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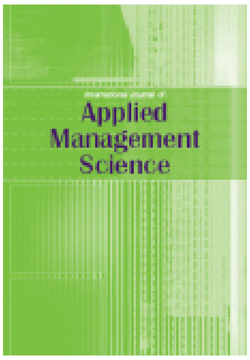

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Use of known population median of study variable for elevated estimation of population mean

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Abstract: In this research, we proposed a new enhanced estimator of population mean of primary variable utilising the acquaintance on the known median of the main variable. We perused the features of the distribution of the proposed estimator till the approximation of order one. The articulated estimator is collated with the estimators in competition of the population mean, and the prerequisites of the suggested estimator to be more efficient over competing are derived. These conditions are put to the proof using the numerical example. The efficiencies are compared in terms of the mean squared errors.

Keywords: main variable; modified ratio estimator; mean squared error; percentage relative efficiency.

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1 Introduction

The utilisation of auxiliary information enhances the estimation of any parameter and so is for the population mean $\bar{Y}$. One of the significant drawbacks of the auxiliary information (X) is that it increases the survey cost. Therefore, we think of the elevated estimation of the survey without raising the survey cost. One of the beautiful uses of the alternative of the auxiliary information is the use of the known population median (M) of the primary variable (Y) which can be found with no increasing survey cost. The median do not require each population unit to be enquired; instead, it is obtained from the information at various intervals. Some examples of enhanced estimation of $\bar{Y}$, utilising known M of Y have been given by Subramani (2016). Subramani (2016) has shown improvement over competing ratio and regression type estimators, which utilise X with increasing survey cost.

Many authors have made use of known $X$for elevated estimation of the $\bar{Y}$. Gupta and Shabbir (2008) suggested an estimator having improvement over other estimators under a simple random sampling scheme while Koyuncu and Kadilar (2009) also proposed some efficient estimators of $\bar{Y}$. Al-Omari et al. (2009) proposed some novel ratio type estimators under simple and ranked set sampling schemes. Shabbir and Gupta (2011) and Singh and Solanki (2012) suggested elevated estimators under different sampling schemes using quantitative and qualitative information on X. Subramani (2013) and Yadav and Kadilar (2013a, 2013b) suggested some generalised estimators of $\bar{Y}$. Subramani and Kumarapandiyan (2012, 2013) proposed some estimators using known parameters of the auxiliary variable. Yadav and Mishra (2015) and Yadav et al. (2016) suggested improved estimators simple and under predictive approach. Many more authors, like Abid et al. (2016), Gupta and Yadav (2017, 2018), Subramani and Ajith (2017), Srija and Subramani (2018), Yadav et al. (2018), Yadav et al. (2019) and Yadav and Zaman (2020) utilised known X as its various parameters and suggested the efficient estimators. One of the drawbacks of the ratio, regression estimators is that both use known X for elevated estimation of the population parameters but this information is gathered on augmented survey cost.

Thus, we search for the estimators that make use of the known parameters of Y which are obtained without augmenting survey cost. Population median is one of those parameters, which do not require information on every unit of the population, but it is calculated if the information is obtained in terms of the intervals. Various authors used the known $M$and suggested the more efficient estimators of $\bar{Y}$ without augmenting the survey cost. Subramani (2016) introduced the concept of using known population median for enhanced estimation of $\bar{Y}$. Later various authors including Yadav et al. (2017), Yadav and Pandey (2017) and Yadav et al. (2020) utilised M and proposed the elevated estimators of $\bar{Y}$.

In this research, adapting Nangsue (2009) estimator and applying the Subramani (2016) scheme to it, we suggest a new generalised ratio type estimator of $\bar{Y}$ utilising known M. The estimator suggested by Subramani (2016) is a special case of the suggested estimator. We also study the sampling properties of the suggested estimator till the approximation of order one. Further, it is compared both theoretically and empirically with different estimators of $\bar{Y}$ in competition. The rest article is presented as follows; Section 2 provides the review of estimators of the $\bar{Y}$. Section 3 is about the proposed estimator of $\bar{Y}$ using M. Section 4 derived the properties of suggested estimator and compared its efficiency with other competitive estimators of $\bar{Y}$. Section 5 presents a numerical study to verify the theoretical conditions of the efficiencies of suggested estimator over the mentioned competing estimators. Section 6 describes the results of this investigation. At last, Section 7 shows the conclusion of the study with future directions.

2 Review of competing estimators

Let N be the population under consideration from which n samples are drawn using simple random sampling scheme resulting ${}^{N}C_{n}$ all possible samples each of size n. Let Y be the primary variable and X be the auxiliary variable having high degree of correlation with Y. $(\bar{Y}, \bar{X})$ are the population means of Y and X respectively while M is the median of Y. Further, let $(\bar{y}, \bar{x})$ represents the sample means of Y and X respectively while m is the sample median of Y along with $\bar{M} = \frac{1}{{}^{N}C_{n}} \sum_{i=1}^{{}^{N}C_{n}} m_i$ as mean of all sample medians of Y. ρ_{yx} and ρ_{ym} represents the coefficients of correlations between Y&X and Y&Mwhereas b_{yx} and b_{ym} are coefficients of regressions of Y on X and Y on M respectively.

Various unbiased and biased estimators along with their variances and MSEs till first degree approximation have been summarised in the following Table 1.

Table 1 Different estimators with their variances and MSEs

S. No.	*Estimator with variance/MSE*
1.	$t_o = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ $V(t_0) = \frac{1-f}{n} S_y^2 = \frac{1-f}{n} \bar{Y}^2 C_y^2$
2.	$t_1 = \bar{y} + \beta(\bar{X} - \bar{x})$: Watson (1937) $V(t_1) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2)$
3.	$t_2 = \bar{y}\frac{\bar{X}}{\bar{x}}$: Cochran (1940) $MSE(t_2) = \frac{1-f}{n} \bar{Y}^2 [C_y^2 + C_x^2 - 2C_{yx}]$
4.	$t_3 = \bar{y}\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right]$: Bahl and Tuteja (1991) $MSE(t_3) = \frac{1-f}{n} \bar{Y}^2 [C_y^2 + \frac{C_x^2}{4} - C_{yx}]$
5.	$t_4 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^2$: Kadilar and Cingi (2003) $MSE(t_4) = \frac{1-f}{n} \bar{Y}^2 [C_y^2 + 4C_x^2 - 4C_{yx}]$
6.	$t_5 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^\alpha$: Srivastava (1967) $MSE_{\min}(t_5) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2)$ for $\alpha_{opt} = -C_{yx} / C_x^2$
7.	$t_6 = \bar{y}\left[\frac{\bar{X}}{\bar{X} + \alpha(\bar{x} - \bar{X})}\right]$: Reddy (1974) $MSE_{\min}(t_6) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2)$ for $\alpha_{opt} = C_{yx} / C_x^2$
8.	$t_7 = \left[\omega_1 \bar{y} + \omega_2 (\bar{X} - \bar{x})\right]\left(\frac{\eta\bar{X} + \delta}{\eta\bar{x} - \delta}\right)$: Gupta and Shabbir (2008) $MSE_{\min}(t_7) = \bar{Y}^2 \left[1 - \nu_1\right]$

Table 1 Different estimators with their variances and MSEs (continued)

S. No.	*Estimator with variance/MSE*
9.	$t_8 = \psi_1 \bar{y} \left(\frac{\eta \bar{X} + \delta}{\eta \bar{x} - \delta} \right) + \psi_2 (\bar{X} - \bar{x}) \left(\frac{\eta \bar{X} + \delta}{\eta \bar{x} - \delta} \right)^2$: Singh and Solanki (2012) $MSE_{\min}(t_8) = \bar{Y}^2 \left[1 - \nu_2 \right]$
10.	$t_9 = \bar{y} \left\{ 2 - \left(\frac{\bar{x}}{\bar{X}} \right)^{\alpha} \exp\left(\frac{\xi (\bar{X} - \bar{x})}{(\bar{X} + \bar{x})} \right) \right\}$: Solanki et al. (2012) $MSE_{\min}(t_9) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2)$ for $(2\alpha + \xi) = 2\kappa$
11.	$t_{10} = \theta_1 \bar{y} + \theta_2 (\bar{X} - \bar{x}) \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right)$: Ekpenyong and Enang (2015) $MSE_{\min}(t_{10}) = \bar{Y}^2 \left[1 - q \right]$
12.	$t_{11} = \bar{y} \left(\frac{\bar{x}}{\bar{X}} \right)^{\delta} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right)$: Kadilar (2016) $MSE_{\min}(t_{11}) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2)$ for $\delta_{opt} = \left(\frac{1}{2} - \rho_{yx} C_y / C_x \right)$
13.	$t_{12} = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{b}$: Nangsue (2009) where $b = b_{yx}$ $MSE_{\min}(t_{12}) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2)$
14.	$t_{13} = \bar{y} \frac{M}{m}$: Subramani (2016) $MSE(t_{13}) = \frac{1-f}{n} \bar{Y}^2 [C_y^2 + R_{13}^2 C_m^2 - 2R_{13} C_{ym}]$
15.	$t_{14} = \bar{y} \left(\frac{\bar{x} + C_x}{\bar{X} + C_x} \right)^{b}$ and $t_{15} = \bar{y} \left(\frac{\bar{x} + \rho}{\bar{X} + \rho} \right)^{b}$: Soponviwatkul and Lawson (2017) where $b = b_{yx}$ $MSE_{\min}(t_{14}) = MSE_{\min}(t_{15}) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2)$

Note: There is no benefit of using additional information on parameters of *X* as the MSEs of both the above estimators are equal to MSE of Nangsue (2009) estimator, which does not use known parameters of *X*.

Where,

$$C_y = \frac{S_y}{\bar{Y}},\ S_y^2 = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})^2 = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(\bar{y}_i - \bar{Y})^2,\ C_{yx} = \rho_{yx}\, C_y C_x,\ f = \frac{n}{N},$$

$$C_x = \frac{S_x}{\bar{X}},\ S_x^2 = \frac{1}{N-1}\sum_{i-1}^{N}(X_i - \bar{X})^2 = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(\bar{x}_i - \bar{X})^2,$$

$$Cov(x,y) = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})(X_i - \bar{X}),\ \rho_{yx} = \frac{Cov(x,y)}{S_x S_y},\ R_{13} = \frac{\bar{Y}}{M},\ C_m = \frac{S_m}{M},$$

$$S_{ym} = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(\bar{y}_i - \bar{Y})(m_i - M),\ C_{ym} = \frac{S_{ym}}{\bar{Y}M},\ S_m^2 = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(m_i - M)^2,$$

$$\alpha_1 = [1 + \lambda\{C_y^2 + \tau C_x^2(3\tau - 4\kappa)\}],\ \alpha_2 = \lambda C_x^2,\ \alpha_3 = \lambda C_x^2(\kappa - 2\tau),\ \alpha_4 = [1 - \lambda\tau C_x^2(\kappa - \tau)],$$

$$\alpha_5 = \lambda\tau C_x^2,\ A = [1 + \lambda\{C_y^2 + \tau C_x^2(3\tau - 4\kappa)\}],\ B = \lambda C_x^2,\ C = \lambda C_x^2(3\tau - \kappa),$$

$$D = [1 + \tau C_x^2(\tau - \kappa)],\ E = \lambda\tau C_x^2,\ \tau = \frac{\eta\bar{X}}{\eta\bar{X} + \delta},\ \omega_1^* = \frac{(\alpha_2\alpha_4 - \alpha_3\alpha_5)}{(\alpha_1\alpha_2 - \alpha_3^2)},$$

$$\omega_1^* = \frac{R(\alpha_1\alpha_5 + \alpha_3\alpha_4)}{(\alpha_1\alpha_2 - \alpha_3^2)},\ R = \frac{\bar{Y}}{\bar{X}},\ \psi_1^* = \frac{(BD - CE)}{(AB - C^2)},\ \psi_2^* = \frac{(AE - CD)}{(AB - C^2)},\ \lambda = \frac{1-f}{n},$$

$$\upsilon_1 = \frac{(\alpha_2\alpha_4^2 + 2\alpha_3\alpha_4\alpha_5 + \alpha_1\alpha_5^2)}{(\alpha_1\alpha_2 - \alpha_3^2)},\ \upsilon_2 = \frac{(BD^2 - 2CDE + AE^2)}{(AB - C^2)},\ q = \frac{(\gamma_4 + 2\gamma_2\gamma_3 + \gamma_1\gamma_3^2)}{(\gamma_1\gamma_4 - \gamma_2^2)},$$

$$\theta_1^* = (\gamma_4 + \gamma_2\gamma_3)/(\gamma_1\gamma_4 - \gamma_2^2),\ \theta_2^* = R(\gamma_2 + \gamma_1\gamma_3)/(\gamma_1\gamma_4 - \gamma_2^2),\ \gamma_1 = 1 + \lambda C_y^2,$$

$$\gamma_2 = \lambda C_x^2(\kappa - \frac{1}{2}),\ \gamma_3 = \frac{\lambda}{2}C_x^2,\ \gamma_4 = \lambda C_x^2.$$

ω_1^*, ω_2^*, ψ_1^* and ψ_2^* are optimum values of ω_1, ω_2, ψ_1 and ψ_2 respectively. η ($\eta \neq 0$), α, δ and ξ are constants or parameters of *X*.

3 Proposed estimator

Motivated by Nangsue (2009) and Subramani (2016), a general ratio estimator of $\bar{Y}$ utilising known *M* as,

$$t_p = \bar{y}\left(\frac{M}{m}\right)^b \tag{1}$$

where $b = b_{ym}$, defined by $b = \frac{s_{ym}}{s_m^2}$.

To study properties of t_p, we use approximations, $\bar{y}=\bar{Y}(1+e_0)$, $m=M(1+e_1)$, $s_{ym}=S_{ym}(1+e_2)$ and $s_m^2=S_m^2(1+e_3)$ such that $E(e_0)=0$, $E(e_1)=\frac{\bar{M}-M}{M}=\frac{Bias(m)}{M}$, $E(e_2)=E(e_3)=0$ and $E(e_0^2)=\frac{1-f}{n}C_y^2$, $E(e_1^2)=\frac{1-f}{n}C_m^2$, $E(e_0e_1)=\frac{1-f}{n}C_{ym}$, $E(e_1e_2)=\frac{1-f}{n}\frac{\mu_{21}}{S_{ym}M}$ and $E(e_1e_3)=\frac{1-f}{n}\frac{\mu_{12}}{S_m^2M}$, where $\mu_{rs}=\frac{1}{N}\sum_{i=1}^{N}(m_i-M)^r(y_i-\bar{Y})^s$.

Expressing t_p in the form of $e_i's$ ($i=0,1$), we get

$$
\begin{aligned}
t_p &= \bar{Y}(1+e_0)\left(\frac{M}{M(1+e_1)}\right)^b \\
&= \bar{Y}(1+e_0)(1+e_1)^{-b} \\
&= \bar{Y}(1+e_0)\left[1-be_1+\frac{b(b+1)}{2}e_1^2\right] \\
&= \bar{Y}\left[1+e_0-be_1-be_0e_1+\frac{b(b+1)}{2}e_1^2\right] \\
&= \bar{Y}\left[1+e_0-\frac{s_{ym}}{s_m^2}e_1-\frac{s_{ym}}{s_m^2}e_0e_1+\frac{s_{ym}}{s_m^2}\left(\frac{s_{ym}}{s_m^2}+1\right)\frac{e_1^2}{2}\right] \\
&= \bar{Y}\left[1+e_0-\frac{S_{ym}(1+e_2)}{S_m^2(1+e_3)}e_1-\frac{S_{ym}(1+e_2)}{S_m^2(1+e_3)}e_0e_1+\frac{S_{ym}(1+e_2)}{S_m^2(1+e_3)}\left(\frac{S_{ym}(1+e_2)}{S_m^2(1+e_3)}+1\right)\frac{e_1^2}{2}\right] \\
&= \bar{Y}\left[\begin{array}{l} 1+e_0-\frac{S_{ym}}{S_m^2}e_1(1+e_2)(1+e_3)^{-1}-\frac{S_{ym}}{S_m^2}e_0e_1(1+e_2)(1+e_3)^{-1} \\ +\frac{S_{ym}}{S_m^2}\frac{e_1^2}{2}(1+e_2)(1+e_3)^{-1}\left(\frac{S_{ym}}{S_m^2}(1+e_2)(1+e_3)^{-1}+1\right) \end{array}\right] \\
&= \bar{Y}\left[\begin{array}{l} 1+e_0-Be_1(1+e_2)(1-e_3+e_3^2)-Be_0e_1(1+e_2)(1-e_3+e_3^2) \\ \qquad +B\frac{e_1^2}{2}(1+e_2)(1-e_3+e_3^2)[B(1+e_2)(1-e_3+e_3^2)+1] \end{array}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
B &= \frac{S_{ym}}{S_m^2} \\
&= \bar{Y}\left[1+e_0-B(e_1+e_1e_3-e_1e_4)-Be_0e_1+B\frac{e_1^2}{2}+B^2\frac{e_1^2}{2}\right] \qquad (2)
\end{aligned}
$$

$$
t_p-\bar{Y}=\bar{Y}\left[e_0-B(e_1+e_1e_3-e_1e_4)-Be_0e_1+B\frac{e_1^2}{2}+B^2\frac{e_1^2}{2}\right]
$$

Taking expectation both sides of (2) and applying values of different expectations, the bias of t_p can be expressed,

$$Bias(t_p) = \overline{Y}\left[(B+B^2)\frac{1-f}{n}\frac{C_m^2}{2} - B\frac{\overline{M}-M}{M} - B\frac{1-f}{n}(C_{ym} + \frac{\mu_{12}}{S_m^2 M} - \frac{\mu_{21}}{S_{ym}M}) \right] \quad (3)$$

From equation (2), till first degree approximation, we have,

$$t_p - \overline{Y} \approx \overline{Y}(e_0 - Be_1) \quad (4)$$

Squaring on both sides of (4) and getting it's expectation on, we get the MSE of t_p as,

$$MSE(t_p) = \overline{Y}^2 E(e_0 - Be_1)^2$$
$$= \overline{Y}^2 E(e_0^2 + B^2 e_1^2 - 2Be_0 e_1)$$

Applying different expectations values, we have,

$$MSE(t_p) = \frac{1-f}{n}\overline{Y}^2[C_y^2 + B^2 C_m^2 - 2BC_{ym}] \quad (5)$$

4 Efficiency comparison

The efficiency conditions for t_p over the estimators in competition are presented in Table 2.

Table 2 Theoretical efficiency of t_p over the competing estimators of $\overline{Y}$

S. No.	*Estimator*	*t_p performs better than competing one if*
1.	$t_o = \overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ Mean per unit estimator	$V(t_0) - MSE_{\min}(t_p) > 0$, or $[\upsilon_1 + r] > 2$
2.	$t_1 = \overline{y} + \beta(\overline{X} - \overline{x})$ Watson (1937) estimator	$MSE(t_1) - MSE_{\min}(t_p) > 0$, or $\lambda C_y^2(1-\rho_{yx}^2) + r > 1$
3.	$t_2 = \overline{y}\frac{\overline{X}}{\overline{x}}$ Cochran (1940) estimator	$MSE(t_2) - MSE_{\min}(t_p) > 0$, or $\lambda[C_y^2 + C_x^2 - 2C_{yx}] + r > 1$
4.	$t_3 = \overline{y}\exp\left[\frac{\overline{X}-\overline{x}}{\overline{X}+\overline{x}}\right]$ Bahl and Tuteja (1991) estimator	$MSE(t_3) - MSE_{\min}(t_p) > 0$, or $\lambda[C_y^2 + 4C_x^2 - 4C_{yx}] + r > 1$
5.	$t_4 = \overline{y}\left(\frac{\overline{x}}{\overline{X}}\right)^2$ Kadilar and Cingi (2003) estimator	$MSE(t_4) - MSE_{\min}(t_p) > 0$, or $\lambda[C_y^2 + 4C_x^2 - 4C_{yx}] + r > 1$

Table 2 Theoretical efficiency of t_p over the competing estimators of $\bar{Y}$ (continued)

S. No.	*Estimator*	*t_p performs better than competing one if*
6.	$t_5 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}$ Srivastava (1967) estimator	$MSE_{\min}(t_5) - MSE_{\min}(t_p) > 0$, or $\lambda C_y^2(1-\rho_{yx}^2) + r > 1$
7.	$t_6 = \bar{y}\left[\frac{\bar{X}}{\bar{X} + \alpha(\bar{x} - \bar{X})}\right]$ Reddy (1974) estimator	$MSE_{\min}(t_6) - MSE_{\min}(t_p) > 0$, or $\lambda C_y^2(1-\rho_{yx}^2) + r > 1$
8.	$t_7 = \left[\omega_1\bar{y} + \omega_2(\bar{X} - \bar{x})\right]\left(\frac{\eta\bar{X} + \delta}{\eta\bar{x} - \delta}\right)$ Gupta and Shabbir (2008) estimator	$MSE_{\min}(t_7) - MSE_{\min}(t_p) > 0$, or $r - v_1 > 0$
9.	$t_8 = \psi_1\bar{y}\left(\frac{\eta\bar{X}+\delta}{\eta\bar{x}-\delta}\right) + \psi_2(\bar{X}-\bar{x})\left(\frac{\eta\bar{X}+\delta}{\eta\bar{x}-\delta}\right)^2$ Singh and Solanki (2012) estimator	$MSE_{\min}(t_8) - MSE_{\min}(t_p) > 0$, or $r - v_2 > 0$
10.	$t_9 = \bar{y}\left\{2 - \left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{\xi(\bar{X} - \bar{x})}{(\bar{X} + \bar{x})}\right)\right\}$ Solanki et al. (2012) estimator	$MSE_{\min}(t_9) - MSE_{\min}(t_p) > 0$, or $\lambda C_y^2(1-\rho_{yx}^2) + r > 1$
11.	$t_{10} = \theta_1\bar{y} + \theta_2(\bar{X} - \bar{x})\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)$ Ekpenyong and Enang (2015) estimator	$MSE_{\min}(t_{10}) - MSE_{\min}(t_p) > 0$, or $r - q > 0$
12.	$t_{11} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\delta} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)$ Kadilar (2016) estimator	$MSE_{\min}(t_{11}) - MSE_{\min}(t_p) > 0$, or $\lambda C_y^2(1-\rho_{yx}^2) + r > 1$
13.	$t_{12} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{b}$ Nangsue (2009)	$MSE_{\min}(t_{12}) - MSE_{\min}(t_p) > 0$, or $\lambda C_y^2(1-\rho_{yx}^2) + r > 1$
14.	$t_{13} = \bar{y}\frac{M}{m}$ Subramani (2016) estimator	$MSE(t_{13}) - MSE_{\min}(t_p) > 0$, or $\lambda[C_y^2 + R_{13}^2 C_m^2 - 2R_{13}C_{ym}] + r > 1$
15.	$t_{14} = \bar{y}\left(\frac{\bar{x} + C_x}{\bar{X} + C_x}\right)^{b}$ and $t_{15} = \bar{y}\left(\frac{\bar{x} + \rho}{\bar{X} + \rho}\right)^{b}$ Soponviwatkul and Lawson (2017)	$MSE(t_i) - MSE_{\min}(t_p) > 0$, $i = 14,15$ or $\lambda C_y^2(1-\rho_{yx}^2) + r > 1$

5 Empirical study

We verify the theoretical conditions of the efficiencies of t_p over the mentioned competing estimators of $\bar{Y}$, and for this, we consider the population given by Subramani (2016). The parameters of the population are given in Table 3 and the MSEs and percentage relative efficiencies (PREs) of t_p over the estimators in competition are shown in Table 4.

Table 3 Population parameters

Parameter	*Value*
N	20
n	5
${}^{N}C_{n}$	15504
$\bar{Y}$	41.5
$\bar{M}$	40.0552
M	40.5
$\bar{X}$	441.95
R_{13}	1.0247
C_y^2	0.008338
C_x^2	0.007845
C_m^2	0.006606
C_{ym}	0.005394
C_{yx}	0.005275
ρ_{yx}	0.6522
ρ_{ym}	0.81543

Table 4 MSE of various estimators and PRE with respect to proposed one

Estimator	*MSE*	*PRE of t_p over*
t_0	2.15	213.442
t_1	1.24	123.101
t_2	1.48	146.927
t_3	1.30	129.058
t_4	1.81	179.688
t_5	1.24	123.101
t_6	1.24	123.101
t_7	1.24	123.101
t_8	1.47	145.935

Table 4 MSE of various estimators and PRE with respect to proposed one (continued)

Estimator	*MSE*	*PRE of t_p over*
t_9	1.24	123.101
t_{10}	1.23	122.109
t_{11}	1.24	123.101
t_{12}	1.24	123.101
t_{13}	1.09	108.210
t_{14}	1.24	123.101
t_{15}	1.24	123.101
t_p	**1.0073**	100.000

Figure 1 represents the graphs of the MSE of t_p and the competing estimators. The MSE proposed estimator has been shown in red colour. The PRE of t_p over the existing estimators of $\overline{Y}$ has been shown in Figure 2.

Figure 1 MSEs of t_p and competing estimators

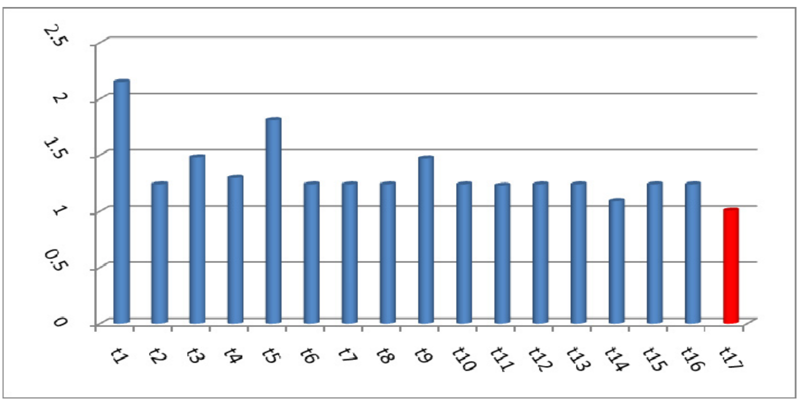
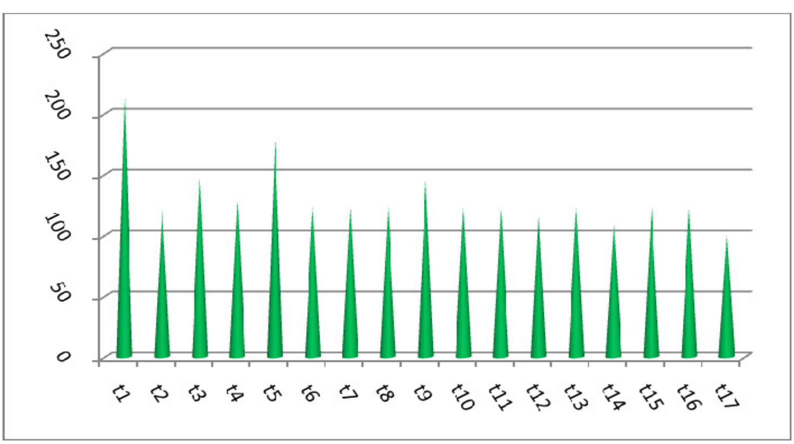


Figure 2 PREs of t_p over competing estimators



6 Results and discussion

From the results, it can be easily observed that the sampling variance of $\bar{y}$ is 2.15 while rest of the estimators that utilised auxiliary information have their MSEs in the interval [1.09 1.81]. The MSE of the t_p is 1.0073 which is least among the class of all presented estimators of $\bar{Y}$. The PRE of t_p over mentioned existing estimators including Subramani (2016) estimators ranges in the interval [108.210 213.442].

7 Conclusion

In present investigation, we advocated a new enhanced general ratio estimator of $\bar{Y}$ utilising known *M*. The sampling features of the proposed estimator are analysed until order one approximation. The expressions for the bias and MSEs are derived. The suggested estimator was then compared with the mentioned fourteen estimators of $\bar{Y}$ in competition that made use of auxiliary information, which was gathered on the augmented survey cost. The proposed estimator was also compared with the Subramani (2016) estimator, which was the source of motivation and made use of known *M*. The proposed estimator was compared theoretically as well as empirically with the above-competing estimators, and it has been shown that t_p has the least MSE among mentioned estimators. Thus, it is the most efficient estimator and hence advocated for elevated estimation of $\bar{Y}$ without raising the survey cost.

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