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Akira Inoue, Takao Sato, Akira Yanou, Tomohiro Henmi

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Suppress of effects in steady state of disturbance and parameter deviations of generalised minimum variance control

Akira Inoue*

Okayama University,
Okayama 700-0002, Japan
Email: inoue-09@t.okadai.jp
*Corresponding author

Takao Sato

Graduate School of Engineering,
University of Hyogo,
Himeji 671-2280, Japan
Email: tsato@eng.u-hyogo.ac.jp

Akira Yanou and Tomohiro Henmi

Faculty of Health Science and Technology,
Kawasaki University of Medical Welfare,
Kurashiki 701-0193, Japan
Email: yanou-a@mw.kawasaki-m.ac.jp
Email: henmi@mw.kawasaki-m.ac.jp

Abstract: This paper proposes a control scheme to suppress the effects in steady state on output caused by unknown disturbances to input and output and deviations of parameters in transfer functions. The scheme is based on generalised minimum variance control (GMVC). In many cases, the disturbances and parameter changes are caused by frictions, backlash, payload changes, parts replacement or aged deterioration and they are slowly changing, such as step-wise or ramp-wise. Hence in this paper, the disturbances and parameter changes are supposed to be slowly changing. First conditions to suppress effects on output of disturbances and parameter changes are obtained. Then the controller to suppress such effects are obtained by selecting parameters to satisfy the conditions of Yola-Kucera generalised stabilising controller. Numerical simulations of a model of two-degree of freedom fourth order system are given to show the effectiveness of the proposed controller.

Keywords: generalised minimum variance control; GMVC; disturbances on input and output; parameter deviations; Youla-Kucela generalised stabilising controller.

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Biographical notes: Akira Inoue has received his PhD from the Department of Applied Mathematics and Physics, Kyoto University, Kyoto, Japan. He retired from Okayama University in 2009 and became Professor Emeritus, Okayama University. He is a Fellow of SICE, senior life member of IEEE, senior member of ISCIE and IEEJ and a life-time member of RSJ. His research areas are in adaptive control, nonlinear control of mechanical systems, process control and safety of plants.

Takao Sato received his PhD in Engineering from the Okayama University, Japan in 2002. He was a Junior Fellow of New Energy and Industrial Technology Development Organization from 2001 to 2002. He joined Graduate School of Engineering, University of Hyogo at 2002, became an Associate Professor at 2008 and was a Professor at 2021. His research area is mainly PID control, multi-rate control system and their application to mechatronics. He is a senior member of IEEJ and a member of IEEE, SICE and ISCIE.

Akira Yanou received his PhD in Engineering from Okayama University, Japan in 2001. He worked with School of Engineering, Kinki University from 2002 to 2008, Graduate School of Natural Science and Technology, Okayama University from 2009 to 2016, and Kawasaki College of Allied Health Professions in 2016. In 2017, he joined the Faculty of Health Science and Technology, Kawasaki University of Medical Welfare, as an Associate Professor and was a Professor at 2021. His research areas are in adaptive control, strong stability systems and estimation of process parameters.

Tomohiro Henmi received his PhD in Engineering from Okayama University, Japan in 2005. He worked with National Institute of Technology, Kagawa College from 2005 to 2019. In 2019, he joined the Faculty of Health Science and Technology, Kawasaki University of Medical Welfare, as an Associate Professor. His research areas are in adaptive control and control of mechatronic systems.

1 Introduction

Mechanical systems are frequently corrupted by unknown disturbances such as frictions or backlash and parameter changes caused by aged deterioration. To obtain high-precision positioning control, it is crucially important to suppress the effects of such unknown disturbances and parameter chances.

For this purpose, generalised minimum variance control (GMVC) (Clarke and Gawthrop, 1979) is suitable by the ability to control unstable and non-minimum phase plants by using the generalised output. Also, GMVC has a simple structure comparing to generalised predictive controller (GPC), that is, GMVC needs to solve only one Diophantine polynomial equation, whereas, GPC has several Diophantine equations. By these reasons, GMVC is applied in industry (Ramos et al., 2004; Fusco and Russo, 2006; Laurinda et al., 2007) and to mechanical systems. The controller of this paper is based on GMVC.

A method to reduce the effects of disturbance and plant parameter deviations from nominal values is to treat these deviations as plant uncertainty and there are many papers to discuss the uncertainty as the robust control (Doyle et al., 1990). But most of papers of robust control consider the conditions for stability of plants including uncertainty and do not give the reductions of the effects of the uncertainty.

Also the effects of disturbance and plant parameter deviations are expressed by sensitivity functions and using the functions, the suppress of these effects are made into model matching problem and the problem is solved by H_∞ technique (Vidyasagar, 1985). But the method needs to solve H_∞ problem.

For GMVC, the controller by polynomial approach is extended to the generalised Youla-Kucera controller (Vidyasagar, 1985) and is applied to a strongly stable controller (Inoue et al., 1999; Inoue and Deng, 2013). To reduce the effects of disturbances to inputs of the plant, the extended controller is used (Inoue et al., 2021). But the paper does not consider disturbance to output and parameter deviations. And a state space controller of GMVC equivalent to the controller by polynomial approach is obtained (Inoue et al., 2022). The equivalent controller is used under the corruption of disturbances, but the paper does not derive the extended controller and also does not consider parameter changes.

As for the applications of the extended controller, there exist several papers for GPC. Kouvaritakis et al. (1992) introduced a Youla parameter to enhance the degree of robustness of the closed loop systems. Cheng et al. (2009) extended the result to GPC with constrains. As for the suppression of the effects of disturbances, Inoue et al. (2018a) used an extended controller with a full-order observer, used a reduced-order observer (Inoue et al., 2018b), also used a disturbance estimating observer (Inoue et al., 2018c). But these papers do not consider the parameter deviations.

This paper proposes a control scheme for the suppress of effects in steady state on output caused by unknown disturbances to input and output and deviations of parameters in transfer function.

Since in many cases, disturbances and parameter deviations are caused by frictions, backlashes, payload changes, parts replacements or aged deterioration in mechanical systems and they change slowly. Hence, in this paper, the disturbances and parameter changes are supposed to be slowly changing.

To suppress the effects of disturbances on output and parameter deviations is original of this paper. And the analysis of the effects is newly unified one.

First the effects on output of disturbances and parameter changes are analysed and conditions to suppress such effects are obtained. Then the controller to suppress these effects are obtained by selecting parameters to satisfy the conditions in an extended controller of GMVC of polynomial approach. The extended controller is derived by using Yola-Kucera generalised stabilising controller.

Numerical simulations of a model of two-degree of freedom fourth order system are given to show the effectiveness of the proposed controller.

2 Problem statement

The controlled plant has single-input single-output and described as

$$(A(z^{-1}) + dA(z^{-1}))y(k) \\ = z^{-d}(B(z^{-1}) + dB(z^{-1}))(u(k) + d_u(k)), \tag{1}$$

$$y_o(k) = y(k) + d_y(k), \ k = 0, 1, ... \tag{2}$$

where z^{-1} denotes time-delay; $z^{-1}y(k) = y(k-1)$.

2.1 Variables in the plant

The variables are:

- $u(k)$: control input
- $d_u(k)$: unknown slowly changing disturbance to input $u(k)$
- $y(k)$: output not observable

- $y_o(k)$: observed output corrupted by $d_y(k)$
- $d_y(k)$: unknown slowly changing disturbance to output $y(k)$
- d: known time delay.

The reason to restrict disturbances and parameter deviations to be slowly changing is explained in Remark at the end of Section 5.

$A(z^{-1})$ and $B(z^{-1})$ are known nominal polynomials of order n, m and $n > m$ also $n > d + m$. $dA(z^{-1})$ and $dB(z^{-1})$ are unknown slowly changing parameter deviations. $A(z^{-1})$ and $B(z^{-1})$ are denoted by

$$A(z^{-1}) = 1 + a_1 z^{-1} + ... + a_n z^{-n}, \tag{3}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + ... + b_m z^{-m}. \tag{4}$$

2.2 Control objective

The control objective is that the output $y(k)$ has a desirable response to the reference input $r(k)$ under the slowly changing unknown disturbances $d_u(k)$ and $d_y(k)$ and the slowly changing unknown parameter deviations $dA(z^{-1})$ and $dB(z^{-1})$.

To this objective, GMVC designs a controller to minimise the following index

$$J = \Phi(k+d)^2 \tag{5}$$

using the generalised output $\Phi(k+d)$,

$$\begin{aligned}
&\Phi(k+d) = P(z^{-1})y(k+d) + Q(z^{-1})u(k)\\
&-R(z^{-1})r(k),\\
&P(z^{-1}) = p_0 + p_1 z^{-1} + ... + p_p z^{-p},\\
&p_0 \neq 0,\ p > d,\\
&Q(z^{-1}) = q_0 + q_1 z^{-1} + ... + q_q z^{-q},\\
&R(z^{-1}) = r_0 + r_1 z^{-1} + ... + r_r z^{-r}.
\end{aligned} \tag{6}$$

where $P(z^{-1})$, $Q(z^{-1})$ and $R(z^{-1})$ are design polynomials. That is, $P(z^{-1})$ and $Q(z^{-1})$ are selected to satisfy the next condition to obtain a given desirable closed-loop characteristic $T(z^{-1})$ (Clarke and Gawthrop, 1979),

$$P(z^{-1})B(z^{-1}) + Q(z^{-1})A(z^{-1}) = T(z^{-1}). \tag{7}$$

Polynomial $R(z^{-1})$ is selected to satisfy the close-loop steady gain to be equal to 1.

3 GMVC controller

First, GMVC is designed for the nominal plant

$$A(z^{-1})y(k) = z^{-d}B(z^{-1})u(k), \tag{8}$$

without disturbances and parameter changes and plant output $y(k)$ is observable.

To minimise J, control input is designed using the estimate $\widehat{\Phi}(k+d|k)$ of $\Phi(t+d)$ to zero;

$$\widehat{\Phi}(k+d|k) = 0. \tag{9}$$

The estimate $\widehat{\Phi}(k+d|k)$ is obtained by using information of the present and the past inputs $u(k)$, $u(k-1)$, ... and outputs $y(k)$, $y(k-1)$, ... Hence, it needs to estimate the only values of the future steps of $y(k+i)$, $(i = d, d-1, ..., 1)$ in the first term $P(z^{-1})y(k+d)$ and the rests of outputs $y(k+i)$, $(i = 0, -1, ..., d-p)$ in the term are measured directly and are not necessary to be estimated. Therefore, to separate the outputs necessary to be estimated and the output not necessary and also to select the value of the present input $u(k)$, polynomials $P(z^{-1})$ and $Q(z^{-1})$ are separated as;

$$P(z^{-1}) = P_1(z^{-1}) + z^{-d}P_2(z^{-1}), \tag{10}$$

$$P_1(z^{-1}) = p_0 + p_1 z^{-1} + ... + p_{d-1} z^{-d+1}, \tag{11}$$

$$P_2(z^{-1}) = p_d + p_{d+1} z^{-1} + ... + p_p z^{-p+d}, \tag{12}$$

$$Q(z^{-1}) = q_0 + z^{-1}Q_2(z^{-1}), \tag{13}$$

$$Q_2(z^{-1}) = q_1 + q_2 z^{-1} + ... + q_q z^{-q+1}. \tag{14}$$

Then the only term necessary to be estimated in the function $\Phi(k+d)$ of equation (6) is $P_1(z^{-1})y(k+d)$ and it is defined as

$$\Phi_1(k+d) \stackrel{\text{def}}{=} P_1(z^{-1})y(k+d). \tag{15}$$

To estimate $\Phi_1(k+d)$, solve Diophantine equation (Clarke and Gawthrop, 1979)

$$P_1(z^{-1}) = A(z^{-1})E(z^{-1}) + z^{-d}F(z^{-1}), \tag{16}$$

and define polynomial $S(z^{-1})$

$$S(z^{-1}) = E(z^{-1})B(z^{-1}) = s_0 + z^{-1}S_1(z^{-1}). \tag{17}$$

where $E(z^{-1})$ is $(d-1)^{\text{th}}$ order and $F(z^{-1})$ is $(n-1)^{\text{th}}$ order polynomials. Multiply $z^d E(z^{-1})$ to plant (8) and substitute $A(z^{-1})E(z^{-1}) = P_1(z^{-1}) - z^{-d}F(z^{-1})$ of equation (16),

$$\begin{aligned}
&z^d(P_1(z^{-1}) - z^{-d}F(z^{-1}))y(k)\\
&= z^d E(z^{-1})z^{-d}B(z^{-1})u(k),\\
&P_1(z^{-1})y(k+d) = F(z^{-1})y(k)\\
&+S(z^{-1})u(k).
\end{aligned} \tag{18}$$

Then define an estimate $\hat{\Phi}_1(k+d|k)$ of $\Phi_1(k+d) = P_1(z^{-1})y(k+d)$ as

$$\hat{\Phi}_1(k+d|k) \stackrel{\text{def}}{=} F(z^{-1})y(k) + S(z^{-1})u(k). \tag{19}$$

$$P(z^{-1})y(k+d) = P_1(z^{-1})y(k+d) + P_2(z^{-1})y(k), \tag{20}$$

using this equation, estimate $\hat{\Phi}(k+d|k)$ is defined as

$$\begin{aligned}
&\hat{\Phi}(k+d|k) \stackrel{\text{def}}{=} \hat{\Phi}_1(k+d|k) + P_2(z^{-1})y(k)\\
&+Q(z^{-1})u(k) - R(z^{-1})r(k).
\end{aligned} \tag{21}$$

And the controller is derived from $\hat{\Phi}(k+d|k) = 0$, then,

$$(F(z^{-1}) + P_2(z^{-1}))y(k) + (S(z^{-1}) + Q(z^{-1}))u(k) \\ -R(z^{-1})r(k) = 0. \tag{22}$$

Using equations (13) and (17), this equation is

$$(F(z^{-1}) + P_2(z^{-1}))y(k) + s_0 u(k) \\ +S_1(z^{-1})u(k-1) + q_0 u(k) + Q_2(z^{-1})u(k-1) \\ -R(z^{-1})r(k) = 0. \tag{23}$$

Then control input is

$$u(k) = [-F(z^{-1} + P_2(z^{-1}))y(k) \\ -(S_1(z^{-1}) + Q_2(z^{-1}))u(k-1) \\ +R(z^{-1})r(k)]/(s_0 + q_0). \tag{24}$$

4 Extended controller

The controller (24) is extended to Youla-Kucela generalised stabilising controllers (Vidyasagar, 1985; Inoue and Deng, 2013).

Let $U_d(z^{-1})$ and $U_n(z^{-1})$ be design parameter polynomials of orders n_d and n_n;

$$U_d(z^{-1}) = u_{d0} + u_{d1}z^{-1} + ... + u_{dn_d}z^{-n_d}, \tag{25}$$
$$U_n(z^{-1}) = u_{n0} + u_{n1}z^{-1} + ... + u_{dn_n}z^{-n_n}. \tag{26}$$

The controller $u(k)$ of equation (24) is extended to $u_e(k)$ by adding an additional term $u_a(k)$ as

$$u_e(k) = u(k) + u_a(k), \tag{27}$$
$$u_a(k) = \frac{U_n(z^{-1})}{U_d(z^{-1})}(z^{-d}B(z^{-1})u_e(k) \\ -A(z^{-1})y(k)). \tag{28}$$

When $U_d(z^{-1})$ and $U_n(z^{-1})$ be chosen as

$$U_d(z^{-1}) = 1, \ U_n(z^{-1}) = 0. \tag{29}$$

Then the additional term $u_a(k) = 0$ disappears and the extended controller reduces to non-extended one.

5 Effects of disturbances and parameter deviations to plant output

This section gives a mathematical expression of the effects of disturbances and parameter deviations to plant output at the steady state. In controllers (22) and (28), the observed output $y_o(k)$ is used. Then the extended controller is

$$(F(z^{-1}) + P_2(z^{-1}))y_o(k) + (S(z^{-1}) + Q(z^{-1}))u(k) \\ -R(z^{-1})r(k) = 0, \tag{30}$$
$$u_a(k) = \frac{U_n(z^{-1})}{U_d(z^{-1})}(z^{-d}B(z^{-1})u_e(k) \\ -A(z^{-1})y_o(k)), \tag{31}$$
$$u_e(k) = u(k) + u_a(k). \tag{32}$$

Substituting equations (31) and (32) into equation (30), the extended controller is rewritten as

$$G_d(z^{-1})u_e(k) = -G_n(z^{-1})y_o(k) + G_r(z^{-1})r(k), \tag{33}$$
$$G_d(z^{-1}) = (S(z^{-1}) + Q(z^{-1})) \\ \times(U_d(z^{-1}) - z^{-d}B(z^{-1}))U_n(z^{-1}), \tag{34}$$
$$G_n(z^{-1}) = (F(z^{-1}) + P_2(z^{-1}))U_d(z^{-1}) \\ +(S(z^{-1}) + Q(z^{-1}))A(z^{-1})U_n(z^{-1}), \tag{35}$$
$$G_r(z^{-1}) = U_d(z^{-1})R(z^{-1}). \tag{36}$$

Substituting $u_e(k)$ of equation (33) into $u(k)$ in equation (1), the closed-loop system is obtained.

$$(T_y(z^{-1}) + G_d(z^{-1})dA(z^{-1}) \\ +G_n(z^{-1})z^{-d}dB(z^{-1}))y(k) = T_{dy}(z^{-1})d_y(k) \\ +T_{du}(z^{-1})d_u(k) + T_r(z^{-1})r(k), \tag{37}$$
$$T_y(z^{-1}) = G_d(z^{-1})A(z^{-1}) + z^{-d}B(z^{-1})G_n(z^{-1}) \\ = U_d(z^{-1})(Q(z^{-1})A(z^{-1}) + P(z^{-1})B(z^{-1})), \tag{38}$$
$$T_{dy}(z^{-1}) = -z^{-d}(B(z^{-1}) + dB(z^{-1}))G_n(z^{-1}), \tag{39}$$
$$T_{du}(z^{-1}) = z^{-d}(B(z^{-1}) + dB(z^{-1}))G_d(z^{-1}), \tag{40}$$
$$T_r(z^{-1}) = z^{-d}(B(z^{-1}) + dB(z^{-1}))G_r(z^{-1}) \\ = z^{-d}(B(z^{-1}) + dB(z^{-1}))U_d(z^{-1})R(z^{-1}). \tag{41}$$

Theorem 1: If polynomials $U_n(z^{-1})$ and $U_d(z^{-1})$ are selected to satisfy the conditions:

1 $U_d(z^{-1})$ is stable

2 at $k \to \infty$, $G_d(z^{-1})$ is 0, that is,

$$G_d(z^{-1})|_{z=1} = 0, \tag{42}$$

then the effects of disturbance $d_u(k)$ and parameter deviation $dA(z^{-1})$ in the closed-loop system (37) at steady state are,

$$T_{du}(z^{-1})d_u(k) \to 0 \text{ at } k \to \infty, \tag{43}$$
$$G_d(z^{-1})dA(z^{-1}) \to 0 \text{ at } k \to \infty. \tag{44}$$

That is, in the closed-loop system (37), the effects of disturbance $d_u(k)$ and parameter deviation $dA(z^{-1})$ at steady state are suppressed.

Proof: From condition (42) and using equation (40), equations (43) and (44) hold.

Theorem 2: If polynomials $U_n(z^{-1})$ and $U_d(z^{-1})$ are selected to satisfy the conditions:

1 $U_d(z^{-1})$ is stable

2 at $k \to \infty$, $G_n(z^{-1})$ is 0, that is,

$$G_n(z^{-1})|_{z=1} = 0, \tag{45}$$

then the effect of disturbance $d_y(k)$ in equation (37) at steady state is,

$$T_{dy}(z^{-1})d_y(k) \to 0 \text{ at } k \to \infty. \tag{46}$$

That is, the effect of disturbance $d_y(k)$ at steady state is suppressed. And when $dA(z^{-1}) = 0$ and $d_u(k) = 0$, the effect of parameter deviation $dB(z^{-1})$ in equation (37) at steady state is

$$\lim_{k\to\infty} \left(\frac{y(k)}{r(k)}|_{dB\neq 0} - \frac{y(k)}{r(k)}|_{dB=0} \right) = \frac{z^{-d}G_r(z^{-1})dB(z^{-1})}{T_y(z^{-1})}\bigg|_{z=1.} \tag{47}$$

That is, if $z^{-d}G_r(z^{-1})/T_y(z^{-1})$ at $z = 1$ is small, then the effect caused by parameter deviation $dB(z^{-1})$ is small at steady state.

Proof: From condition (45) and using equation (39), equation (46) holds. Equation (47) is proved from equations (37) and (45) by

$$\lim_{k\to\infty} \frac{y(k)}{r(k)}\bigg|_{dB\neq 0} = \frac{z^{-d}(B(z^{-1}) + dB(z^{-1}))G_r(z^{-1})}{T_y(z^{-1}) + G_n(z^{-1})z^{-d}dB(z^{-1})}\bigg|_{z=1} = \frac{z^{-d}(B(z^{-1}) + dB(z^{-1}))G_r(z^{-1})}{T_y(z^{-1})}\bigg|_{z=1,} \tag{48}$$

$$\lim_{k\to\infty} \frac{y(k)}{r(k)}\bigg|_{dB=0} = \frac{z^{-d}B(z^{-1})G_r(z^{-1})}{T_y(z^{-1})}\bigg|_{z=1.} \tag{49}$$

Remark: When there are no disturbances $d_u(k) = 0$ and $d_y(k) = 0$ and no parameter destinations $dA(z^{-1}) = 0$ and $dB(z^{-1}) = 0$, then $U_d(z^{-1})$ in then numerator (41) and $U_d(z^{-1})$ in denominator (38) in the closed-loop system (37) are canceled and the closed-loop systems (37) shows that responses from reference $r(k)$ to output $y(k)$ are independent to parameters $U_n(z^{-1})$ and $U_d(z^{-1})$. This means that the extended controllers do not improve the transient responses and is only useful in the steady state. This is the reason that disturbances and parameter deviations are supposed to be slowly changing. When a change occurs, then the system goes in transient state and if the next change comes quickly, then the system goes in transient state again, before goes in steady state. That is, if the changes occur quickly, then the system does not go in steady state and the extended controller does not realise its usefulness. This fact is confirmed by simulations in the next section. To improve the transient responses is a future work.

6 Simulations

Simulated plant is a model of a two degree of freedom mechanical system. The system has two wheels connected by a spring. It is derived by torque applied at the end of another spring attached at the centre of the first wheel. This torque τ is the control input u. The wheels are affected by viscous friction and elastic forces from springs. The output y to be controlled is the rotation angle θ_2 of the second wheel. The system is depicted in Figure 1. Symbols and their values are listed in Table 1.

Figure 1 Mechanical system (see online version for colours)

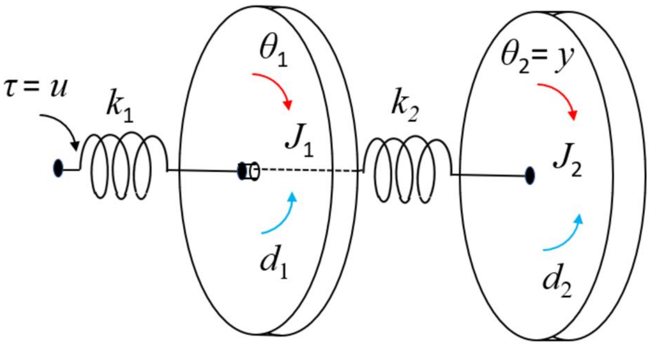


Table 1 Symbols and their values

	First wheel	*Second wheel*
Rotation angle	θ_1 rad	θ_2 rad
Inertia moment	J_1 0.2 kgm^2	J_2 0.1 kgm^2
Viscous friction	d_1 0.1 Nms/rad	d_2 0.21 Nms/rad
Spring constant	k_1 50 Nm/rad	k_2 40 Nm/rad

Equations of motions of the system are

$$J_1\ddot{\theta}_1 = -d_1\dot{\theta}_1 - k_1\theta_1 - k_2(\theta_1 - \theta_2) + \tau, \tag{50}$$

$$J_2\ddot{\theta}_2 = -d_2\dot{\theta}_2 + k_2(\theta_1 - \theta_2). \tag{51}$$

These equations are transformed into discrete-time systems with sampling time 0.05 sec and the numerator of the discrete-time system is approximated by two steps time delay. The approximated discrete time system is

$$(A(z^{-1}) + dA(z^{-1}))y(k) = z^{-d}(B(z^{-1}) + dB(z^{-1}))(u(k) + d_u(k)), \tag{52}$$

$$y_o(k) = y(k) + d_y(k),\ y = \theta_2,\ u = \tau, \tag{53}$$

$$A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4}, \tag{54}$$

$$B(z^{-1}) = b_0,\ d = 2, \tag{55}$$

where $d_u(k)$ and $d_y(k)$ are slowly changing disturbances. Parameters $a_1 \sim a_4$ and b_0 are

$$a_1 = -0.9870,\ a_2 = 1.1111,\ a_3 = -0.9321,\ a_4 = 0.8458,\ b_0 = 0.0208. \tag{56}$$

Figure 2 compares the impulse responses of the original continues-time equations of motion (50) and (51) and the approximated discrete-time system (52). The figure shows the discrete-time system approximates the continuous-time equations well. Also it shows the system oscillates hard.

The generalised output $\Phi(k + d)$ is defined by

$$P(z^{-1)} = p_0 + p_1z^{-1} + p_2z^{-2} + p_3z^{-3} + p_4z^{-4}, \tag{57}$$

$$P_1(z^{-1}) = p_0 + p_1z^{-1}, P_2(z^{-1}) = p_2 + p_3z^{-1} + p_4z^{-2},$$

$$Q(z^{-1}) = q_0,\ Q_2(z^{-1}) = 0,\ R(z^{-1}) = r_0 + r_1z^{-1}. \tag{58}$$

The coefficients of these polynomials are set by

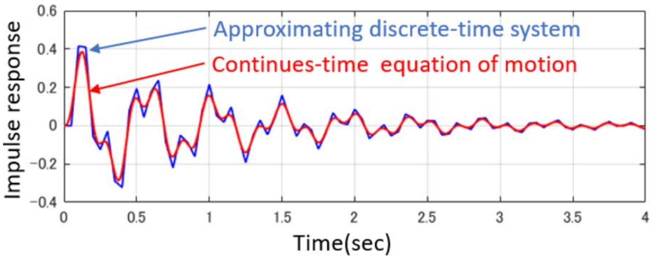
$$p_0 = 1,\ p_1 = -1.5,\ p_2 = 0.8375,\ p_3 = -0.2062,$$
$$p_4 = 0.0189,\ q_0 = 0,\ r_0 = 1,\ r_1 = -0.8498, \quad (59)$$

so that the poles of the closed-loop system are stable and

$$\text{Poles: } 0.3,\ 0.35,\ 0.4,\ 0.45. \quad (60)$$

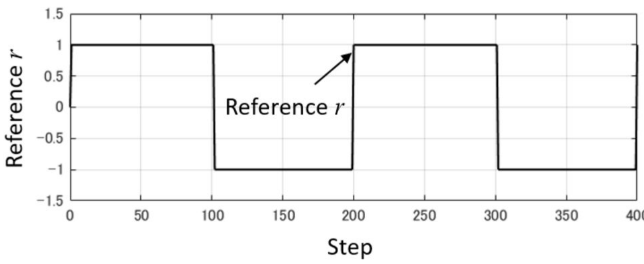
And r_0 and r_1 are selected so that the closed loop steady state gain is 1.

Figure 2 Impulse responses of continuous-time equation of motion and approximating discrete-time system (see online version for colours)



In simulations, reference input $r(k)$ is a rectangular wave with period 200 steps and amplitude 1.0 and is shown in Figure 3.

Figure 3 Reference input



Disturbances and plant parameter changes are step-wise and ramp-wise and are shown in Table 2, Figures 4 and 5.

Table 2 Disturbances and parameter deviations

Disturbance		*Amplitude*	*Start*	*End*	*Figure*
Step to $y(k)$	-	0∼1.0	100	350	Figure 4
Step to $u(k)$	-	0∼1.0	50	300	Figure 4
Ramp to $y(k)$	-	–3.9∼0	50	330	Figure 4
Ramp to $u(k)$	-	0∼3.9	30	310	Figure 4
Parameter	*Nominal*	*Deviation*	*Start*	*End*	*Figure*
Step a_1	–0.987	–0.987 × 0.1	20	250	Figure 5
Step a_2	1.1111	1.1111 × 0.1	150	350	Figure 5
Step b_0	0.0208	0.0208 × 0.1	20	300	Figure 5
Ramp a_1	–0.987	–0.987 × 0.2	70	350	Figure 5
Ramp a_2	1.1111	1.1111 × 0.2	90	370	Figure 5
Ramp b_0	0.0208	0.0208 × 0.2	110	390	Figure 5

Simulations are conducted in eight cases. Details are shown Table 3. In simulations #2 and #6, polynomials $U_n(z^{-1})$ and $U_d(z^{-1})$ are selected to satisfy condition (42) and in simulations #4 and #8, to satisfy (45).

Figure 4 Disturbances (see online version for colours)

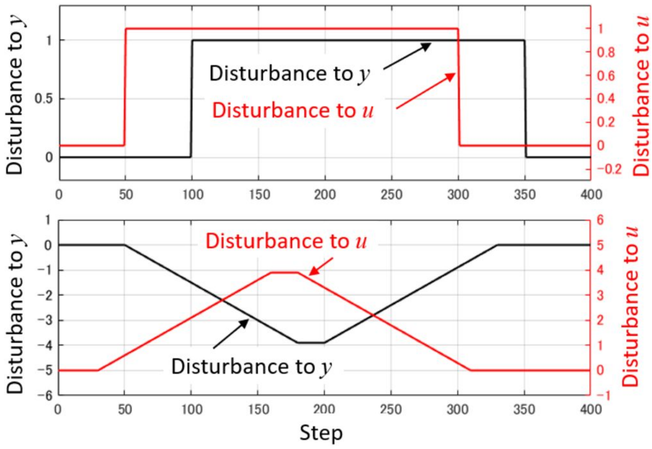


Figure 5 Parameters a_1, a_2 and b_0 (see online version for colours)

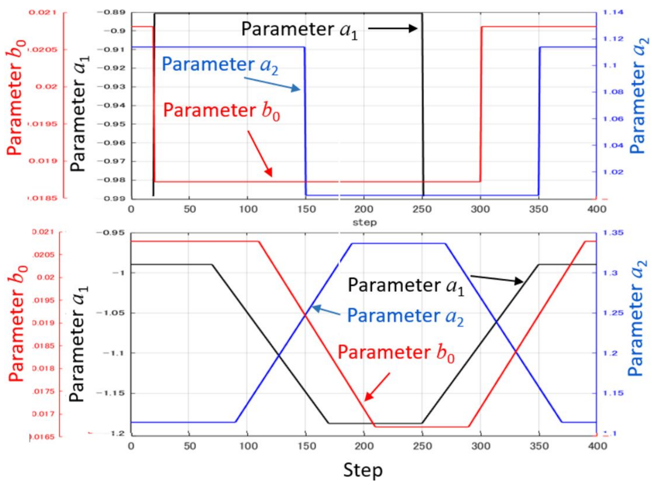


Table 3 Simulation cases

#	*Controller*	$U_n(z^{-1})$ and $U_n(z^{-1})$	*Disturbance*	*Para. devai.*	*Figure*
#1	Non-extended	$U_n = 0$ $U_d = 1$	Step to $u(k)$	Step a_1 and a_2	Figure 6
#2	Extended	$U_n = 9.612$ $U_d = 1 - 0.8z^{-1}$			
#3	Non-extended	$U_n = 0$ $U_d = 1$	Step to $y(k)$	Step b_0	Figure 7
#4	Extended	$U_n = 16.946$ $U_d = 1 - 0.5z^{-1}$			
#5	Non-extended	$U_n = 0$ $U_d = 1$	Ramp to $u(k)$	Ramp a_1 and a_2	Figure 8
#6	Extended	$U_n = 9.612$ $U_d = 1 - 0.8z^{-1}$			
#7	Non-extended	$U_n = 0$ $Ud = 1$	Ramp to $y(k)$	Ramp b_0	Figure 9
#8	Extended	$U_n = 16.946$ $U_d = 1 - 0.5z^{-1}$			

In Figures 6∼9 show that non-extended controller does not follow the given reference under the disturbance and parameter changes, but the extended controllers suppress the offsets at steady state.

Figure 6 Outputs of simulation #1 and #2 with step disturbance to $u(k)$ and parameter step deviations of a_1 and a_2 (see online version for colours)

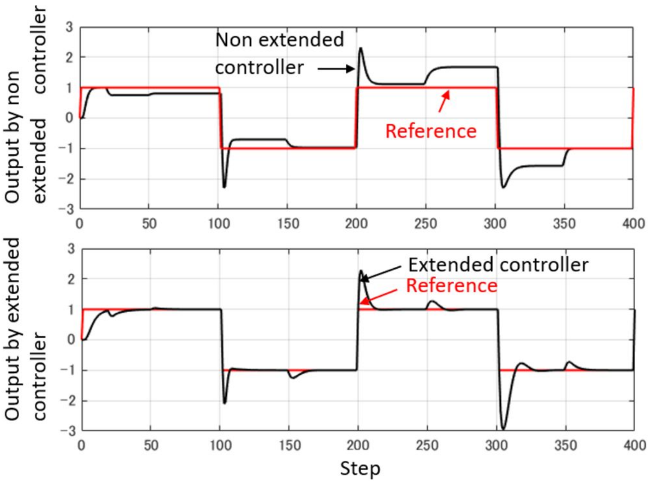
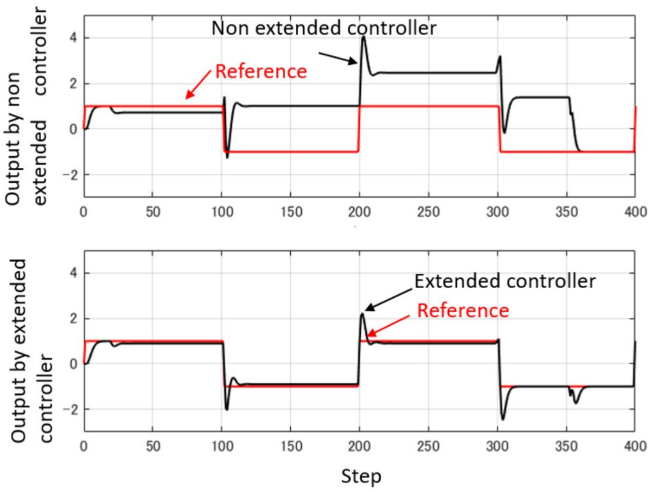


Figure 7 Outputs of simulation #3 and #4 with step disturbance to $y(k)$ and parameter step deviation of b_0 (see online version for colours)



Remark: In simulations #2 and #4, different parameters of $U_n(z^{-1})$ and $U_d(z^{-1})$ are used. This means for different type of disturbances and parameter changes, different values of parameters in the extended controller are required. To make effective for different types by using single value in the controller is remains a future work.

7 Conclusions

This paper proposed an extended controller to suppress slowly changing disturbances such as friction, backlash or caused by changes of payload in mechanical systems and parameter deviations caused by aged deterioration or parts replacement in steady state for high precise positioning control. The extended controller is obtained by applying Youla-Kucela generalised stabilising controller. Conditions for suppression of effects of disturbances and parameter changes in steady state are derived. The suppression is attained by selecting the parameters in the extended controller satisfying the conditions.

Figure 8 Outputs of simulation #5 and #6 with ramp disturbance to $u(k)$ and parameter ramp deviations of a_1 and a_2 (see online version for colours)

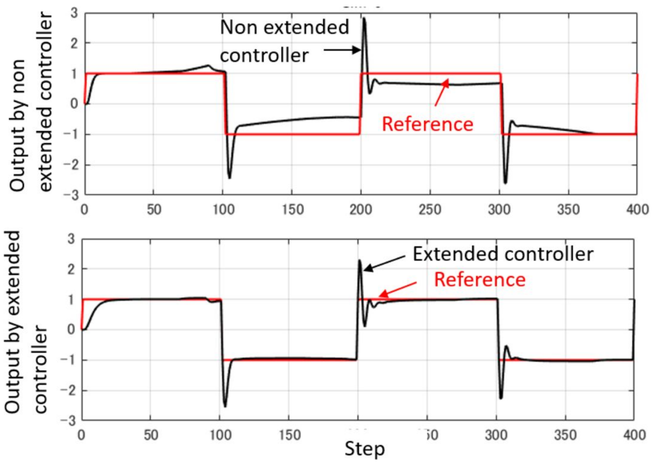
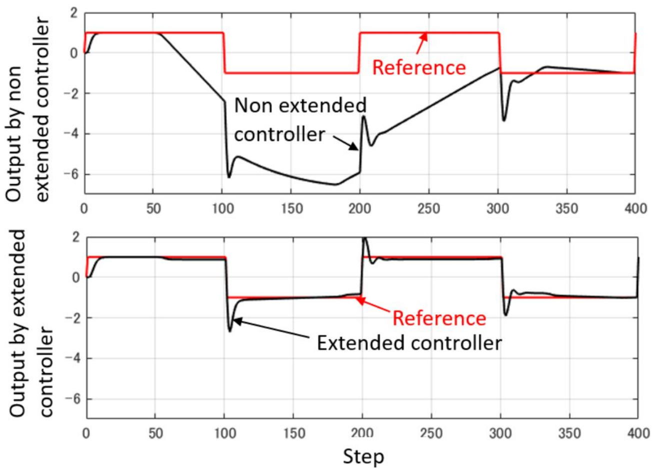


Figure 9 Outputs of simulation #7 and #8 with ramp disturbance to $y(k)$ and parameter ramp deviation of b_0 (see online version for colours)



For suppression of different disturbances to input and output and different parameter changes in transfer functions, different values of parameters in the extended controller are necessary. To make effective for different types by using single value in the controller is remains as a future work.

The derived closed-loop system shows that responses from reference $r(k)$ to output $y(k)$ are independent to parameters of the extended controller. This means that the extended controllers do not improve the transient responses. This fact is confirmed by simulations. To improve the transient responses is also a future work.

This paper considers slowly changing disturbances and parameter changes because such cases occur frequently in mechanical systems. Also, to consider randomly changing disturbances or parameter deviations is important. To consider such cases is also an open problem.

This paper gives numerical simulations of using fourth order mechanical model. To confirm the effectiveness of the proposed controller by experiments is important and is a future work.

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