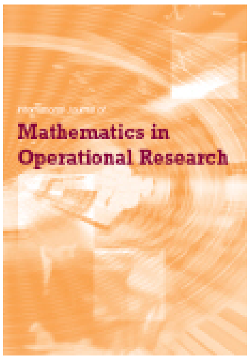

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Reliability and performance analysis of a series-parallel photovoltaic system with human operators using Gumbel-Hougaard family copula

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Abstract: This study offers a complete technique for evaluating the reliability efficiency of medium grid-connected photovoltaic (PV) power systems with two out of three PV panels, one out of two charge controller, two out of two batteries and one out of two inverters. State enumeration is used to examine real-world grid-connected PV systems. A set of reliability indices is defined to assess the dependability performance of PV systems. The system's dependability and availability were analysed and visually shown, as were the sensitivity parameters. The Gumbel-Haugaard family copula approach is used to create and solve Chapmen-Kolmogorov differential equations. There include numerical numbers for availability, reliability, mean time to failure (MTTF), cost analysis, and sensitivity analysis. The implications of failure rates on different solar photovoltaic subsystems were studied. To demonstrate the acquired results and to assess the influence of various system characteristics, numerical examples are provided. The current study may help companies and their repairers overcome some of the problems that repairers of specific manufacturing and industrial systems confront.

Keywords: sensitivity; subsystems, reliability, charge controller, availability.

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1 Introduction

Over the last decade, there has been a significant increase in the use of solar energy for electricity and heat energy sources. Photovoltaic (PV) systems, in particular, have established a vital role in the electricity energy sectors, accounting for more than 10% of current electricity energy supply. The percentage growth in linked PV can be attributed to a variety of factors. Such as low installation costs and quick energy and investment payback, which may include consumer stimulation. Continuous output energy production must be demonstrated in this instance in order to satisfy the cost benefit analysis of PV systems. Because of the rapid expansion of PV system capacity on grid systems across the world, PV system technology is maturing and becoming more competitive in the power market. As a result, PV system engineers will prioritise the optimisation of PV operations in terms of reliability, efficiency, and maintenance, including fault management.

The sun's energy is one of the oldest and cheapest types of primary energy, and it has historically been utilised for preservation and fabric drying. Drying agricultural commodities and it is still employed for this purpose in most impoverished nations today (solar energy as thermal).

System reliability is a measure of how well a system performs in bad situations. According to the specifications, most complex systems are made up of components and subsystems that are linked in series, parallel, standby, or a combination of these. A solar system analysis can help users make timely decisions to guarantee the system's optimal operation. Since antiquity, the terms ‘reliable’ and ‘reliability’ have been used interchangeably. The importance of reliability and performance evaluation in various industrial, manufacturing and production settings have caught the attention of many researchers. To mention few, Singh and Lado (2019) studied the cost assessment of complex repairable system consisting two subsystems in series configuration using Gumbel-Hougaard family copula. Singh et al. (2020b) discusses the performance analysis of a complex repairable system with two subsystems in series configuration with an imperfect switch. Abubakar and Singh (2019) analyse the performance assessment of an industrial system (African Textile Manufactures Ltd.) through copula linguistic approach. Gahlot et al. (2018) dealt with performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics. Singh et al. (2020a) performed the reliability analysis of repairable network system of three computer labs connected with a server under 2-out-of-3: G configuration. Yusuf et al. (2020) analysed the reliability of communication network with redundant relay station under partial and complete failure. Abdilahi et al. (2014) presented feasibility study of renewable energy-based microgrid system in Somaliland’s urban centres. Cristaldi et al. (2015) dealt with Markov analysis reliability model for photovoltaic module encapsulation failures. Ferreira et al. (2016) analysed the reliability of distribution networks with dispersed generation. Patelli and Beer (2017) analysed of complex systems with common cause failures, safety, reliability, risk, resilience and sustainability of structures and infrastructure. Singh and Singh (2015) analysed the performance of three unit redundant system with switch and human failure. Singh et al. (2016) analysed the performance of complex system in series configuration under different failure and repair discipline using copula. Singh et al. (2018) analysed the performance of repairable system in series configuration under different types of failure and repair policies using copula linguistics. Singh et al. (2020a) dealt with reliability analysis of repairable network system of three computer labs connected with a server under 2-out-of-3: G configuration. Singh et al. (2020b) discussed the performance of a complex repairable system with two subsystems in series configuration with imperfect switch. Temraz (2019) studied the availability and reliability of a parallel system under imperfect repair and replacement. Velasco and Guerra (2016) dealt with reliability analysis of distribution systems with photovoltaic generation using a power flow simulator using Monte Carlo approach. Wang et al. (2021) analysed the reliability of a two-dissimilar-unit warm standby repairable system with priority in use. Xu et al. (2016) developed the copula-based slope reliability using the failure domain defined by the *g*-line. Yang and Tsao (2019) discuss the reliability and availability of standby systems with working vacations and retrial of failed components. Zhang et al. (2013) discussed the reliability assessment of photovoltaic power systems. Kumar et al. (2020) analysed the reliability of a redundant system with ‘FCFS’ repair policy subject to weather conditions. Sayed et al. (2019) dealt with study of reliability, availability and maintainability analysis for grid-connected solar photovoltaic systems.

The copula of a multivariate distribution describes not only the correlations of the random variables, but also the dependence structure. It is uncoupled from the marginal distributions which can be modelled as empirical distributions or fitted standard distributions as usual Nelsen (2006).

Many writers, including Nelsen (2006), have researched the notion of copula. Copulas are multivariate distribution functions with uniform margins over the interval [0, 1]. The Gumbel-Hougaard family copula is the bivariate distribution given by:

$$C_\theta\left(u_1, u_2\right) = e^{\left(-\left((-\log u_1)^\theta + (-\log u_2)\theta\right)^{\frac{1}{\theta}}\right)}$$

In this case, resides in the interval and determines the dependence between u_1 and u_2. Gumbel-Hougaard copula is a member of the Archimedean copula family. For $\theta = 1$ and $\theta \to \infty$, respectively, independence copula and comonotonicity copula are special examples of Gumbel-Hougaard family copula. The Gumbel-Hougaard family copula is not symmetric and has greater reliance at the right tails Nelsen (2006).

The copula approach is a technique for calculating joint distributions using marginal distributions in which the variables are non-normal. Copulas can also be used to analyse pairs of random variables in a non-parametric way. Sklar (1973) is the one who first introduced copula. Since then, copula analysis has taken on new dimensions and analyses. Numerous researchers have previously presented copula methods in the field of reliability and performance analysis of systems by examining system performance under various conditions. To name a few, relating a multivariate distribution to a one-dimensional marginal distribution function by employed copula was captured by Nelsen (2006). Abubakar and Singh (2019) analysed the performance of industrial system using copula linguistics. Gulati et al. (2016) focus on performance of complex system in series configuration with different failure and repair. Gahlot et al. (2018) presented performance assessment of system in serial configuration. Tyagi et al. (2021) presented copula analysis of parallel system with fault coverage. Chopra and Ram (2021) presented reliability measures of two dissimilar units in parallel using Gumbel-Hougaard copula. Sha (2021) presented copula reliability analysis for hybrid systems. The conditional copula and its application in time series analysis were introduced by Patton (2009). The application of copula in financial management is the topic of Rodriguez (2007). The application of copula in multivariate distributions was captured by Trivedi and Zimmer (2007). Chopra and Ram (2019) analysed the reliability measures of dissimilar parallel system with two units using Gumbel-Hougaard family copula. Ram and Singh (2008) dealt with availability and cost analysis of complex system configured in parallel subject to two types of failures and preemptive resume repair under Gumbel-Hougaard family copula. Ram and Singh (2010) analysed the MTTF, cost and availability of a system under preemtive repair using Gumbel-Hougaard family copula.

The present paper is organised as follows: Section 2 gives a brief on literature review in the area. Section 3 presents notations, assumptions and description of states of the system. The formulation of the model and model solution are presented in Section 4. Section 5 describes the analysis of the system for particular cases. Section 6 discussed the results of this work. Conclusions of the present paper and the direction of future work are presented at length in Section 7.

2 Literature review

The following material was studied in order for us to have an understanding of the modelling, photovoltaic, and Gumbel-Haugaard family copula. Firstly, Fashina et al. (2018a) study the status quo of rural and renewable energy development in Liberia:

policy and implementation. The study regarding the accurate sizing of residential stand-alone photovoltaic systems considering system reliability was carried out by Quiles et al. (2020). Wang et al. (2017) examined the driving factors of energy related carbon emissions using the extended STIRPAT model. Also, the Motivation for incorporation of microgrid technology in rooftop solar photovoltaic deployment to enhance energy economics was a research conducted by Rengasamy et al. (2020). Salah and Fashina (2019) designed a hybrid solar photovoltaic system for Gollis University's administrative block, Somaliland. Uswarman and Rushdi (2021) discussed the reliability evaluation of rooftop solar photovoltaic using coherent threshold systems. Fashina et al. (2018b) studied the drivers and barriers of renewable energy applications and development in Uganda. Sayed et al. (2019) studied the reliability, availability and maintainability analysis for grid-connected solar photovoltaic systems. Baschel et al. (2018) discuss the impact of component reliability on large-scale photovoltaic systems performance. Dynamic performance evaluation of photovoltaic power plant by stochastic hybrid fault tree automaton model was studied by Chiacchio et al. (2018). Reliability, maintainability and sensitivity analysis of physical processing unit of sewage treatment plant is research conducted by Goyal et al. (2019). Gahlot et al. (2018) discussed the performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics. Lado et al. (2018) studied the performance and cost assessment of repairable complex system with two subsystems connected in series. Yusuf and Musa (2021) analysed the availability study of a complicated system composed of two subsystems operating in parallel, with replacement in the event of a breakdown. Manocha et al. (2019) presented modelling and analysis of hot standby two-unit database system with random inspection of standby unit. Panwar and Kumar (2021) dealt with performance analysis and modelling of feeding unit in paper plane. Gupta and Agarwal (2021) focus on cost analysis of system with N-policy vacation time. Singh and Ayagi (2018) dealt with stochastic analysis of a system under preemptive resume repair.

Researchers mentioned above have made significant contributions to improving the efficiency and performance of various systems, as well as investigating the variables that impede photovoltaic system performance. Little is known about the dependability metrics used to assess the strength, efficacy, and performance of solar systems. More research is needed on the dependability measure of measuring the strength, effectiveness, and performance enhancement of solar systems. Due to the lack of PV system data, the present work developed a reliability modelling technique to investigate the overall performance of the PV system. In this study, a novel solar system model comprised of four subsystems: panel, inverter, battery bank, and control charger is considered in which the system of partial differential equations is constructed and solved using the transition diagram to yield system strong reliability characteristics such as reliability, availability, mean time to failure (MTTF), sensitivity analysis, and profit function. The objective of this work is to obtain reliability metrics such as availability, reliability, mean time to failure (MTTF) and cost function in order to assess the strength of the PV system. The findings of this study will be useful to home, commercial, and industrial plant managers, as well as industries and manufacturing systems that propose to employ photovoltaic as energy and power sources.

3 Notations, assumptions and state of the system

3.1 Notations

t	time variable on a time scale
s	Laplace transform variable for all expressions
π_1	failure rate of units in the subsystem 1
π_2	failure rate of units in the subsystem 2
π_3	failure rate of units in the subsystem 3
π_4	failure rate of units in the subsystem 4
π_{H1}	failure rate of subsystem 2 due to human mistake 1
π_{H1}	failure rate of subsystem 2 due to human mistake 2
$\beta(x)$	repair rate of units in subsystem 1
$\beta(y)$	repair rate of units in subsystem 2
$\beta(z)$	repair rate of units in subsystem 3
$\beta(m)$	repair rate of units in subsystem 4
$\beta(v)$	repair rate for complete failed state of subsystem 2 due to human mistake 1
$\beta(w)$	repair rate for complete failed state of subsystem 4 due to human mistake 2
$p_i(t)$	the probability that the system is in S_i state at instants for $i = 0$ to 10
$\overline{P}(s)$	Laplace transformation of state transition probability $p(t)$
$P_i(x, t)$	the probability that a system is in state S_i for $i = 1, \ldots, 8$, the system under repair and elapse repair time is (x, t) with repair variable x and time variable t
$P_i(y, t)$	the probability that a system is in state S_i for $i = 1, \ldots, 8$, the system under repair and elapse repair time is (y, t) with repair variable x and time variable t
$E_p(t)$	expected profit during the time interval $[0, t)$
K_1, K_2	revenue and service cost per unit time, respectively.

Gumbel-Hougaard copula is defined as

$$C_\delta(\eta_1, \eta_2) = Exp\left[-\left((-\log\eta_1)^\delta + (-\log\eta_2)\delta\right)^{\frac{1}{\delta}}\right], \qquad 1 \le \delta \le \infty$$

The value of $\delta = 1$ corresponds to independence copula and as $\delta \to \infty$, its correspond to the comonotonicity copula.

3.2 Assumptions

Throughout the model's explanation, the following assumptions are made:

1 At first, all subsystems are in good functioning order.

2 For operational mode, two units from subsystem 1 and two units from subsystem 3 must be used consecutively.

3 For operational mode, just one unit in subsystem 2 is required. In addition, one unit out of one in subsystem 4 is required for operating mode.

4 If one of the units in subsystem 1 fails, the system will be rendered inoperable. Also, if one of the units in subsystem 3 fails.

5 The system will also be rendered inoperable if all two components from subsystems 2 and 4 fail.

6 A system's failing unit can be fixed when it is in an inoperable or failed state. Copula repair follows a total failure of a unit in subsystem.

7 It is believed that a copula-repaired system acts like a new system and that no damage occurs during repair. As soon as the failed the failed unit gets repaired, it is ready to perform the task.

3.3 Description of the model

The model depicted in Figure 1 consists of four subsystems A, B, C and D configured as series-parallel. Subsystem A (the solar panel) has three identical units working as 2-out-of-3, subsystem B (the charge controller) has two units working as 1-out-of-2, subsystem C (batteries) has two parallel units and subsystem D (the inverter) working as 1-out-of-2. The system has two types of failures which are partial and complete failure. Partial failure is a failure of a unit in a subsystem which allow the system to continue working, while complete failure occurs at the failure of any of the subsystem. When complete failure occurs, the system is repaired using copula. The system has eleven states in which four states are operational and seven states are complete failure states (see Figure 2). Brief description of the states is given in Table 1.

Figure 1 Reliability block diagram of the system (see online version for colours)

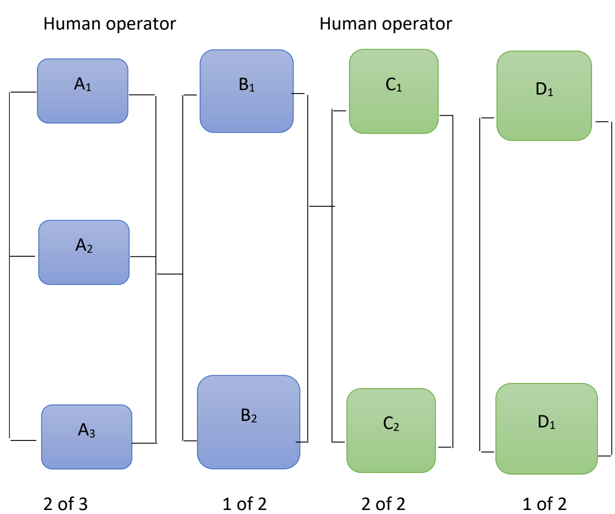
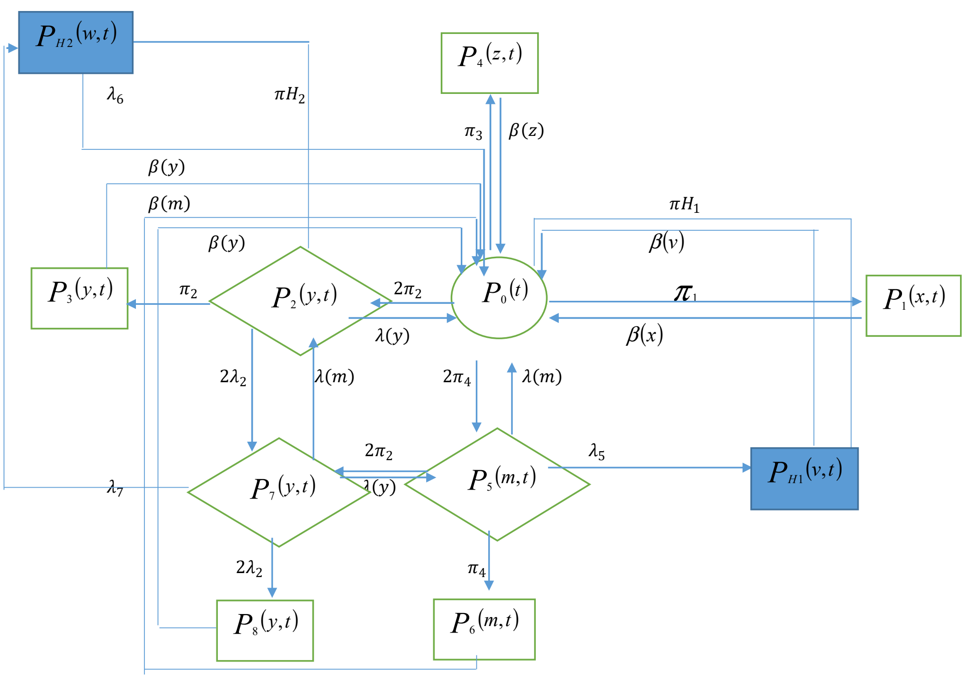


Table 1 A description of the system's states

State	*Description*
S_0	Initial state, units A_1, A_2 are working. And the system is in operational condition. Unit B_1 in the subsystem 2 is in working state. In subsystem 3, the units C_1 and C_2 are in operation, while A_3 and B_2 in hot standby in subsystems 1 and 2, respectively.
S_1	In this state, the units A_1 and B_1 failed and under repair. And the elapsed repair time is (x, t). While the units B_2, A_2, A_3, C_1, C_2, and D_1, are on operation and D_2 is on standby.
S_2	The units D_1 has failed. While the units A_2, A_3, B_2, C_1, C_2, and D_2, and on operation.
S_3	The state S_4 is complete failed state due to the failure of subsystem 1.
S_4	The state S_4 is complete failed state due to the failure of two units in subsystem 2.
S_5	The state S_5 is complete failed state due to the failure of unit in subsystem 3.
S_6	The state S_6 is complete failed state due to the failure of two units in subsystem 4.
S_7	The partial failure state due to failure of unit in subsystem B
S_8	Complete failure state due to failure of subsystem B
S_{H1}	Complete failure state due to first human operator
S_{H2}	Complete failure state due to second human operator

Figure 2 State transition diagram of model (see online version for colours)



Note: – complete failure, – partial failure state, – state of perfection,
– complete failure due to human operator.

4 Formulation of the model and solution

4.1 Model formulation

By the probability of considerations and continuity of arguments as in Nelsen (2006), Gulati et al. (2016), Singh and Ayagi (2017), Gahlot et al. (2018), Lado et al. (2018), Lado and Singh (2019), and Singh and Poonia (2019), the system of partial differential difference equations generated from Figure 2 is shown below:

$$\left[\frac{\partial}{\partial t}+\pi_1+2\pi_1+\pi_3+2\pi_4+\pi_{H1}+\pi_{H1}\right]P_0(t)$$
$$=\int_0^\infty \beta(x)P_1(x,t)dx+\int_0^\infty \lambda(y)P_2(y,t)dy+\int_0^\infty \beta(z)P_4(z,t)dz+\int_0^\infty \lambda \tag{1}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial t}{\partial x}+\beta(x)\right]P_1(x,t)=0 \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\pi_2+2\pi_4+\lambda_6+\lambda(y)\right]P_2(y,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\beta(y)\right]P_3(y,t)=0 \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\beta(z)\right]P_4(z,t)=0 \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial m}+\pi_4+2\pi_2+\lambda_5+\lambda(m)\right]P_5(m,t)=0 \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial m}+\beta(m)\right]P_6(m,t)=0 \tag{7}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\pi_4+\lambda_7+\lambda(m)+\lambda(y)\right]P_7(y,t)=0 \tag{8}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\beta(y)\right]P_8(y,t)=0 \tag{9}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial v}+\beta(v)\right]P_{H1}(v,t)=0 \tag{10}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial w}+\beta(w)\right]P_{H2}(w,t)=0 \tag{11}$$

Boundary conditions:

$$P_1(0,t)=\pi_1 P_0(t) \tag{12}$$

$$P_2(0,t)=2\pi_2 P_0(t) \tag{13}$$

$$P_3(0,t) = 2\pi_2^2 P_0(t) \tag{14}$$

$$P_4(0,t) = \pi_3 P_0(t) \tag{15}$$

$$P_5(0,t) = 2\pi_4 P_0(t) \tag{16}$$

$$P_6(0,t) = 2\pi^4 P_0(t) \tag{17}$$

$$P_7(0,t) = 8\pi_2\pi_4 P_0(t) \tag{18}$$

$$P_8(0,t) = 8\pi_2\pi_4^2 P_0(t) \tag{19}$$

$$P_{H1}(0,t) = \pi_{H1}\left(P_0(t) + P_2(0,t) + P_5(0,t) + P_7(0,t)\right) \tag{20}$$

$$P_{H2} = (0,t) = \pi_{H2}\left(P_0(t) + P_2(0,t) + P_5(0,t) + P_7(0,t)\right) \tag{21}$$

4.2 Solution of the model

And with the exception of the starting condition $P_0(0) = 1$, all other transition probabilities are zero at t = 0. We derived the following results by applying Laplace transforms on equations (1) to (21):

$$\begin{aligned}&[s + \pi_1 + 2\pi_1 + \pi_3 + 2\pi_4 + \pi_{H1} + \pi_{H1}]\overline{P}_0(s) \\ &= \int_0^\infty \beta(x)\overline{P}_1(x,s)dx + \int_0^\infty \lambda(y)\overline{P}_2(y,s)dy + \int_0^\infty \beta(z)\overline{P}_4(z,s)dz \\ &+ \int_0^\infty \lambda(m)\overline{P}_5(m,s)dm + \int_0^\infty \beta(v)\overline{P}_{H1}(v,s)dv + \int_0^\infty \beta(w)\overline{P}_{H2}(w,s)dw\end{aligned} \tag{22}$$

$$\left[s + \frac{\partial}{\partial x} + \beta(x)\right]\overline{P}_1(x,s) = 0 \tag{23}$$

$$\left[s + \frac{\partial}{\partial y} + \pi_2 + 2\pi_4 + \lambda_6 + \lambda(y)\right]\overline{P}_2(y,s) = 0 \tag{24}$$

$$\left[s + \frac{\partial}{\partial y} + \beta(y)\right]\overline{P}_3(y,s) = 0 \tag{25}$$

$$\left[s + \frac{\partial}{\partial z} + \beta(z)\right]\overline{P}_4(z,s) = 0 \tag{26}$$

$$\left[s + \frac{\partial}{\partial m} + \pi_4 + 2\pi_2 + \lambda_5 + \lambda(m)\right]\overline{P}_5(m,s) = 0 \tag{27}$$

$$\left[s + \frac{\partial}{\partial m} + \beta(m)\right]\overline{P}_6(m,s) = 0 \tag{28}$$

$$\left[s + \frac{\partial}{\partial y} + \pi_4 + \lambda_7 + \lambda(m) + \lambda(y)\right]\overline{P}_7(y,s) = 0 \tag{29}$$

$$\left[s+\frac{\partial}{\partial y}+\beta(y)\right]\overline{P}_9(y,s)=0 \tag{30}$$

$$\left[s+\frac{\partial}{\partial v}+\beta(v)\right]\overline{P}_{H1}(v,s)=0 \tag{31}$$

$$\left[s+\frac{\partial}{\partial w}+\beta(w)\right]P_{H2}(w,s)=0 \tag{32}$$

Laplace boundary conditions

$$\overline{P}_1(0,s)=\pi_1\overline{P}_0(s) \tag{33}$$

$$\overline{P}_2(0,s)=2\pi_2\overline{P}_0(s) \tag{34}$$

$$\overline{P}_3(0,s)=2\pi_2^2\overline{P}_0(s) \tag{35}$$

$$\overline{P}_4(0,s)=\pi_3\overline{P}_0(s) \tag{36}$$

$$\overline{P}_5(0,s)=2\pi_4\overline{P}_0(s) \tag{37}$$

$$\overline{P}_6(0,s)=2\pi_4^2\overline{P}_0(s) \tag{38}$$

$$\overline{P}_7(0,s)=8\pi_2\pi_4\overline{P}_0(s) \tag{39}$$

$$\overline{P}_8(0,s)=8\pi_2\pi_4^2\overline{P}_0(s) \tag{40}$$

$$\overline{P}_{H1}(0,s)=\pi_{H1}\left(\overline{P}_0(s)+\overline{P}_2(0,s)+\overline{P}_5(0,s)+\overline{P}_7(0,s)\right) \tag{41}$$

$$\overline{P}_{H2}(0,s)=\pi_{H2}\left(\overline{P}_0(s)+\overline{P}_2(0,s)+\overline{P}_5(0,s)+\overline{P}_7(0,s)\right) \tag{42}$$

Equations (12) to (21) are used to obtained the boundary Laplace conditions in equations (33) to (42) which are substituted in equations (22) to (32) and with the help of initial condition and shifting properties in equations (43) and (44) to have equations (55) to (64).

$$\int_0^\infty\left[e^{-sx}\cdot e^{\int_0^x f(x)dx}\right]dx=L\left\{\frac{1-\overline{S}_f(x)}{S}\right\}=\frac{1-\overline{S}_f(x)}{S} \tag{43}$$

$$\int_0^\infty\left[e^{-sx}\cdot f(x)e^{-\int_0^x f(x)dx}\right]dx=L\left\{\overline{S}_f(x)\right\}=\overline{S}_f(x) \tag{44}$$

We have

$$\overline{P}_1(s)=\overline{P}_1(0,s)=\left\{\frac{1-\overline{S}_\beta(s)}{S}\right\} \tag{45}$$

$$\overline{P}_2(s)=\overline{P}_2(0,s)\left\{\frac{1-\overline{S}_\lambda\left(s+\pi_2+2\pi_4+\lambda_6\right)}{S+\pi_2+2\pi_4+\lambda_6}\right\} \tag{46}$$

$$\overline{P}_3(s)=\overline{P}_3(0,s)=\left\{\frac{1-\overline{S}_\beta(x)}{S}\right\} \tag{47}$$

$$\overline{P}_4(s)=\overline{P}_4(0,s)=\left\{\frac{1-\overline{S}_\beta(s)}{S}\right\} \tag{48}$$

$$\overline{P}_5(s)=\overline{P}(0,s)\left\{\frac{1-\overline{S}_\lambda\left(s+\pi_4+2\pi_2+\lambda_5\right)}{S+\pi_4+2\pi_2+\lambda_5}\right\} \tag{49}$$

$$\overline{P}_6(s)=\overline{P}_6(0,s)\left\{\frac{1-\overline{S}_\beta(s)}{S}\right\} \tag{50}$$

$$\overline{P}_7(s)=\overline{P}_7(0,s)\left\{\frac{1-\overline{S}_{2\lambda}\left(s+\pi_4+\lambda_7\right)}{s+\pi_4+\lambda_7}\right\} \tag{51}$$

$$\overline{P}_8(s)=\overline{P}_8(0,s)\left\{\frac{1-\overline{S}_\beta(s)}{S}\right\} \tag{52}$$

$$\overline{P}_{H1}(s)=\overline{P}_{H1}(0,s)\left\{\frac{1-\overline{S}_\beta(s)}{S}\right\} \tag{53}$$

$$\overline{P}_{H2}(s)=\overline{P}_{H2}(0,s)\left\{\frac{1-\overline{S}_\beta(s)}{S}\right\} \tag{54}$$

Substituting the Laplace's boundary conditions, i.e., equations (33) to (42) into equations (45) to (54), we have:

$$\overline{P}_1(s)=\pi_1\left\{\frac{1-\overline{S}_\beta(s)}{S}\right\}\overline{P}_0(s) \tag{55}$$

$$\overline{P}_2(s)=2\pi_2\left\{\frac{1-\overline{S}_f\left(s+\pi_2+2\pi_4+\lambda_6\right)}{S+\pi_2+2\pi_4+\lambda_6}\right\}\overline{P}_0(s) \tag{56}$$

$$\overline{P}_3(s)=2\pi_2^2\left\{\frac{1-\overline{S}_\beta(s)}{S}\right\} \tag{57}$$

$$\overline{P}_4(s)=\pi_3\left\{\frac{1-\overline{S}_\beta(s)}{S}\right\}\overline{P}_0(s) \tag{58}$$

$$\overline{P}_5(s)=2\pi_4\left\{\frac{1-\overline{S}_\lambda\left(s+\pi_4+2\pi_2+\lambda_5\right)}{S+\pi_4+2\pi_2+\lambda_5}\right\}\overline{P}_0(s) \tag{59}$$

$$\overline{P}_6(s)=2\pi_4^2\left\{\frac{1-\overline{S}_\beta(s)}{S}\right\}\overline{P}_0(s) \tag{60}$$

$$\overline{P}_7(s)=8\pi_2\pi_4\left\{\frac{1-\overline{S}_{2\lambda}\left(s+\pi_4+\lambda_7\right)}{s+\pi_4+\lambda_7}\right\}\overline{P}_0(s) \tag{61}$$

$$\overline{P}_8(s) = 8\pi_2\pi_4^2 \left\{ \frac{1-\overline{S}_\beta(s)}{S} \right\} \overline{P}_0(s) \tag{62}$$

$$\overline{P}_{H1}(s) = \pi_{H1}\left(1+2\pi_2+2\pi_4+8\pi_2\pi_4\right)\left\{ \frac{1-\overline{S}_\beta(s)}{S} \right\} \overline{P}_0(s) \tag{63}$$

$$\overline{P}_{H2}(s) = \pi_{H2}\left(1+2\pi_2+2\pi_4+8\pi_2\pi_4\right)\left\{ \frac{1-\overline{S}_\beta(s)}{S} \right\} \overline{P}_0(s) \tag{64}$$

For $\overline{P}_{(0)}(s) = \dfrac{1}{D(s)}$ (65)

And
$$\begin{aligned} D(s) = &\left(s+\pi_1+2\pi_1+\pi_3+2\pi_4+\pi_{H1}+\pi_{H1}\right. \\ &-\left[\pi_1\overline{S}_\beta(s)+2\pi_2\overline{S}_\lambda\left(s+\pi_2+2\pi_4+\lambda_6\right)+\pi_3\overline{S}_\beta(s)\right. \\ &+2\pi_4\overline{S}_\lambda\left(s+\pi_4+2\pi_2+\lambda_5\right) \\ &+\pi_{H1}\left(1+2\pi_2+2\pi_4+8\pi_2\pi_4\right)\overline{S}_\beta(s) \\ &+\pi_{H2}\left(1+2\pi_2+2\pi_4+8\pi_2\pi_4\right)\overline{S}_\beta(s) \end{aligned} \tag{66}$$

The probabilities that the system is working (sum of all operational states for the system) is obtained as:

$$\overline{P}_{up}(s) = \overline{P}_0(s)+\overline{P}_2(s)+\overline{P}_5(s)+\overline{P}_7(s) \tag{67}$$

By substituting equations (56), (59) and (61) into equation (67), we have the $\overline{P}_{up}(s)$ which is the probability that the system is in operational condition.

$$\begin{aligned} \overline{P}_{up}(s) = &\left[1+2\pi_2\left\{\frac{1-\overline{S}_\lambda\left(s+\pi_2+2\pi_4+\lambda_6\right)}{S+\pi_2+2\pi_4+\lambda_6}\right\}+2\pi_4\left\{\frac{1-\overline{S}_\lambda\left(s+\pi_4+2\pi_2+\lambda_5\right)}{S+\pi_4+2\pi_2+\lambda_5}\right\}\right. \\ &\left.+8\pi_2\pi_4\left\{\frac{1-\overline{S}_{2\lambda}\left(s+\pi_4+\lambda_7\right)}{s+\pi_4+\lambda_7}\right\}\right]\overline{P}_0(s) \end{aligned} \tag{68}$$

Equation (68) can also be interpreted by applying equation (65) as equation (69) below:

$$\begin{aligned} \overline{P}_{up}(s) = \frac{1}{D(s)}&\left[1+2\pi_2\left\{\frac{1-\overline{S}_\lambda\left(s+\pi_2+2\pi_4+\lambda_6\right)}{S+\pi_2+2\pi_4+\lambda_6}\right\}\right. \\ &+2\pi_4\left\{\frac{1-\overline{S}_\lambda\left(s+\pi_4+2\pi_2+\lambda_5\right)}{S+\pi_4+2\pi_2+\lambda_5}\right\} \\ &\left.+8\pi_2\pi_4\left\{\frac{1-\overline{S}_{2\lambda}\left(s+\pi_4+\lambda_7\right)}{s+\pi_4+\lambda_7}\right\}\right] \end{aligned} \tag{69}$$

$$\overline{P}_{up}(s) = 1-\overline{P}_{down}(s) \tag{70}$$

5 Analysis of the system for particular cases

5.1 Availability analysis

Taking $S_{\alpha_0}(s) = \bar{S}_{\exp[x^\theta + \{\log\varphi(x)\}^\theta]^{1/\theta}}(s) = \dfrac{\exp[x^\theta + \{\log\varphi(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{\log\varphi(x)\}^\theta]^{1/\theta}}$, $\bar{P}_\phi(s) = \dfrac{\phi}{s+\phi}$ but $\phi = 1$ and $\pi_1 = 0.001$, $\pi_2 = 0.002$, $\pi_3 = 0.003$, $\pi_4 = 0.004$, $\lambda_5 = 0.005$, $\lambda_6 = 0.006$, $\lambda_7 = 0.007$, $\pi_{H1} = 0.008$, $\pi_{H2} = 0.009$.

Table 2 Availability variance with respect to time

Time (in days)	$\bar{P}_{up}(t)$
0	1.0000000
10	0.7588687
20	0.5698407
30	0.4278978
40	0.3213117
50	0.2412755
60	0.1811756
70	0.1360461
80	0.1021581
90	0.0767113
100	0.0576031

Table 3 Variation in reliability as a function of time

Time (in days)	*Reliability*
0	0.9999999
10	0.8144529
20	0.6686089
30	0.5529455
40	0.4603992
50	0.3857042
60	0.3249114
70	0.2750407
80	0.2338273
90	0.1995368
100	0.1708303

Table 4 Variation of MTTF with failure rates π_k

Failure rate	*MTTF*								
	(a)	*(b)*	*(c)*	*(d)*	*(e)*	*(f)*	*(g)*	*(h)*	*(i)*
0.001	67.9152	82.6736	63.3291	68.9259	72.2032	67.6369	64.1268	72.1616	78.8481
0.002	66.0353	72.3643	61.5688	64.8916	67.5096	67.1778	69.0621	73.6741	74.1616
0.003	56.0343	64.3235	63.9152	69.1502	69.3057	67.7893	72.0159	78.3643	78.6741
0.004	64.9222	60.7917	61.3589	65.9152	68.4692	67.4563	65.9813	74.2138	79.3643
0.005	63.7081	52.3194	57.8915	67.3446	69.9152	76.1677	67.9544	69.2067	78.2138
0.006	71.4000	52.6224	40.5056	69.5509	66.5832	68.9152	69.9328	67.3291	67.2067
0.007	58.0051	47.5112	56.1946	68.6130	61.4288	65.6924	70.9152	71.5688	69.3291
0.008	52.5300	46.8545	57.9527	53.5864	71.4187	78.4943	73.9005	64.9152	61.5688
0.009	53.98055	42.55796	54.77443	58.50983	61.52746	66.31715	64.88811	69.35890	51.91523

Table 5 Expected profit as a function of time

Failure rate	$E_p(t)$								
	(a)	*(b)*	*(c)*	*(d)*	*(e)*	*(f)*	*(g)*	*(h)*	*(i)*
0.001	3,284.387	−2,182.18	−2,917.064	2,503.639	−4,092.81	−610.8763	−88.578	−3,683.8	−3,893.99
0.002	3,157.784	−9,110.955	−2,805.273	681.795	−3,524.28	−530.8762	−64.87203	−3,495.5	−3,683.17
0.003	3,041.876	−7,292.321	−2,703.492	−29.98786	−3,103.52	−468.6151	−50.57941	−3,327.296	−3,495.32
0.004	2,935.492	−6,044.073	−2,610.551	−534.870	−2,783.51	−449.2147	−41.30302	−3,176.343	−3,327.26
0.005	2,837.617	−5,146.804	−2,525.479	−807.724	−2,534.49	−379.3602	−34.94311	−3,040.222	−3,176.36
0.006	2,747.367	−4,478.243	−2,447.377	−986.730	−2,336.85	−346.7429	−30.39391	−2,917.062	−3,040.28
0.007	2,663.960	−3,965.638	−2,375.535	−1,019.912	−2,277.49	−359.7093	−27.02807	−2,805.26	−2,917.03
0.008	2,586.749	−3,563.305	−2,309.284	−1,216.968	1,056.964	−297.0558	−34.46803	−2,703.2	−2,805.21
0.009	2,515.109	−3,241.312	−2,248.067	1,201.719	−948.839	−277.8832	−22.47578	−2,610.9	−2,703.45

And repair rates $\lambda(y) = \lambda(m) = \beta(x) = \beta(y) = \beta(z) = \beta(v) = \beta(w) = \beta(m) = 1$ in equation (69), and applying the inverse Laplace transform to equation (69), the expression for system availability is

$$\begin{aligned}\overline{P}_{up}(t) = \{&-0.00004904770570\mathrm{e}^{-1.011000000t} + 0.091603099809\mathrm{e}^{-2.722649460it} \\ &+(-0.006078301855 - 0.001417810992\mathrm{I})\,\mathrm{e}^{(-1.014501644-0.00099381113161)t} \\ &+(-0.006078301855 + 0.001417810992\mathrm{I})\mathrm{e}^{(-1.014501644-0.00099381113161)t} \\ &+1.010602552\mathrm{e}^{-0.02864725083t}\end{aligned} \tag{71}$$

Taking $t = 0, 10, \ldots, 100$, availability of the system is obtained and presented in Table 2.

5.2 Reliability analysis

Letting all repair rates, $\lambda(y) = \lambda(m) = \beta(x) = \beta(y) = \beta(z) = \beta(v) = \beta(w) = \beta(m) = 0$ in equation (69), and taking the values of failure rates and employing inverse Laplace transformation, the expression is reliability relation.

$$\begin{aligned}R(t) = \{&0.002909090909e^{-0.01100000000t} + 0.2352941176e^{-0.01600000000t} \\ &+0.3617967914e^{-0.03300000000t} + 0.4000000000e^{-0.01300000000t}\}\end{aligned} \tag{72}$$

Using $t = 0, 10, \ldots, 100$ as time units in equation (72), reliability is determined and shown in Table 3.

5.3 Mean time to failure analysis

Setting repairs to zero in equation (69), the expression for MTTF is defined as follows: fixing $\pi_1 = 0.001$, $\pi_2 = 0.002$, $\pi_3 = 0.003$, $\pi_4 = 0.004$, $\lambda_5 = 0.005$, $\lambda_6 = 0.006$, $\lambda_7 = 0.007$, $\pi_{H1} = 0.008$, $\pi_{H2} = 0.009$, varying, π_k in equation (60), The MTTF is calculated in relation to the failure rate, as shown in Table 4.

5.4 Cost benefit analysis

The phrase indicating the anticipated profit in $[0, t)$

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \tag{73}$$

Taking fixed values of parameters of equation (69), the subsequent equation (74) follows:

$$\begin{aligned}E_p(t) = \{&-0.0000259e^{-1.002t} + 0.00251703e^{-3.728442t} + 0.00543761e^{-1.035024t} \\ &-0.0004791e^{-2.0148015t} + 0.76636171e^{-0.000031151} - 0.00214955e^{-2.0060301t}\} \\ &-K_2 t\end{aligned} \tag{74}$$

With $K_1 = 1$ and $K_2 = 0.1, 0.2, \ldots, 0.5$, respectively, and $t = 0, 10, 20, \ldots, 100$, units of time, the predicted profit is calculated in Table 5.

6 Results discussion

From Table 2 and the graph presented in Figure 3, it is evident that availability of the system decreases drastically as the time passes and after a long time that the availability of the system decreases as an increment in the time variable. Where the system availability decreases as time passes, the quality of the product, the production output as well as revenue mobilisation will tend to decreases for the fact that maintenance action to be impose to restore the system to its position prior to failure may tend to be costly which will be detrimental to plant management. The plant management may as well device a mean to control such system failure through different adequate preventive and corrective maintenance strategies in order to maintain the system effectiveness at highest level. Preventive and corrective maintenance measures such as adding fault tolerant units, invoking perfect repair in the event of an incomplete failure, replacing the affected subsystem with a new one in the event of a complete failure, regular inspection, etc. can assist in maintaining the highest level of system effectiveness.

Figure 3 Availability as a time function (see online version for colours)

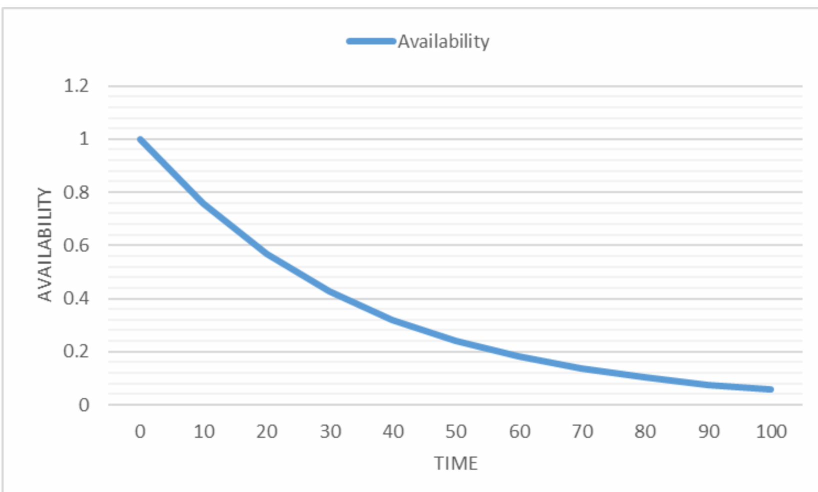


Table 3 and the simulation depicted in Figure 4 presented the system's reliability with respect to time *t*. Table 3 and Figure 4 demonstrated that the reliability values fluctuate between 0 and 1 over time. Table 3 and Figure 4 show that reliability decreases dramatically with the passage of time. To improve system reliability, techniques responsible for reducing failure and improving repair can be used to extend the system's reliability and lifespan.

The MTTF of the system is depicted in Table 4 and the chart in Figure 5 with respect to various failure rates. According to Table 4 and Figure 5, the MTTF decreases as the failure rate increases. The MTTF decreases with each increase in failure rate. Where the MTTF is decreasing, so is the system lifespan.

The graph in Figure 6 and Table 5 shows that the system's expected profit/cost decreases over time. This can be attributed to a decrease in system availability over time. To maximise profit, plant management may devise a method to control such system failure via various preventive and corrective maintenance strategies. Results presented in

Table 5 and Figure 6 show profit against time for different values of K_2. From Table 5 and Figure 6, the predicted profit decreases with increase in time for any value of K_2. However, the predicted profit increases as the value of K_2 decreases. The expected profit can be increased by implementing the above-mentioned replacement and redundancy suggestions.

Figure 4 Reliability as a function of time (see online version for colours)

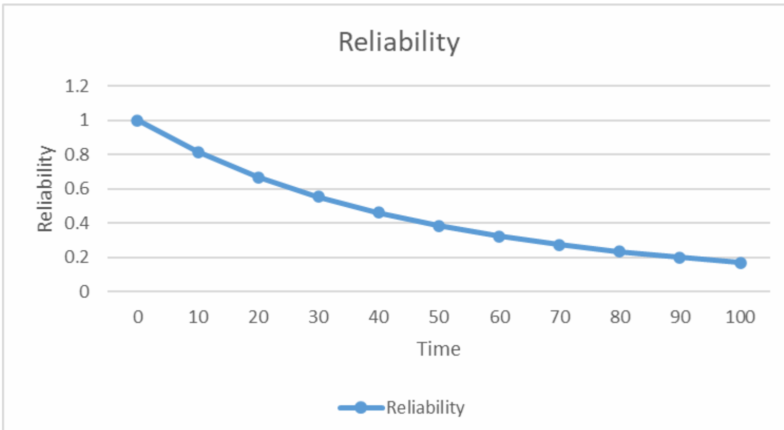
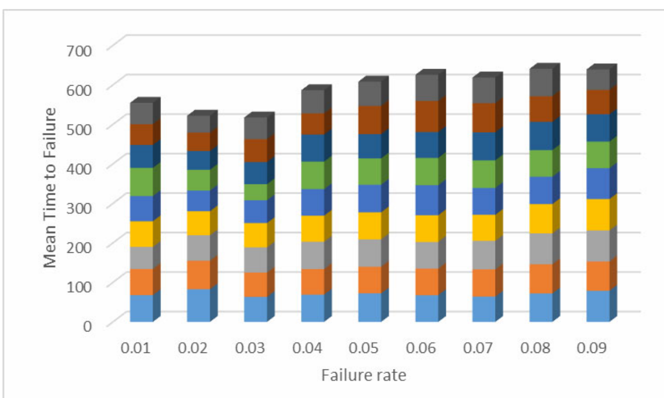
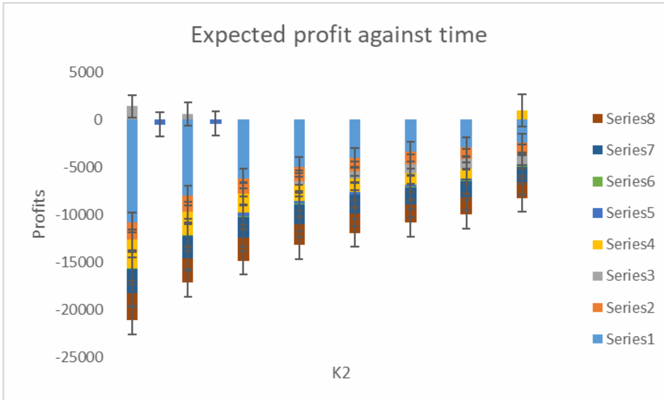


Figure 5 Variation of MTTF with failure rates (see online version for colours)



From the analysis above, it is evident that the performance of the system can be improved by incorporating more units on standby, invoking perfect repair in the event of an incomplete failure, replacing the affected subsystem with a new one in the event of a complete failure, regular inspection and preventive maintenance, employing more repair machines, and so on. To improve the system performance, it is worthwhile to utilise fault tolerant components to boost the system's reliability, availability, MTTF, profit and longevity.

Figure 6 Expected profit against time (see online version for colours)



7 Conclusions

The performance of a photovoltaic (PV) system comprised of four subsystems A, B, C, and D arranged in series-parallel configuration was investigated in this paper. Subsystem A (the solar panel) consists of three identical units that operate as 2-out-of-3, subsystem B (the charge controller) consists of two units that operate as 1-out-of-2, subsystem C (batteries) consists of two parallel units, and subsystem D (the inverter) operates as 1-out-of-2. The system has two types of failures: partial failure and complete failure. Partial failure occurs when a unit in a subsystem fails, allowing the system to continue operating, whereas complete failure occurs when any of the subsystems fails. When the system fails completely, copula is used to repair it. The Markovian process, Laplace transformation, and supplementary variable techniques are used to calculate the system's transient probabilities and reliability measures of system performance and strength. According to the findings of the study, the reliability measures discussed are time and failure sensitive.

By performing numerical experiments, expressions of reliability metrics for testing the strength and performance of the system, such as availability, reliability, mean time to failure, and cost function, are derived and validated. MATLAB was used to simulate the effect of time and various system parameters on reliability metrics. Where the system's reliability strength is strong, it may help the system to withstand some of the obstacles such as bird dropping deposition, wind speed, and dust, thereby hindering the system's performance and increasing the system's life span. These are the paper's main contributions.

The findings of the paper suggest that reliability modelling can be used to evaluate the strength, efficiency, and performance of a PV system. When the PV system's strength, efficiency, and performance are determined, users will be able to serve the cost of kerosene, gasoline, diesel, and other fuels that expose human hearts to air and land pollution for their household and commercial uses. As a result, the model's graphical representation demonstrates that one can confidently predict the future behaviour of a complex system at any point in time for any given set of parametric parameters. After the failure of the second unit in each subsystem, this research will include both online and offline preventive maintenance. This will be investigated further in our future research.

References

Abdilahi, A.M., Abdul, H.M.Y., Mohd, W.M., Omar, T.K., Alshammari, F.S and Faizah, M.N. (2014) 'Feasibility study of renewable energy-based microgrid system in Somaliland's urban centers', *Renewable and Sustainable Energy Reviews*, Vol. 40, pp.1048–1059 [online] https://doi.org/10.1016/j.rser.2014.07.150.

Abubakar, M.I. and Singh, V.V. (2019) 'Performance assessment of an industrial system (African Textile Manufactures Ltd.) through copula linguistic approach', *Operation Research and Decisions*, Vol. 29, No. 4, pp.1–18.

Baschel, S., Koubli, E., Roy, J. and Gottschalg, R. (2018) 'Impact of component reliability on large scale photovoltaic systems performance', *Energies*, Vol. 11, p.1579.

Chiacchio, F., Famoso, F., D'Urso, D., Brusca, S., Aizpurua, J.I. and Cedola, L. (2018) 'Dynamic performance evaluation of photovoltaic power plant by stochastic hybrid fault tree automaton model', *Energies*, Vol. 11, p.306.

Chopra, G. and Ram, M. (2019) 'Reliability measures of two dissimilar units parallel system using Gumbel-Hougaard family copula', *International Journal of Mathematical, Engineering and Management Sciences*, Vol. 4, No. 1, pp.116–130, DOI: 10.33889/IJMEMS.2019.4.1-011.

Cristaldi, L., Khalil, M., Faifer, M. and Soulatiantork, P. (2015) 'Markov process reliability model for photovoltaic module encapsulation failures', *Proceedings of the 2015 International Conference on Renewable Energy Research and Applications (ICRERA)*, 22–25 November, Palermo, Italy, pp.203–208.

Fashina, A., Mundu, M., Akiyode, O., Abdullah, L., Sanni, D and Ounyesiga, L. (2018a) 'The drivers and barriers of renewable energy applications and development in Uganda: a review', *Clean Technologies*, Vol. 1, No. 1, pp.1–31 [online] https://doi.org/10.3390/cleantech.

Fashina, A.A., Akiyode, O.O. and Sanni, D.M. (2018b) 'Study the status quo of rural and renewable energy development in Liberia: policy and implementation', *SPC Journal of Energy*, Vol. 1, No. 1, pp.9–20, DOI: 10.14419/e.v1i1.14518.

Ferreira, H., Faas, H., Fulli, G., Kling, W.L. and Lopes, P.J. (2016) *Reliability Analyses on Distribution Networks with Dispersed Generation: A Review of the State of the Art*, Joint Research Centre – Institute for Energy and Transport [online] http://ses.jrc.ec.europa.eu/publications/conference-papers (accessed on 1 February 2016).

Gahlot, M., Singh, V.V., Ayagi, H.I. and Goel, C.K. (2018) 'Performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics', *International Journal of Reliability and Safety*, Vol. 12, No. 4, pp.348–374.

Goyal, D., Kumar, A., Saini, M. and Joshi. H. (2019) 'Reliability, maintainability and sensitivity analysis of physical processing unit of sewage treatment plant', *SN Appl. Sci.*, Vol. 1, p.1507 [online] https://doi.org/10.1007/s42452-019-1544-7.

Gulati, J., Singh, V.V., Rawal, D.K. and Goel, C.K. (2016) 'Performance analysis of complex system in series configuration under different failure and repair discipline using copula', *International Journal of Reliability, Quality and Safety Engineering*, Vol. 23, No. 2, pp.812–832.

Gupta, R and Agarwal, D. (2021) 'Cost analysis of N-policy vacation machine repair problem with optional repair', *International Journal of Mathematics in Operational Research*, Vol. 19, No. 3, pp.354–374, DOI: 10.1504/IJMOR.2021.116964.

Kumar, A., Pawar, D and Malik, S.C. (2020) 'Reliability analysis of a redundant system with 'FCFS' repair policy subject to weather conditions', *International Journal of Advanced Science and Technology*, Vol. 29, No. 3, pp.7568–7578.

Lado, A and Singh, V.V. (2019) 'Cost assessment of complex repairable system consisting of two subsystems in the series configuration using Gumbel-Hougaard family copula', *International Journal of Quality and Reliability Management*, Vol. 36, No. 10, pp.1683–1698.

Lado, A.K., Singh, V.V., Kabiru, H.I. and Yusuf, I. (2018) 'Performance and cost assessment of repairable complex system with two subsystems connected in series configuration', *International Journal Reliability and Applications*, Vol. 19, No. 1, pp.27–42.

Manocha, A., Taneja, G., Singh, S. and Rishi, R. (2019) 'Modelling and analysis of two-unit hot standby database system with random inspection of standby unit', *International Journal of Mathematics in Operational Research*, Vol. 15, No. 2, pp.156–180, DOI: 10.1504/IJMOR.2019.101618.

Nelsen, R.B. (2006) *An Introduction to Copulas*, 2nd ed., Springer Publisher, New York.

Panwar, N and Kumar, S. (2021) 'Stochastic modelling and performance analysis of feeding unit in paper plant', *International Journal of Mathematics in Operational Research*, Vol. 19, No. 3, pp.302–316, DOI: 10.1504/IJMOR.2021.116965.

Patelli, E. and Beer, M. (2017) 'Reliability analysis on complex systems with common cause failures, safety, reliability, risk, resilience and sustainability of structures and infrastructure', *12th Int. Conf. on Structural Safety and Reliability*, Vienna, Austria, 6–10 August [online] https://www.researchgate.net/publication/319077308.

Patton, A.J. (2009) 'Copula-based models for financial time series', *Handbook of Financial Time Series*, pp.767–785, Springer, Berlin, Heidelberg.

Quiles, E., Roldán-Blay, C., Guillermo, E. and Roldán-Porta, E.C. (2020) 'Accurate sizing of residential stand-alone photovoltaic systems considering system reliability', *Sustainability*, Vol. 12, p.1274, DOI: 10.3390/su12031274.

Ram, M. and Singh, S.B. (2008) 'Availability and cost analysis of a parallel redundant complex system with two types of failure under preemptive-resume repair discipline using Gumbel-Hougaard family copula in repair', *International Journal of Reliability, Quality and Safety Engineering*, Vol. 15, No. 4, pp.341–365.

Ram, M. and Singh, S.B. (2010) 'Availability, MTTF and cost analysis of complex system under preemptive-repeat repair discipline using Gumbel-Hougaard family copula', *International Journal of Quality & Reliability Management*, Vol. 27, No. 5, pp.576–595.

Rengasamy, M., Gangatharan, S., Elavarasan, R.M. and Popa, L.M. (2020) 'Motivation for incorporation of microgrid technology in rooftop solar photovoltaic deployment to enhance energy economics', *Sustainability*, Vol. 12, p.10365, DOI: 10.3390/su122410365.

Rodriguez, J.C. (2007) 'Measuring financial contagion: a copula approach', *Journal of Empirical Finance*, Vol. 14, No. 3, pp.401–423.

Salah, J.A. and Fashina, A.A. (2019) 'Design of a hybrid solar photovoltaic system for Gollis University's administrative block, Somaliland', *International Journal of Physical Research*, Vol. 7, No. 2, pp.37–47, DOI: 10.14419/ijpr.v7i2.28949.

Sayed, A., El-Shimy, M., El-Metwally, M. and Elshahed, M. (2019) 'Reliability, availability and maintainability analysis for grid-connected solar photovoltaic systems', *Energies*, Vol. 12, No. 7, pp.2–18 [online] https://doi.org/10.3390/en12071213.

Sha, N. (2021) 'A copula approach of reliability analysis for hybrid systems', *Reliability: Theory & Applications*, Vol. 16, No. 1, pp.231–242.

Singh, V. and Lado, A. (2019) 'Cost assessment of complex repairable system consisting two subsystems in series configuration using Gumbel-Hougaard family copula', *International Journal of Quality & Reliability Management*, Vol. 36, No. 10, pp.1683–1698 [online] https://doi.org/10.1108/IJQRM-12-2018-0322.

Singh, V.V. and Ayagi, H.I. (2017) 'Study of reliability measures of system consisting of two subsystems in series configuration using copula', *Palestinian Journal of Mathematics*, Vol. 6, No. 1, pp.1–10.

Singh, V.V. and Ayagi, H.I. (2018) 'Stochastic analysis of a complex system under preemptive resume repair policy using Gumbel-Hougaard family copula', *International Journal of Mathematics in Operational Research*, Vol. 12, No. 2, pp.273–291.

Singh, V.V. and Poonia, P.K. (2019) 'Probabilistic assessment of two-unit parallel system with correlated lifetime under inspection using regenerative point technique', *IJRRS*, Vol. 2, No. 1, pp.5–14.

Singh, V.V. and Singh, N.P. (2015) 'Performance analysis of three unit redundant system with switch and human failure', *Math. Eng. Sci. Aerosp.*, Vol. 6, No. 2, pp.295–308.

Singh, V.V., Gahlot, M., Ayagi, H.I. and Goel, C.K. (2018) 'Performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics', *International Journal of Reliability and Safety*, Vol. 12, No. 4, pp.348–374.

Singh, V.V., Gulati, J., Rawal, D.K and Goel, C.K. (2016) 'Performance analysis of complex system in series configuration under different failure and repair discipline using copula', *International Journal of Reliability Quality and Safety Engineering*, Vol. 23, No. 2 [online] https://doi.org/10.1142/S0218539316500108.

Singh, V.V., Poonia, P.K and Abdullahi, A.H. (2020a) 'Performance analysis of a complex repairable system with two subsystems in series configuration with imperfect switch', *Journal of Mathematical and Computational Science*, Vol. 10, No. 2, pp.359–383.

Singh, V.V., Poonia, P.K. and Rawal, D.K, (2020b) 'Reliability analysis of repairable network system of three computer labs connected with a server under 2-out-of-3: G configuration', *Cycle Reliability, and Safety Engineering*, DOI: 10.1007/s41872-020-00129-w.

Sklar, A. (1973) 'Random variables, joint distribution functions, and copulas', *Kybernetika*, Vol. 9, No. 6, pp.449–460.

Temraz, N.S.Y. (2019) 'Availability and reliability of a parallel system under imperfect repair and replacement: analysis and cost optimization', *Int. J. Syst. Assur. Eng. Manag.*, Vol. 10, pp.1002–1009 [online] https://doi.org/10.1007/s13198-019-00829-2.

Trivedi, P.K. and Zimmer, D.M. (2007) *Copula Modeling: An Introduction for Practitioners*, Now Publishers Inc., USA.

Tyagi, V., Arora, R. and Ram, M. (2021) 'Copula based measures of repairable parallel system with fault coverage', *International Journal of Mathematical, Engineering and Management Sciences*, Vol. 6, No. 1, pp.322–344.

Uswarman, R. and Rushdi, A.M. (2021) 'Reliability evaluation of rooftop solar photovoltaic using coherent threshold systems', *Journal of Engineering Research and Reports*, Vol. 20, No. 2, pp.32–44.

Velasco, J.A.M and Guerra, G. (2016) 'Reliability analysis of distribution systems with photovoltaic generation using a power flow simulator and a parallel Monte Carlo approach', *Energies*, Vol. 9, p.537, DOI: 10.3390/en9070537.

Wang, C., Fei, W., Xinlin, Z., Yu, Y., Yongxian, S., Yuyao, Y. and Hongou, Z. (2017) 'Examining the driving factors of energy related carbon emissions using the extended STIRPAT model based on IPAT identity in Xinjiang', *Renewable and Sustainable Energy Reviews*, Vol. 67, pp.51–61 [online] https://doi.org/10.1016/j.rser.2016.09.006.

Wang, J., Xie, N. and Yang, N. (2021) 'Reliability analysis of a two-dissimilar-unit warm standby repairable system with priority in use', *Communications in Statistics – Theory and Methods*, Vol. 50, No. 4, pp.792–814.

Xu, X., Jianlin, L., Gong, J., Deng, H and Wan, L. (2016) 'Copula-based slope reliability analysis using the failure domain defined by the *g*-line', *Mathematical Problems in Engineering*, Vol. 2016, Article ID 6141838 [online] http://dx.doi.org/10.1155/2016/6141838.

Yang, D-Y and Tsao, C-L. (2019) 'Reliability and availability analysis of standby systems with working vacations and retrial of failed components', *Reliability Engineering & System Safety*, Vol. 182, pp.46–55 [online] https://doi.org/10.1016/j.ress.2018.09.020.

Yusuf, I. and Musa, I.M. (2021) 'Availability analysis of a complex system consisting of two subsystems in parallel configuration with replacement at failure', *International Journal of Mathematics in Operational Research*, Vol. 19, No. 4, pp.530–550, DOI: 10.1504/IJMOR.2021.117629.

Yusuf, I., Ismail, A. L., and Ali, U.A. (2020) 'Reliability analysis of communication network with redundant relay station under partial and complete failure', *Journal of Mathematical and Computational Science*, Vol. 10, No. 4, pp.863–880.

Zhang, P., Li, W., Li, S., Wang, Y and Xiao, W. (2013) 'Reliability assessment of photovoltaic power systems: review of current status and future perspectives', *Appl. Energy*, Vol. 104, pp.822–833.