}} &= \mathbf{C}_{3,i}\bar{\mathbf{x}}(t) + \mathbf{D}_{31,i}\boldsymbol{\omega}(t), \qquad (i = 1, \ldots, N) \end{aligned} \tag{10}$$

The corresponding H_2/H_∞-DPDC dynamic output feedback controller for each vertex model $\mathbf{S}_{I_n}$ is expressed as:

$$\begin{aligned} \dot{\mathbf{x}}_c(t) &= \mathbf{A}_{c,i}\mathbf{x}_c(t) + \mathbf{B}_{c,i}\tilde{\mathbf{y}}(t) \\ \mathbf{u}(t) &= \mathbf{C}_{c,i}\mathbf{x}_c(t) + \mathbf{D}_{c,i}\tilde{\mathbf{y}}(t); \qquad i = 1, \cdots, N. \end{aligned} \tag{11}$$

$\boldsymbol{A}_{c,i}$, $\boldsymbol{B}_{c,i}$ and $\boldsymbol{C}_{c,i}$ is the controller gain matrices, Combining the vertex LTI equation (10) and the equation (11), the augmented system for each vertex is:

$$\begin{aligned} \dot{\boldsymbol{x}}_{cl}(t) &= \boldsymbol{A}_{cl,i}\boldsymbol{x}_{cl}(t) + \boldsymbol{B}_{cl,i}\boldsymbol{\omega}(t) \\ \boldsymbol{z}_1 &= \boldsymbol{C}^1_{cl,i}\boldsymbol{x}_{cl}(t) + \boldsymbol{D}^1_{cl,i}\boldsymbol{\omega}(t) \\ \boldsymbol{z}_2 &= \boldsymbol{C}^2_{cl,i}\boldsymbol{x}_{cl}(t) + \boldsymbol{D}^2_{cl,i}\boldsymbol{\omega}(t) \end{aligned} \tag{12}$$

The augmented system matrix is:

$$\begin{aligned} \boldsymbol{A}_{cl,i} &= \begin{bmatrix} \boldsymbol{A}_i + \boldsymbol{B}_{2,i}\boldsymbol{D}_{c,i}C_{3,i} & \boldsymbol{B}_{2,i}C_{c,i} \\ \boldsymbol{B}_{c,i}C_{3,i} & \boldsymbol{A}_{c,i} \end{bmatrix} \\ \boldsymbol{B}_{cl,i} &= \begin{bmatrix} \boldsymbol{B}_{1,i} + \boldsymbol{B}_{2,i}\boldsymbol{D}_{c,i}\boldsymbol{D}_{31,i} \\ \boldsymbol{B}_{c,i}\boldsymbol{D}_{31,i} \end{bmatrix} \\ \boldsymbol{C}^1_{cl,i} &= \begin{bmatrix} \boldsymbol{C}_{1,i} + \boldsymbol{D}_{12,i}\boldsymbol{D}_{c,i}\boldsymbol{C}_{3,i} & \boldsymbol{D}_{12,i}\boldsymbol{C}_{c,i} \end{bmatrix} \\ \boldsymbol{D}^1_{cl,i} &= \boldsymbol{D}_{11,i} + \boldsymbol{D}_{12,i}\boldsymbol{D}_{c,i}\boldsymbol{D}_{31,i} \\ \boldsymbol{C}^2_{cl,i} &= \begin{bmatrix} \boldsymbol{C}_{2,i} + \boldsymbol{D}_{22,i}\boldsymbol{D}_{c,i}\boldsymbol{C}_{3,i} & \boldsymbol{D}_{22,i}\boldsymbol{C}_{c,i} \end{bmatrix} \\ \boldsymbol{D}^2_{cl,n} &= \boldsymbol{D}_{21,n} + \boldsymbol{D}_{22,n}\boldsymbol{D}_{k,n}\boldsymbol{D}_{31,n} \end{aligned} \tag{13}$$

The augmented system state vector is: .

Lemma 1: (Jiang et al., 2016)

1 $(\boldsymbol{A}_{1,,i}, \boldsymbol{B}_{2,i})$ has stabilisability, $(\boldsymbol{C}_{2,i}, \boldsymbol{A}_{1,i})$ is detectable, which is established

2 the condition for the γ_1 optimal H_2 control problem to be solved is that there are matrices $\mathbf{P} = \mathbf{P}^{-1}$, $\mathbf{P} > 0$ and $\mathbf{W} = \mathbf{W}^T$ satisfying the following conditions.

$$\begin{bmatrix} \boldsymbol{P}\boldsymbol{A}_{cl,i} + \boldsymbol{A}^T_{cl,i}\boldsymbol{P} & \left(\boldsymbol{C}^2_{cl,i}\right)^T \\ \boldsymbol{C}^2_{cl,i} & -\boldsymbol{I} \end{bmatrix} < 0, i = 1, 2, \ldots N \tag{14}$$

$$\begin{bmatrix} \boldsymbol{W} & \boldsymbol{B}^T_{cl,i}\boldsymbol{P} \\ \boldsymbol{P}\boldsymbol{B}_{cl,i} & \boldsymbol{P} \end{bmatrix} < 0 \tag{15}$$

$$Tr(\mathbf{W}) < \gamma_1^2 \tag{16}$$

Lemma 2: (Jiang et al., 2016) (Bounded real lemma) Let the continuous-time transfer function matrix be $\mathbf{G}(s) = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$, then for a given positive real number γ_2, the following statements are equivalent:

1 the matrix $\mathbf{A}$ is stable, and $\|\boldsymbol{G}(s)\|_\infty < \gamma_2$

(2) There is a positive definite symmetric matrix $\mathbf{P} = \mathbf{P}^{-1}$, $\mathbf{P} > 0$, so that LMI (17) is established.

$$\begin{bmatrix} \boldsymbol{A}^T_{cl,i}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_{cl,i} & \boldsymbol{P}\boldsymbol{B}_{cl,i} & \left(\boldsymbol{C}^1_{cl,i}\right)^T \\ \boldsymbol{B}^T_{cl,i}\boldsymbol{P} & -\gamma_1 I & \left(\boldsymbol{D}^1_{cl,i}\right)^T \\ \boldsymbol{C}^1_{cl,i} & \boldsymbol{D}^1_{cl,i} & -\gamma_2\boldsymbol{I} \end{bmatrix} < 0 \tag{17}$$

Theorem 1: There exists a dynamic output feedback controller (11) such that the closed-loop control system (12) is robust asymptotically stable. The H_∞ performance index of each vertex closed-loop control system with TP uncertain model is γ_2. There exists a common positive definite symmetric matrix $\mathbf{P} = \mathbf{P}^{-1}$, $\mathbf{P} > 0$ and a common symmetric matrix $\mathbf{W} = \mathbf{W}^T$ such that H_2 optimal γ_1 control can satisfy the following LMIs necessary and sufficient conditions on all vertex closed-loop control systems with TP uncertain model.

$$\begin{bmatrix} \boldsymbol{A}^T_{cl,i}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_{cl,i} & \boldsymbol{P}\boldsymbol{B}_{cl,i} & \left(\boldsymbol{C}^1_{cl,i}\right)^T \\ \boldsymbol{B}^T_{cl,i}\boldsymbol{P} & -\gamma_1\boldsymbol{I} & \left(\boldsymbol{D}^1_{cl,i}\right)^T \\ \boldsymbol{C}^1_{cl,i} & \boldsymbol{D}^1_{cl,i} & -\gamma_2\boldsymbol{I} \end{bmatrix} < 0, i = 1, 2, \ldots N \tag{18}$$

$$\begin{bmatrix} \boldsymbol{P}\boldsymbol{A}_{cl,i} + \boldsymbol{A}^T_{cl,i}\boldsymbol{P} & \left(\boldsymbol{C}^2_{cl,i}\right)^T \\ \boldsymbol{C}^2_{cl,i} & -\boldsymbol{I} \end{bmatrix} < 0, i = 1, 2, \ldots N \tag{19}$$

$$\begin{bmatrix} \boldsymbol{W} & \boldsymbol{B}^T_{cl,i}\boldsymbol{P} \\ \boldsymbol{P}\boldsymbol{B}_{cl,i} & \boldsymbol{P} \end{bmatrix} < 0, i = 1, 2, \ldots N \tag{20}$$

$$Tr(\boldsymbol{W}) < \gamma_1^2 \tag{21}$$

Let $He(\mathbf{A}) = \mathbf{A} + \mathbf{A}^T$, and defining matrix $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$: $\mathbf{\Pi}_1 = \begin{bmatrix} \mathbf{X} & \mathbf{I} \\ \mathbf{M}^T & \mathbf{0} \end{bmatrix}$, $\mathbf{\Pi}_2 = \mathbf{P}_{cl}\mathbf{\Pi}_1\begin{bmatrix} \mathbf{I} & \mathbf{Y} \\ \mathbf{0} & \mathbf{N}^T \end{bmatrix}$, $\mathbf{X} = \mathbf{X}^T$, $\mathbf{Y} = \mathbf{Y}$, $\mathbb{A}_i$, $\mathbb{B}_i$, $\mathbb{C}_i$ and $\mathbb{D}_i$. The definition of variable $\mathbb{A}_i$, $\mathbb{B}_i$, $\mathbb{C}_i$ and $\mathbb{D}_i$ are as follows:

$$\begin{aligned} \mathbb{A}_i &= \mathbf{N}\mathbf{A}_{c,i}\mathbf{M}^T + \mathbf{N}\mathbf{B}_{c,i}\mathbf{C}_{2,i}\mathbf{X} + \mathbf{Y}\mathbf{B}_{2,i}\mathbf{C}_{c,i}\mathbf{M}^T \\ &\quad + \mathbf{Y}(\mathbf{A}_i + \mathbf{B}_{2,i}\mathbf{D}_{c,i}\mathbf{C}_{3,i})\mathbf{X} \\ \mathbb{B}_n &= \mathbf{N}\mathbf{B}_{c,i} + \mathbf{Y}\mathbf{B}_{2,i}\mathbf{D}_{c,i} \\ \mathbb{C}_n &= \mathbf{C}_{c,i}\mathbf{M}^T + \mathbf{D}_{c,i}\mathbf{C}_{2,i}\mathbf{X} \\ \mathbb{D}_n &= \mathbf{D}_{c,i} \qquad (i = 1, 2, \ldots, N) \end{aligned} \tag{22}$$

By changing equations (18), (19) and (20) with the substitution method, we can finally get:

$$
He\left\{\begin{bmatrix} \mathbf{A}_i\mathbf{X}+\mathbf{B}_{2,i}\mathbb{C}_i & \mathbf{A}_i+\mathbf{B}_{2,i}\mathbb{D}_i\mathbf{C}_{3,i} & \mathbf{B}_{1,i}+\mathbf{B}_{2,i}\mathbb{D}_i\mathbf{D}_{31,i} & 0 \\ \mathbb{A}_i & \mathbf{Y}\mathbf{A}_i+\mathbb{B}_i\mathbf{C}_{3,i} & \mathbf{Y}\mathbf{B}_{1,i}+\mathbb{B}_i\mathbf{D}_{31,i} & 0 \\ 0 & 0 & -(\gamma_1/2)\mathbf{I} & 0 \\ \mathbf{C}_{1,i}\mathbf{X}+\mathbf{D}_{12,i}\mathbb{C}_i & \mathbf{C}_{1,i}+\mathbf{D}_{12,i}\mathbb{D}_i\mathbf{C}_{3,i} & \mathbf{D}_{11,i}+\mathbf{D}_{12,i}\mathbb{D}_i\mathbf{D}_{31,i} & -(\gamma_2/2)\mathbf{I} \end{bmatrix}\right\}<0 \tag{23}
$$

$$
\begin{bmatrix} \mathbf{X} & \mathbf{I} \\ \mathbf{I} & \mathbf{Y} \end{bmatrix}>0 \tag{24}
$$

$$
He\left\{\begin{bmatrix} \mathbf{A}_i\mathbf{X}+\mathbf{B}_{2,i}\mathbb{C}_i & \mathbf{A}_i+\mathbf{B}_{2,i}\mathbb{D}_i\mathbf{C}_{3i} & \mathbf{0} \\ \mathbb{A}_i & \mathbf{Y}\mathbf{A}_i+\mathbb{B}_i\mathbf{C}_{3,i} & \mathbf{0} \\ \mathbf{C}_{2,i}\mathbf{X}+\mathbf{D}_{22,i}\mathbb{C}_i & \mathbf{C}_{2,i}+\mathbf{D}_{22,i}\mathbb{D}_i\mathbf{C}_{3,i} & -(\gamma_1/2)\mathbf{I} \end{bmatrix}\right\}<0 \tag{25}
$$

$$
\begin{bmatrix} \mathbf{W} & \mathbf{B}_{1,i}^T+\left(\mathbf{B}_{2,i}\mathbb{D}_i\mathbf{D}_{31,i}\right)^T & \left(\mathbf{Y}\mathbf{B}_{1,i}+\mathbb{B}_i\mathbf{D}_{31,i}\right)^T \\ \mathbf{B}_{1,i}+\mathbf{B}_{2,i}\mathbb{D}_i\mathbf{D}_{31,i} & \mathbf{X} & \mathbf{I} \\ \mathbf{Y}\mathbf{B}_{1,i}+\mathbb{B}_i\mathbf{D}_{31,i} & \mathbf{I} & \mathbf{Y}v \end{bmatrix}>0 \tag{26}
$$

$$
Tr(\mathbf{W})<\gamma_1^2 \tag{27}
$$

From LMIs (24) and (26), $\begin{bmatrix} \mathbf{X} & \mathbf{I} \\ \mathbf{I} & \mathbf{Y} \end{bmatrix}>0$ can be obtained, which is equivalent to $\mathbf{XY–I} > 0$, $\mathbf{I\text{-}XY} = \mathbf{MN}^T$. The matrices M and N can be obtained using QR decomposition. After solving the above LMIs, the gain matrix $\mathbf{A}_{c,i}$, $\mathbf{B}_{c,i}$, $\mathbf{C}_{c,i}$ and $\mathbf{D}_{c,i}$ of the controller can be obtained.

$$
\begin{aligned}
\mathbf{A}_{c,i} &= \mathbf{N}^{-1}\begin{pmatrix} \mathbb{A}_i-\mathbf{N}\mathbf{B}_{c,i}\mathbf{C}_{2,i}\mathbf{X}-\mathbf{Y}\mathbf{B}_{2,i}\mathbf{C}_{c,i}\mathbf{M}^T \\ -\mathbf{Y}(\mathbf{A}+\mathbf{B}_{2,i}\mathbf{D}_{c,i}\mathbf{C}_{3,i})\mathbf{X} \end{pmatrix}(\mathbf{M}^T)^{-1} \\
\mathbf{B}_{c,i} &= \mathbf{N}^{-1}\left(\mathbb{B}_i-\mathbf{Y}\mathbf{B}_{2,i}\mathbf{D}_{c,i}\right) \\
\mathbf{C}_{c,i} &= \left(\mathbb{C}_i-\mathbf{D}_{c,i}\mathbf{C}_{2,i}\mathbf{X}\right)(\mathbf{M}^T)^{-1} \\
\mathbf{D}_{c,i} &= \mathbb{D}_i \qquad (i=1,2,\ldots N)
\end{aligned}
$$

The H_2/H_∞-DPDC dynamic output feedback controller designed in this paper can be written in the form of a TP, as follows.

$$
\begin{aligned}
\dot{\mathbf{x}}_c(t) &= \bar{\mathrm{A}}_c^n \bigotimes_{n=1}^{N} \bar{\mathbf{w}}_n(p_n(t))\mathbf{x}_c(t)+\bar{\mathrm{B}}_c^n \bigotimes_{n=1}^{N} \bar{\mathbf{w}}_n(p_n(t))\tilde{\mathbf{y}}(t) \\
\mathbf{u}(t) &= \bar{\mathrm{C}}_c^n \bigotimes_{n=1}^{N} \bar{\mathbf{w}}_n(p_n(t))\mathbf{x}_c(t)+\bar{\mathrm{D}}_c^n \bigotimes_{n=1}^{N} \bar{\mathbf{w}}_n(p_n(t))\tilde{\mathbf{y}}(t)
\end{aligned} \tag{28}
$$

4 Numerical simulation and analysis

Assuming that the statically unstable missile is in a landing state, the range of flight altitude is $h\in[5000\sim0]$ m and the range of flight speed is $v\in$ [900~1200] m/s. Set the initial speed of the missile to be v_0 = 900 m/s and the initial flight altitude to be h_0 = 5000 m. The flight envelope composed of variable parameter height $h(t)$ and velocity $v(t)$ can be equally divided into 100 × 100 grids, denoted by closed space $\mathbf{\Omega} = [5000, 900]_1 \times\ldots\times [0, 1200]_{100\times100}$. According to the closed space, the LTI models of 100 × 100 missiles can be calculated. All LTI system models are composed of tensor $\wp^s(v, h) = \mathbf{S}_{1,2,\ldots,100\times100}$, and the HOSVD of tensor $\wp^s$ is carried out to obtain the vertex system and weight function. The TP algebraic form of the statically unstable missile uncertainty equation (7) is shown in equation (29).

$$
\begin{aligned}
\dot{\bar{\mathbf{x}}}(t) &\approx \sum_{i=1}^{N}\bar{w}_i(\mathbf{p}(t))\left(\mathbf{A}_i\bar{\mathbf{x}}(t)+\mathbf{B}_{1,i}\boldsymbol{\omega}(t)+\mathbf{B}_{2,i}\mathbf{u}(t)\right) \\
\mathbf{z}_1(t) &\approx \sum_{i=1}^{N}\bar{w}_i(\mathbf{p}(t))\left(\mathbf{C}_{1,i}\bar{\mathbf{x}}(t)+\mathbf{D}_{11,i}\boldsymbol{\omega}(t)+\mathbf{D}_{12,i}\mathbf{u}(t)\right) \\
\mathbf{z}_2(t) &\approx \sum_{i=1}^{N}\bar{w}_i(\mathbf{p}(t))\left(\mathbf{C}_{2,i}\bar{\mathbf{x}}(t)+\mathbf{D}_{21,i}\boldsymbol{\omega}(t)+\mathbf{D}_{22,i}\mathbf{u}(t)\right) \\
\tilde{\mathbf{y}}(t) &\approx \sum_{i=1}^{N}\bar{w}_i(\mathbf{p}(t))(\mathbf{C}_{3,i}\bar{\mathbf{x}}(t)+\mathbf{D}_{31,i}\boldsymbol{\omega}(t)),\ N=6
\end{aligned} \tag{29}
$$

The disturbance in the uncertain model is divided into two parts, ω_{ay} is the measurement noise of overload in the pitching direction, ω_{dn} is the measurement noise of angular velocity in the pitching direction. According to the method of the third section, the dynamic output feedback gain of each vertex model can be obtained. Combined with the weight function obtained in the second section, the robust H_2/H_∞-DPDC dynamic output feedback controller based on TP model can be obtained. The designed controller is simulated and the simulation results are shown as follows.

Figure 1 Schematic diagram of weight function of missile uncertain TP model (see online version for colours)

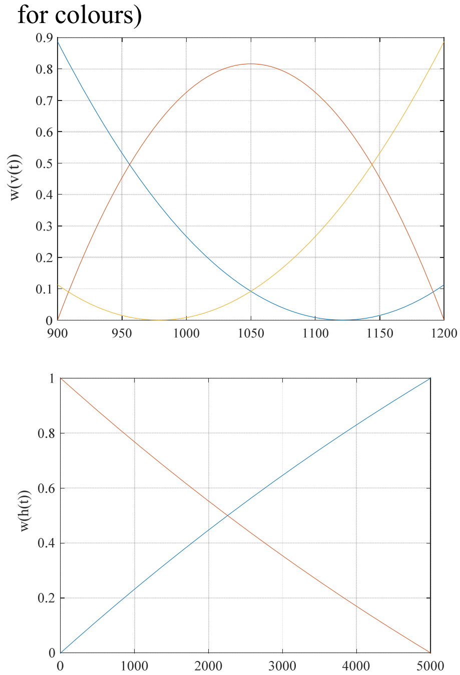


Figure 2 Dynamic curve of angle of attack α affected by disturbance

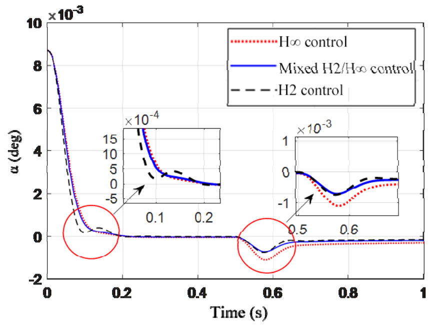


Figure 3 Dynamic curve of pitch rate $\dot{\vartheta}$ affected by disturbance (see online version for colours)

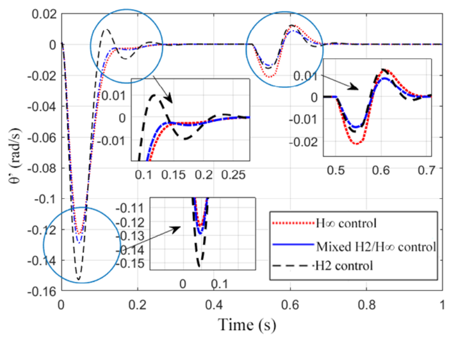
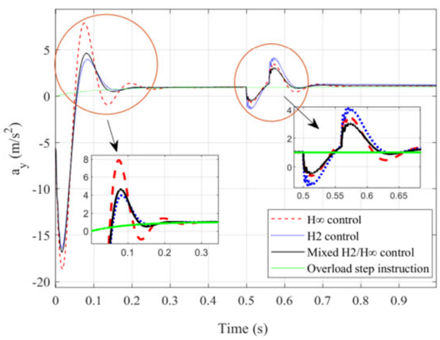


Figure 4 Dynamic curve of overload a_y affected by disturbance



The initial angle of attack of is $\alpha = \pi*0.5/180$. The initial pitch rate is $\dot{\vartheta} = 0$. The input square wave is used as the overload interference of the system to test the robustness, stability and anti-interference ability of the system. The longitudinal overload input instruction is input by unit step signal. The above three MATLAB numerical simulation results show that the mixed H_2/H_∞ robust variable gain dynamic output feedback controller combines the advantages of H_∞ performance and H2 performance controller. The designed robust variable gain control system has good robustness, can achieve stability in 0.2 seconds. Low overshoot and strong anti-interference ability in 0.5 to 0.7 seconds. Under the effect of interference input, the output fluctuation of the control system is smaller and smoother, and the control system itself maintains good command tracking.

5 Conclusions

Aiming at the strong nonlinear, time-varying and uncertain characteristics of statically unstable missile model, the missile model is converted into TP model by TP transformation method. In order to make the missile model more suitable for practical engineering application, the model filtering method is used to optimise the missile model, and the optimised missile model meets the basic requirements of the model state space in the subsequent control system design. The controller designed in this paper has strong practicability, which avoids the problem that the state feedback controller cannot measure part of the state in practical application. The DPDC dynamic output feedback controller used only in fuzzy systems is extended to TP model. A robust variable gain mixed H_2/H_∞-DPDC dynamic output feedback controller based on static unstable missile TP uncertain model is designed. The global stability control and anti-interference control of the closed-loop control system can be realised by weighted combination of vertex controller gain matrix. The response speed is fast and the robustness is good.

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