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A goal programming strategy for bi-level decentralised multi-objective linear programming problem with neutrosophic numbers

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Abstract: This paper develops a goal programming (GP) algorithm to evaluate bi-level decentralised multi-objective linear programming problem (BLDMOLPP) in neutrosophic number (NN) environment. In a BLDMOLPP, a single decision maker (DM) is present at the upper level and multiple decision makers at the lower level. Here the parameters of the problem are considered to be NNs in the form of $[P+QI]$, where P and Q are real numbers and indeterminacy is represented through the symbol I. I is expressed in the form of a real interval as agreed upon by the DMs. The BLDMOLPP with NNs then gets converted into an interval BLDMOLPP. Using interval programming, the target intervals for the objective functions are identified and subsequently, the goal achievement functions are constructed. The upper level DM provides some possible relaxation on the decision variables under his/her control to cooperate with the lower level DMs to attain a compromise optimal solution. Thereafter, goal programming (GP) models are formulated by minimising the deviational variables and thereby obtaining the most satisfactory solution for all DMs. Finally, a numerical example demonstrates the feasibility and simplicity of the proposed strategy.

Keywords: neutrosophic number; bilevel decentralised programming; multi-objective programming; goal programming.

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1 Introduction

Bi-level decentralised multi-objective linear programming problem (BLDMOLPP) consists of one upper level DM (ULDM) and multiple lower level DMs (LLDMs) where each DM has to optimise multiple objective functions over a common feasible region. Such problems occur frequently in hierarchical organisations such as logistic companies, business organisations, manufacturing units, etc. Each DM is assigned a unique set of objective functions, a unique set of decision variables, and a set of constraints that is common to all DMs. Here the objective functions and the constraints are considered as linear functions with the coefficients and the constants considered as NNs.

Anandalingam (1988) has employed Stackelberg solution concept to solve multi-level programming problem (MLPP) and also discussed procedures to solve bilevel decentralised programming problem (BLDPP). Ahlatcioglu and Tiryaki (2007) used analytical hierarchy process for solving a decentralised bilevel linear fractional programming problem. With the advent of fuzzy sets (Zadeh, 1965), new methods were proposed to solve MLPP and BLDPP. Some of them are fuzzy mathematical programming (Sinha, 2003a, 2003b), interactive fuzzy programming (Sakawa et al., 1998; Sakawa and Nishizaki, 2002), compensatory fuzzy operator (Shih and Lee, 2000), and fuzzy goal programming (FGP) (Pramanik and Roy, 2007).

GP (Chang, 2007; Charnes and Cooper, 1961; Ignizio, 1976; Lee, 1972) is an important mathematical apparatus which has significant use in solving multi-objective programming problem with conflicting objectives to attain an optimal compromise solution. The concept of interval GP was introduced by Inuiguchi and Kume in 1991. In fuzzy environment, GP is termed as fuzzy goal programming (FGP). The relation between GP and FGP was formed by Mohamed (1997) and it was of much help to solve multi-objective programming problem. Baky (2009) solved BLDMOLPP using fuzzy goal programming algorithm.

To deal with complex decision making problems which involved incomplete and indeterminate information, Smarandache in 1998 introduced the concept of neutrosophic sets. Further, the concept of neutrosophic number (NN) which is useful to formulate real life problems involving imprecise and indeterminate information was incorporated by Smarandache (2014, 2015) and preliminaries of NN were proposed. Multi-objective linear programming problem was solved using neutrosophic optimisation procedure by Roy and Das (2015). Multi-objective programming problem with neutrosophic numbers

was solved with the help of first order Taylor series by Hezam et al. (2016). GP with neutrosophic numbers was proposed by Abdel-Baset et al. (2016) and was employed to solve an industrial design problem. Pramanik (2016) discussed multi-objective linear programming problems involving uncertainty and indeterminacy by employing neutrosophic GP technique. A ranking method was developed by Deli and Şubaş (2017) for single valued neutrosophic numbers and the concept was applied to a multi-attribute decision making problem. Ye (2018) proposed NN function and neutrosophic number linear programming method to handle optimisation problems involving NN and used it to solve a production planning problem. Ye et al. (2018) proposed solutions of non-linear optimisation models with NN for unconstrained and constrained problems. In recent times, Banerjee and Pramanik (2018) furnished a GP technique for linear programming problem with single objective with coefficients as NNs. Pramanik and Banerjee (2018) formulated a GP strategy for linear programming problem with multiple objectives involving neutrosophic numbers. GP models were proposed for bi-level programming problem with NN by minimising deviational variables by Pramanik and Dey (2018).

In this paper, we propose a GP methodology to solve BLDMOLPP in NN environment. Firstly, a minimisation type BLDMOLPP with NN is formulated. For specific *I*, the NNs are converted into interval numbers and thus the BLDMOLPP with NN converts into a BLDMOLPP with interval parameters. The target interval for each objective function is determined by calculating the best and worst solutions of each DM which helps to formulate the goal achievement functions. The optimal solution for the ULDM is obtained separately. The upper level DM allows some possible relaxation on the decision variables under his/her control to cooperate with the lower level DMs. GP models are then developed to solve BLDMOLPP. We obtain the optimal solution in the form of an interval which is more relevant for practical decision making problems. The novelty of the proposed method is demonstrated with the help of a numerical example.

The rest of the paper is organised in the following way. We present some basic notions and operations on interval numbers and NNs in Section 2. In Section 3, the mathematical formulation of BLDMOLPP with NN parameters is presented. The GP model for BLDMOLPP with NNs is developed in Section 4. Section 5 contains a numerical example to illustrate the applicability of the developed method. The article ends with the concluding remarks in Section 6.

2 Neutrosophic numbers and Interval numbers

2.1 Preliminaries on neutrosophic numbers

Smarandache (2014, 2015) introduced the concept of NN to deal with problems in indeterminate environment. The mathematical expression of NN is defined in the form $Z = P + QI$, where P, Q are real numbers and I denotes indeterminacy. Here P represents the determinate part and QI is the indeterminate part. I is usually considered as some specified interval as per the requirements of the problem. Let $I \in [I^L, I^U]$

Therefore, Z can be explicitly written as follows:

$$Z = [P + QI^L, P + QI^U] = [Z^L, Z^U]$$

Example: Suppose $Z = 7 + 3I$ is a neutrosophic number where the determinate part is 7 and the indeterminate part is 3*I*. Suppose it is considered that $I \in [0.1, 0.6]$. Then, *Z* is equivalent to an interval number $Z = [7.3, 8.8]$.

Let, $Z_1 = [P_1 + Q_1 I_1] = [P_1 + Q_1 I_1^{\,L}, P_1 + Q_1 I_1^{\,U}] = [Z_1^{\,L}, Z_1^{\,U}]$ and $Z_2 = [P_2 + Q_2 I_2] = [P_2 + Q_2 I_2^{\,L}, P_2 + Q_2 I_2^{\,U}] = [Z_2^{\,L}, Z_2^{\,U}]$ be two neutrosophic numbers where $I_1 \in [I_1^L, I_1^U]$, $I_2 \in [I_2^L, I_2^U]$, then basic operations of two neutrosophic numbers are given below:

(i) $Z_1 + Z_2 = [Z_1^L + Z_2^L, Z_1^U + Z_2^U]$,

(ii) $Z_1 - Z_2 = [Z_1^L - Z_2^U, Z_1^U - Z_2^L]$,

(iii) $Z_1 \cdot Z_2 = [\text{Min}\{Z_1^L \cdot Z_2^L, Z_1^L \cdot Z_2^U, Z_1^U \cdot Z_2^L, Z_1^U \cdot Z_2^U\},$
$\text{Max}\{Z_1^L \cdot Z_2^L, Z_1^L \cdot Z_2^U, Z_1^U \cdot Z_2^L, Z_1^U \cdot Z_2^U\}]$

(iv) $Z_1 / Z_2 = [\text{Min}\{Z_1^L / Z_2^L, Z_1^L / Z_2^U, Z_1^U / Z_2^L, Z_1^U / Z_2^U\},$
$\text{Max}\{Z_1^L / Z_2^L, Z_1^L / Z_2^U, Z_1^U / Z_2^L, Z_1^U / Z_2^U\}]$, if $0 \notin Z_2$.

2.2 Preliminaries on interval number (Moore, 1966)

An interval number on the real line *R* is represented in the form $E = [E^-, E^+] = \{e : E^- \le e \le E^+; e \in R\}$, where E^-, E^+ denote left and right limit respectively of *E* on *R*.

Definition 2.2.1: The midpoint and the width of *E*, denoted respectively by $\gamma(E)$ and $\delta(E)$ can be defined as:

$$\gamma(E) = \frac{1}{2}(E^- + E^+)$$

$$\delta(E) = [E^+ - E^-]$$

Definition 2.2.2: The scalar multiplication on *E* can be defined as follows:

$$\mu E = \begin{cases} [\mu E^-, \mu E^+], \mu \ge 0 \\ [\mu E^+, \mu E^-], \mu \le 0 \end{cases}$$

Definition 2.2.3: The absolute value of *E*, denoted by $|E|$ is defined as given below:

$$|E| = \begin{cases} [E^-, E^+], E^- \ge 0 \\ [0, \max\{-E^-, E^+\}], E^- < 0 < E^+ \\ [-E^+, -E^-], E^+ \le 0 \end{cases}$$

Definition 2.2.4: Binary operation * between 2 NNs $E_1 = [E_1^-, E_1^+]$ and $E_2 = [E_2^-, E_2^+]$ is defined as follows:

$$E_1 * E_2 = \{e_1 * e_2 : E_1^- \le e_1 \le E_1^+, E_2^- \le e_2 \le E_2^+; e_1, e_2 \in R\}$$

3 Problem formulation

It is assumed that there are two levels in a hierarchical decision making structure with the ULDM denoted as DM_0 and k number of DMs at the lower level denoted as $LLDM_j$ or DM_j, $j = 1,2,...,k$. Let $w = (w_0, w_1, ..., w_k) \in R^n$ be the vector of decision variables. Let the ULDM exercises control over the vector $w_0 \in R^{n_0}$ and $LLDM_j$, $j = 1,2,...,k$, controls the vector $w_j \in R^{n_j}$, where $n = n_0 + n_1 + ... + n_k, w_j = (w_{j1}, w_{j2}, ..., w_{jn_j}), j = 0,1,...,k$. Also it is assumed that $G_j(w_0, w_1, ..., w_k) \equiv G_j(w) : R^{n_0} \times R^{n_1} \times ... \times R^{n_k} \to R^{m_j}$, $j = 0,1,...,k$ are the vector of objective functions corresponding to the DM_j, $j = 1,2,...,k$.

Thus, the BLDMOLPP with NN of minimisation type objective function can be formulated in the following manner (Ahlatcioglu and Tiryaki, 2007; Anandalingam, 1988; Sakawa and Nishizaki, 2002; Shih and Lee, 2000; Sinha, 2003b):

Upper level:

$$[\mathrm{DM}_0] : \min_{w_0} G_0(w) = \min_{w_0} (g_{01}(w), g_{02}(w), ..., g_{0m_0}(w)) \tag{1}$$

where $w_1, w_2, ..., w_k$ solves.

Lower level:

$$[\mathrm{DM}_1] : \min_{w_1} G_1(w) = \min_{w_1} (g_{11}(w), g_{12}(w), ..., g_{1m_1}(w)) \tag{2}$$

$$[\mathrm{DM}_2] : \min_{w_2} G_2(w) = \min_{w_2} (g_{21}(w), g_{22}(w), ..., g_{2m_2}(w)) \tag{3}$$

$$\vdots$$

$$[\mathrm{DM}_k] : \min_{w_k} G_k(w) = \min_{w_k} (g_{k1}(w), g_{k2}(w), ..., g_{km_k}(w)) \tag{4}$$

Subject to

$$\sum_{j=0}^{k} (A_{rj} + I_{rj} E_{rj}) w_j \geq \eta_r + I_r \beta_r, \quad r = 1,2,...,m \tag{5}$$

$$w = (w_0, w_1, ..., w_k) \geq 0 \tag{6}$$

where

$$g_{is}(w) = \sum_{j=0}^{k} (f_{isj} + I_{isj} b_{isj}) w_j + (\xi_{is} + I_{is} \lambda_{is}) \qquad i = 0,1,...,k;\ s = 1,2,...,m_i \tag{7}$$

Here $I_{rj} \in [I_{rj}^L, I_{rj}^U], I_{isj} \in [I_{isj}^L, I_{isj}^U], I_{is} \in [I_{is}^L, I_{is}^U]$ and $A_{rj}, E_{rj}, \eta_r, \beta_r, f_{isj}, b_{isj}, \xi_{is}, \lambda_{is}, I_{rj}^L, I_{rj}^U, I_{isj}^L, I_{isj}^U, I_{is}^L, I_{is}^U$ are real numbers.

Equation (7) can be rewritten in the following form:

$$\begin{aligned} g_{is}(w) &= \sum_{j=0}^{k}(f_{isj}+I_{isj}b_{isj})w_j+(\xi_{is}+I_{is}\lambda_{is}) \\ &= \sum_{j=0}^{k}[(f_{isj}+I_{isj}^{L}b_{isj})w_j,(f_{isj}+I_{isj}^{U}b_{isj})w_j]+[(\xi_{is}+I_{is}^{L}\lambda_{is}),(\xi_{is}+I_{is}^{U}\lambda_{is})] \\ &= [\sum_{j=0}^{k}(f_{isj}+I_{isj}^{L}b_{isj})w_j+(\xi_{is}+I_{is}^{L}\lambda_{is}),\sum_{j=0}^{k}(f_{isj}+I_{isj}^{U}b_{isj})w_j+(\xi_{is}+I_{is}^{U}\lambda_{is})] \\ &= [C_{is}^{L},C_{is}^{U}] \end{aligned} \tag{8}$$

where $\sum_{j=0}^{k}(f_{isj}+I_{isj}^{L}b_{isj})w_j+(\xi_{is}+I_{is}^{L}\lambda_{is})=C_{is}^{L}$ and $\sum_{j=0}^{k}(f_{isj}+I_{isj}^{U}b_{isj})w_j+(\xi_{is}+I_{is}^{U}\lambda_{is})=C_{is}^{U}$.

Similarly, the constraints can be rewritten as

$$\begin{aligned} &\sum_{j=0}^{k}(A_{rj}+I_{rj}E_{rj})w_j \geq \eta_r+I_r\beta_r \qquad r=1,2,...,m \\ &\Rightarrow \left[\sum_{j=0}^{k}(A_{rj}+I_{rj}^{L}E_{rj})w_j,\sum_{j=0}^{k}(A_{rj}+I_{rj}^{U}E_{rj})w_j\right] \geq \left[\eta_r+I_r^{L}\beta_r,\eta_r+I_r^{U}\beta_r\right] \\ &\qquad\qquad \geq [\mathrm{N}_r^{L},\mathrm{N}_r^{U}] \end{aligned} \tag{9}$$

where $\eta_r+I_r^{L}\beta_r=\mathrm{N}_r^{L}$ and $\eta_r+I_r^{U}\beta_r=\mathrm{N}_r^{U}$.

4 A goal programming formulation to BLDMOLPP with neutrosophic numbers

The minimisation type BLDMOLPP with the parameters as neutrosophic numbers can be formulated in the following way:

Upper level:

$$[\mathrm{DM}_0]:\min_{w_0} g_{0s}(w)=[C_{0s}^{L},C_{0s}^{U}] \qquad s=1,2,...,m_0 \tag{10}$$

where $w_1,w_2,...,w_k$ solves

Lower level:

$$[\mathrm{DM}_1]:\min_{w_1} g_{1s}(w)=[C_{1s}^{L},C_{1s}^{U}] \qquad s=1,2,...,m_1 \tag{11}$$

$$[\mathrm{DM}_2]:\min_{w_2} g_{2s}(w)=[C_{2s}^{L},C_{2s}^{U}] \qquad s=1,2,...,m_2 \tag{12}$$

$$\vdots$$

$$[\mathrm{DM}_k]:\min_{w_k} g_{ks}(w)=[C_{ks}^{L},C_{ks}^{U}] \qquad s=1,2,...,m_k \tag{13}$$

Subject to

$$\left[\sum_{j=0}^{k}(A_{rj}+I_{rj}^{L}E_{rj})w_j,\sum_{j=0}^{k}(A_{rj}+I_{rj}^{U}E_{rj})w_j\right]\geq[\mathrm{N}_r^L,\mathrm{N}_r^U]\quad r=1,2,...,m$$
$$w=(w_0,w_1,...,w_k)\geq 0$$

Proposition 1 (Shaocheng, 1994): Let an interval inequality be written in the form $\sum_{k=1}^{n}[e_1^k,e_2^k]z_k\geq[f_1,f_2]$. Then the maximum value range and the minimum value range can be obtained by solving the inequalities $\sum_{k=1}^{n}[e_2^k]z_k\geq f_1$ and $\sum_{k=1}^{n}[e_1^k]z_k\geq f_2$ respectively.

As in Ramadan (1996), to acquire the best optimal solution of $g_{is}(w)$, $(i=0,1,...,k;\ s=1,2,...,m_i)$ the following problem is solved.

$$\min g_{is}(w)=C_{is}^{L},\qquad i=0,1,...,k;s=1,2,...,m_i \tag{14}$$

Subject to

$$\begin{aligned}&\sum_{j=0}^{k}(A_{rj}+I_{rj}^{U}E_{rj})w_j\geq\mathrm{N}_r^L\qquad r=1,2,...,m\\&w_j\geq 0\end{aligned} \tag{15}$$

We solve the above problem and let $w_{is}^{B}=(w_{is0}^{B},w_{is1}^{B},...,w_{isk}^{B})$, $(i=0,1,...,k;\ s=1,2,...,m_i)$ be the best solution of each objective function when solved individually and g_{is}^{B} be the best objective value of $g_{is}(w)$.

Also to acquire the worst optimal solution of $g_{is}(w)$, $(i=0,1,...,k;\ s=1,2,...,m_i)$ the following problem is solved according to Ramadan (1996).

$$\min g_{is}(w)=C_{is}^{U},\qquad i=0,1,...,k;s=1,2,...,m_i \tag{16}$$

Subject to

$$\begin{aligned}&\sum_{j=0}^{k}(A_{rj}+I_{rj}^{L}E_{rj})w_j\geq\mathrm{N}_r^U\qquad r=1,2,...,m\\&w_j\geq 0\end{aligned} \tag{17}$$

We solve the above problem and let $w_{is}^{W}=(w_{is0}^{W},w_{is1}^{W},...,w_{isk}^{W})$, $(i=0,1,...,k;\ s=1,2,...,m_i)$ be the worst solution of each objective function when solved individually and g_{is}^{W} be the worst objective value of $g_{is}(w)$.

So the optimal interval range of $g_{is}(w)$ is $[g_{is}^{B},g_{is}^{W}]$.

Let the objective function $g_{is}(w)$ has its target interval assigned as $[g_{is}^{*B},g_{is}^{*W}]$ by the DMs.

Then the target level of the objective function $g_{is}(w)$ appears as:

$$C_{is}^{U}\geq g_{is}^{*B} \tag{18}$$

$$C_{is}^{L} \le g_{is}^{*W} \qquad (i = 0,1,...,k;\ s = 1,2,...,m_i) \tag{19}$$

Hence formulation of the goal achievement functions takes place in the following way:

$$-C_{is}^{U} + d_{is}^{U} = -g_{is}^{*B} \tag{20}$$

$$C_{is}^{L} + d_{is}^{L} = g_{is}^{*W} \qquad (i = 0,1,...,k;\ s = 1,2,...,m_i) \tag{21}$$

where $d_{is}^{U} > 0,\ d_{is}^{L} > 0$ represent the deviational variables.

The optimal solution for the ULDM is separately determined first. Solving the following GP model the optimal solution for the ULDM is obtained.

$$\text{Min } \theta \tag{22}$$

Subject to

$$-C_{0s}^{U} + d_{0s}^{U} = -g_{0s}^{*B}$$
$$C_{0s}^{L} + d_{0s}^{L} = g_{0s}^{*W}$$
$$\sum_{j=0}^{k} (A_{rj} + I_{rj}^{U} E_{rj}) w_j \ge \mathrm{N}_r^{L}$$
$$\sum_{j=0}^{k} (A_{rj} + I_{rj}^{L} E_{rj}) w_j \ge \mathrm{N}_r^{U}$$
$$\theta \ge d_{0s}^{U}, \theta \ge d_{0s}^{L}$$
$$d_{0s}^{U}, d_{0s}^{L}, w_j \ge 0$$
$$s = 1,2,...,m_0;\ r = 1,2,...,m;\ j = 0,1,...,k$$

Let $w^{0*} = (w_0^*, w_1^*, ..., w_k^*)$ be the optimal solution of the ULDM. Let t_p^L and $t_p^R,\ p = 1,2,...,n_0$ be the negative and positive tolerance values (preference bounds) on the decision vector $w_0^* = (w_{01}^*, w_{02}^*, ..., w_{0n_0}^*)$ which is controlled by the ULDM. t_p^L and t_p^R may not necessarily be the same (Dey et al., 2014; Dey and Pramanik, 2011; Pramanik, 2012; Pramanik and Dey, 2011a, 2011b; Pramanik et al., 2011a, 2011b, 2011c). These preference bounds are decided by the ULDM in order to cooperate with the LLDMs to attain a compromise optimal solution of the BLDMOLPP. The preference bounds on the decision vector w_0^* can be written as:

$$w_{0p}^* - t_p^L \le w_{0p}^* \le w_{0p}^* - t_p^U, \qquad p = 1,2,...,n_0 \tag{23}$$

Finally, the BLDMOLPP with neutrosophic numbers can be solved with the help of GP model which can be formulated as below:

GP model:

$$\text{Min } S = \sum_{s=1}^{m_0} (\tau_{0s}^{U} d_{0s}^{U} + \tau_{0s}^{L} d_{0s}^{L}) + \sum_{s=1}^{m_1} (\tau_{1s}^{U} d_{1s}^{U} + \tau_{1s}^{L} d_{1s}^{L}) + ... + \sum_{s=1}^{m_k} (\tau_{ks}^{U} d_{ks}^{U} + \tau_{ks}^{L} d_{ks}^{L}) \tag{24}$$

Subject to

$$-C_{is}^{U} + d_{is}^{U} = -g_{is}^{*B}$$
$$C_{is}^{L} + d_{is}^{L} = g_{is}^{*W}$$
$$\sum_{j=0}^{k} (A_{rj} + I_{rj}^{U} E_{rj}) w_j \geq \mathrm{N}_r^{L}$$
$$\sum_{j=0}^{k} (A_{rj} + I_{rj}^{L} E_{rj}) w_j \geq \mathrm{N}_r^{U}$$
$$w_{0p}^{*} - t_p^{L} \leq w_{0p}^{*} \leq w_{0p}^{*} - t_p^{U},$$
$$w_j, d_{is}^{U}, d_{is}^{L}, \tau_{is}^{U}, \tau_{is}^{L} \geq 0$$
$$i = 0,1,...,k; s = 1,2,...,m_i; r = 1,2,...,m; j = 0,1,...,k; p = 1,2,...,n_0$$

Here $\tau_{is}^{U}, \tau_{is}^{L}$ $(i = 0,1,...,k; s = 1,2,...,m_i)$ represent the numerical weights associated with the corresponding deviational variables as decided by the DMs.

4.1 Algorithm to solve BLDMOLPP with NNs

Step 1: The original BLDMOLPP with NNs is converted into interval BLDMOLPP as given in equations (10–13) along with the transformed constraints (9).

Step 2: The best and worst solutions for each objective function are obtained using equations (14–17).

Step 3: The goal achievement function for the objectives are formed using equations (20–21).

Step 4: Solution of GP model (22) provides the best solution for ULDM.

Step 5: ULDM assigns upper and lower tolerance limits to his/her controlled decision variable according to equation (23).

Step 6: Solving GP model (24) the optimal compromise values of the decision variables are obtained.

5 Numerical illustration

The applicability of the proposed strategy to solve BLDMOLPP with NN coefficients is illustrated with the help of a numerical example. We consider $I \in [0,1]$.

Upper level:

$$\mathrm{DM}_0 : \min_{x_0} \begin{cases} g_1(x) = [2+3I]x_0 + [5+9I]x_1 + [4+5I]x_2 + [1+2I] \\ g_2(x) = [5+4I]x_0 + [6+9I]x_1 + [10+I]x_2 + [7+2I] \end{cases}$$

Lower level:

$$\mathrm{DM}_1 : \min_{x_1} \begin{cases} g_3(x) = [2+5I]x_0 + [4+7I]x_1 + [8+9I]x_2 + [5+2I] \\ g_4(x) = [4-3I]x_0 + [9-5I]x_1 + [1+2I]x_2 \end{cases}$$

$$\mathrm{DM}_2 : \min_{x_2} \begin{cases} g_5(x) = [5-4I]x_0 + [6+7I]x_1 + [2+8I]x_2 + [9-5I] \\ g_6(x) = [2-I]x_0 + [9-4I]x_1 + [7-5I]x_2 + [3+7I] \end{cases}$$

Subject to

$$[4+2I]x_0 + [3+7I]x_1 + [1+5I]x_2 \geq [15+10I]$$
$$[6+I]x_0 + [-2+4I]x_1 + [6+2I]x_2 \geq [5+3I]$$
$$x_0, x_1, x_2 \geq 0$$

The transformed problems to obtain the best and worst solutions for ULDM are shown in Table 1. The best and worst solutions that are obtained solving the problems in Table 1 are presented in Table 2.

Table 1 Transformed problems to obtain best and worst solutions for ULDM

Objective function	*Problem to obtain the best solution*	*Problem to obtain the worst solution*
g_1	Min $2x_0 + 5x_1 + 4x_2 + 1$	Min $5x_0 + 14x_1 + 9x_2 + 3$
	Subject to	Subject to
	$6x_0 + 10x_1 + 6x_2 \geq 15$	$4x_0 + 3x_1 + x_2 \geq 25$
	$7x_0 + 2x_1 + 8x_2 \geq 5$	$6x_0 - 2x_1 + 6x_2 \geq 8$
	$x_0, x_1, x_2 \geq 0$	$x_0, x_1, x_2 \geq 0$
g_2	Min $5x_0 + 6x_1 + 10x_2 + 7$	Min $9x_0 + 15x_1 + 11x_2 + 9$
	Subject to	Subject to
	$6x_0 + 10x_1 + 6x_2 \geq 15$	$4x_0 + 3x_1 + x_2 \geq 25$
	$7x_0 + 2x_1 + 8x_2 \geq 5$	$6x_0 - 2x_1 + 6x_2 \geq 8$
	$x_0, x_1, x_2 \geq 0$	$x_0, x_1, x_2 \geq 0$

Table 2 The best and worst solutions obtained for ULDM

Objective function	*Best solution with solution point*	*Worst solution with solution point*
g_1	6 at (2.5,0,0)	34.25 at (6.25,0,0)
g_2	16.4827 at (0.3448,1.2931,0)	65.25 at (6.25,0,0)

Hence the ULDM's target objective functions can be formed as:

$$2x_0 + 5x_1 + 4x_2 + 1 \leq 34$$
$$5x_0 + 14x_1 + 9x_2 + 3 \geq 6$$
$$5x_0 + 6x_1 + 10x_2 + 7 \leq 65$$
$$9x_0 + 15x_1 + 11x_2 + 9 \geq 16.5$$

Accordingly, the goal functions with specified targets can be written as:

$$2x_0 + 5x_1 + 4x_2 + 1 + d_{01}^L = 34$$
$$-5x_0 - 14x_1 - 9x_2 - 3 + d_{01}^U = -6$$
$$5x_0 + 6x_1 + 10x_2 + 7 + d_{02}^L = 65$$
$$-9x_0 - 15x_1 - 11x_2 - 9 + d_{02}^U = -16.5$$

GP model (22) provides the best solution of ULDM as $x_0 = 6.25, x_1 = 0, x_2 = 0$.

The transformed problems to obtain the best and worst solutions for DM_1 are shown in Table 3. The solutions that are obtained solving the problems in Table 3 are presented in Table 4.

Table 3 Transformed problems for obtaining the best and worst solutions for DM_1

Objective function	*Problem to obtain the best solution*	*Problem to obtain the worst solution*
g_3	Min $2x_0 + 4x_1 + 8x_2 + 5$	Min $7x_0 + 11x_1 + 17x_2 + 7$
	Subject to	Subject to
	$6x_0 + 10x_1 + 6x_2 \geq 15$	$4x_0 + 3x_1 + x_2 \geq 25$
	$7x_0 + 2x_1 + 8x_2 \geq 5$	$6x_0 - 2x_1 + 6x_2 \geq 8$
	$x_0, x_1, x_2 \geq 0$	$x_0, x_1, x_2 \geq 0$
g_4	Min $x_0 + 4x_1 + x_2$	Min $4x_0 + 9x_1 + 3x_2$
	Subject to	Subject to
	$6x_0 + 10x_1 + 6x_2 \geq 15$	$4x_0 + 3x_1 + x_2 \geq 25$
	$7x_0 + 2x_1 + 8x_2 \geq 5$	$6x_0 - 2x_1 + 6x_2 \geq 8$
	$x_0, x_1, x_2 \geq 0$	$x_0, x_1, x_2 \geq 0$

Table 4 Best and worst solutions obtained for DM_1

Objective function	*Best solution with solution point*	*Worst solution with solution point*
g_3	10 at (2.5,0,0)	50.75 at (6.25,0,0)
g_4	2.5 at (2.5,0,0)	25 at (6.25,0,0)

Hence the target objective functions for DM_1 can be formed as:

$$2x_0 + 4x_1 + 8x_2 + 5 \leq 50$$
$$7x_0 + 11x_1 + 17x_2 + 7 \geq 10$$
$$x_0 + 4x_1 + x_2 \leq 25$$
$$4x_0 + 9x_1 + 3x_2 \geq 3$$

Accordingly, the goal functions with specified targets can be written as:

$$2x_0 + 4x_1 + 8x_2 + 5 + d_{11}^L = 50$$
$$-7x_0 - 11x_1 - 17x_2 - 7 + d_{11}^U = -10$$

$$x_0 + 4x_1 + x_2 + d_{12}^L = 25$$
$$-4x_0 - 9x_1 - 3x_2 + d_{12}^U = -3$$

The transformed problems to obtain the best and worst solutions for DM_2 are shown in Table 5. The solutions that are obtained solving the problems in table 5 are presented in Table 6.

Table 5 Transformed problems for obtaining the best and worst solutions for DM_2

Objective function	*Problem to obtain the best solution*	*Problem to obtain the worst solution*
g_5	Min $x_0 + 6x_1 + 2x_2 + 4$	Min $5x_0 + 13x_1 + 10x_2 + 9$
	Subject to	Subject to
	$6x_0 + 10x_1 + 6x_2 \geq 15$	$4x_0 + 3x_1 + x_2 \geq 25$
	$7x_0 + 2x_1 + 8x_2 \geq 5$	$6x_0 - 2x_1 + 6x_2 \geq 8$
	$x_0, x_1, x_2 \geq 0$	$x_0, x_1, x_2 \geq 0$
g_6	Min $x_0 + 5x_1 + 2x_2 + 3$	Min $2x_0 + 9x_1 + 7x_2 + 10$
	Subject to	Subject to
	$6x_0 + 10x_1 + 6x_2 \geq 15$	$4x_0 + 3x_1 + x_2 \geq 25$
	$7x_0 + 2x_1 + 8x_2 \geq 5$	$6x_0 - 2x_1 + 6x_2 \geq 8$
	$x_0, x_1, x_2 \geq 0$	$x_0, x_1, x_2 \geq 0$

Table 6 Best and worst solutions obtained for DM_2

Objective function	*Best solution with solution point*	*Worst solution with solution point*
g_5	6.5 at (2.5,0,0)	40.25 at (6.25,0,0)
g_6	5.5 at (2.5,0,0)	22.5 at (6.25,0,0)

Hence the target objective functions for DM_2 can be formed as:

$$x_0 + 6x_1 + 2x_2 + 4 \leq 40$$
$$5x_0 + 13x_1 + 10x_2 + 9 \geq 7$$
$$x_0 + 5x_1 + 2x_2 + 3 \leq 22$$
$$2x_0 + 9x_1 + 7x_2 + 10 \geq 6$$

Accordingly, the goal functions with specified targets can be written as:

$$x_0 + 6x_1 + 2x_2 + 4 + d_{21}^L = 40$$
$$-5x_0 - 13x_1 - 10x_2 - 9 + d_{21}^U = -7$$
$$x_0 + 5x_1 + 2x_2 + 3 + d_{22}^L = 22$$
$$-2x_0 - 9x_1 - 7x_2 - 10 + d_{22}^U = -6$$

Suppose the preference bounds assigned by ULDM on the decision variable x_0 is considered as $6.25 - 0.75 \leq x_0 \leq 6.25 + 1.25$.

Solution of the following GP model provides the optimal values of the decision variables of the original BLDMOLPP.

GP model:

$$\text{Min } \frac{1}{12}(d_{01}^{L}+d_{01}^{U}+d_{02}^{L}+d_{02}^{U}+d_{11}^{L}+d_{11}^{U}+d_{12}^{L}+d_{12}^{U}+d_{21}^{L}+d_{21}^{U}+d_{22}^{L}+d_{22}^{U}) \tag{25}$$

Subject to

$$2x_0+5x_1+4x_2+1+d_{01}^{L}=34$$
$$-5x_0-14x_1-9x_2-3+d_{01}^{U}=-6$$
$$5x_0+6x_1+10x_2+7+d_{02}^{L}=65$$
$$-9x_0-15x_1-11x_2-9+d_{02}^{U}=-16.5$$
$$2x_0+4x_1+8x_2+5+d_{11}^{L}=50$$
$$-7x_0-11x_1-17x_2-7+d_{11}^{U}=-10$$
$$x_0+4x_1+x_2+d_{12}^{L}=25$$
$$-4x_0-9x_1-3x_2+d_{12}^{U}=-3$$
$$x_0+6x_1+2x_2+4+d_{21}^{L}=40$$
$$-5x_0-13x_1-10x_2-9+d_{21}^{U}=-7$$
$$x_0+5x_1+2x_2+3+d_{22}^{L}=22$$
$$-2x_0-9x_1-7x_2-10+d_{22}^{U}=-6$$
$$6x_0+10x_1+6x_2\geq 15$$
$$7x_0+2x_1+8x_2\geq 5$$
$$4x_0+3x_1+x_2\geq 25$$
$$6x_0-2x_1+6x_2\geq 8$$
$$5.5\leq x_0\leq 7.5$$
$$x_0,x_1,x_2\geq 0$$
$$d_{0i}^{L},d_{0i}^{U},d_{1i}^{L},d_{1i}^{U},d_{2i}^{L},d_{2i}^{U}\geq 0$$
$$i=1,2$$

Solution of GP model (25) provides the decision vector as (6.25, 0, 0). Using it, the obtained optimal range of the objective functions is presented in Table 7.

Table 7 Optimal range of the objective functions

Objective functions	*Optimal range*
g_1	[13.5, 34.25]
g_2	[38.25, 65.25]
g_3	[17.5, 50.75]
g_4	[6.25, 25]
g_5	[10.25, 40.25]
g_6	[9.25, 22.5]

6 Conclusion

A GP strategy is proposed in this paper to solve BLDMOLPP with the parameters as neutrosophic numbers. The problem gets converted into a BLDMOLPP with interval numbers when the NNs represented as [P+Q*I*] are converted into intervals. The target interval of each objective function is obtained using interval programming technique. Then goal achievement functions are established to attain the target goals of the objectives. The optimal solution of the ULDM is obtained separately and preference bounds are provided on the decision variables controlled by him/her. The compromise optimal solution of the BLDMOLPP is then obtained using GP strategy. The efficiency and applicability of the strategy is explained through a numerical example.

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