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Distribution and inventory planning in multi-echelon supply chains under demand uncertainty

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Abstract: Distribution and inventory planning in a multi-echelon system are studied under an uncertain demand context. To deal with this problem a mixed integer linear programming (MILP) model is proposed. This considers a multi-echelon system formed by N-warehouses and M-retailers. The problem consists on determining the optimal reordering plan for the operating network, which minimises the overall system's operation cost. The uncertain demand faced by retailers is addressed by defining the optimal safety stock that guarantees a given service level at each regional warehouse and each retailer. Also, the risk pooling effect is taken into account when determining inventory levels in each entity. A case study based on a real retailer distribution chain is presented and solved.

Keywords: supply chain management; inventory planning; mixed integer linear programming; MILP; guaranteed service approach; demand uncertainty; risk pooling.

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1 Introduction

The distribution operation within different industries faces uncertainties that cover a wide range of factors such as demands, prices and lead times for the supply of products. Triki and Al-Hinai (2016) researched the optimisation techniques for multi-period planning horizon and Omrani and Ghiasi (2017) studied optimisation problems with data uncertainty. Demand uncertainty and bullwhip effect phenomenon are important drivers that all managers must to take into account (Vicente et al., 2018). Between these, demand uncertainty may well have the dominant impact on profits and service level. This can lead to excess inventories or inability to meet service level. Excess inventory results in unnecessary holding costs, while the inability to meet the customer needs results in both loss of profit with the possibility, on long-term, loss of customers (Jung et al., 2004).

Inventory optimisation in a multi-echelon supply chain network, characterised by an uncertain demand, is a real world problem (Amiri-Aref et al., 2018). In this context, optimal inventory planning has become a major goal of the companies in order to simultaneously reduce costs and improve service level in today's increasingly competitive business environment (Daskin et al., 2002; Axsater, 2003; Yadollahi et al., 2017). A high service level can be obtained by maintaining increased inventory levels to hedge against demand uncertainties. Although additional inventory improves service level, it increases inventory holding cost. It is then necessary a trade-off between service level and inventory holding cost. This can be achieved through the solution of stochastic optimisation problems where the inventory levels are the key optimisation variables (Stephan et al., 2010). One such approach implies the use of the safety stock as a lower bound on the inventory level which is chosen such as to absorb some level of the demand uncertainty (Graves and Willems, 2000).

There exists a large number of works on estimating safety stock levels based on classical inventory theory. However, they fail to address the key features of realistic supply chain problems, namely, multiple products sharing multiple facilities with capacity constraints and demands originating from multiple customers. In addition, in real world supply chains, safety stock levels are dependent on factors such as the probabilistic distribution of the demands, the demand to capacity ratio, service level on meeting the demands for multiple products and transportation lead times among facilities. Such factors introduce complexities that classical inventory models simply do not accommodate (Porteus, 2002; Chopra and Meindl, 2004).

The main objective of this paper is to explore this opportunity by adapting the concept of safety stock into a network inventory planning model. Within this context, the goal of the present research is to develop a model that includes lower bounds on the inventory levels of various products and through different entities. Additionally, the approach entails the definition of the safety stock as a model variable and a guaranteed service level as a model parameter to reduce the shortage in inventory levels. The model also considers risk pooling effect, first referred by Eppen (1979), which states that significant safety stock cost can be saved by grouping in one central location the demand of multiple stocking locations.

The system studied in this paper and where the proposed approach is tested considers several typical real world conditions, such as multi-echelon, multi-warehouse, multi-retailer, multi-product, multi-period, limited transportation, limited storage, transshipment, lead-time and uncertain demand; so as to find the optimal solution by minimising total operation costs under a certain guaranteed service level.

The remainder of this paper is organised as follows. Section 2 includes a literature review on mathematical optimisation approaches to model demand uncertainty and the guaranteed service approach to model the multi-echelon distribution and inventory planning system. The problem definition is given in Section 3. Section 4 describes the distribution and inventory planning mathematical model. The case study is present in Section 5. Section 6 presents the results and analysis. Finally the conclusions are drawn in Section 7.

2 Literature review

Mathematical optimisation approaches applied to the modelling of inventory planning in supply chains considering uncertain demand has been researched over the last years, but the inventory management is usually considered without detailed inventory planning supply chain policies (Inderfurth, 1991; Minner, 2001; Simchi-Levi and Zhao, 2011; Hu et al., 2017). O'Driscoll (2017) proposed a two-stage stochastic programming model for a competitive oil refinery with stochastic crude and fuel prices. Some research revealed that the nature of demand uncertainty was the key differentiator between the various supply chain optimisation techniques (Cole and Bradshaw, 2016; Zaman and Saha, 2018). In the published models, the safety stock is often given as a parameter and it usually is treated as a lower bound of the total inventory level (Relvas et al., 2006; Schulz et al., 2005; Paterson et al., 2011). This approach cannot optimise the safety stock levels, especially when considering demand uncertainty. Thus, it can only provide an approximation of the inventory cost and may lead to suboptimal solutions. Jung et al. (2004) use a simulation-optimisation framework to determine the optimal safety stocks levels of a supply chain with consideration of production capacity.

On the other hand, most of the existing literature focuses on single-echelon systems. The uncertain demand is addressed by defining the optimal amount of safety stock that guarantees certain service level at a given customer. Daskin et al. (2002) introduced a model in which supply chains design decisions integrate inventory considerations. It is assumed that no limitation in storage capacity is considered and all lead times from supplier to distribution centres are the same. Thus, given these assumptions, the inventory structure is considered as a single-echelon system. A similar research can be found in Shen et al. (2003). Ozsen et al. (2008, 2009) extend the model of Daskin et al. (2002) and Shen et al. (2003) to include capacities on the inventory held. Bossert and Willems (2007) extend the guaranteed service modelling framework in order to optimise the inventory policy in a supply chain.

You and Grossmann (2010) propose an optimisation model of a multi-echelon supply chain design and inventory management under demand uncertainty. The supply chain involves only one product, transshipment is not allowed and the model assumes only one planning period.

When dealing with uncertainty in multi-echelon inventory planning two main approaches have been explored in the literature: the stochastic service approach and the guaranteed service approach (Humair and Willems, 2006).

The first one uses a stochastic programming model where uncertainty is considered directly using a scenario-based approach (Tsiakis et al., 2001; Sahinidis, 2004). Each scenario is associated with a certain probability of occurrence and represents one possible realisation for the uncertain parameter. In general, two decision stages are considered. In the first stage, ‘here and now’ decisions have to be made before the uncertain parameter realisation is known. In the second stage, ‘wait and see’ decisions are considered which are associated with a recourse action because they can be made after the random parameter is known. The main disadvantage of this method is that the model size tends to increase rapidly with the number of scenarios considered. In addition, it is not always feasible to explicitly enumerate all possible discrete values of the uncertain parameter.

The second one consists of using the chance constraint approach in which each uncertain parameter is treated as a random variable with a given probability distribution, which is applied in several cases to model demand uncertainty (Gupta and Maranas, 2003; You and Grossmann, 2008; Rodriguez and Vecchietti, 2011; Humair and Willems, 2011). The guaranteed service approach aims at determining the optimal placement and amount of safety stocks in a multi-echelon system to ensure the overall target service level at the lowest cost (Eruguz et al., 2014). Recently, Hong et al. (2018) study a supply chain configuration problem to optimise the service time and option selection decisions to minimise the overall cost of the supply chain. Generally, in a supply chain, most of the parameters are not deterministic, for this reason is better to consider demand and service time as uncertain parameters (Rashid et al., 2018). When applying this approach demand uncertainty is considered by specifying a demand level above the mean that must be satisfied. One strategy explored by You and Grossmann (2008) is to define the safety stock as a decision variable and a guaranteed service level as a parameter in the model to reduce the shortage in the inventories.

In this work, the second approach is chosen as it allows determining a safety stock level at supply chain entities in order to guarantee a certain service level and avoids the creation of multiple scenarios in a single model, which increases largely the model size.

The guaranteed service approach has been addressed in several problems in multi-echelon stochastic inventory planning and supply chain optimisation (Eruguz et al.,

2016) but it has not yet been treated on short-term inventory planning problems (Graves and Willems, 2000, 2005, 2008; Neale and Willems, 2009). On another hand, integrating stochastic inventory planning into the operational planning supply chain is nontrivial, and it has not been addressed in the existing literature.

The concept of guaranteed service approach, which is used in this work is based on the works by Graves and Willems (2000) and You and Grossmann (2010, 2011). Such concept is here applied to multi-echelon networks where when comparing to single-echelon inventory must consider explicitly the presence of lead time, which may include material handling time and transportation time. Within single-echelon systems, the ones already addressed in the literature, lead time is exogenous and generally can be treated as a parameter. However, within multi-echelon systems, lead time of a downstream node depends on the upstream node's inventory level and on the supply chain demand uncertainty. Therefore, lead time and internal service level are uncertain and simply propagating the single-echelon inventory planning system to multi-echelon inventory planning system leads to suboptimal solutions.

The goal of the present research consists on determining the optimal reordering plan for the operating supply chain network, which minimises the overall system's operation cost. The uncertain demand faced by retailers is addressed by defining the optimal safety stock that guarantees a given service level at each regional warehouse and each retailer. Also, the risk pooling effect is taken into account when determining inventory levels in each entity. This research develop an inventory planning policy within a multi-period, multi-warehouse, multi-retailer, multi-product distribution supply chain using exact optimisation methods.

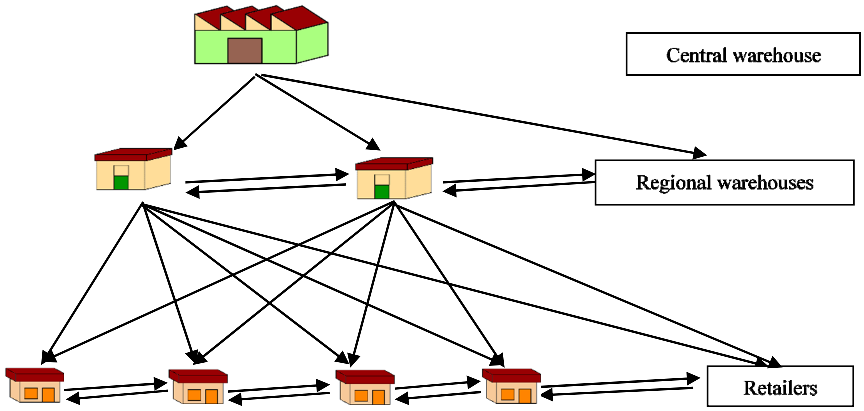
It was incorporated in this present paper the uncertainty across the guaranteed service model approach. The present mixed integer linear programming (MILP) model considers the safety stock level as a variable to be optimised and the service level as a parameter so as to reduce shortage occurrence in inventories. A safety stock level at different supply chain entities is considered, which guarantees a certain desired service level. This approach avoids the creation of multiple scenarios so as to determine optimal expected costs, and which contribute to increase drastically the model size. It was solved the nonlinearisation of the model. The risk pooling effect is also taken into account in the model by relating the probability distribution functions of the demands in the downstream nodes to their upstream nodes.

The research objectives of the paper were presented and an adequate methodology is then required. To this end, the guaranteed service approach for multi-period, multi-product and multi-echelon supply chain is applied to the problem in next section.

3 Problem definition

This section is an extended version that complements a previous one presented in a conference by Vicente et al. (2015). A generic supply chain under product demand uncertainty is considered in this study, across the guaranteed service model approach. It comprises one central warehouse, multiple regional warehouses and multiple retailers as depicted in Figure 1, where multiple products are distributed over a given time horizon of multiple time periods.

Figure 1 Supply chain structure (see online version for colours)



The structure assumes that retailers replenish their inventories from the regional warehouses, these replenish their inventories from a central warehouse and customer demand is observed at the retailers. Each retailer faces a normally distributed demand with mean μ and variance σ^2, which is independent of the other retailers' demands. For this supply chain structure, single sourcing is assumed, e.g., each retailer can only be served by one regional warehouse as each regional warehouse can only be served by one warehouse (central warehouse). Lateral transshipment between regional warehouses and between retailers is allowed. The corresponding deterministic order processing times, which include the material handling time and transportation time, are given. The guaranteed service time of the central warehouse and the guaranteed service time of each retailer are known. The safety stock factors for regional warehouses and retailers are also given. This corresponds to the standard normal deviate of the maximum amount of demand that the node will satisfy from its safety stock. All entities use a variable order quantity covering the demand of variable length time periods. All storage and transportation capacities are limited and transportation occurs after orders have been placed. If the demand in a given time period and at a given retailer is not satisfied, this is assumed as a lost sale. Note that this lost sale is basically related with initial inventory and the order processing time on the first time periods of planning time horizon, but not with the uncertainty related with demand. Lost sales related with uncertainty are not considered. In terms of costs, different cost types are included. These are related to the ordering process; holding in stock; holding in safety stock (net lead time dependent); holding in-transit (for shipping and transshipping operations); transportation (for shipping and transshipping operations) and lost sale. Fixed ordering costs occur each time a regional warehouse or a retailer places an order and are related to the ordering activity, being although independent of the quantity ordered. Holding costs are defined for both stock and safety stock and in-transit inventory. The first and second ones (stock and safety stock) are defined by unit stored and by time period on each regional warehouse or retailer. The third ones are defined by unit of product transported and are dependent of the order processing time. Transportation costs are considered by unit of material transported between the different stages of the supply chain. Related to these are the transshipping costs that represent the lateral transportation costs by unit that occurs within each stage between two identical entities. These can occur between regional warehouses or between retailers. Finally, lost sales costs are associated to the demand that cannot be satisfied and are defined by unit of product.

Thus, it is important to effectively represent and optimise the products' flows through the entire supply chain so as to minimise operation costs. These aspects are considered into the problem in study and the relevant decision that needs to be modelled is then to determine the shipping quantity to be sent from the regional warehouses to each retailer in each time period so as to minimise the total system costs while guaranteeing the required demand. The problem in study can then be defined as follows:

3.1 Given

- The planning time horizon and the defined discrete time scale.
- The number of regional warehouses and retailers.
- The number of products.
- Initial inventory by product in each regional warehouse and retailer.
- Mean and standard deviation of demand for each product on a time period basis (the product demand is normally distributed and occurs in retailers).
- Storage capacities in each regional warehouse and retailer by time period.
- Transportation capacities between entities.
- Order processing time between entities.
- Ordering costs by order of each product at each regional warehouse and retailer (independent of order quantity).
- Guaranteed service time by product of central warehouse and retailers.
- Safety stock factor by product and by regional warehouse and retailer.
- Unitary holding cost by time period by product at each regional warehouse or retailer.
- Unitary holding in transit cost by time period by product (dependent of order processing time).
- Unitary transportation (shipping and transshipping) cost by product.
- Unitary lost sale cost by product and by each time period.

3.2 Determine

- The inventory profiles by product throughout the planning time horizon at each regional warehouse and retailer in each time period.
- Safety stock by product for the planning time horizon in each regional warehouse and retailer.

- The flows of products across the supply chain for each time period. These involve shipping quantities between entities on different supply chain levels and transshipment quantities between entities on the same supply chain level.
- Lost sale quantities by product at each retailer in each time period.

So as to minimise an objective function that consists on the minimisation of the total operational costs for the time horizon considered. Note that the guaranteed service time is a variable for the regional warehouses and thus the net lead times are variables to determine the safety stock for regional warehouses and retailers.

This problem is modelled through a mathematical programming model, which will be presented in the subsequent section.

4 Distribution and inventory planning mathematical model

The supply chain distribution and inventory planning problem presented is formulated as a MILP model, as an extended version that complements a previous one presented in a conference by Vicente et al. (2015). This model uses a variable order quantity covering the demand of variable length time periods. It considers time represented through a discretised time scale, where the time periods have equal durations.

The indices, sets, parameters and variables (non-negative continuous and binary) used in the model formulation are defined using the following notation:

4.1 Indices

i	product
j, k, l, m	entity node
t	time period.

4.2 Constants

NP	number of products
NW	number of regional warehouses
NR	number of retailers
NT	number of time periods.

4.3 Sets

$i \in P = \{1, 2, \ldots, NP\}$	products
$j, k, l, m \in I = \{0, 1, 2, \ldots, NW, NW + 1, NW + 2, \ldots, NW + \mathrm{NR}\}$	supply chain nodes.
$t \in T = \{1, 2, \ldots, NT\}$	time periods
$W = \{1, 2, \ldots, NW\}$, $W \subset I$	warehouses
$R = \{1, 2, \ldots, NR\}$, $\mathrm{R} \subset \mathrm{I}$	retailers

$W_o = \{0\}, W_o \subset I$	central warehouse
$DN = \{1, 2, \ldots, NW, NW + 1, NW + 2, \ldots, NW + NR\}, DN \subset I$	demand nodes (regional warehouses and retailers)
$SN = \{0, 1, 2, \ldots, NW\}, SN \subset I$	supply nodes (central warehouse and regional warehouses).

Note that $W_0 \cup W \cup R = I$.

4.4 Parameters

BGM	a large positive number
PD_{ijt}	product demand of product i at entity j in time period t (note that product demand is a random value normally distributed that occurs at the retailers), defined as $N(\mu_{ikt}, SDCD_{ikt}^2)$
HOC_{ij}	unitary holding cost of the product i at entity j
HTC_{ijk}	unitary holding in transit cost of the product i from entity j to entity k (note that holding in transit is for shipping operations and for transshipping operations)
Ito_{ij}	initial inventory level of the product i at entity j
LSC_{ijt}	unitary lost sale cost of the product i at entity j in time period t
NLT_{ij}^U	upper bound of net lead time of product i of entity j
OC_{ij}	ordering cost of the product i at entity j (note that ordering cost is independent of quantity of product i)
R_{ikt}	guaranteed service time of product i of entity k in time period t
$SDCD_{ij}^2$	variance of product demand of the product i at entity j (it occurs only at the retailers but not at the warehouses)
S_{ij}^U	upper bound of guaranteed service time of product i of entity j
SI_{i0}	guaranteed service time of product i of central warehouse
SSF_{ij}	safety stock factor of the product i at entity j
STC_{jt}	storage capacity at entity j in time period t
$T1_{ijk}$	order processing time of product i of entity k served by entity j, including material handling time in entity k and transportation time from entity j to entity k
$TRACMAX_{jk}$	maximum transportation capacity from entity j to entity k
$TRACMIN_{jk}$	minimum transportation capacity from entity j to entity k

TRC_{ijk} unitary transportation cost of the product i from entity j to entity k (note that transportation is for shipping operations and for transshipping operations)

XV_{ij}^{U} upper bound of auxiliary variable XV_{ijt} of product i of entity j

μ_{ikt} mean product demand of the product i at entity k in time period t (it occurs only at the retailers but not at the warehouses).

4.5 Non-negative continuous variables

FI_{ijt} inventory of product i at entity j at the end of the time period t (at the regional warehouses and retailers)

LS_{ijt} lost sales of product i at entity j at the end of the time period t (note that lost sales only occur at the retailers)

NLT_{ijt} net lead time of product i at entity j in time period t (at the regional warehouses and retailers)

S_{ijt} guaranteed service time of product i of entity j in time period t (note that the guaranteed service time is only a variable for the regional warehouses)

SQ_{ijkt} shipping quantity of product i from entity j to entity k during time period t (among all entities of the supply chain)

SS_{ij} safety stock level of the product i at entity j (at the regional warehouses and retailers).

4.6 Non-negative auxiliary continuous variables

X_{ijkt} auxiliary variable for linearisation of the bilinear term $NLW_{ijt} \times BV2_{ijkt}$

XV_{ijt} auxiliary variable for reformulation of the term $\sum_{k\in R} SDCD_{ikt}^2 \times X_{ijkt}$

Y_{ijkt} auxiliary variable for linearisation of the bilinear term $S_{ijt} \times BV2_{ijkt}$.

4.7 Binary variables

$BV1_{ijt} = 1$ if an order of product i is placed by entity j in time period t, 0 otherwise

$BV2_{ijkt} = 1$ if there is a shipping quantity of product i from entity j to entity k in time period t, 0 otherwise.

Our aim is then to minimise the expected value of the total cost. This leads to the objective function (1).

$$\text{Minimise total expected cost} \\ = \sum_{i\in P}\sum_{j\in I}\sum_{t\in T}\left(OC_{ij} \times BV1_{ijt} + HOC_{ij} \times \left(FI_{ijt} + SS_{ij}\right) + LSC_{ijt} \times LS_{ijt}\right) \\ + \sum_{i\in P}\sum_{j\in I}\sum_{k\in I}\sum_{t\in T}\left(\left(HTC_{ijk} \times T1_{jk} + TRC_{ijk}\right) \times SQ_{ijkt}\right) \tag{1}$$

The first term of the objective function (1) represents the ordering costs, holding costs (in stock and in safety stock) and lost sale costs. Note that ordering costs are product quantity independent. The holding in transit costs and the transportation costs are the second term. Note that holding in transit costs are order processing time dependent.

4.8 Constraints

The model developed consists of different types of constraints. These are grouped into: inventory; shipping; link flow; single sourcing; storage capacities; transportation capacities; net lead time; safety stock policy and non-negativity and binary conditions.

Note that we use the deterministic order processing time on the determination of the inventory. However, for compute the safety stock we must use the net lead time to deal with the uncertainty. Constraints will be presented grouped by type.

4.9 Inventory constraints

Inventory constraints have to be defined for both warehouses and retailers, taking into account all inputs and outputs at each time period.

4.9.1 Regional warehouses

The total incoming quantity at each regional warehouse j is equal to the shipping quantity from the central warehouse, plus the sum of the transshipping quantities from the other regional warehouses l, at time period t, considering the order processing time. The total outgoing quantity at each regional warehouse j is equal to the sum of shipping quantities to the retailers k plus the sum of the transshipping quantities to the other regional warehouses l, at time period t. For $t = 1$ the inventory of product i at the end of this time period t at regional warehouses j is given by constraints (2), which takes into account the initial inventory level of product i at each regional warehouse j (Ito_{ij}).

$$\begin{aligned} FI_{ij1} &= Ito_{ij} + SQ_{i,0,j,t-T1_{0j}|T1_{oj}=0} - \sum_{k\in R} SQ_{ijk1} \\ &- \sum_{l\in W \wedge l\neq j} SQ_{ijl1} + \sum_{l\in W \wedge l\neq j} SQ_{i,l,j,t-T1_{lj}|T1_{lj}=0}, \qquad i\in P,\ j\in W,\ t=1 \end{aligned} \tag{2}$$

For the remaining time periods the inventory at the end of these time periods at regional warehouses is given by constraint (3).

$$\begin{aligned} FI_{ijt} &= FI_{i,j,t-1} + SQ_{i,0,j,t-T1_{0j}|T1_{oj}<t} - \sum_{k\in R} SQ_{ijkt} \\ &- \sum_{l\in W \wedge l\neq j} SQ_{ijlt} + \sum_{l\in W \wedge l\neq j} SQ_{i,l,j,t-T1_{lj}|T1_{lj}<t}, \qquad i\in P,\ j\in W,\ t\in T\setminus\{1\} \end{aligned} \tag{3}$$

4.9.2 Retailers

At each retailer k, the incoming quantity is equal to the sum of the shipping quantity from the regional warehouses j, plus the sum of the transshipment quantities from the others retailers m, at time period t, considering the order processing time. At each retailer k, the outgoing quantity is equal to the product demand minus the lost sale of that retailer k plus the sum of the transshipping quantities to the others retailers m, at time period t. For $t = 1$ the inventory of product i at the end of this time period t at the retailers k is given by constraint (4), which accounts for the initial inventory level of product i at retailer k (Ito_{ik}) whereas constraint (5) is applicable for the remaining time periods.

$$\begin{aligned} FI_{ik1} &= Ito_{ik} + \sum_{j \in W} SQ_{i,j,k,t-T1_{jk}|T1_{jk}=0} - (PD_{ik1} - LS_{ik1}) \\ &- \sum_{m \in R \wedge m \neq k} SQ_{ikm1} + \sum_{m \in R \wedge m \neq k} SQ_{i,m,k,t-T1_{mk}|T1_{mk}=0}, \qquad i \in P, k \in R, t = 1 \end{aligned} \tag{4}$$

$$\begin{aligned} FI_{ikt} &= FI_{i,k,t-1} + \sum_{j \in W} SQ_{i,j,k,t-T1_{jk}|T1_{jk}<t} - (PD_{ikt} - LS_{ikt}) \\ &- \sum_{m \in R \wedge m \neq k} SQ_{ikmt} + \sum_{m \in R \wedge m \neq k} SQ_{i,m,k,t-T1_{mk}|T1_{mk}<t}, \qquad i \in P, k \in R, t \in T \setminus \{1\} \end{aligned} \tag{5}$$

4.10 Shipping constraints

Since transportation occurs after an order has been placed from a destination to its source, it is assumed that the fixed ordering cost is always incurred when the transportation occurs. Hence, if the transportation amount is not zero the binary variable $BV1_{ijt}$ equals 1, as implied in constraint (6). The left hand side of this constraint represents the quantity received by a regional warehouse j, which can come from the central warehouse (first term) or any other regional warehouse l (second term).

$$SQ_{i0jt} + \sum_{l \in W \wedge l \neq j} SQ_{iljt} \leq BGM \leq BV1_{ijt}, \qquad i \in P, j \in W, t \in T \tag{6}$$

Equivalent constraints are defined for retailers, constraint (7). The BGM value will have a value that is valid as an upper bound of any quantity that can be ordered by a regional warehouse or retailer.

$$\sum_{j \in W} SQ_{ijkt} + \sum_{m \in R \wedge m \neq k} SQ_{imkt} \leq BGM \times BV1_{ikt}, \qquad i \in P, k \in R, t \in T \tag{7}$$

4.11 Link flow

When $BV2_{ijkt}$ equals to 1, it represents the link flow for product i between entity j and entity k at time period t, otherwise the entity j is not linked to entity k. Thus, if there is not a link flow between two entities, the binary variable $BV2_{ijkt}$ equals 0, then SQ_{ijkt} also equals 0, as implied in constraint (8). The left hand side of this constraint represents the shipping quantity of product i between entity j and entity k in each time period t.

$$SQ_{ijkt} \leq BGM \times BV2_{ijkt}, \qquad i \in P, j \in I, k \in I, j \neq k, t \in T \tag{8}$$

4.12 Single sourcing

Each regional warehouse is served by the central warehouse [constraint (9)] and each retailer is only served, by one regional warehouse [constraint (10)], by product *i* and by time period *t*.

$$BV2_{i0jt} = 1, \qquad i \in P, j \in W, t \in T \tag{9}$$

$$\sum_{j \in W} BV2_{ijkt} = 1, \qquad i \in P, k \in R, t \in T \tag{10}$$

4.13 Storage capacities

The total inventory stored at any node, given by the sum of the inventory level of each product *i*, must respect the storage capacity in each demand node *j* at any time period *t* which is enforced by constraint (11). The capacity is shared among products.

$$\sum_{i \in P} FI_{ijt} \le STC_{jt}, \qquad j \in DN, t \in T \tag{11}$$

4.14 Transportation capacities

At any time period *t*, the sum of the shipping quantity of each product *i* must respect the transportation lower and maximum limits between each two nodes *j* and *k*, as stated in constraints (12) and (13).

$$\sum_{i \in P} SQ_{ijkt} \le TRACMAX_{jk}, \qquad j \in DN, k \in DN, j \ne k, t \in T \tag{12}$$

$$TRACMIN_{jk} \le \sum_{i \in P} SQ_{ijkt}, \qquad j \in DN, k \in DN, j \ne k, t \in T \tag{13}$$

4.15 Net lead time

The next constraints (14) and (15) are used to define the net lead time of the regional warehouses and retailers, following the concept and approach used on the works by You and Grossmann (2010, 2011). The replenishment lead time of warehouses should be equal to the guaranteed service time SI_{i0} of product *i* of central warehouse, which serves regional warehouse *j*, plus the order processing time $T1_{i0j}$. Note that the guaranteed service time SI_{i0} of product *i* of central warehouse is treated as parameter and represents the worst case supply uncertainty delay of central warehouse. When $BV2_{i0jt}$ equals to 1, it represents the link flow for product *i* between central warehouse and regional warehouse *j* at time period *t*, otherwise the regional warehouse *j* is not linked to central warehouse. Thus, the net lead time of product *i* of regional warehouse *j* at time period *t* (NLT_{ijt}) should be greater than its replenishment lead time minus its guaranteed service time to its successor retailers, S_{ijt}, which is a variable.

Net lead time of regional warehouses for shipment operation is given by constraint (14).

$$NLT_{ijt} \geq \left(SI_{i0} + T1_{i0j}\right) \times BV2_{i0jt} - S_{ijt}, \qquad i \in P, j \in W, t \in T \tag{14}$$

Similarly, the net lead time of product i of retailer k at time period t (NLT_{ikt}) is greater than its replenishment lead time minus its maximum guaranteed service time, R_{ikt}, which is given by the next nonlinear inequality (15). Note that we consider R_{ikt} as a parameter.

Net lead time of retailers for shipment operation is given by constraint (15).

$$NLT_{ikt} \geq \left(S_{ijt} + T1_{ijk}\right) \times BV2_{ijkt} - R_{ikt}, \qquad i \in P, j \in W, k \in R, t \in T \tag{15}$$

There is a nonlinearity on constraint (15) – term $S_{ijt} \times BV2_{ijkt}$. In order to solve the nonlinearity on the right end-side of constraint (15), we define the auxiliary variable Y_{ijkt} (non-negative continuous variable) that represents the guaranteed service time of regional warehouse j to its successive retailers k by product i at each time period t.

Thus, the nonlinear term is replaced by the non-negative continuous variable Y_{ijkt} in constraint (15). The value of this auxiliary variable is given by equation (16).

$$Y_{ijkt} = S_{ijt} \times BV2_{ijkt}, \qquad i \in P, j \in W, k \in R, t \in T \tag{16}$$

Using the definition of variable Y_{ijkt} it is possible to impose the logical conditions (17) and (18). If the binary variable $BV2_{ijkt}$ is 0, then the auxiliary variable Y_{ijkt} is also 0 [condition (17)]. If, on the other hand, the binary variable $BV2_{ijkt}$ is equal to 1, we want to ensure that the new auxiliary variable takes the value of the guaranteed service time in the current time interval (S_{ijt}), as expressed in condition (18).

$$BV2_{ijkt} = 0 \Rightarrow Y_{ijkt} = 0, \qquad i \in P, j \in W, k \in R, t \in T \tag{17}$$

$$BV2_{ijkt} = 1 \Rightarrow Y_{ijkt} = S_{ijt}, \qquad i \in P, j \in W, k \in R, t \in T \tag{18}$$

To translate these logical conditions into the MILP model, we need to add the extra constraints (19)–(22).

$$Y_{ijkt} - S_{ij}^{U} \times BV2_{ijkt} \leq 0, \qquad i \in P, j \in W, k \in R, t \in T \tag{19}$$

$$-S_{ijt} + Y_{ijkt} \leq 0, \qquad i \in P, j \in W, k \in R, t \in T \tag{20}$$

$$S_{ijt} - Y_{ijkt} + S_{ij}^{U} \times BV2_{ijkt} \leq S_{ij}^{U}, \qquad i \in P, j \in W, k \in R, t \in T \tag{21}$$

$$Y_{ijkt} \geq 0, \qquad i \in P, j \in W, k \in R, t \in T \tag{22}$$

where S_{ij}^{U} is the upper bound for S_{ijt} (and hence also for Y_{ijkt}).

Constraints (19) and (20) ensure that the auxiliary variable takes the value of 0 if the binary variable is equal to 0 [constraint (19)]. If this variable is equal to 1, then the auxiliary variable takes, at most, the value of S_{ijt} [constraint (20)]. In order to ensure that in this situation the auxiliary variable takes exactly the value of S_{ijt}, we add constraint (21). Note that this equation only becomes active whenever $BV2_{ijkt} = 1$. Thus, Constraint (15) is replaced by constraints (19)–(23).

$$NLT_{ikt} \geq Y_{ijkt} + T1_{ijk} \times BV2_{ijkt} - R_{ikt}, \qquad i \in P, j \in W, k \in R, t \in T \tag{23}$$

Note that, for the net lead time constraints we only use the shipping operations (links between entities on different echelons of the supply chain). Because of the single sourcing assumption, we do not take into account the transshipping operations (links between entities on the same echelon of the supply chain).

4.16 Safety stock policy

The product demand at retailer $k \in R$ follows a given normal distribution with mean μ_{ikt} and variance $SDCD_{ikt}^2$. Because of the risk pooling effect proposed by Eppen (1979), the product demand over the net lead time NLT_{ijt} at regional warehouse $j \in W$ is also normally distributed with a mean of $NLT_{ijt} \times \sum_{k \in R} \mu_{ikt} \times BV2_{ijkt}$ and a variance $NLT_{ijt} \times \sum_{k \in R} SDCD_{ikt}^2 \times BV2_{ijkt}$, obtained considered the demand of all retailers assigned to this regional warehouse. Thus, the safety stock required in the regional warehouse $j \in W$ with a safety stock factor SSF_{ij} is given by constraint (24).

$$SS_{ij} \geq SSF_{ij} \times \sqrt{\sum_{k \in R} SDCD_{ikt}^2 \times BV2_{ijkt} \times NLT_{ijt}}, \qquad i \in P, j \in W, t \in T \tag{24}$$

There are two nonlinearities on the right end-side of constraint (24): a bilinear term and a square root term. First we solve the nonlinearity related to the bilinear term and later on the square root term. In order to solve the bilinear term we define the auxiliary variable X_{ijkt} (non-negative continuous variable) that represents the net lead time of regional warehouse *j* to its successive retailers *k* by product *i* at each time period *t*.

Thus, the nonlinear term $NLT_{ijt} \times BV2_{ijkt}$ is replaced by the non-negative continuous variable X_{ijkt} in constraint (24). The value of this auxiliary variable is given by equation (25).

$$X_{ijkt} = NLT_{ijt} \times BV2_{ijkt}, \qquad i \in P, j \in W, k \in R, t \in T \tag{25}$$

Using a similar approach for the previous nonlinearity for the net lead time, we need to add the extra constraints (26)–(29).

$$X_{ijkt} - NLT_{ij}^U \times BV2_{ijkt} \leq 0, \qquad i \in P, j \in W, k \in R, t \in T \tag{26}$$

$$-NLT_{ij} + X_{ijkt} \leq 0, \qquad i \in P, j \in W, k \in R, t \in T \tag{27}$$

$$NLT_{ijt} - X_{ijkt} + NLT_{ij}^U \times BV2_{ijkt} \leq NLT_{ij}^U, \qquad i \in P, j \in W, k \in R, t \in T \tag{28}$$

$$X_{ijkt} \geq 0, \qquad i \in P, j \in W, k \in R, t \in T \tag{29}$$

where NLT_{ij}^U is the upper bound for NLT_{ijt} (and hence also for X_{ijkt}).

Thus, constraint (24) is replaced by constraints (26)–(30)

$$SS_{ij} \geq SSF_{ij} \times \sqrt{\sum_{k \in R} SDCD_{ikt}^2 \times X_{ijkt}}, \qquad i \in P, j \in W, t \in T \tag{30}$$

To further reduce the nonlinear terms in the constraint (30), the term $\sum_{k \in R} SDCD_{ikt}^2 \times X_{ijkt}$ is replaced by a new non-negative continuous variable XV_{ijt}, in constraint (31).

$$XV_{ijt} = \sum_{k \in R} SDCD_{ikt}^2 \times X_{ijkt}, \qquad i \in P, j \in W, t \in T \tag{31}$$

$$XV_{ijt} \geq 0, \qquad i \in P, j \in W, t \in T \tag{32}$$

So, constraint (30) now becomes as constraint (33).

$$SS_{ij} \geq SSF_{ij} \times \sqrt{XV_{ijt}}, \qquad i \in P, j \in W, t \in T \tag{33}$$

Similarly, the product demand over the net lead time NLT_{ikt} of retailer $k \in R$ is normally distributed with a mean of $NLT_{ijt} \times \mu_{ikt}$ and a variance of $NLT_{ijt} \times SDCD_{ikt}^2$. Thus, the safety stock required in the retailer $k \in R$ with a safety stock factor SSF_{ik} is given by constraint (34).

$$SS_{ik} \geq SSF_{ik} \times SDCD_{ikt} \times \sqrt{NLT_{ikt}}, \qquad i \in P, k \in R, t \in T \tag{34}$$

In order to solve the nonlinearity on the right hand side of constraints (33) and (34), we use a similar approach as introduced by Falk and Soland (1969) for a univariate square root term $\sqrt{x}$, in which the variable x has lower bound 0 and upper bound x^U, its secant $\frac{x}{\sqrt{x^U}}$ represents the convex envelope and provides a valid lower bound of the square root term. As our model is a minimisation problem, replacing the univariate square root terms with their secants in constraints (33) and (34) will lead to the following linear constraints (35) and (36).

$$SS_{ij} \geq SSF_{ij} \times \frac{XV_{ijt}}{\sqrt{XV_{ij}^U}}, \qquad i \in P, j \in W, t \in T \tag{35}$$

$$SS_{ik} \geq SSF_{ik} \times SDCD_{ikt} \times \frac{NLT_{ikt}}{\sqrt{NLT_{ik}^U}}, \qquad i \in P, k \in R, t \in T \tag{36}$$

4.17 Non-negativity and binary conditions

As defined above, the model uses both non-negative continuous variables (37)–(39) and binary variables (40)–(41).

$$SQ_{ijkt}, FI_{ijt}, LS_{ijt} \geq 0, \qquad i \in P, j \in I, k \in I, t \in T \tag{37}$$

$$S_{ijt} \geq 0, \qquad i \in P, j \in W, t \in T \tag{38}$$

$$NLT_{ijt}, SS_{ij} \geq 0, \qquad i \in P, j \in DN, t \in T \tag{39}$$

$$BV1_{ijt} \in \{0,1\}, \qquad i \in P, j \in DN, t \in T \tag{40}$$

$$BV2_{ijkt} \in \{0,1\}, \qquad i \in P, j \in I, k \in I, t \in T \tag{41}$$

The above model formed by constraints (2)–(14), (19)–(23), (26)–(29), (31), (32), and (35)–(41) using the objective function (1) describes the proposed multi-echelon distribution and inventory planning model under demand uncertainty.

5 Case study

In this section we present a case study based on a retail company. Due to confidentiality reasons the data provided has been changed but still describes the real operation.

The model was implemented in GAMS 24.2 modelling language and solved using CPLEX 12.3 solver in an Intel Core i7 CPU 3.40 GHz and 8GB RAM. The stopping criteria were either a computational time limit of 3,600 seconds or the determination of the optimal solution.

Table 1 General case study parameters

Parameters	*Values (€)*
Ordering cost ($OC_{ij}, j \in DN, \forall_i$)	20
Holding cost ($HOC_{ij}, j \in W, \forall_i$)	0.2
Holding cost ($HOC_{ik}, k \in R, \forall_i$)	0.6
Holding in transit cost ($HTC_{i0j}, j \in W, \forall_i$)	0.3
Holding in transit cost ($HTC_{ijl}, j \in W, l \in W, j \neq l, \forall_i$)	0.3
Holding in transit cost ($HTC_{ijk}, j \in W, k \in R, \forall_i$)	0.9
Holding in transit cost ($HTC_{ikm}, k \in R, m \in R, k \neq m, \forall_i$)	0.9
Lost-sales cost ($LSC_{ikt}, k \in R, \forall_{it}$)	25

Table 2 Unitary products transportation costs (euro)

		Warehouse 1	*Warehouse 2*	*Retailer 1*	*Retailer 2*	*Retailer 3*	*Retailer 4*
TRC	Warehouse 0	0.55	0.22	0	0	0	0
	Warehouse 1	0	0.7	0.22	0.2	0.32	0.38
	Warehouse 2	0.7	0	0.68	0.52	0.34	0.1
	Retailer 1	0	0	0	0.2	0.8	1.3
	Retailer 2	0	0	0.2	0	0.3	1.0
	Retailer 3	0	0	0.8	0.3	0	0.36
	Retailer 4	0	0	1.3	1.0	0.36	0

Table 3 Initial inventory level (Ito) on warehouses and retailers (unit)

		Warehouse 1	*Warehouse 2*	*Retailer 1*	*Retailer 2*	*Retailer 3*	*Retailer 4*
Ito	Product 1	45	30	24	22	20	18
	Product 2	15	11	16	14	12	10
	Product 3	11	9	8	4	6	9

Table 4 Product demand parameters (PD) data for product 1/product 2/product 3 (unit)

		Mean demand	*Standard deviation*
PD	Retailer 1	12/8/6	4/4/4
	Retailer 2	11/7/9	4/4/4
	Retailer 3	10/6/7	3/3/3
	Retailer 4	9/5/6	3/3/3

Table 5 Order processing time (T1) of all products (time period)

		Warehouse 1	*Warehouse 2*	*Retailer 1*	*Retailer 2*	*Retailer 3*	*Retailer 4*
T1	Warehouse 0	2	1	0	0	0	0
	Warehouse 1	0	1	1	1	1	1
	Warehouse 2	1	0	2	2	1	0
	Retailer 1	0	0	0	1	1	1
	Retailer 2	0	0	1	0	1	1
	Retailer 3	0	0	1	1	0	1
	Retailer 4	0	0	1	1	1	0

The supply chain considered involves one central warehouse, two regional warehouses and four retailers. Three main types of product families are considered. The safety stock factors (SSF_{ij}) for regional warehouses and retailers were considered the same and equal to 1.96, which corresponds to 97.5% service level considering that the product demand is normally distributed. This service level is common in the industry sector of the company in study. The guaranteed service time of the central warehouse (SI_{i0}) is 1 time period. As the last echelon, representing the retailers, is an exogenous input (which can be treated as a parameter), the guaranteed service time of retailers (R_{ikt}) are set to 0 in order to have an immediate response. The maximum storage capacity for warehouses is of 5,000 units and for retailers is of 500 units. The transportation quantity limit between entities is considered between 0 and 500 units. A seven time period planning horizon was assumed to test our model (modelled in Section 4), which uses a variable order quantity covering the demand of variable length time periods. Tables 1 to 5 present the parameters' values considered for this case study, including general model parameters, transportation costs, initial inventory levels, product demand and order processing time. Customer demands at retailers are random values of the normal distribution (Table 4), generated in GAMS 24.2 modelling language.

6 Results and analysis

The retail company wants to compare two options for order management in the supply chain:

- Option A: Regional warehouse order and retailer order fulfilment flows per product can be formed by several flows of that product from any entity of the supply chain (e.g., on a per product perspective, each retailer could be served by any of the regional warehouses (a single combination) and by all others retailers).
- Option B: Regional warehouse order and retailer order fulfilment flows by product are formed by only one flow of that product from only one entity in a different echelon of the supply chain (e.g., on a per product perspective, each retailer is only served by one regional warehouse and transshipment is not allowed).

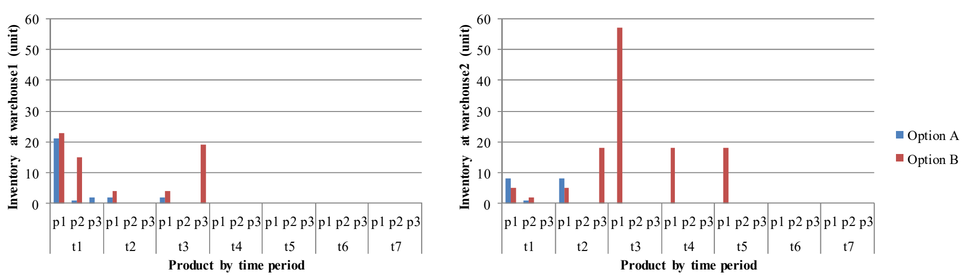
As results, Table 6 provides the total costs and costs per nature for both options, for the complete planning horizon of seven-time periods considered for this case study. The most significant costs are the lost sale costs for option B. This is due to the fact that transshipment is not allowed and the initial inventory is not sufficient to accommodate the demand in the first time periods and the order processing times on the first periods of time. Regarding to total costs, the option B (transshipment is not allowed) presents a higher value, it is at least 23% higher that option A (transshipment is allowed). Also, in option B note that, for transshipping operation, the costs are zero as transshipping is not allowed. In case A, the highest cost is related to ordering.

Table 6 Total costs and by nature of product 1/product 2/product 3 for seven-time period planning horizon (euro)

Order management	*Option A*		*Option B*	
	Value	*Percentage*	*Value*	*Percentage*
Ordering	600.00	27.55	520.00	19.40
Holding in stock	330.00	15.16	361.60	13.50
Holding in safety stock	229.99	10.56	229.99	8.58
Holding in transit shipment	446.70	20.52	469.80	17.53
Holding in transit transshipment	49.50	2.27	0	0
Transportation shipment	221.08	10.15	223.62	8.34
Transportation transshipment	50.28	2.31	0	0
Lost sale	250.00	11.48	875.00	32.65
Total	2,177.55	100	2,680.01	100

The inventory profiles by product at the end of each time period at regional warehouses and retailers for option A and option B are respectively shown in Figures 2 and 3. As we can see, for both options of order management by product and for all planning time horizon, the retailers echelon (Figure 3) hold more inventory (aggregated by echelon and by all time periods) than the regional warehouses echelon (Figure 2).

Figure 2 Inventory profile at regional warehouses by product and by time period of planning horizon (see online version for colours)



The safety stocks by product required in regional warehouses and retailers throughout the planning time horizon for both options are showed in Table 7. This safety stock is the same for all time periods of the planning time horizon and for both options of order management. This is due to the fact that safety stock is independent of order management option being only dependent of the safety stock factor, standard deviation of product demand and the net lead time. This is materialised by incurring in the same holding costs related with safety stock for both order management options (Table 6).

Figure 3 Inventory profile at retailers by product and by time period of planning horizon (see online version for colours)

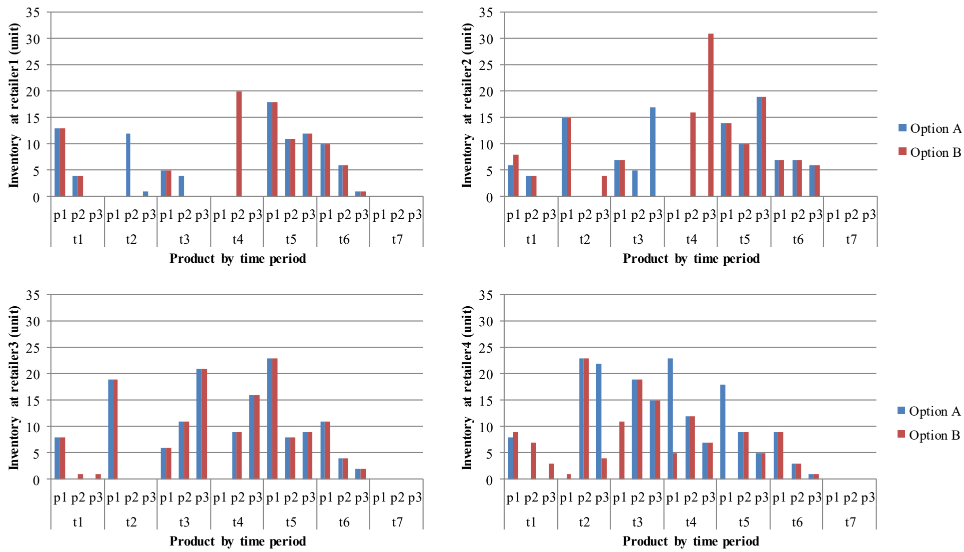


Table 7 Safety stock by product at regional warehouses and retailers throughout the planning time horizon for both options (unit)

Product	*P1*	*P2*	*P3*
Regional warehouse 1	15	15	15
Regional warehouse 2	7	7	7
Retailer 1	4	4	4
Retailer 2	4	4	4
Retailer 3	3	3	3
Retailer 4	0	0	0

Table 8 Net led time by product at regional warehouses and retailers throughout the planning time horizon for both options (time period)

Product	*P1*	*P2*	*P3*
Regional warehouse 1	3	3	3
Regional warehouse 2	2	2	2
Retailer 1	1	1	1
Retailer 2	1	1	1
Retailer 3	1	1	1
Retailer 4	0	0	0

For all products, the guaranteed service time of regional warehouses is zero and the net lead time by product of regional warehouses and retailers are shown in Table 8, throughout the planning time horizon for both options. They are determined by the model and they are constant for all time periods. The regional warehouses have a higher net lead time and hold the most level of safety stock to ensure their guaranteed service time are zero, so they work as a 'push' echelon.

Lost sales for both options at retailers are shown in Table 9. In option A, there are not lost sales for product 1 and product 2. For option B, product 1 is the only one that has not lost sales. Note that retailer 4 does not present any lost sales for both options.

Service levels for both options at retailers are shown in Table 10. As we can see, option A presents a better service level. Retailer 4 has a 100% of service level for all products in both options.

These lost sales and related service levels occur because of the insufficient initial inventory and the order processing time on the first time periods of the planning horizon. When transshipment operation is allowed (only on option A), it is also possible to have less lost sales on the first periods of the time horizon, given the higher flexibility of the network to react to demand widening the number or sourcing nodes.

Table 9 Lost sales at retailers by product aggregate on seven-time period planning horizon (unit)

Order management	*Option A*			*Option B*		
Product	*P1*	*P2*	*P3*	*P1*	*P2*	*P3*
Retailer 1	0	0	6	0	9	10
Retailer 2	0	0	4	0	10	5
Retailer 3	0	0	0	0	0	1
Retailer 4	0	0	0	0	0	0

Table 10 Service level at retailers by product aggregated for the seven-time period planning horizon (percentage)

Order management	*Option A*			*Option B*		
Product	*P1*	*P2*	*P3*	*P1*	*P2*	*P3*
Retailer 1	100	100	83.8	100	81.6	73.0
Retailer 2	100	100	94.1	100	83.3	92.6
Retailer 3	100	100	100	100	100	97.7
Retailer 4	100	100	100	100	100	100

Table 11 Computational statistics of product1/product2/product3 for seven-time period planning horizon

Order management	*Option A*	*Option B*
MIP solution	2,177.55	2,680.01
Best possible	2,177.55	2,680.01
Relative gap	0%	0%
Single equations	2,437	2,731
Single variables	1,828	1,828
Discrete variables	357	357
Computational time used (s)	26.27	14.88

After analysing the obtained results it can be said that the company should decide by operating under option A. This option presents lower total costs and a higher service level. Although option B presents zero holding in transit and transportation transshipment costs, however it presents high lost sale costs.

Table 11 shows the computational statistics for option A and option B. We obtain both optimal solutions (0% of relative gap), in less than 30 seconds.

7 Conclusions

This paper addresses an inventory planning model to determine the optimal inventory and distribution plan over a multi-echelon and multi-period planning time horizon under product demand uncertainty to support the decision making process in short-term process planning. The guaranteed service approach policy is selected to deal with uncertainty and is used to model the safety stock inventory system of a distribution company. The risk pooling effect is also taken into account in the model by relating the probability distribution functions of the demands in the downstream nodes to their upstream nodes.

The proposed MILP model considers the safety stock level as a variable to be optimised and the service level as a parameter so as to reduce shortage occurrence in inventories.

The theoretical implications are as follows. In the proposed uncertainty management policy, safety stock level at different supply chain entities is considered, which guarantees a certain desired service level. This approach avoids the creation of multiple scenarios so as to determine optimal expected costs, and which contribute to increase drastically the model size.

As the unique contributions of the present paper enhances the usage of the guaranteed service level approach in a supply chain inventory planning problem treated at an operational and multi-period setting, which was lacking in the related literature.

As managerial implications, a case study is explored and the inventories, safety stocks, lost sales and service levels obtained by the model optimisation under uncertain product demands for two options of order management area analysed. The difference between options is related to the transshipment operation occurrence. Order management with transshipment operation resulted in lower total costs and a higher service level. There are some limitations of the research, namely on the validation of the proposed model with more complex distribution supply chain structures.

Future research directions of the present work could be considered. Firstly the guaranteed service time of central warehouse and retailers could be studied as variables. Also the study of the transshipment operation, in terms of net lead time and its influence on the safety stocks, among all entities in the supply chain should be analysed. Finally, lost sales related with the demand uncertainty could be also explored.

References

Amiri-Aref, M., Klibi, W. and Babai, M.Z. (2018) 'The multi-sourcing location inventory problem with stochastic demand', *European Journal of Operational Research*, Vol. 266, No. 1, pp.72–87.

Axsater, S. (2003) 'Supply chain operations: serial and distribution inventory systems', in de Kok, A.G. and Graves, S.C. (Eds.): *Supply Chain Management: Design, Coordination and Operation, Handbooks in Operations Research and Management Science*, Vol. 11, pp.525-559, Elsevier, North Holland, Amsterdam.

Bossert, J.M. and Willems, S.P. (2007) 'A periodic review modeling approach for guaranteed service supply chains', *Interfaces*, Vol. 37, No. 5, pp.420–435.

Chopra, S. and Meindl, P. (2004) *Supply Chain Management: Strategy, Planning and Operation*, 2nd ed., Prentice-Hall, Upper Saddle River, New Jersey.

Cole, B.M. and Bradshaw, S. (2016) 'The planning and optimisation of a supply chain network under conditions of uncertainty', *International Journal of Operational Research*, Vol. 27, No. 3, pp.411–436.

Daskin, M.S., Coullard, C.R. and Shen, Z.J. (2002) 'An inventory-location model: formulation, solution algorithm and computational results', *Annals of Operations Research*, Vol. 110, Nos. 1–4, pp.83–106.

Eppen, G. (1979) 'Effects of centralization on expected costs in a multi-echelon newsboy problem', *Management Science*, Vol. 25, No. 5, pp.498–501.

Eruguz, A.S., Jemai, Z., Sahin, E. and Dallery, Y. (2014) 'Optimising reorder intervals and order-up-to levels in guaranteed service supply chains', *International Journal of Production Research*, Vol. 52, No. 1, pp.149–164.

Eruguz, A.S., Sahin, E., Jemai, Z. and Dallery, Y. (2016) 'A comprehensive survey of guaranteed-service models for multi-echelon inventory optimization', *International Journal of Production Economics*, Vol. 172, pp.110–125.

Falk, J.E. and Soland, R.M. (1969) 'An algorithm for separable non-convex programming problems', *Management Science*, Vol. 15, pp.550–569.

Graves, S.C. and Willems, S.P. (2000) 'Optimizing strategic safety stock placement in supply chains', *Manufacturing and Service Operations Management*, Vol. 2, No. 1, pp.68–83.

Graves, S.C. and Willems, S.P. (2005) 'Optimizing the supply chain configuration for new products', *Management Science*, Vol. 51, No. 8, pp.1165–1180.

Graves, S.C. and Willems, S.P. (2008) 'Strategic inventory placement in supply chains: nonstationary demand', *Manufacturing and Service Operations Management*, Vol. 10, No. 2, pp.278–287.

Gupta, A. and Maranas, C.D. (2003) 'Managing demand uncertainty in supply chain planning', *Computers and Chemical Engineering*, Vol. 27, Nos. 8–9, pp.1219–1227.

Hong, Z., Dai, W., Luh, H. and Yang, C. (2018) 'Optimal configuration of a green product supply chain with guaranteed service time and emission constraints', *European Journal of Operational Research*, Vol. 266, No. 2, pp.663–677.

Hu, X., Demeulemeester, E., Cui, N., Wang, J. and Tian, W. (2017) 'Improved critical chain buffer management framework considering resource costs and schedule stability', *Flexible Services and Manufacturing Journal*, Vol. 29, No. 2, pp.159–183.

Humair, S. and Willems, S.P. (2006) 'Optimizing strategic safety stock placement in supply chains with clusters of commonality', *Operations Research*, Vol. 54, No. 4, pp.725–742.

Humair, S. and Willems, S.P. (2011) 'Optimizing strategic safety stock placement in general acyclic networks', *Operations Research*, Vol. 59, No. 3, pp.781–787.

Inderfurth, K. (1991) 'Safety stock optimization in multistage inventory systems', *International Journal of Production Economics*, Vol. 24, Nos. 1–2, pp.103–113.

Jung, J.Y., Blau, G., Pekny, J.F., Reklaitis, G.V. and Eversdyk, D. (2004) 'A simulation based optimization approach to supply chain management under demand uncertainty', *Computers and Chemical Engineering*, Vol. 28, No. 10, pp.2087–2106.

Minner, S. (2001) 'Strategic safety stocks in reverse logistics supply chains', *International Journal of Production Economics*, Vol. 71, Nos. 1–3, pp.417–428.

Neale, J.J. and Willems, S.P. (2009) 'Managing inventory in supply chains with nonstationary demand', *Interfaces*, Vol. 39, No. 5, pp.388–399.

O'Driscoll, P.J. (2017) 'A modern two-stage stochastic programming portfolio model for an oil refinery with financial risk management', *International Journal of Operational Research*, Vol. 28, No. 1, pp.121–140.

Omrani, H. and Ghiasi, F.G. (2017) 'Facility location decisions in supply chain design under uncertainty: a robust optimisation approach', *International Journal of Operational Research*, Vol. 30, No. 3, pp.391–406.

Ozsen, L., Coullard, C.R. and Daskin, M.S. (2008) 'Capacitated warehouse location model with risk pooling', *Naval Research Logistics*, Vol. 55, No. 4, pp.295–312.

Ozsen, L., Daskin, M.S. and Coullard, C.R. (2009) 'Facility location modeling and inventory management with multisourcing', *Transportation Science*, Vol. 43, No. 4, pp.455–472.

Paterson, C., Kiesmuller, G., Teunter, R. and Glazebrook, K. (2011) 'Inventory models with lateral transshipments: a review', *European Journal of Operational Research*, Vol. 210, No. 2, pp.125–136.

Porteus, E.L. (2002) *Foundations of Stochastic Inventory Theory*, Stanford University Press, Stanford.

Rashid, R., Arani, S.D., Hoseini, S.F. and Omran, M.M. (2018) 'A new supply chain network design approach, regarding retailer's inventory level and supplier's response time', *International Journal of Operational Research*, Vol. 31, No. 4, pp.421–441.

Relvas, S., Matos, H.A., Barbosa-Póvoa, A.P., Fialho, J. and Pinheiro, A.S. (2006) 'Pipeline scheduling and inventory management of a multiproduct distribution oil system', *Industrial and Engineering Chemistry Research*, Vol. 45, No. 23, pp.7841–7855.

Rodriguez, M.A. and Vecchietti, A. (2011) 'Multicriteria optimization model for supply process problem under provision and demand uncertainty', *Industrial and Engineering Chemistry Research*, Vol. 50, No. 18, pp.10630–10642.

Sahinidis, N.V. (2004) 'Optimization under uncertainty: state-of-the-art and opportunities', *Computers and Chemical Engineering*, Vol. 28, Nos. 6–7, pp.971–983.

Schulz, E.P., Diaz, M.S. and Bandoni J.A. (2005) 'Supply chain optimization of large-scale continuous processes', *Computers and Chemical Engineering*, Vol. 29, No. 6, pp.1305–1316.

Shen, Z.J., Coullard, C.R. and Daskin, M.S. (2003) 'A join location-inventory model', *Transportation Science*, Vol. 37, No. 1, pp.40–55.

Simchi-Levi, D. and Zhao, Y. (2011) 'Performance evaluation of stochastic multi-echelon inventory systems: a survey', *Advances in Operations Research*, Vol. 2012, pp.1–34.

Stephan, H.A., Gschwind, T. and Minner, S. (2010) 'Manufacturing capacity planning and the value of multi-stage stochastic programming under Markovian demand', *Flexible Services and Manufacturing Journal*, Vol. 22, Nos. 3–4, pp.143–162.

Triki, C. and Al-Hinai, N. (2016) 'Optimisation techniques for planning the petrol replenishment to retail stations over a multi-period horizon', *International Journal of Operational Research*, Vol. 27, Nos. 1–2, pp.341–355.

Tsiakis, P., Shah, N. and Pantelides, C.C. (2001) 'Design of multi-echelon supply chain networks under demand uncertainty', *Industrial and Engineering Chemistry Research*, Vol. 40, No. 16, pp.3585–3604.

Vicente, J.J., Relvas, S. and Barbosa-Póvoa, A.P. (2015) 'Multi-period and multi-product inventory management model with lateral transshipments', *16th National Conference of the APDIO, Operational Research*, Bragança, Portugal, 3–5 June, Vol. 4, pp.425–444.

Vicente, J.J., Relvas, S. and Barbosa-Póvoa, A.P. (2018) 'Effective bullwhip metrics for multi-echelon distribution systems under order batching policies with cyclic demand', *International Journal of Production Research*, Vol. 56, No. 4, pp.1593–1619.

Yadollahi, E., Aghezzaf, E-H. and Raa, B. (2017) 'Managing inventory and service levels in a safety stock-based inventory routing system with stochastic retailer demands', *Applied Stochastic Models in Business and Industry*, Vol. 33, No. 4, pp.369–381.

You, F. and Grossmann, I.E. (2008) 'Design of responsive supply chains under demand uncertainty', *Computers and Chemical Engineering*, Vol. 32, No. 12, pp.3090–3111.

You, F. and Grossmann, I.E. (2010) 'Integrated multi-echelon supply chain design with inventories under uncertainty: MINLP models, computational strategies', *American Institute of Chemical Engineers Journal*, Vol. 56, No. 2, pp.419–440.

You, F. and Grossmann, I.E. (2011) 'Stochastic inventory management for tactical process planning under uncertainties: MINLP models and algorithms', *American Institute of Chemical Engineers Journal*, Vol. 57, No. 5, pp.1250–1277.

Zaman, K. and Saha, S.K. (2018) 'An efficient methodology for robust assignment problem', *International Journal of Operational Research*, Vol. 33, No. 2, pp.239–255.