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A novel inverse DEA model for restructuring DMUs with negative data

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Abstract: One of the important issues in generalised restructuring of a set of units is the identification of the input and output levels from a new set of post-restructuring units. This paper deals with the generalised restructuring of decision making units (DMUs) in the presence of negative data for achieving efficiency targets. A novel inverse DEA model is proposed for modelling the generalised restructuring of a set of DMUs in the presence of negative data. Sufficient conditions are given for estimation of inputs and outputs of the new set of post-restructuring units to realise efficiency targets. A numerical example is employed to illustrate the developed theory.

Keywords: data envelopment analysis; DEA; inverse DEA; multiple objective programming; MOP; generalised restructuring; efficiency; negative data.

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1 Introduction

Traditional data envelopment analysis (DEA) is a mathematical programming-based assessment tool for a set of decision making units (DMUs). In DEA, it is assumed that all inputs and outputs have non-negative values. Nevertheless, some of the inputs or outputs of the DMUs may be negative (Emrouznejad et al., 2010). For example, financial statements and growth rates could have positive or negative values. The DEA is initiated by the Farrell studies (Farrell, 1957) and later developed by many scholars, see, e.g., Charnes et al. (1978), Cooper et al. (1999), Zanboori et al. (2014), Emrouznejad and Yang (2018), Ghobadi et al. (2018) and Moonesian et al. (2019) for some reviews. It is worth noting, recently, in the published work of Hosseinzadeh Lotfi et al. (2020), R codes related to DEA models have been presented.

DEA models have been utilised for estimating the efficiency scores of the DMUs with certain input-output levels. However, in the last two decades, various studies have been concentrated on the inverse DEA as an analytical framework of DEA to find the required inputs and outputs levels for achieving a predetermined efficiency target (Lim, 2016; Hadi-Vencheh and Foroughi, 2006; Jahanshahloo et al., 2014; Wei et al., 2000). The first inverse DEA has been employed to estimate the input increments of a DMU for its given output increments, under preserving the constant returns to scale (CCR) efficiency index (Zhang and Cui, 1999). The inverse DEA has been employed to preserve the performance index under input or output levels variations (Hadi-Vencheh and Foroughi, 2006). In a special case, it has been utilised to solve the resource allocation problem (Wei et al., 2000). The inverse DEA problem with frontier change (Lim, 2016), under inter-temporal dependence (Ghobadi, 2019; Jahanshahloo et al., 2015) and in the presence of fuzzy data (Ghobadi, 2018; Ghobadi and Jahangiri, 2015; Ghobadi et al., 2019) has been studied in the literature.

According to the concept of inverse DEA, Amin and Oukil (2019) proposed a new approach in the merging problem of units under flexible targets setting. This approach allows that the decision maker to pursue different objectives in the merging problem of units, for example saving more inputs from a particular unit. Also, a novel method for target setting in mergers is proposed based on combining goal programming (GP) and inverse DEA (Amin et al., 2019). This method allows managers to save desired resources. Recently, the merging problem studied under temporal dependence of data by Zeinodin and Ghobadi (2020). They used inverse DEA and multi-objective programming for inputs/outputs-estimation of the merged unit under the inter-temporally dependence assumption. Resource allocation (Ghobadi and Jahangiri, 2019; Hadi-Vencheh et al., 2008), sensitivity analysis (Jahanshahloo et al., 2005), preserving or improving efficiency values (Ghobadi, 2017; Jahanshahloo et al., 2014; Lertworasirikul et al., 2011; Yan et al., 2002; Wei et al., 2000), setting revenue targets (Lin, 2010), banks merging (Amin et al., 2017; Gattoufi et al., 2014; Zeinodin and Ghobadi, 2019), and firms' restructuring (Amin et al., 2017) are some analytical and practical frameworks where the inverse DEA is employed. Synergies through mergers/acquisitions and reverse synergies through split DMUs have been combined in a general framework called restructuring DMUs (Amin et al., 2017). More precisely, the restructuring DMUs for realising the efficiency targets could happen in two situations: In the first situation, a new set of post-restructuring with synergies through mergers/acquisitions is produced by a homogeneous set of DMUs or a set of pre-restructuring DMUs. In the second one, a new set of post-restructuring with synergies is created through splitting.

To the authors' knowledge, no study has been reported for inverse DEA-based DMUs restructuring without any limitation in the number of pre and post restructuring units except the recent work of Amin et al. (2017). The inverse DEA idea has been employed by them to propose a new model for handling generalised restructuring situations called generalised inverse DEA (GInvDEA). To attain the pre-defined targets, the simultaneous redistribution of input and output levels inherited from pre-restructuring units between post-restructuring units has been suggested.

The main drawback of the mentioned model is that it could not be employed for targets setting of the post-restructuring units in the presence of negative data. In this paper, a novel inverse DEA model is proposed that could work in the presence of input and output levels with negative values. As a result, a new set of post-restructuring entities could adjust its inputs and outputs to realise predetermined efficiency targets. Sufficient conditions are given for estimation of inputs and outputs of the new set of post-restructuring units to realise efficiency targets. The validity of the proposed method is demonstrated through a numerical example.

This paper is organised as follows. Section 2 contains a brief literature review of DEA models in the presence of negative data. Section 3 presents a novel inverse

DEA model for targets setting after restructuring in the presence of negative data. The performance of the proposed inverse DEA is investigated through a numerical example in Section 4. Section 5 gives a brief conclusion.

2 DEA with negative data

There are different approaches to dealing with the negative data in the literature. Data transformations have been utilised to convert negative data to positive ones (Lovell, 1995; Pastor, 1994; Seiford and Zhu, 2002). As another approach, the absolute values of negative inputs and outputs have been considered by Scheel (2001) as outputs and inputs, respectively. Portela et al. (2004) proposed a model for measuring the efficiency of a set of DMUs in the presence of negative data, called range directional measures (RDM). This method could offer efficiency scores for each of DMUs, similar to radial methods in DEA, while negative data are used without any transformations. A semi-oriented radial measure (SORM) has been developed by Emrouznejad et al. (2010) for efficiency measurement of a set of DMUs with both negative inputs and outputs. In this paper, by considering the RDM model proposed by Portela et al. (2004) as the basic DEA model, a novel inverse DEA model is presented for adjusting the restructuring DMUs targets.

Let us to consider a set of n DMU_s, $\{DMU_j : j = 1, \ldots, n\}$, in which DMU_j produce multiple outputs $y_{rj} (r = 1, \ldots, s)$, by utilising multiple inputs $x_{ij} (i = 1, \ldots, m)$. Let input and output for DMU_j be denoted by $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})^t$ and $Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})^t$, respectively. To measure the efficiency score of DMU_o, $o \in \{1, 2, \ldots, n\}$, the following model is proposed by Portela et al. (2004):

$$
\begin{aligned}
\rho_o^* = 1 - \max\ & \varphi \\
\text{s.t.}\ & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \varphi R_{io}, && \forall i \in I, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \varphi R_{ro}, && \forall r \in O, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \varphi \geq 0, \lambda_j \geq 0, && j = 1, 2, \ldots, n,
\end{aligned} \tag{1}
$$

where

$$
R_{io} = x_{io} - \min\{x_{ij} \mid j = 1, 2, \ldots, n\}, \qquad \forall i \in I,
$$

$$
R_{ro} = \max\{y_{rj} \mid j = 1, 2, \ldots, n\} - y_{ro}, \qquad \forall r \in O.
$$

Here, (R_{io}, R_{ro}) is called the range of possible improvement of DMU_o. $\rho_o^* = 1 - \varphi^*$ is called the efficiency score of DMU_o. It is not difficult to see that $\rho_o^* \leq 1$. This model is translation invariant and units invariant, two important characteristics in DEA models that can deal with negative data.

3 Restructuring DMUs based on the concept of inverse DEA

The inverse DEA models proposed by Amin et al. (2017) could not be employed to reach the pre-defined targets in the restructuring DMUs in the presence of negative input and output levels. This could be due to the limitations of the basic DEA models utilised in their modelling. Now, in this section a new inverse DEA model is given to handle with the restructuring DMUs in the presence of negative data. To attain this goal, consider that the set of DMUs, $J = \{1, 2, \ldots, n\}$ is divided into two subsets: Λ (set of selected pre-restructuring DMUs) and Π (other DMUs), where $\Pi, \Lambda \subset J$, $\Lambda \cap \Pi = \emptyset$, and $\Lambda \cup \Pi = J$. Now, suppose that there is a set of $\bar{p}$ selected pre-restructuring DMUs, indexed in Λ, for generating $\bar{q}$ post-restructuring DMUs, indexed in Γ, to reach the predetermined efficiency targets ($\overline{\rho}_q$ for all $q \in \Gamma$). In fact, the input and output vectors (x_q, y_q) of a new set of post-restructuring DMUs (DMU_q for all $q \in \Gamma$) should be estimated to reach the desired efficiency targets $\overline{\rho}_q$.

After restructuring DMUs, we use the following model to measure the efficiency of DMU_q; $q \in \Gamma$:

$$\begin{aligned}
&\rho_q^* = 1 - \max\ \varphi_q \\
&\text{s.t.} \sum_{j\in\Pi} \lambda_j^q x_{ij} + \sum_{q\in\Gamma} \lambda^q x_{iq} \le x_{iq} - \varphi_q R_{iq}, && \forall i \in I, \\
&\sum_{j\in\Pi} \lambda_j^q y_{rj} + \sum_{q\in\Gamma} \lambda^q y_{rq} \ge y_{rq} + \varphi_q R_{rq}, && \forall r \in O, \\
&\sum_{j\in\Pi} \lambda_j^q + \sum_{q\in\Gamma} \lambda^q = 1, \\
&\varphi_q \ge 0, \lambda_j^q \ge 0;\ \forall j \in \Pi, \lambda^q \ge 0; && \forall q \in \Gamma,
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
&R_{iq} = x_{iq} - \min\{x_{ij} \mid j \in \Pi \cup \Gamma\}, && \forall i \in I, \\
&R_{rq} = \max\{y_{rj} \mid j \in \Pi \cup \Gamma\} - y_{rq}, && \forall r \in O.
\end{aligned}$$

The variable vector of the above model is $(\lambda_j^q, \lambda_q, \varphi_q)$. If the optimal value of models (2) is equal $\bar{\rho}_q$, we say that the efficiency score of DMU_q is $\bar{\rho}_q$. To estimate the input and output vectors, the following multiple-objective nonlinear programming (MONLP) model is proposed;

$$\begin{aligned}
&\min\ (\alpha_{ij}^q;\ \ \forall j \in \Lambda,\ \forall q \in \Gamma, \forall i \in I), \\
&\max\ (\beta_{rj}^q;\ \ \forall j \in \Lambda,\ \forall q \in \Gamma, \forall r \in O), \\
&\text{s.t.} \sum_{j\in\Pi} \lambda_j^q x_{ij} + \sum_{q\in\Gamma}\sum_{j\in\Lambda} \lambda^q \alpha_{ij}^q \le \sum_{j\in\Lambda} \alpha_{ij}^q - \bar{\varphi}_q R_{iq}, && \forall i \in I,\ \forall q \in \Gamma, \\
&\sum_{j\in\Pi} \lambda_j^q y_{rj} + \sum_{q\in\Gamma}\sum_{j\in\Lambda} \lambda^q \beta_{rj}^q \ge \sum_{j\in\Lambda} \beta_{rj}^q + \bar{\varphi}_q R_{rq}, && \forall r \in O,\ \forall q \in \Gamma, \\
&\sum_{j\in\Pi} \lambda_j^q + \sum_{q\in\Gamma} \lambda^q = 1, && \forall q \in \Gamma, \\
&\sum_{q\in\Gamma} \alpha_{ij}^q \le x_{ij}, && \forall i \in I,\ \forall j \in \Gamma,
\end{aligned}$$

$$\sum_{q\in\Gamma}\beta_{rj}^{q} \geq y_{rj}, \qquad \forall r \in O,\ \forall j \in \Gamma,$$

$$\sum_{j\in\Lambda}\alpha_{ij}^{q} \geq x_{i}^{Ideal}, \qquad \forall i \in I,\ \forall q \in Q,$$

$$\sum_{j\in\Lambda}\beta_{rj}^{q} \leq y_{r}^{Ideal}, \qquad \forall r \in O,\ \forall q \in Q,$$

$$\lambda_j^q \geq 0, \qquad \forall j \in \Pi,\ \forall q \in \Gamma,\ \alpha_{ij}^{q} \in \mathbb{R};\ \forall j \in \Lambda,\ \forall q \in \Gamma, \forall i \in I,$$

$$\beta_{rj}^{q} \in \mathbb{R}, \qquad \forall j \in \Lambda,\ \forall q \in \Gamma, \forall r \in O, \tag{3}$$

where $\mathbb{R}$ is the set of real numbers and

$$x_i^{Ideal} = \min\{x_{ij} \mid j \in \Pi \cup \Gamma\}, \qquad \forall i \in I,$$

$$y_r^{Ideal} = \max\{y_{rj} \mid j \in \Pi \cup \Gamma\}, \qquad \forall r \in O.$$

$(\lambda_j^q, \lambda^q, \alpha_{ij}^q, \beta_{rj}^q)$ is the variables vector in MONLP (3). Using the weight-sum method (Ehrgott, 2005), model (3) can be converted to the following single-objective nonlinear programming problem:

$$\min \sum_{i\in I}\sum_{q\in\Gamma}\sum_{j\in\Lambda} w_{ij}^q \alpha_{ij}^q - \sum_{r\in O}\sum_{q\in\Gamma}\sum_{j\in\Lambda} w_{rj}^q \beta_{rj}^q,$$

$$\text{s.t. The constraints of model (3).} \tag{4}$$

In the real world, the most common reconstructions happen between DMUs to improve their respective performances. Therefore, we can assume that the restructuring did not change the pre-restructuring efficiency boundary. More precisely, DMU_j for $j \in \Pi$ are can display all produced post-restructuring DMUs using a convex combination of their own. Accordingly, $\lambda_q^* = 0$ for all $q \in \Gamma$ in each optimal solution of model (4). Then, model (4) can be converted to the following LP model if and only if the corresponding pre and post-restructuring efficiency frontiers are identical.

$$\min \sum_{i\in I}\sum_{q\in\Gamma}\sum_{j\in\Lambda} w_{ij}^q \alpha_{ij}^q - \sum_{r\in O}\sum_{q\in\Gamma}\sum_{j\in\Lambda} w_{rj}^q \beta_{rj}^q,$$

$$\text{s.t.} \sum_{j\in\Pi}\lambda_j^q x_{ij} \leq \sum_{j\in\Lambda}\alpha_{ij}^q - \bar{\varphi}_q R_{iq}, \qquad \forall i \in I,\ \forall q \in \Gamma,$$

$$\sum_{j\in\Pi}\lambda_j^q y_{rj} \geq \sum_{j\in\Lambda}\beta_{rj}^q + \bar{\varphi}_q R_{rq}, \qquad \forall r \in O,\ \forall q \in \Gamma,$$

$$\sum_{j\in\Pi}\lambda_j^q = 1, \qquad \forall q \in \Gamma,$$

$$\sum_{q\in\Gamma}\alpha_{ij}^q \leq x_{ij}, \qquad \forall i \in I,\ \forall j \in \Lambda,$$

$$\sum_{q\in\Gamma}\beta_{rj}^q \geq y_{rj}, \qquad \forall r \in O,\ \forall j \in \Lambda,$$

$$\sum_{j\in\Lambda}\alpha_{ij}^q \geq x_i^{Ideal}, \qquad \forall i \in I,\ \forall q \in Q,$$

$$\sum_{j\in\Lambda}\beta_{rj}^{q}\le y_{r}^{Ideal}, \qquad \forall r\in O,\ \forall q\in Q,$$
$$\lambda_{j}^{q}\ge 0, \quad \forall j\in\Pi,\ \forall q\in\Gamma,\ \alpha_{ij}^{q}\in\mathbb{R},\ \forall j\in\Lambda,\ \forall q\in\Gamma,\forall i\in I,$$
$$\beta_{rj}^{q}\in\mathbb{R}, \quad \forall j\in\Lambda,\ \forall q\in\Gamma,\forall r\in O. \tag{5}$$

Theorem 3.1 shows how the above LP can be used to estimate inputs and outputs of post-restructuring DMUs.

Theorem 3.1: Suppose that the corresponding pre and post-restructuring efficiency frontiers are identical. Let $\Delta=(\lambda_{j}^{q*},\alpha_{ij}^{q*},\beta_{rj}^{q*})$ be an optimal solution to model (5). If

$$x_{iq}=\sum_{j\in\Lambda}\alpha_{ij}^{q*}, \qquad \forall i\in I,\ \forall q\in\Gamma,$$
$$y_{rq}=\sum_{j\in\Lambda}\beta_{rj}^{q*}, \qquad \forall r\in O,\ \forall q\in\Gamma, \tag{6}$$

such that $x_{q}=(x_{1q},...,x_{mq})\neq(x_{1}^{Ideal},...,x_{m}^{Ideal})=x^{Ideal}$ or $y_{rq}=(y_{1q},...,y_{sq})\neq(y_{1}^{Ideal},...,y_{s}^{Ideal})=y^{Ideal}$ for all $q\in\Gamma$, then $\rho_{q}^{*}=1-\bar{\varphi}_{q}$ for each $q\in\Gamma$.

Proof: Feasibility of Δ for model (5), implies:

$$\sum_{j\in\Pi}\lambda_{j}^{q*}x_{ij}\le\sum_{j\in\Lambda}\alpha_{ij}^{q*}-\bar{\varphi}_{q}R_{iq}=x_{iq}-\bar{\varphi}_{q}R_{iq}, \quad \forall i\in I,\ \forall q\in\Gamma, \tag{7}$$

$$\sum_{j\in\Pi}\lambda_{j}^{q*}y_{rj}\ge\sum_{j\in\Lambda}\beta_{rj}^{q*}+\bar{\varphi}_{q}R_{iq}=y_{rq}+\bar{\varphi}_{q}R_{iq}, \quad \forall r\in O,\ \forall q\in\Gamma, \tag{8}$$

$$\sum_{j\in\Pi}\lambda_{j}^{q*}=1, \qquad \forall q\in\Gamma, \tag{9}$$

$$\sum_{q\in\Gamma}\alpha_{ij}^{q*}\le x_{ij}, \qquad \forall i\in I,\ \forall j\in\Lambda, \tag{10}$$

$$\sum_{q\in\Gamma}\beta_{rj}^{q*}\ge y_{rj}, \qquad \forall r\in O,\ \forall j\in\Lambda, \tag{11}$$

$$x_{iq}=\sum_{j\in\Lambda}\alpha_{ij}^{q*}\ge x_{i}^{Ideal}, \qquad \forall i\in I,\ \forall q\in\Gamma, \tag{12}$$

$$y_{rq}=\sum_{j\in\Lambda}\beta_{rj}^{q*}\le y_{r}^{Ideal}, \qquad \forall r\in O,\ \forall q\in\Gamma, \tag{13}$$

$$\lambda_{j}^{q*}\ge 0; \qquad \forall j\in\Pi,\ \forall q\in\Gamma. \tag{14}$$

By equations (7)–(9) and (14), $(\lambda_{j}^{q}=\lambda_{j}^{q*};\forall j\in\Pi,\ \lambda^{q}=0;\forall q\in\Gamma,\ \varphi_{q}=\bar{\varphi}_{q};\ \forall q\in\Gamma, x_{iq}=\sum_{j\in\Lambda}\alpha_{ij}^{q*}, y_{rq}=\sum_{j\in\Lambda}\beta_{rj}^{q*})$ is obviously a feasible solution to problem (2). Therefore, $\rho_{q}^{*}\le 1-\bar{\varphi}_{q}$ for each $q\in\Gamma$.

Since $\varphi_q R_{iq}$ and $\varphi_q R_{rq}$ for all i, r, q, then by equations (7) and (8), we get

$$\sum_{j\in\Pi} \lambda_j^{q*} x_{ij} \leq x_{iq}, \qquad \forall i \in I,\ \forall q \in \Gamma, \tag{15}$$

$$\sum_{j\in\Pi} \lambda_j^{q*} y_{rj} \geq y_{rq}, \qquad \forall r \in O,\ \forall q \in \Gamma. \tag{16}$$

Considering $\Omega = (\tilde{\lambda}_j^q; \forall j \in \Pi, \tilde{\lambda}^q,\ \tilde{\varphi}_q;\ \forall q \in \Gamma)$ as an optimal solution to LP (2) and equations (15) and (16), we have

$$x_{iq} - \tilde{\varphi}_q R_{iq} \geq \sum_{j\in\Pi} \tilde{\lambda}_j^q x_{ij} + \sum_{q\in\Gamma} \tilde{\lambda}^q x_{iq} \geq \sum_{j\in\Pi} \tilde{\lambda}_j^q x_{ij} + \sum_{q\in\Gamma} \tilde{\lambda}^q \left(\sum_{j\in\Pi} \lambda_j^{q*} x_{ij} \right)$$
$$= \sum_{j\in\Pi} \left(\tilde{\lambda}_j^q + \sum_{q\in\Gamma} \tilde{\lambda}^q \lambda_j^{q*} \right) x_{ij}, \qquad \forall i \in I. \tag{17}$$

$$y_{rq} + \tilde{\varphi}_q R_{iq} \leq \sum_{j\in\Pi} \tilde{\lambda}_j^q y_{rj} + \sum_{q\in\Gamma} \tilde{\lambda}^q y_{rq} \leq \sum_{j\in\Pi} \tilde{\lambda}_j^q y_{rj} + \sum_{q\in\Gamma} \tilde{\lambda}^q \left(\sum_{j\in\Pi} \lambda_j^{q*} y_{rj} \right)$$
$$= \sum_{j\in\Pi} \left(\tilde{\lambda}_j^q + \sum_{q\in\Gamma} \tilde{\lambda}^q \lambda_j^{q*} \right) y_{rj}, \qquad \forall r \in O. \tag{18}$$

By equations (17), (18), and defining $\bar{\lambda}_j^q := \tilde{\lambda}_j^q + \sum_{q\in\Gamma} \tilde{\lambda}^q \lambda_j^{q*}$ for each $j \in \Pi$, we get

$$\sum_{j\in\Pi} \bar{\lambda}_j^q x_{ij} \leq x_{iq} - \tilde{\varphi}_q R_{iq}, \qquad \forall i \in I, \tag{19}$$

$$\sum_{j\in\Pi} \bar{\lambda}_j^q y_{rj} \geq y_{rq} + \tilde{\varphi}_q R_{rq}, \qquad \forall r \in O. \tag{20}$$

In addition, since Ω is an optimal solution to LP (2) and equation (9),

$$\sum_{j\in\Pi} \bar{\lambda}_j^q = \sum_{j\in\Pi} \left(\tilde{\lambda}_j^q + \sum_{q\in\Gamma} \tilde{\lambda}^q \lambda_j^{q*} \right) = \sum_{j\in\Pi} \tilde{\lambda}_j^q + \sum_{q\in\Gamma} \tilde{\lambda}^q \left(\sum_{j\in\Pi} \lambda_j^{q*} \right)$$
$$= \sum_{j\in\Pi} \tilde{\lambda}_j^q + \sum_{q\in\Gamma} \tilde{\lambda}^q = 1. \tag{21}$$

By contradiction assume that there exists at least one $k \in \Gamma$ such that $\rho_k^* = 1 - \tilde{\varphi}_k < 1 - \bar{\varphi}_k$. In other words, $\tilde{\varphi}_k > \bar{\varphi}_k$. By equations (19) and (20), we obtained:

$$\sum_{j\in\Pi} \bar{\lambda}_j^k x_{ij} \leq x_{ik} - \tilde{\varphi}_k R_{ik} < x_{ik} - \bar{\varphi}_k R_{ik}, \qquad \forall i \in I, \tag{22}$$

$$\sum_{j\in\Pi}\bar{\lambda}_j^k y_{rj} \geq y_{rk} + \tilde{\varphi}_k R_{rk} > y_{rk} + \bar{\varphi}_k R_{rk}, \qquad \forall r \in O. \tag{23}$$

According to assumptions of Theorem, we have $x_k \neq x^{Ideal}$ or $y_k \neq y^{Ideal}$. Without loss of generality, we assume that $x_k \neq x^{Ideal}$. Then, there exists at least one $l \in I$ such that $x_{lk} = \sum_{j\in\Lambda}\alpha_{lj}^{k*} > x_l^{Ideal}$ by equation (12).

Considering equation (22), we define

$$\mu = \min\left\{\frac{\sum_{j\in\Pi}\bar{\lambda}_j^k x_{lj} - (1-\bar{\varphi}_k)\sum_{j\in\Lambda}\alpha_{lj}^{k*} - \bar{\varphi}_k x_l^{Ideal}}{-(1-\bar{\varphi}_k)},\right.$$
$$\left.\sum_{j\in\Lambda}\alpha_{lj}^{k*} - x_l^{Ideal}\right\}. \tag{24}$$

It is obvious that $\mu > 0$. Now define $\bar{\beta}_{rj}^q = \beta_{rj}^{q*}$ for all $j \in \Lambda$, $q \in \Gamma$, $r \in O$, and

$$\bar{\alpha}_{ij}^q = \begin{cases} \alpha_{ij}^{q*} - \mu_j & \text{if } i = l, q = k, \\ \alpha_{ij}^{q*} & \text{otherwise,} \end{cases} \tag{25}$$

in which $\sum_{j\in\Lambda}\mu_j = \mu$.

By equation (24), we have

$$\sum_{j\in\Pi}\bar{\lambda}_j^k x_{ij} \leq \left(\sum_{j\in\Lambda}\alpha_{lj}^{k*} - \mu\right)(1-\bar{\varphi}_k) + \bar{\varphi}_k x_l^{Ideal} = \sum_{j\in\Lambda}\bar{\alpha}_{lj}^k - \bar{\varphi}_k R_{lq}, \tag{26}$$

$$\sum_{j\in\Lambda}\bar{\alpha}_{lj}^k = \sum_{j\in\Lambda}\alpha_{lj}^{k*} - \mu \geq x_l^{Ideal}, \tag{27}$$

By equations (11), (13), (19), (20), (26), and (27), it is obvious that $(\bar{\lambda}_j^q, \bar{\alpha}_{ij}^q, \bar{\beta}_{rj}^q)$ is a feasible solution to problem (5) in which, the value of the objective function of LP (5) at this feasible point is equal:

$$\begin{aligned}
&\sum_{i\in I}\sum_{q\in\Gamma}\sum_{j\in\Lambda}w_{ij}^q\bar{\alpha}_{ij}^q - \sum_{r\in O}\sum_{q\in\Gamma}\sum_{j\in\Lambda}w_{rj}^q\bar{\beta}_{rj}^q = \sum_{i\in I-\{l\}}\sum_{q\in\Gamma-\{k\}}\sum_{j\in\Lambda}w_{ij}^q\bar{\alpha}_{ij}^q \\
&+\sum_{j\in\Lambda}w_{lj}^k\bar{\alpha}_{ij}^q - \sum_{r\in O}\sum_{q\in\Gamma}\sum_{j\in\Lambda}w_{rj}^q\bar{\beta}_{rj}^q = \sum_{i\in I-\{l\}}\sum_{q\in\Gamma-\{k\}}\sum_{j\in\Lambda}w_{ij}^q\alpha_{ij}^{q*} \\
&+\sum_{j\in\Lambda}w_{lj}^k(\alpha_{lj}^{q*} - \mu_j) - \sum_{r\in O}\sum_{q\in\Gamma}\sum_{j\in\Lambda}w_{rj}^q\bar{\beta}_{rj}^q < \sum_{i\in I-\{l\}}\sum_{q\in\Gamma-\{k\}}\sum_{j\in\Lambda}w_{ij}^q\alpha_{ij}^{q*} \\
&+\sum_{j\in\Lambda}w_{lj}^k\alpha_{lj}^{q*} - \sum_{r\in O}\sum_{q\in\Gamma}\sum_{j\in\Lambda}w_{rj}^q\bar{\beta}_{rj}^q = \sum_{i\in I}\sum_{q\in\Gamma}\sum_{j\in\Lambda}w_{ij}^q\alpha_{ij}^{q*} - \sum_{r\in O}\sum_{q\in\Gamma}\sum_{j\in\Lambda}w_{rj}^q\beta_{rj}^{q*}.
\end{aligned}$$

This contradicts the assumption and completes the proof. □

4 An illustrative example

In this section, a numerical example is given to verify the realisation of the research goals. The data employed in this section are adapted from those provided by Emrouznejad et al. (2010) (see Table 1). This table shows the input and output levels for 10 DMU. Each DMU has an input, x, to produce two outputs y_1 and y_2. The model (1) is employed to obtain the efficiency score of DMUs (see Table 1).

Table 1 Inputs, outputs, and efficiency

$DMUs$	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}
x	12	35	25	22	40	50	35	40	25	16
y_1	15	18	20	12	–10	–8	–18	–10	–7	26
y_2	11	6	13	20	25	27	6	22	19	8
ρ^*	1.000	0.682	0.992	1.000	1.000	1.000	0.384	0.767	0.842	1.000

Table 1 shows the inefficiency of DMU_7, DMU_8, and DMU_9. Suppose that these three DMUs are merged to generate two new DMUs. In other words, the decision maker combines these three pre-restructuring DMUs to produce two post-restructuring DMUs with pre-specified efficiency goals. Then, the sets of pre and post restructuring DMUs indices are denoted by $\Lambda = \{7,\ 8,\ 9\}$, $\Gamma = \{1,\ 2\}$ respectively and $\Pi = J - \Lambda$. At first, let DMU_{new1} and DMU_{new2} be the two DMUs generated by the consolidation with a predetermined efficiency targets $\bar{\rho}_{new1}$ = 0.950 ($\bar{\varphi}_{new1}$= 0.050) and $\bar{\rho}_{new2}$ = 0.900 ($\bar{\varphi}_{new2}$ = 0.100), respectively. To estimate inputs and outputs DMU_{new1} and DMU_{new2} using the model (5), the following model is considered.

$$
\begin{aligned}
\min\ & \sum_{p\in\Lambda}\sum_{q\in\Gamma}\bar{w}_p^q\alpha_p^q - \sum_{p\in\Lambda}\sum_{q\in\Gamma}\bar{w}_{p1}^q\beta_{p1}^q - \sum_{p\in\Lambda}\sum_{q\in\Gamma}\bar{w}_{p2}^q\beta_{p2}^q,\\
\text{s.t.}\ & \sum_{j\in\Pi}\lambda_j^1 x_j \le \alpha_7^1+\alpha_8^1+\alpha_9^1-\bar{\varphi}_{new1}R_1,\\
& \sum_{j\in\Pi}\lambda_j^2 x_j \le \alpha_7^2+\alpha_8^2+\alpha_9^2-\bar{\varphi}_{new2}R_2,\\
& \sum_{j\in\Pi}\lambda_j^1 y_{1j} \le \beta_{r7}^1+\beta_{r8}^1+\beta_{r9}^1+\bar{\varphi}_{new1}R_{r1}, \qquad r=1,2,\\
& \sum_{j\in\Pi}\lambda_j^2 y_{rj} \le \beta_{r7}^2+\beta_{r8}^2+\beta_{r9}^2+\bar{\varphi}_{new2}R_{r2}, \qquad r=1,2,\\
& \sum_{j\in\Pi}\lambda_j^q = 1, \qquad q\in\Gamma,\\
& \alpha_7^1+\alpha_7^2\le 35,\ \alpha_8^1+\alpha_8^2\le 40,\ \alpha_9^1+\alpha_9^2\le 25,\\
& \beta_{17}^1+\beta_{17}^2\ge -18,\ \beta_{18}^1+\beta_{18}^2\ge -10,\ \beta_{19}^1+\beta_{19}^2\ge -7,\\
& \beta_{27}^1+\beta_{27}^2\ge 6,\ \beta_{28}^1+\beta_{28}^2\ge 22,\ \beta_{29}^1+\beta_{29}^2\ge 19,\\
& \alpha_7^1+\alpha_8^1+\alpha_9^1\ge 12 = x^{Ideal},\ \alpha_7^2+\alpha_8^2+\alpha_9^2\ge 12 = x^{Ideal},\\
& \beta_{17}^1+\beta_{18}^1+\beta_{19}^1\le 26 = y_1^{Ideal},\ \beta_{17}^2+\beta_{18}^2+\beta_{19}^2\le 26 = y_1^{Ideal},\\
& \beta_{27}^1+\beta_{28}^1+\beta_{29}^1\le 27 = y_2^{Ideal},\ \beta_{27}^2+\beta_{28}^2+\beta_{29}^2\le 27 = y_2^{Ideal},
\end{aligned}
$$

$$\begin{aligned} &\alpha_j^q \in \mathbb{R}, && \forall j \in \Lambda, q \in \Gamma, \\ &\beta_{rj}^q \in \mathbb{R}, && \forall j \in \Lambda, q \in \Gamma, r = 1, 2, \\ &\lambda_j^1 \geq 0, \lambda_j^2 \geq 0, && \forall j \in \Pi. \end{aligned} \tag{28}$$

Considering different weights for each of the inputs and outputs, two optimal solutions could be obtained (two created scenarios for DMU_{new1} and DMU_{new2}) for this LP that are reported in Table 2.

Table 2 Inputs and outputs inherited of DMU_7, DMU_8, and DMU_9

Optimal solutions	*First*						*Second*					
	α_j^{1*}	α_j^{2*}	β_{1j}^{1*}	β_{1j}^{2*}	β_{2j}^{1*}	β_{2j}^{2*}	α_j^{1*}	α_j^{2*}	β_{1j}^{1*}	β_{1j}^{2*}	β_{2j}^{1*}	β_{2j}^{2*}
DMU_7	24.00	11.00	10.21	–11.78	–21.00	27.00	35.00	0.00	–5.35	–11.78	11.44	0.00
DMU_8	0.00	40.00	0.00	0.00	22.00	0.00	0.00	40.00	0.00	0.00	14.00	8.00
DMU_9	0.00	3.22	0.00	0.00	19.00	0.00	10.78	14.22	0.00	0.00	0.00	19.00

Therefore, DMU_{new1} and DMU_{new2} should have the input and output levels as shown in Table 3. This leads to predetermined efficiency targets ($\bar{\rho}_{new1}$ = 0.950, $\bar{\rho}_{new2}$ = 0.900).

Table 3 Inputs and outputs of DMU_{new1} and DMU_{new2}

The first scenario create	DMU_{new1}	$\alpha_7^{1*} + \alpha_8^{1*} + \alpha_9^{1*}$ (x)	24.00
new DMUs based on the		$\beta_{17}^{1*} + \beta_{18}^{1*} + \beta_{19}^{1*}$ (y_1)	10.21
first optimal solution		$\beta_{27}^{1*} + \beta_{28}^{1*} + \beta_{29}^{1*}$ (y_2)	20.00
	DMU_{new2}	$\alpha_7^{2*} + \alpha_8^{2*} + \alpha_9^{2*}$ (x)	54.22
		$\beta_{17}^{2*} + \beta_{18}^{2*} + \beta_{19}^{2*}$ (y_1)	–11.78
		$\beta_{17}^{2*} + \beta_{18}^{2*} + \beta_{19}^{2*}$ (y_2)	27.00
		$\alpha_7^{1*} + \alpha_8^{1*} + \alpha_9^{1*}$ (x)	45.78
The second scenario create	DMU_{new1}	$\beta_{17}^{1*} + \beta_{18}^{1*} + \beta_{19}^{1*}$ (y_1)	–5.35
new DMUs based on the		$\beta_{27}^{1*} + \beta_{28}^{1*} + \beta_{29}^{1*}$ (y_2)	25.44
second optimal solution		$\alpha_7^{2*} + \alpha_8^{2*} + \alpha_9^{2*}$ (x)	54.22
	DMU_{new2}	$\beta_{17}^{2*} + \beta_{18}^{2*} + \beta_{19}^{2*}$ (y_1)	–11.78
		$\beta_{17}^{2*} + \beta_{18}^{2*} + \beta_{19}^{2*}$ (y_2)	27.00

Tables 2 and 3 show that if the second scenario is selected to generate DMU_{new1} and DMU_{new2}, then:

1 The contributions of DMU_7, DMU_8, and DMU_9 in the input of DMU_{new1} are approximately 76%, 0%, and 24%, respectively. Moreover, the corresponding contributions of DMU_7, DMU_8, and DMU_9 in the input of DMU_{new2} are approximately 0%, 74%, and 26%, respectively.

2 The first outputs of DMU_{new1} and DMU_{new2} are fully supplied by DMU_7 while DMU_8 and DMU_9 have no contribution.

3 The contributions of DMU_7 and DMU_8 in the second output of DMU_{new1} are approximately 45%, 55%, respectively while DMU_9 has no contribution. Also,

the contributions of DMU_7, DMU_8, and DMU_9 in the input of DMU_{new2} are approximately 0%, 30%, and 70%, respectively.

As the second case, suppose that these three DMUs (DMU_7, DMU_8, and DMU_9) combine their activities by generating two new DMUs (DMU_{new1} and DMU_{new2}), such that DMU_{new1} and DMU_{new2} are fully efficient, $\bar{\rho}_q = 1$ ($\bar{\varphi}_q = 0$), $q \in \Gamma$. To estimate the input and output levels DMU_{new1} and DMU_{new2}, model (28) is employed. As a result, two optimal solutions are obtained as shown in Table 4.

Table 4 Inputs and outputs inherited of DMU_7, DMU_8, and DMU_9

Optimal solutions	*First*						*Second*					
	α_j^{1*}	α_j^{2*}	β_{1j}^{1*}	β_{1j}^{2*}	β_{2j}^{1*}	β_{2j}^{2*}	α_j^{1*}	α_j^{2*}	β_{1j}^{1*}	β_{1j}^{2*}	β_{2j}^{1*}	β_{2j}^{2*}
DMU_7	50.00	–15.00	–8.00	12.00	–14.00	20.00	35.00	0.00	12.00	–8.00	–21.00	27.00
DMU_8	0.00	37.00	0.00	0.00	22.00	0.00	–13.00	50.00	0.00	0.00	22.00	0.00
DMU_9	0.00	0.00	0.00	0.00	19.00	0.00	0.00	0.00	0.00	0.00	19.00	0.00

Therefore, DMU_{new1} and DMU_{new2} should have the input and output levels as shown in Table 5 to achieve fully efficient ($\bar{\rho}_{new1} = 1$ and $\bar{\rho}_{new2} = 1$).

Table 5 Inputs and outputs of DMU_{new1} and DMU_{new2}

The first scenario create new DMUs based on the first optimal solution	DMU_{new1}	$\alpha_7^{1*} + \alpha_8^{1*} + \alpha_9^{1*}$ (x)	50.00
		$\beta_{17}^{1*} + \beta_{18}^{1*} + \beta_{19}^{1*}$ (y_1)	–8.00
		$\beta_{27}^{1*} + \beta_{28}^{1*} + \beta_{29}^{1*}$ (y_2)	27.00
	DMU_{new2}	$\alpha_7^{2*} + \alpha_8^{2*} + \alpha_9^{2*}$ (x)	22.00
		$\beta_{17}^{2*} + \beta_{18}^{2*} + \beta_{19}^{2*}$ (y_1)	12.00
		$\beta_{17}^{2*} + \beta_{18}^{2*} + \beta_{19}^{2*}$ (y_2)	20.00
The second scenario create new DMUs based on the second optimal solution	DMU_{new1}	$\alpha_7^{1*} + \alpha_8^{1*} + \alpha_9^{1*}$ (x)	22.00
		$\beta_{17}^{1*} + \beta_{18}^{1*} + \beta_{19}^{1*}$ (y_1)	12.00
		$\beta_{27}^{1*} + \beta_{28}^{1*} + \beta_{29}^{1*}$ (y_2)	20.00
	DMU_{new2}	$\alpha_7^{2*} + \alpha_8^{2*} + \alpha_9^{2*}$ (x)	50.00
		$\beta_{17}^{2*} + \beta_{18}^{2*} + \beta_{19}^{2*}$ (y_1)	–8.00
		$\beta_{17}^{2*} + \beta_{18}^{2*} + \beta_{19}^{2*}$ (y_2)	27.00

According to Tables 4 and 5, selecting the first scenario for generating DMU_{new1} and DMU_{new2} gives the following results:

1 The input of DMU_{new1} is entirely supplied by DMU_7 while DMU_8 and DMU_9 have no contribution. Also, the contributions of DMU_7, DMU_8, and DMU_9 in the input of DMU_{new1} and DMU_{new2} are approximately –68%, 168%, and 0%, respectively. This means that DMU_7 has a negative effect for generating DMU_{new1} and DMU_{new2}.

2 The first outputs of DMU_{new1} and DMU_{new2} are totally supplied by DMU_7 while DMU_8 and DMU_9 have no role.

3 The contributions of DMU_7, DMU_8, and DMU_9 in second output of DMU_{new1} are approximately –52%, 82%, and 70%, respectively. It is clear that DMU_7 has

a negative effect on generating the new DMU 1. Also, the second output of DMU_{new2} is fully supplied by DMU_7 while DMU_8 and DMU_9 have no quota.

As we know, the problem of target setting for a generated entity from a merger in the presence of negative data has been studied by Amin and Al-Muharrami (2018). The common form of merger happens when at least two DMUs combine their activities to create a superior merged DMU. Thus, there is no available approach to compare our results with that. It is clear that the inverse DEA model proposed in this paper is more general than the corresponding one proposed by Amin and Al-Muharrami (2018). On the other hand, the mathematical model proposed for a merger by them changes the number of the model constraints that leads to increase in computational complexity. However, the mentioned drawbacks could not be observed in the current study.

5 Conclusions

In this paper, the targets setting problem for a set of DMUs generated from a generalised restructuring has been discussed. For this purpose, a novel extended inverse DEA method is proposed to deal with the negative data. Consider that the traditional inverse DEA methods could not be effective in the presence of negative data. This leads to obtaining maximum benefit from restructures among the restructuring DMUs. The obtained results could help managers for choosing the best alternative among potential restructuring DMUs or equivalently selecting the right DMUs to restructure. Accordingly, the maximum benefit could be obtained by minimising inputs and maximising outputs.

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