International Journal of Operational Research

ISSN online: 1745-7653 - ISSN print: 1745-7645
https://www.inderscience.com/ijor

Optimising fully fuzzy interval integer transshipment problems

A. Akilbasha, G. Natarajan, P. Pandian

DOI: 10.1504/IJOR.2020.10030761

Article History:

Received:	20 November 2019
Accepted:	20 February 2020
Published online:	26 January 2023

Optimising fully fuzzy interval integer transshipment problems

A. Akilbasha*, G. Natarajan and P. Pandian

Department of Mathematics,
School of Advanced Sciences,
VIT, Vellore-14, Tamil Nadu, India
Email: bashaakil@gmail.com
Email: gnatarajan@vit.ac.in
Email: ppandian@vit.ac.in
*Corresponding author

Abstract: In this paper, we focus on solving fully fuzzy interval integer transshipment (FIIT) problems where the unit shipping costs, available supply capacities, and required destination demands are triangular fuzzy interval integers. An innovative method namely, back order sequence method has been developed for finding an optimal solution of the fully FIIT problem. The proposed method provides that the optimal values of decision variables and objective function value for the fully FIIT problem are fuzzy interval integers. A numerical example is presented to illustrate the solution procedure of optimising fully FIIT problems. The optimal solution to the transshipment problem by the proposed method can help the managers to take an appropriate decision regarding transshipments.

Keywords: interval; fuzzy set; fuzzy interval; transshipment problem; optimal solution; back order sequence method.

Reference to this paper should be made as follows: Akilbasha, A., Natarajan, G. and Pandian, P. (2023) 'Optimising fully fuzzy interval integer transshipment problems', *Int. J. Operational Research*, Vol. 46, No. 1, pp.1–19.

Biographical notes: A. Akilbasha is working as a Senior Research Fellowship (SRF) in the Department of Mathematics, VIT, Vellore in India. He has obtained his Bachelor's and Master's from the Thiruvalluvar University, Vellore. He has published several research papers in national and international journals in the field of operations research. He has selected by the UGC Maulana Azad National Fellowship for Minority Students (MANF-UGC, ID No.: 201516-MANF-2015-17-TAM-51388) in 2016. He has published around ten research articles in national and international journals.

G. Natarajan is working as a Professor in the Department of Mathematics VIT, Vellore in India. He received his PhD from the VIT, Vellore. He has guided two students for Doctoral and one for MPhil respectively. He has published several research articles in national and international journals. He has more than 35 years of teaching experience.

P. Pandian is working as a Professor in the Department of Mathematics VIT, Vellore in India. He received his PhD from the Madurai Kamaraj University, Madurai. He has guided eight students for doctoral degrees. He has published around 90 research articles in reputed national and international journals. He has attended many conferences and given many lectures on various topics throughout India. He has more than 30 years of teaching experience.

1 Introduction

In a transportation problem (TP) shipment of the commodity takes place among sources and destinations. But instead of direct shipments to destinations, the commodity can be transported to a particular destination through one or more intermediate or transshipment (TS) points. Each of these points in turn supply to other points. Thus, when the shipments pass from destination to destination and from source to source, we have a transshipment problem. Since the transshipment problem is a particular case of TP hence to solve the transshipment problem, we first convert the transshipment problem into an equivalent TP and then solve it to obtain an optimal solution by using the MODI method of TP. In a TP, shipments are allowed only between source-sink pairs. In many applications, this assumption is too strong. For example, it is often the case that shipments may be allowed between sources and between sinks. Moreover, some points may exist through which units of a product can be transshipped from a source to a sink. Models with these additional features are called transshipment problems. Interestingly, it turns out that any given transshipment problem can be converted easily into an equivalent TP. The availability of such a conversion procedure significantly broadens the applicability of our algorithm for solving TPs.

2 Literature review

The transshipment problem as an extension to the classical TP with the inclusion of the possibility of the transshipment was introduced by Orden (1956). Rhody (1963) studied the transshipment problem as a reduced matrix model. Judge et al. (1965), King and Logan (1964) and Hurt and Tramel (1965) studied the transshipment problem as a general linear programming model. Jafari (2019) inclined to solve the asymmetric travelling salesman problem (ATSP), a particular computational advantage of this model is that it has a rapid convergence to optimality in contrast with other assignment problem (AP)-based models, they also showed that the method can be extended easily to solve other variants of ATSPs such as the multiple TSP and the Selective TSP. Determinants of Indian banks efficiency: a two-stage Approach is suggested by Jayaraman and Srinivasan (2019) this study seeks to examine the cost, revenue and profit efficiency of Indian banks from 2004 to 2013 using data envelopment analysis (DEA) and identifies the determinants of efficiency using Tobit regression. A mixed-integer programming model is proposed (Khatibi et al., 2019), the aim is simultaneous to minimise the total cost of tardiness, earliness, delay, and compression as well as expansion costs of job processing time; and minimise the passengers overcrowding on the gate. Ozdemir et al. (2006) studied the multi-location transshipment problem with capacitated production and lost sales. Pandian and Natarajan (2010a, 2010b) developed new methods for solving TPs involving fixed and imprecise parameters. Two new methods are proposed by Kumar et al. (2011b) to find the fuzzy optimal solution of fuzzy TPs with some additional transshipments. NagoorGani et al. (2011) and Kumar et al. (2011a) developed methods for finding an optimal solution for a fully fuzzy transshipment problem based on fuzzy linear programming and ranking approach.

An advanced method namely, the slice-sum method is used for determining an optimal solution, to fully rough interval integer TPs and have been developed by Pandian et al. (2016) for pharmaceutical sciences (Pandian et al., 2018). Kumar et al. (2019) given an attempt is made to find an optimal ordering of three machines for *n* jobs involving processing times, transportation time, breakdown interval and weights of jobs by using the genetic algorithm (GA) approach. The produced algorithm protects the best schedule of the jobs which has the minimum mean weighted flow time at each iteration. Transshipment problems having crisp, interval and fuzzy parameters were studied and solved using the zero-point method (Pandian and Natarajan, 2010a, 2010b) with few modifications by Rajendran and Pandian (2012). Gong and Yücesan (2012) examined the multi-location transshipment problem with positive replenishment lead times using infinitesimal perturbation analysis by combining with a stochastic approximation method. The vehicle routing problem with trailers and transshipments was studied by Drexl (2013). Das et al. (2017) established an innovative solid TP that intends to maximise profit under the rough interval approximation methodology. Rais et al. (2014) addressed and studied the pickup-and-delivery problem with transshipments as a mixed-integer programming model. Akilbasha et al. (2018) proposed an innovative exact method for solving fully interval integer TPs. Meissner and Senicheva (2018) discussed the multi-location inventory systems under periodic review with multiple opportunities for proactive transshipments within one order cycle. A behavioural study in supply chain models through transshipment strategies were discussed by Villa and Castañeda (2018).

Aguilar-Chinea et al. (2019) have proposed automatic learning techniques to obtain a predictor of the robustness of transshipment schedules. A new transshipment contract approach has been proposed in order to coordinate such a supply chain and guarantee the members' profit by Aslani and Heydari (2019). Bushuev et al. (2018) investigated the strategies for improving supply chain delivery performance when the cumulative density function of the delivery time distribution exists in closed form. Maggioni et al. (2019) have proposed and studied two-stage and multistage stochastic optimisation models for a bike-sharing problem with transshipment to determine the optimal number of bikes to assign to each station at the beginning of the service. Maity and Roy (2019) studied the multi-item multi-choice TP in the ground of inventory optimisation by using the concept of basic inventory optimisation and developed a methodology for integrated optimisation in inventory transportation (IOIT) to reduce the logistic cost of a system. Li et al. (2019) have solved the single-crane scheduling problem at rail transshipment yards, in which gantry cranes move containers between trains, trucks and a storage area. Lmariouh et al. (2019) the objective is to minimise the sum of the production, transportation, and inventory costs; also they have proposed a mixed-integer linear program for a variant of the multi-vehicle, multi-product production routing problem.

The rest of this paper is formatted as follows: a few known definitions, basic arithmetic operators and partial ordering related to intervals, fuzzy sets, and fuzzy intervals are presented in Section 3. In Section 4, presents the mathematical formulation of the fully FIIT problems. The proposed method namely, back order sequence method for optimising the fully FIIT problem is discussed. The numerical example is illustrated in Section 5. In Section 6, the results and discussion part have been included then finally the conclusion of the article has been provided.

3 Preliminaries

The following few definitions, basic arithmetic operators and partial ordering related to real intervals and fuzzy sets are used in this paper which can be found in Akilbasha et al. (2018), Klir and Yuan (2008), Moore (1979) and Pandian and Natarajan (2010a).

Let D denote the set of all closed bounded intervals on the real line R. That is, $D = \{[a, b]: a \le b, a \text{ and } b \text{ are in } R\}$.

Definition 3.1: Let $A = [a, b]$ and $B = [c, d]$ be in D. Then,

1 $A \oplus B = [a + c, b + d]$

2 $A \Theta B = [a - d, b - c]$

3 $kA = [ka, kb]$ if k is a positive real number

4 $kA = [kb, ka]$ if k is a negative real number

5 $A \otimes B = [p, q]$ where $p = \min\{ac, ad, bc, bd\}$ and $q = \max\{ac, ad, bc, bd\}$.

Definition 3.2: Let $A = [a, b]$ and $B = [c, d]$ be in D. Then,

1 $A \le B$ if and only if $a \le c$ and $b \le d$

2 $A = B$ if and only if $a = c$ and $b = d$.

Definition 3.3: Let $A = [a, b]$ be in D. Then,

1 A is said to be non-negative, that is, $A \ge 0$ if $a \ge 0$

2 A is said to be an integer if a and b are integers.

Definition 3.4: Let A be a classical set and $\mu_A(x)$ be a membership function from A to [0, 1]. A fuzzy set A^* with the membership function $\mu_A(x)$ is defined by:

$$A^* = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0, 1]\}$$

Definition 3.5: A fuzzy number $\tilde{a}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where a_1, a_2 and a_3 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below.

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & \text{for} \quad x \le a_1 \\ (x - a_1)/(a_2 - a_1) & \text{for} \quad a_1 \le x < a_2 \\ 1 & \text{for} \quad x = a_2 \\ (a_3 - x)/(a_3 - a_2) & \text{for} \quad a_2 < x \le a_3 \\ 0 & \text{for} \quad x > a_3 \end{cases}$$

Let $F(R)$ be a set of all triangular fuzzy numbers over R, a set of real numbers.

Definition 3.6: Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be in $F(R)$. Then,

1 $\tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

2 $\tilde{a} \,\Theta\, \tilde{b} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

3 $k\tilde{a} = (ka_1, ka_2, ka_3)$, for $k \geq 0$

4 $k\tilde{a} = (ka_3, ka_2, ka_1)$, for $k < 0$

5 $\tilde{a} \otimes \tilde{b} = (t_1, t_2, t_3)$.

where t_1 = minimum$\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$; $t_2 = \{a_2b_2\}$ and t_3 = maximum$\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$.

Definition 3.7: Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be in $F(R)$. Then,

1 $\tilde{a}$ and $\tilde{b}$ are said to be equal if $a_i = b_i$, $i = 1, 2, 3$

2 $\tilde{a}$ is said to be less than or equal $\tilde{b}$ if $a_i \leq b_i$, $i = 1, 2, 3$.

Definition 3.8: Let $\tilde{a} = (a_1, a_2, a_3)$ be in $F(R)$. Then,

1 $\tilde{a}$ is said to be positive if $a_1 \geq 0$

2 $\tilde{a}$ is said to be an integer if a_i, $i = 1, 2, 3$ are integers.

Now, we define the following definitions of membership function, the basic arithmetic operators and partial ordering on closed bounded fuzzy intervals (Judge et al., 1965) based on the definitions in real interval sets and fuzzy sets.

Let $\tilde{D}$ denote the set of all closed bounded fuzzy intervals over $F(R)$.

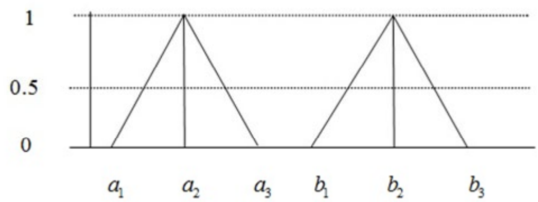
That is, $\tilde{D} = \{[\tilde{a}, \tilde{b}], \tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$, $a_3 \leq b_1$ and a_i's and b_i's are in $R\}$.

The membership function $\mu_{[\tilde{a},\tilde{b}]}(x)$ of the fuzzy interval number $[\tilde{a}, \tilde{b}]$ is given below.

$$\mu_{[\tilde{a},\tilde{b}]}(x) = \begin{cases} (x - a_1)/(a_2 - a_1): & a_1 \leq x \leq a_2 \\ (a_3 - x)/(a_3 - a_2): & a_2 \leq x \leq a_3 \\ (x - b_1)/(b_2 - b_1): & b_1 \leq x \leq b_2 \\ (b_3 - x)/(b_3 - b_2): & b_2 \leq x \leq b_3 \\ 0: & \text{otherwise} \end{cases}$$

The graph of $\mu_{[\tilde{a},\tilde{b}]}(x)$ is given in Figure 1.

Figure 1 Membership of an interval-typed triangular function



Definition 3.9: Let $\tilde{A} = [\tilde{a}, \tilde{b}]$ and $\tilde{B} = [\tilde{c}, \tilde{d}]$ be in $\tilde{D}$. Then,

1 $\tilde{A} \oplus \tilde{B} = \left[\tilde{a} + \tilde{c}, \tilde{b} + \tilde{d}\right]$

2 $\tilde{A} \,\Theta\, \tilde{B} = \left[\tilde{a} - \tilde{d}, \tilde{b} - \tilde{c}\right]$

3 $k\tilde{A} = [k\tilde{a}, k\tilde{b}]$ if k is a positive real number

4 $k\tilde{A} = [k\tilde{b}, k\tilde{a}]$ if k is a negative real number

5 $\tilde{A} \otimes \tilde{B} = [\tilde{p}, \tilde{q}]$ where $\tilde{p} = (p_1, p_2, p_3), \tilde{q} = (q_1, q_2, q_3),$ $p_1 = \min\{a_1c_1, a_1d_1, b_1c_1, b_1d_1\}, q_1 = \max\{a_1c_1, a_1d_1, b_1c_1, b_1d_1\}, p_2 = \min\{a_2c_2, a_2d_2, b_2c_2, b_2d_2\},$ $q_2 = \max\{a_2c_2, a_2d_2, b_2c_2, b_2d_2\}, p_3 = \min\{a_3c_3, a_3d_3, b_3c_3, b_3d_3\}$ and $q_3 = \max\{a_3c_3, a_3d_3, b_3c_3, b_3d_3\}.$

Definition 3.10: Let $\tilde{A} = [\tilde{a}, \tilde{b}]$ and $\tilde{B} = [\tilde{c}, \tilde{d}]$ be in $\tilde{D}$. Then,

1 $\tilde{A} \le \tilde{B}$ if and only if $\tilde{a} \le \tilde{c}$ and $\tilde{b} \le \tilde{d}$

2 $\tilde{A} = \tilde{B}$ if and only if $\tilde{a} = \tilde{c}$ and $\tilde{b} = \tilde{d}$.

Definition 3.11: Let $\tilde{A} = [\tilde{a}, \tilde{b}]$ be in $\tilde{D}$. Then,

1 $\tilde{A}$ is said to be positive if $\tilde{a} \ge 0$

2 $\tilde{A}$ is said to be an integer if $\tilde{a}$ and $\tilde{b}$ are integers.

4 Fully fuzzy interval integer transshipment (FIIT) problems

Consider a transshipment problem with m origins and n destinations. Any origin or any destination can ship items to any other origin or any destination by using the transshipment problem; it would be more suitable to number them successively so that the origins and destinations are numbered from 1 to m and from $m + 1$ to $m + n$ respectively. Let $[\tilde{a}_i, \tilde{p}_i]$ be the fuzzy interval quantity available at the i^{th} origin and $[\tilde{b}_i, \tilde{q}_j]$ be the fuzzy interval requirement at the j^{th} destinations. Let $[\tilde{x}_{ij}, \tilde{y}_{ij}]$ $(i, j = 1, 2, ..., m + n; i \neq j)$ be the fuzzy interval quantity shipped from the i^{th} station to the j^{th} station and $[\tilde{c}_{ij}, \tilde{d}_{ij}]$ $(i, j = 1, 2, ..., m + n)$ be the fuzzy interval unit shipping cost from the i^{th} station to j^{th} station where $[\tilde{c}_{ij}, \tilde{d}_{ij}]$ need not be same as $[\tilde{c}_{ji}, \tilde{d}_{ji}]$. The main objective is to obtain an optimal transshipping pattern such that the total fuzzy interval cost of transportation is minimum.

Now, the mathematical formulation of the above said fully FIIT problem, (P) is given below:

$$\text{(P) Minimise } [\tilde{z}_1, \tilde{z}_2] = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} [\tilde{c}_{ij}, \tilde{d}_{ij}] \otimes [\tilde{x}_{ij}, \tilde{y}_{ij}].$$

Subject to:

$$\sum_{\substack{j=1 \\ i \neq j}}^{m+n} [\tilde{x}_{ij}, \tilde{y}_{ij}] = [\tilde{a}_i, \tilde{p}_i] + \sum_{\substack{j=1 \\ i \neq j}}^{m+n} [\tilde{x}_{ji}, \tilde{y}_{ji}], i = 1, 2, ..., m \tag{1}$$

$$\sum_{\substack{i=1 \\ i \neq j}}^{m+n} [\tilde{x}_{ij}, \tilde{y}_{ij}] = [\tilde{b}_j, \tilde{q}_j] + \sum_{\substack{i=1 \\ i \neq j}}^{m+n} [\tilde{x}_{ji}, \tilde{y}_{ji}], j = m+1, m+2, ..., m+n \tag{2}$$

$$\tilde{x}_{ij}, \tilde{y}_{ij} \geq 0, \text{ for } i, j = 1, 2, ..., m+n \text{ and are integers} \tag{3}$$

where m denotes the number of supply points; n denotes the number of demand points; $[\tilde{x}_{ij}, \tilde{y}_{ij}]$ is the fuzzy interval number of units shipped from supply point i to demand point j with $\tilde{x}_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^3)$ and $\tilde{y}_{ij} = (y_{ij}^1, y_{ij}^2, y_{ij}^3)$; $[\tilde{c}_{ij}, \tilde{d}_{ij}]$ is the fuzzy interval cost of shipping one unit from supply point i to the demand point j with $\tilde{c}_{ij} = (c_{ij}^1, c_{ij}^2, c_{ij}^3)$ and $\tilde{d}_{ij} = (d_{ij}^1, d_{ij}^2, d_{ij}^3)$; $[\tilde{a}_i, \tilde{p}_i]$ is the fuzzy interval supply at supply point i with $\tilde{a}_i = (a_i^1, a_i^2, a_i^3)$ and $\tilde{p}_i = (p_i^1, p_i^2, p_i^3)$ and $[\tilde{b}_j, \tilde{q}_j]$ is the fuzzy interval demand at demand point j with $\tilde{b}_j = (b_j^1, b_j^2, b_j^3)$ and $\tilde{q}_j = (q_j^1, q_j^2, q_j^3)$.

If $\left[\sum_{i=1}^{m} \tilde{a}_i, \sum_{i=1}^{m} \tilde{p}_i\right] = \left[\sum_{j=m+1}^{m+n} \tilde{b}_j, \sum_{j=m+1}^{m+n} \tilde{q}_j\right]$, the fully FIIP problem, (P) is said to be balanced. Otherwise, it is called unbalanced.

Definition 4.1: A set of fuzzy intervals $\{[(x_{ij}^1, x_{ij}^2, x_{ij}^3), (y_{ij}^1, y_{ij}^2, y_{ij}^3)],$ for all i and $j\}$ is said to be a feasible solution to the problem (P) if it satisfies the equations (1), (2) and (3).

Definition 4.2: A feasible solution $\{[(x_{ij}^1, x_{ij}^2, x_{ij}^3), (y_{ij}^1, y_{ij}^2, y_{ij}^3)],$ for all i and $j\}$ to the problem (P) is said to be an optimal solution of the problem (P) if the feasible solution minimises the objective function of the problem (P), that is:

$$\sum_{i=1}^{m}\sum_{\substack{j=1\\i\neq j}}^{m+n}\left[\left(c_{ij}^{1},c_{ij}^{2},c_{ij}^{3}\right),\left(d_{ij}^{1},d_{ij}^{2},d_{ij}^{3}\right)\right]\otimes\left[\left(\overline{x}_{ij}^{1},\overline{x}_{ij}^{2},\overline{x}_{ij}^{3}\right),\left(\overline{y}_{ij}^{1},\overline{y}_{ij}^{2},\overline{y}_{ij}^{3}\right)\right]$$

$$\leq\sum_{i=1}^{m}\sum_{\substack{j=1\\i\neq j}}^{m+n}\left[\left(c_{ij}^{1},c_{ij}^{2},c_{ij}^{3}\right),\left(d_{ij}^{1},d_{ij}^{2},d_{ij}^{3}\right)\right]\otimes\left[\left(m_{ij}^{1},m_{ij}^{2},m_{ij}^{3}\right),\left(n_{ij}^{1},n_{ij}^{2},n_{ij}^{3}\right)\right]$$

For all feasible $\{[(m_{ij}^1, m_{ij}^2, m_{ij}^3), (n_{ij}^1, n_{ij}^2, n_{ij}^3)],$ for all i and $j\}$ to the problem (P).

Now, from the given fully FIIT problem (P), we construct six IT problems namely, IT6 problem, IT5 problem, IT4 problem, IT3 problem, IT2 problem, and IT1 problem are given below:

(IT6) Minimise $z_2^3 = \sum_{i=1}^{m}\sum_{\substack{j=1\\i\neq j}}^{m+n} d_{ij}^3 y_{ij}^3$

Subject to:

$$\sum_{\substack{j=1\\i\neq j}}^{m+n} y_{ij}^3 = p_i^3 + \sum_{\substack{j=1\\i\neq j}}^{m+n} y_{ji}^3,\ i = 1, 2, ..., m$$

$$\sum_{\substack{i=1\\i\neq j}}^{m+n} y_{ij}^3 = q_j^3 + \sum_{\substack{i=1\\i\neq j}}^{m+n} y_{ji}^3,\ j = m+1, m+2, ..., m+n$$

$y_{ij}^3 \geq 0,$ for $i, j = 1, 2, ..., m+n$ and are integers

(IT5) Minimise $z_2^2 = \sum_{i=1}^{m}\sum_{\substack{j=1\\i\neq j}}^{m+n} d_{ij}^2 y_{ij}^2$

Subject to:

$$\sum_{\substack{j=1\\i\neq j}}^{m+n} y_{ij}^2 = p_i^2 + \sum_{\substack{j=1\\i\neq j}}^{m+n} y_{ji}^2,\ i = 1, 2, ..., m$$

$$\sum_{\substack{i=1\\i\neq j}}^{m+n} y_{ij}^2 = q_j^2 + \sum_{\substack{i=1\\i\neq j}}^{m+n} y_{ji}^2,\ j = m+1, m+2, ..., m+n$$

$y_{ij}^2 \leq \overline{y}_{ij}^3$, for $i,\ j = 1, 2, ..., m+n$

$y_{ij}^2 \geq 0$, for $i,\ j = 1, 2, ..., m+n$ and are integers

where $\{\overline{y}_{ij}^3; i,\ j = 1, 2, ..., m+n\}$ is an optimal solution to the problem (IT6).

(IT4) Minimise $z_2^1 = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} d_{ij}^1 y_{ij}^1$

Subject to:

$$\sum_{\substack{j=1 \\ i \neq j}}^{m+n} y_{ij}^1 = p_i^1 + \sum_{\substack{j=1 \\ i \neq j}}^{m+n} y_{ji}^1,\ i = 1, 2, ..., m$$

$$\sum_{\substack{i=1 \\ i \neq j}}^{m+n} y_{ij}^1 = q_j^1 + \sum_{\substack{i=1 \\ i \neq j}}^{m+n} y_{ji}^1,\ j = m+1, m+2, ..., m+n$$

$y_{ij}^1 \leq \overline{y}_{ij}^2$, for $i,\ j = 1, 2, ..., m+n$

$y_{ij}^1 \geq 0$, for $i,\ j = 1, 2, ..., m+n$ and are integers

where $\{\overline{y}_{ij}^2; i,\ j = 1, 2, ..., m+n\}$ is an optimal solution of the problem (IT5).

(IT3) Minimise $z_1^3 = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} c_{ij}^3 x_{ij}^3$

Subject to:

$$\sum_{\substack{j=1 \\ i \neq j}}^{m+n} x_{ij}^3 = a_i^3 + \sum_{\substack{j=1 \\ i \neq j}}^{m+n} x_{ji}^3,\ i = 1, 2, ..., m$$

$$\sum_{\substack{i=1 \\ i \neq j}}^{m+n} x_{ij}^3 = b_j^3 + \sum_{\substack{i=1 \\ i \neq j}}^{m+n} x_{ji}^3,\ j = m+1, m+2, ..., m+n$$

$x_{ij}^3 \leq \overline{y}_{ij}^1$, for $i,\ j = 1, 2, ..., m+n$

$x_{ij}^3 \geq 0$, for $i,\ j = 1, 2, ..., m+n$ and are integers

where $\{\overline{y}_{ij}^1; i,\ j = 1, 2, ..., m+n\}$ is an optimal solution of the problem (IT4).

(IT2) Minimise $z_1^2 = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} c_{ij}^2 x_{ij}^2$

Subject to:

$$\sum_{\substack{j=1 \\ i \neq j}}^{m+n} x_{ij}^2 = a_i^2 + \sum_{\substack{j=1 \\ i \neq j}}^{m+n} x_{ji}^2,\ i = 1, 2, ..., m$$

$$\sum_{\substack{i=1\\ i\neq j}}^{m+n} x_{ij}^2 = b_j^2 + \sum_{\substack{i=1\\ i\neq j}}^{m+n} x_{ji}^2,\ j = m+1, m+2, \ldots, m+n$$

$x_{ij}^2 \le \overline{x}_{ij}^3$, for $i,\ j = 1, 2, \ldots, m+n$

$x_{ij}^2 \ge 0$, for $i,\ j = 1, 2, \ldots, m+n$ and are integers

where $\{\overline{x}_{ij}^3; i,\ j = 1, 2, \ldots, m+n\}$ is an optimal solution to the problem (IT3) and,

(IT1) Minimise $z_1^1 = \sum_{i=1}^{m} \sum_{\substack{j=1\\ i\neq j}}^{m+n} c_{ij}^1 x_{ij}^1$

Subject to:

$$\sum_{\substack{j=1\\ i\neq j}}^{m+n} x_{ij}^1 = a_i^1 + \sum_{\substack{j=1\\ i\neq j}}^{m+n} x_{ji}^1,\ i = 1, 2, \ldots, m$$

$$\sum_{\substack{i=1\\ i\neq j}}^{m+n} x_{ij}^1 = b_j^1 + \sum_{\substack{i=1\\ i\neq j}}^{m+n} x_{ji}^1,\ j = m+1, m+2, \ldots, m+n$$

$x_{ij}^1 \le \overline{x}_{ij}^2$, for $i,\ j = 1, 2, \ldots, m+n$

$x_{ij}^1 \ge 0$, for $i,\ j = 1, 2, \ldots, m+n$ and are integers

where $\{\overline{x}_{ij}^2; i,\ j = 1, 2, \ldots, m+n\}$ is an optimal solution to the problem (IT2).

Now, we establish a relationship between an optimal solution of the fully FIIT problem (P) and the constructed six IT problems, (IT1) to (IT6). This established relationship among the IT problems is employed in the proposed method namely, back order sequence method.

Theorem 4.1: If the set $\{\overline{y}_{ij}^3,$ for all i and j and $i \neq j\}$ is an optimal solution of the problem (IT6) with the minimum transportation cost $\overline{z}_2^3$, the set $\{\overline{y}_{ij}^2,$ for all i and j and $i \neq j\}$ is an optimal solution of the problem (IT5) with the minimum transportation cost $\overline{z}_2^2$, the set $\{\overline{y}_{ij}^1,$ for all i and j and $i \neq j\}$ is an optimal solution of the problem (IT4) with the minimum transportation cost $\overline{z}_2^1$, the set $\{\overline{x}_{ij}^3,$ for all i and j and $i \neq j\}$ is an optimal solution of the problem (IT3) with the minimum transportation cost $\overline{z}_1^3$, the set $\{\overline{x}_{ij}^2,$ for all i and j and $i \neq j\}$ is an optimal solution of the problem (IT2) with the minimum transportation cost $\overline{z}_1^2$, the set $\{\overline{x}_{ij}^1,$ for all i and j and $i \neq j\}$ is an optimal solution for the problem (IT1) with the minimum transportation cost $\overline{z}_1^1$, then the set of fuzzy integer intervals $\{[(\overline{x}_{ij}^1, \overline{x}_{ij}^2, \overline{x}_{ij}^3), (\overline{y}_{ij}^1, \overline{y}_{ij}^2, \overline{y}_{ij}^3)],$, for all i and j and $i \neq j\}$ is an optimal solution for the fully FIIT problem (P) with the minimum transportation cost $[(\overline{z}_1^1, \overline{z}_1^2, \overline{z}_1^3), (\overline{z}_2^1, \overline{z}_2^2, \overline{z}_2^3)]$.

Proof: Now, since $\{\bar{x}_{ij}^1,$ for all i, j and $i \neq j\}$, $\{\bar{x}_{ij}^2,$ for all i, j and $i \neq j\}$, $\{\bar{x}_{ij}^3,$ for all i, j and $i \neq j\}$, $\{\bar{y}_{ij}^1,$ for all i, j and $i \neq j\}$, $\{\bar{y}_{ij}^2,$ for all i, j and $i \neq j\}$ and $\{\bar{y}_{ij}^3,$ for all i, j and $i \neq j\}$ are optimal solutions of the problems (IT2), (IT3), (IT4), (IT5) and (IT6) respectively with $\bar{x}_{ij}^1 \leq \bar{x}_{ij}^2 \leq \bar{x}_{ij}^3 \leq \bar{y}_{ij}^1 \leq \bar{y}_{ij}^2 \leq \bar{y}_{ij}^3$ for all i, j and $i \neq j$, we can conclude that the set of fuzzy interval integers $\{[(\bar{x}_{ij}^1, \bar{x}_{ij}^2, \bar{x}_{ij}^3), (\bar{y}_{ij}^1, \bar{y}_{ij}^2, \bar{y}_{ij}^3)],$ for all i, j and $i \neq j\}$ is a feasible solution to the fully FIIT problem (P).

Let $\{[(m_{ij}^1, m_{ij}^2, m_{ij}^3), (n_{ij}^1, n_{ij}^2, n_{ij}^3)],$ for all i, j and $i \neq j\}$ be a feasible solution to the fully FIIT problem (P). This implies that $\{m_{ij}^1,$ for all i, j and $i \neq j\}$, $\{m_{ij}^2,$ for all i, j and $i \neq j\}$, $\{m_{ij}^3,$ for all i, j and $i \neq j\}$, $\{n_{ij}^1,$ for all i, j and $i \neq j\}$, $\{n_{ij}^2,$ for all i, j and $i \neq j\}$ and $\{n_{ij}^3,$ for all i, j and $i \neq j\}$ are feasible to the problems (IT1), (IT2), (IT3), (IT4), (IT5) and (IT6) respectively.

Now, since $\{\bar{x}_{ij}^1,$ for all i, j and $i \neq j\}$, $\{\bar{x}_{ij}^2,$ for all i, j and $i \neq j\}$, $\{\bar{x}_{ij}^3,$ for all i, j and $i \neq j\}$, $\{\bar{y}_{ij}^1,$ for all i, j and $i \neq j\}$, $\{\bar{y}_{ij}^2,$ for all i, j and $i \neq j\}$ and $\{\bar{y}_{ij}^3,$ for all i, j and $i \neq j\}$ are optimal solutions of the problems (IT1), (IT2), (IT3), (IT4), (IT5) and (IT6) respectively, we have:

$$\bar{z}_1^1 = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} c_{ij}^1 \bar{x}_{ij}^1 \leq \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} c_{ij}^1 m_{ij}^1$$

$$\bar{z}_1^2 = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} c_{ij}^2 \bar{x}_{ij}^2 \leq \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} c_{ij}^2 m_{ij}^2$$

$$\bar{z}_1^3 = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} c_{ij}^3 \bar{x}_{ij}^3 \leq \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} c_{ij}^3 m_{ij}^3$$

$$\bar{z}_2^1 = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} d_{ij}^1 \bar{y}_{ij}^1 \leq \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} d_{ij}^1 n_{ij}^1$$

$$\bar{z}_2^2 = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} d_{ij}^2 \bar{y}_{ij}^2 \leq \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} d_{ij}^2 n_{ij}^2$$

and

$$\bar{z}_2^3 = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} d_{ij}^3 \bar{y}_{ij}^3 \leq \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} d_{ij}^3 n_{ij}^3$$

This implies that:

$$\left[\left(\overline{z}_1^1, \overline{z}_1^2, \overline{z}_1^3\right), \left(\overline{z}_2^1, \overline{z}_2^2, \overline{z}_2^3\right)\right]$$
$$= \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} \left[\left(c_{ij}^1, c_{ij}^2, c_{ij}^3\right), \left(d_{ij}^1, d_{ij}^2, d_{ij}^3\right)\right] \otimes \left[\left(\overline{x}_{ij}^1, \overline{x}_{ij}^2, \overline{x}_{ij}^3\right), \left(\overline{y}_{ij}^1, \overline{y}_{ij}^2, \overline{y}_{ij}^3\right)\right]$$

and

$$\sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} \left[\left(c_{ij}^1, c_{ij}^2, c_{ij}^3\right), \left(d_{ij}^1, d_{ij}^2, d_{ij}^3\right)\right] \otimes \left[\left(\overline{x}_{ij}^1, \overline{x}_{ij}^2, \overline{x}_{ij}^3\right), \left(\overline{y}_{ij}^1, \overline{y}_{ij}^2, \overline{y}_{ij}^3\right)\right]$$
$$\leq \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} \left[\left(c_{ij}^1, c_{ij}^2, c_{ij}^3\right), \left(d_{ij}^1, d_{ij}^2, d_{ij}^3\right)\right] \otimes \left[\left(m_{ij}^1, m_{ij}^2, m_{ij}^3\right), \left(n_{ij}^1, n_{ij}^2, n_{ij}^3\right)\right]$$

Therefore, the set of fuzzy interval integers $\{[(\overline{x}_{ij}^1, \overline{x}_{ij}^2, \overline{x}_{ij}^3), (\overline{y}_{ij}^1, \overline{y}_{ij}^2, \overline{y}_{ij}^3)],$ for all i, j and $i \neq j\}$ is an optimal solution of the fully FIIT problem (P) with the minimum transportation cost $[(\overline{z}_1^1, \overline{z}_1^2, \overline{z}_1^3), (\overline{z}_2^1, \overline{z}_2^2, \overline{z}_2^3)]$.

Hence, the theorem is proved.

5 Back order sequence method

We, now propose a new method namely, back order sequence method for finding an optimal solution to the fully FIIT problem.

The back order sequence method has the following steps.

Step 1 Convert the given fully FIIT problem into a balanced one if it is not.

Step 2 Construct six-level integer transshipment (IT) problems of the given fully FIIT problem (P).

Step 3 Solve the (IT6) problem using the zero-point method for transshipment problems (Rajendran and Pandian, 2012). Let $\{\overline{y}_{ij}^3,$ for all i, j and $i \neq j\}$ be an optimal solution of the (IT6) problem with the minimum transportation cost $\overline{z}_2^3$.

Step 4 Solve the (IT5) problem using the zero-point method for transshipment problems (Rajendran and Pandian, 2012). Let $\{\overline{y}_{ij}^2,$ for all i, j and $i \neq j\}$ be an optimal solution of the (IT5) problem with the minimum transportation cost $\overline{z}_2^2$.

Step 5 Solve the (IT4) problem using the zero-point method for transshipment problems (Rajendran and Pandian, 2012). Let $\{\overline{y}_{ij}^1,$ for all i, j and $i \neq j\}$ be an optimal solution of the (IT4) problem with the minimum transportation cost $\overline{z}_2^1$.

Step 6 Solve the (IT3) problem using the zero-point method for transshipment problems (Rajendran and Pandian, 2012). Let $\{\overline{x}_{ij}^3,$ for all i, j and $i \neq j\}$ be an optimal solution of the (IT3) problem with the minimum transportation cost $\overline{z}_1^3$.

Step 7 Solve the (IT2) problem using the zero-point method for transshipment problems (Rajendran and Pandian, 2012). Let $\{\overline{x}_{ij}^2,$ for all i, j and $i \neq j\}$ be an optimal solution of the (IT2) problem with the minimum transportation cost $\overline{z}_1^2$.

Step 8 Solve the (IT1) problem using the zero-point method for transshipment problems (Rajendran and Pandian, 2012). Let $\{\overline{x}_{ij}^1,$ for all i, j and $i \neq j\}$ be an optimal solution of the (IT1) problem with the minimum transportation cost $\overline{z}_1^3$.

Step 9 The optimal solution of the given fully FIIT problem (P) is $\{[(\overline{x}_{ij}^1, \overline{x}_{ij}^2, \overline{x}_{ij}^3), (\overline{y}_{ij}^1, \overline{y}_{ij}^2, \overline{y}_{ij}^3)],$ for all i, j and $i \neq j\}$ with the minimum transportation cost $[(\overline{z}_1^1, \overline{z}_1^2, \overline{z}_1^3), (\overline{z}_2^1, \overline{z}_2^2, \overline{z}_2^3)]$ (by Theorem 4.1.).

5.1 Numerical example

The solution procedure of the back order sequence method for solving the fully FIIT problem is illustrated by the following numerical example.

Example 5.1: Consider the following fully FIIT problem involving two sources and two destinations. The available items at the source S1 and the source S2 are [(3, 7, 10), (15, 17, 19)] units and [(6, 8, 11), (16, 18, 20)] units, respectively. The required demands at the destinations D1 and D2 are [(2, 6, 9), (14, 16, 18)] units and [(7, 9, 12), (17, 19, 21)] units, respectively.

The transportation cost per unit between different sources and destinations are fuzzy interval integers are summarised in the following table as given in Table 1.

Table 1 The transportation cost per unit between different sources and destinations

	S1	*S2*	*D1*	*D2*
S1	-	[(1, 4, 6), (9, 11, 13)]	[(5, 9, 11), (14, 16, 18)]	[(3, 5, 7), (10, 12, 14)]
S2	[(2, 4, 6), (9, 11, 13)]	-	[(7, 11, 13), (16, 18, 20)]	[(3, 5, 7), (10, 12, 14)]
D1	[(1, 3, 5), (8, 10, 12)]	[(8, 10, 12), (15, 17, 19)]	-	[(4, 6, 8), (11, 13, 15)]
D2	[(2, 4, 6), (9, 11, 13)]	[(1, 3, 5), (8, 10, 12)]	[(2, 4, 6), (9, 11, 13)]	-

The given problem can be structured as a fully FIIT problem as follows (Table 2).

Table 2 A fully FIIT problem

	S1	*S2*	*D1*	*D2*	*Supply*
S1	-	[(1, 4, 6), (9, 11, 13)]	[(5, 9, 11), (14, 16, 18)]	[(3, 5, 7), (10, 12, 14)]	[(3, 7, 10), (15, 17, 19)]
S2	[(2, 4, 6), (9, 11, 13)]	-	[(7, 11, 13), (16, 18, 20)]	[(3, 5, 7), (10, 12, 14)]	[(6, 8, 11), (16, 18, 20)]
D1	[(1, 3, 5), (8, 10, 12)]	[(8, 10, 12), (15, 17, 19)]	-	[(4, 6, 8), (11, 13, 15)]	

Table 2 A fully FIIT problem (continued)

	S1	*S2*	*D1*	*D2*	*Supply*
D2	[(2, 4, 6), (9, 11, 13)]	[(1, 3, 5), (8, 10, 12)]	[(2, 4, 6), (9, 11, 13)]	-	
Demand			[(2, 6, 9), (14, 16, 18)]	[(7, 9, 12), (17, 19, 21)]	

Now, since the total supply = [(9, 15, 21), (31, 35, 39)] = the total demand. Therefore the given problem is balanced.

Now, by using step 2, the (IT6) problem of the given fully FIIT problem is obtained as given in Table 3.

Table 3 The optimal solution allotted table of the (IT6) problem

	S1	*S2*	*D1*	*D2*	*Supply*
S1	-	13	18	14	19
S2	13	-	20	14	20
D1	12	19	-	15	
D2	13	12	13	-	
Demand			18	21	

Now, by using step 3, the optimal solution of the (IT6) problem is $\overline{y}_{13}^{3} = 18; \overline{y}_{14}^{3} = 1$ and $\overline{y}_{24}^{3} = 20$ with the minimum transshipment cost is 618.

Now, by using step 2, the (IT5) problem of the given fully FIIT problem is obtained as given in Table 4.

Table 4 The optimal solution allotted table of the (IT5) problem

	S1	*S2*	*D1*	*D2*	*Supply*
S1	-	11	16	12	17
S2	11	-	18	12	18
D1	10	17	-	13	
D2	11	10	11	-	
Demand			16	19	

with $y_{ij}^{2} \leq \overline{y}_{ij}^{3}, i, j = 1, 2, 3, 4.$

Now, by using step 4, the optimal solution to the (IT5) problem is $\overline{y}_{13}^{2} = 16; \overline{y}_{14}^{2} = 1$ and $\overline{y}_{24}^{2} = 18$ with the minimum transshipment cost 484.

Now, using step 2, the (IT4) problem of the given fully FIIT problem is obtained as given in Table 5.

Table 5 The optimal solution allotted table of the (IT4) problem

	S1	*S2*	*D1*	*D2*	*Supply*
S1	-	9	14	10	15
S2	9	-	16	10	16
D1	8	15	-	11	
D2	9	8	9	-	
Demand			14	17	

with $y_{ij}^1 \leq \overline{y}_{ij}^2, i, j = 1, 2, 3, 4.$

Now, by step 5, the optimal solution to the (IT4) problem is $\overline{y}_{13}^1 = 14; \overline{y}_{14}^1 = 1$ and $\overline{y}_{24}^1 = 16$ with the minimum transshipment cost 366.

Now, by using step 2, the (IT3) problem of the given fully FIIT problem is obtained as given in Table 6.

Table 6 The optimal solution allotted table of the (IT3) problem

	S1	*S2*	*D1*	*D2*	*Supply*
S1	-	6	11	7	10
S2	6	-	13	7	11
D1	5	12	-	8	
D2	6	5	6	-	
Demand			9	12	

with $x_{ij}^3 \leq \overline{y}_{ij}^1, i, j = 1, 2, 3, 4.$

Now, using step 6, the optimal solution to the (IT3) problem is $\overline{x}_{13}^3 = 9; \overline{x}_{14}^3 = 1$ and $\overline{x}_{24}^3 = 11$ with the minimum transshipment cost 183.

Now, by using step 2, the (IT2) problem of the given fully FIIT problem is given in Table 7.

Table 7 The optimal solution allotted table of the (IT2) problem

	S1	*S2*	*D1*	*D2*	*Supply*
S1	-	4	9	5	7
S2	4	-	11	5	8
D1	3	10	-	6	
D2	4	3	4	-	
Demand			6	9	

with $x_{ij}^2 \leq \overline{x}_{ij}^3, i, j = 1, 2, 3, 4.$

Now, using step 7, the optimal solution to the (IT2) problem is $\overline{x}_{13}^2 = 6; \overline{x}_{14}^2 = 1$ and $\overline{x}_{24}^2 = 8$ with the minimum transshipment cost 99.

Now, by using step 2, the (IT1) problem of the given fully FIIT problem is given in Table 8.

Table 8 The optimal solution allotted table of the (IT1) problem

	S1	*S2*	*D1*	*D2*	*Supply*
S1	-	1	5	3	3
S2	2	-	7	3	6
D1	1	8	-	4	
D2	2	1	2	-	
Demand			2	7	

with $x_{ij}^1 \leq \overline{x}_{ij}^2, i, j = 1, 2, 3, 4.$

Now, by step 8, the optimal solution to the (IT1) problem is $\overline{x}_{13}^1 = 2; \overline{x}_{14}^1 = 1$ and $\overline{x}_{24}^1 = 6$ with the minimum transshipment cost 31.

Now, by step 9, the optimal solution of the given fully FIIT problem is given below:

$$\left[\left(\overline{x}_{13}^1, \overline{x}_{13}^2, \overline{x}_{13}^3\right), \left(\overline{y}_{13}^1, \overline{y}_{13}^2, \overline{y}_{13}^3\right)\right] = [(2, 6, 9), (14, 16, 18)]$$

$$\left[\left(\overline{x}_{14}^1, \overline{x}_{14}^2, \overline{x}_{14}^3\right), \left(\overline{y}_{14}^1, \overline{y}_{14}^2, \overline{y}_{14}^3\right)\right] = [(1, 1, 1), (1, 1, 1)]$$

and

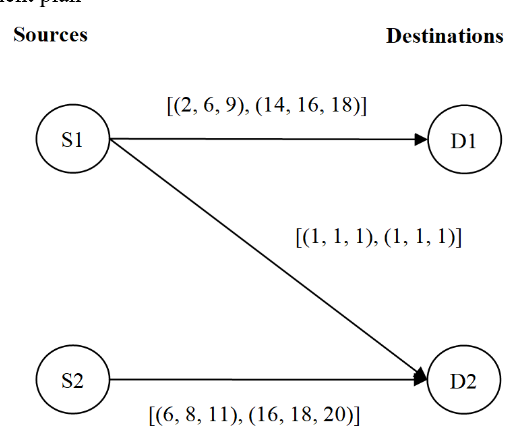
$$\left[\left(\overline{x}_{24}^1, \overline{x}_{24}^2, \overline{x}_{24}^3\right), \left(\overline{y}_{24}^1, \overline{y}_{24}^2, \overline{y}_{24}^3\right)\right] = [(6, 8, 11), (16, 18, 20)]$$

with the minimum transshipment cost [(31, 99, 183), (366, 484, 618)].

6 Results and discussion

From the above result we observed that the transshipment plan is [(2, 6, 9), (14, 16, 18)] fuzzy interval integer units are shipped from source S1 to the destination D1; [(1, 1, 1), (1, 1, 1)] fuzzy interval integer units are shipped from source S1 to the destination D2; and [(6, 8, 11), (16, 18, 20)] fuzzy interval integer units are shipped from source S2 to the destination D2.

Figure 2 Transshipment plan



According to the literature study, there is no research work done in transshipment problems with the fuzzy interval integer environment. This study entirely inclined by using fuzzy interval integer values, this approach adequate for the decision-makers can have a chance to select an appropriate decision regarding transhipments according to their financial situation and time requirement.

6.1 Objectives

- The main objective of this study is to find an optimal transshipment plan for the given problem.
- This will give the best transshipment plan to decision-makers when they are handling logistic problems with fuzzy interval integer parameters.
- This will helps the upcoming researchers to extend this problem into the other problems with parameters like trapezoidal fuzzy sets, rough sets and so on.

7 Conclusions

In this paper, we have considered fully FIIT problems where the cost coefficients of the objective functions and the source and destination parameters have been expressed as fuzzy integer interval values. For this, we have developed an innovative method namely, back order sequence method for finding an optimal solution of fully FIIT problem. Under the proposed method, the optimal values of decision variables and the objective function of the fully FIIT problem are fuzzy interval integers. A numerical example has been given to illustrate the optimal solution procedure for solving the fully FIIT problem. The back order sequence method helps the decision-makers to take an appropriate decision regarding transshipments based on their situations when they are handling logistic models of real-life situations having the supply and demands are in fuzzy interval integer parameters.

Future research will involve considering options other than direct shipment. We would also like to include additional constraints related to driver breaks, time requirements, and conveyance. We also plan to produce robust solutions by including rough sets in the demand and supply.

References

Aguilar-Chinea, R.M., Rodriguez, I.C., Expósito, C., Melian-Batista, B. and Moreno-Vega, J.M. (2019) 'Using a decision tree algorithm to predict the robustness of a transshipment schedule', *Procedia Computer Science*, Vol. 149, pp.529–536.

Akilbasha, A., Pandian, P. and Natarajan, G. (2018) 'An innovative exact method for solving fully interval integer transportation problems', *Informatics in Medicine Unlocked*, Vol. 11, pp.95–99.

Aslani, A. and Heydari, J. (2019) 'Transshipment contract for coordination of a green dual-channel supply chain under channel disruption', *Journal of Cleaner Production*, Vol. 223, pp.596–609.

Bushuev, M.A., Guiffrida, A.L. and Rudchenko, T. (2018) 'Supply chain delivery performance improvement for several delivery time distributions', *Int. J. Operational Research*, Vol. 33, No. 4, pp.538–558.

Das, A., Kumar Bera, U. and Maiti, M. (2017) 'A profit maximizing solid transportation model under a rough interval approach', in *IEEE Transactions on Fuzzy Systems*, Vol. 25, No. 3, pp.485–498.

Drexl, M. (2013) 'Applications of the vehicle routing problem with trailers and transshipments', *European Journal of Operational Research*, Vol. 227, No. 2, pp.275–283.

Gong, Y. and Yücesan, E. (2012) 'Stochastic optimization for transshipment problems with positive replenishment lead times', *International Journal of Production Economics*, Vol. 135, No. 1, pp.61–72.

Hurt, V. and Tramel, T. (1965) 'Alternative formulations of the transshipment problem', *Journal of Farm Economics*, Vol. 47, No. 3, pp.763–773.

Jafari, N. (2019) 'A non-assignment problem-based formulation for the asymmetric travelling salesman problem and its variation', *Int. J. Operational Research*, Vol. 35, No. 4, pp.575–585.

Jayaraman, A.R. and Srinivasan, M.R. (2019) 'Determinants of Indian banks efficiency: a two-stage approach', *Int. J. Operational Research*, Vol. 36, No. 2, pp.270–291.

Judge, G., Havlicek, J. and Rizek, R. (1965) 'An interregional model: its formulation and application to the livestock industry', *Agriculture and Economy and Revision*, Vol. 17, No. 1, pp.1–9.

Khatibi, S., Bafruei, M.K. and Rahmani, M. (2019) 'Modelling a bi-objective airport gate scheduling with controllable processing time using hybrid NSGA-II and VNS algorithm', *Int. J. Operational Research*, Vol. 34, No. 1, pp.1–27.

King, G. and Logan, S. (1964) 'Optimum location, number and size of processing plants with raw product and final product shipments', *Journal of Farm Economics*, Vol. 46, No. 1, pp.94–108.

Klir, G.J. and Yuan, B. (2008) *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, Prentice-Hall, New Jersey.

Kumar, A., Kaur, A. and Gupta, A. (2011a) 'Fuzzy linear programming approach for solving fuzzy transportation problems with transshipment', *J. Math. Model Algor.*, Vol. 10, No. 2, pp.163–180.

Kumar, A., Kaur, A. and Gupta, A. (2011b) 'New methods for solving fuzzy transportation problems with some additional transhipments', *ASOR Bulletin*, Vol. 30, No. 1, pp.42–61.

Kumar, H., Kumar, P. and Sharma, M. (2019) 'A genetic algorithm for a flow shop scheduling problem with breakdown interval, transportation time and weights of jobs', *Int. J. Operational Research*, Vol. 35, No. 4, pp.470–483.

Li, X., Otto, A. and Pesch, E. (2019) 'Solving the single crane scheduling problem at rail transshipment yards', *Discrete Applied Mathematics*, Vol. 264, pp.134–147.

Lmariouh, J., El Hachemi, N., Jamali, M.A., Bouami, D. and Rousseau, L.M. (2019) 'An integrated production and distribution problem with direct shipment: a case from Moroccan bottled-water market', *Int. J. Operational Research*, Vol. 34, No. 1, pp.144–160.

Maggioni, F., Cagnolari, M., Bertazzi, L. and Wallace, S.W. (2019) 'Stochastic optimization models for a bike-sharing problem with transshipment', *European Journal of Operational Research*, Vol. 276, No. 1, pp.272–283.

Maity, G. and Roy, S.K. (2019) 'Multi-item multi-choice integrated optimisation in inventory transportation problem with stochastic supply', *Int. J. Operational Research*, Vol. 35, No. 3, pp.318–339.

Meissner, J. and Senicheva, O.V. (2018) 'Approximate dynamic programming for lateral transshipment problems in multi-location inventory systems', *European Journal of Operational Research*, Vol. 265, No. 1, pp.49–64.

Moore, R.E. (1979) *Method and Applications of Interval Analysis*, SLAM, Philadelphia, PA.

NagoorGani, A., Baskaran, R. and Mohamed Assarudeen, S.N. (2011) 'Transshipment problem in fuzzy environment', *International J. of Math. Sci. &Engg. Applns.*, Vol. 5, No. 3, pp.57–74.

Orden, A. (1956) 'Transshipment problem', *Management Science*, Vol. 2, No. 3, pp.276–285.

Ozdemir, D., Yucesan, E. and Herer, Y. (2006) 'Multilocation transshipment problem with capacitated production and lost sales', in *Proceedings of the 2006 Winter Simulation Conference*, pp.1470–1476.

Pandian, P. and Natarajan, G. (2010a) 'A new method for finding an optimal solution for transportation problems', *International Journal of Mathematical Sciences and Engineering Applications*, Vol. 4, pp.59–65.

Pandian, P. and Natarajan, G. (2010b) 'A new method for finding an optimal solution of fully interval integer transportation problems', *Applied Mathematical Sciences*, Vol. 4, No. 37, pp.1819–1830.

Pandian, P., Natarajan, G. and Akilbasha, A. (2016) 'Fully rough integer interval transportation problems', *International Journal of Pharmacy and Technology*, Vol. 8, No. 2, pp.13866–13876.

Pandian, P., Natarajan, G. and Akilbasha, A. (2018) 'Fuzzy interval integer transportation problems', *International Journal of Pure and Applied Mathematics*, Vol. 119, No. 9, pp.133–142.

Rais, A., Alvelos, F. and Carvalho, M.S. (2014) 'New mixed integer-programming model for the pickup-and-delivery problem with transhipment', *European Journal of Operational Research*, Vol. 235, No. 3, pp.530–539.

Rajendran, P. and Pandian, P. (2012) 'Solving fully interval transhipment problems', *Internal Mathematical Forum*, Vol. 7, No. 41, pp.2027–2035.

Rhody, D. (1963) *Interregional Competitive Position of the Hog-Pork Industry in the Southeast United State*, PhD thesis, Iowa State University, Ames.

Villa, S. and Castañeda, J.A. (2018) 'Transshipments in supply chains: a behavioral investigation', *European Journal of Operational Research*, Vol. 269, No. 2, pp.715–729.