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Experimental parameter estimation methodology based on equivalent output injection

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Abstract: This work proposes a methodology to estimate parameters for linear and nonlinear dynamical systems, with partial state measurement, that satisfy the property of parameter linearity. This methodology is experimental, offline, and recursive. It uses discontinuous state observers to estimate all state variables and the disturbance terms needed in the estimation processes. Because the equivalent output injection corresponds to the disturbances produced by the parameter uncertainties, the methodology allows us to obtain the best parameter estimation by minimising an index related to the power of the equivalent output injection; a smaller value represents a better estimation. With this parameter estimation, we can establish a model that facilitates the design and implementation of many control algorithms, including robust controllers. We validate the methodology through numerical simulations and experiments with linear, nonlinear, and discontinuous systems. Based on the experimental results, we conclude that the proposed algorithm's performance is better than other methodologies.

Keywords: identification; modelling; equivalent output injection; discontinuous observers; least squares algorithm.

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1 Introduction

In the design and implementation of many control algorithms, it is necessary to know the plant's mathematical model. This model must represent, as well as possible,

the dynamics of the actual system. However, there are differences between the model and the actual system due to, mainly, parameter uncertainties and unmodelled dynamics, which in many cases produce terms that have the role of disturbances in the plant.

There are several control algorithms to achieve control objectives in plants with many types of disturbances: some examples are sliding modes control (Utkin et al., 2009), adaptive control (Najim and Saad, 1991), \mathcal{H}_∞ control (Burl, 1998), and the active disturbance rejection control structure (Huang and Xue, 2014). However, if we have a mathematical model that represents the dynamics of interest in the actual system, as well as its parameters, more precisely, the processes of design and experimental implementation of these robust control algorithms are more efficient. See for example, Tudoroiu et al. (2020), where the authors propose an ARMAX model to capture the complex dynamics of a centrifugal chiller plant. This model has simulation purposes of proving the efficiency of the closed-loop control strategy.

There are two stages to obtain a mathematical model of an actual system. The first stage defines the equations which describe its dynamics, and the second one is to estimate the parameters included in these equations. Both stages are essential, but this work addresses the second one, also known as system identification. Zhang and Zhao (2019) is a survey of the methodologies in modelling and identification more significant and recent applied to control systems.

Xu and Hashimoto (1993, 1996) propose two methodologies, called VSS-based direct and indirect methods, to solve the parameter identification problem for linear and nonlinear systems. These methods are applicable to systems with a normal form and linear in parametric space. They use the equivalent control phenomenon, which appears in discontinuous control systems, to estimate the plant's parameters. These methods have two main advantages: they are applicable to systems in which a linear algebraic relationship cannot be obtained by conventional identification methods, as well as to those systems that may not be stable in open-loop. However, the main disadvantages are that these methods assume a full state measurement, and the incorporation of a state observer degrades their performance. Only numerical examples are presented in the papers mentioned. Also, there is no procedure to validate the results experimentally.

A practical method to identify the stick-slip friction model applied to hydraulic actuators is in Márton et al. (2011). This method can estimate friction model parameters based on velocity and pressure measurements with a piecewise linear approximation method. Its performance was evaluated experimentally using the velocity signals of the model and the actual plant. The main advantage of that proposal is its low computational cost, and the main disadvantage is that it cannot estimate the rest of the plant's parameters.

Liu and Wu (2015) proposes an identification method for the structure and friction parameters of a feed servo system. They use a direct graphical method to identify the friction parameters and the nonlinear least squares algorithm to identify the structural parameters. The method yields good results, but it is applied to a particular system, and the authors only present a numerical simulation to show its performance. Also, for an experimental implementation,

there is no a procedure to validate the identified parameters. Imine et al. (2015) present the estimation of the vertical forces and dynamic parameters of a vehicle using the sliding mode observers approach. In particular, they identify the suspension stiffness and unsprung masses parameters by the least squares algorithm. This is an important paper because it shows the efficiency of the equivalent output injection, in discontinuous observers, to estimate disturbances and parameters in an experimental context. However, it presents only a particular case and, to validate the results, the authors install many sensors to compare the estimated signals with the actual ones.

From the experimental point of view, the identification of hydraulic servo-systems is an important research topic. The least squares algorithm and the derivation free particle swarm optimisation method are used in Maier et al. (2019) and Feng et al. (2019) to estimate the parameters for an automated clutch actuation system and a robotic excavator, respectively. They obtain significant results. However, in both cases, there is no criterion to evaluate the quality of the parameter estimation in an experimental context.

Another relevant work is Aggoun et al. (2020), which proposes an adaptive linear neuron to estimate the state of charge of lithium-ion batteries based on parameter estimation. The neuron provides a linear combination of the inputs based on an online identification of the open-circuit voltage. The main contribution of this approach is its adaptable capability and the execution speed of the algorithm. However, the authors only present simulation results.

Xu (2017) mentions that for the linear problem, the least squares method is effective, but for the nonlinear problem, we must use nonlinear optimisation methods.

Discontinuous state observers and extended observers have been used extensively to estimate the disturbances of a plant. Some important works on this topic are Bu et al. (2015), Almeida et al. (2007), Davila et al. (2006), Chen (2004), Wang et al. (2015), Ren et al. (2018), and the references mentioned there. Some of them guarantee finite-time convergence; others have asymptotic convergence; some need a low pass filter to recover the equivalent output injection; and others do not need it. Some of these proposals are given only with a numerical illustration of their performance or with well-controlled experiments. However, all of them have restrictions on their design and operation, and their experimental performance depends on the hardware platform and on the sampling time used to execute the experiments (Rosas et al., 2017). Therefore, if we want to estimate disturbances in a plant, we need to select the best state observer according to the characteristics of the plant and the disturbances in it, and the hardware available.

The present paper proposes a methodology to estimate the parameters for linear and nonlinear dynamical systems, with partial state measurement, that satisfy the property of parameter linearity. This methodology is experimental, offline, and recursive. It uses discontinuous state observers to estimate all state variables and the disturbance terms needed in the estimation processes. The methodology

consists of a recursive process with four steps in each iteration. First, we estimate the non-measured states and the disturbance terms using the equivalent output injection concept. In the next step, we obtain an index related to the power of the equivalent output injection. In the third step, we estimate the parameters of the plant using a least squares algorithm. Then we update the parameters of the plant in the observer. Because the equivalent output injection corresponds to the disturbances produced by the parameter uncertainties, the methodology allows us to obtain the best parameter estimation by minimising an index (for which a smaller value represents a better estimation). With this parameter estimation, we can establish a model that facilitates the design and implementation of many control algorithms, including robust controllers. We validate the methodology through numerical simulations and experiments involving linear, nonlinear, and discontinuous systems.

The main contribution of this paper is the integration of several well-known results on robust discontinuous state observers, equivalent output injection theory, and the least-squares method to produce an experimental methodology to identify the parameters for a broad class of dynamical systems, including linear, nonlinear, and discontinuous systems. This proposal is more general than other previously published methodologies because it is not limited to the observers developed at this time; in the future, we can incorporate new observers for new kinds of systems or improve their performance.

The organisation of this paper is as follows. Section 2 presents preliminary definitions and the problem statement. Section 3 presents the process for estimating the disturbances using discontinuous state observers based on the concept of equivalent output injection. Also, in this section, we present two observer structures for first- and second-order systems. Section 4 presents the strategy proposed in Davila et al. (2006) and Almeida et al. (2007), based on the least squares algorithm, to identify the parameters using the equivalent output injection.

Section 5 presents the proposed methodology of parameter identification, and several numerical and experimental examples of its performance are in Section 6. Section 7 presents a performance comparison of our proposed strategy with two strategies proposed previously: that of Xu and Hashimoto (1996) and that provided by the system identification toolbox in MATLAB. Section 8 presents the overall conclusions of this work as well as some final comments.

2 Problem statement and preliminary definitions

Consider the class of dynamical systems described by

$$\begin{aligned}\dot{x} &= f(x, \theta, u), \\ y &= h(x),\end{aligned}\tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $\theta \in \mathbb{R}^k$ is a parameter vector, $u \in \mathbb{R}^m$ is the input vector, $y \in \mathbb{R}^l$ is the output

vector, $f(\cdot) : \mathbb{R}^n \times \mathbb{R}^k \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ is a vector field, linear or nonlinear, and $h(\cdot) : \mathbb{R}^n \times \mathbb{R}^k \times \mathbb{R}^m \rightarrow \mathbb{R}^l$ is a vector function which defines the system output. It is important to note that system (1), and systems defined later, may satisfy the Lipschitz condition, which guarantees the existence and uniqueness of solutions in the usual form. However, it also may have a discontinuous right-hand side; in this case, its solutions are defined in the Filippov sense.

Now, consider the class of systems, with the form (1), which are linear or affine to parameters θ . Therefore, system (1) may be rewritten as

$$\begin{aligned}\dot{x} &= \varphi(x, u) \theta, \\ y &= h(x),\end{aligned}\tag{2}$$

where $\varphi(x, u)$ is called a regressor. Then, if the parameter vector θ is the sum of a nominal parameter vector θ_0 and an uncertain parameter vector Δ_θ , system (2) may be rewritten as

$$\begin{aligned}\dot{x} &= f(x, \theta_0, u) + \varphi(x, u) \Delta_\theta, \\ y &= h(x).\end{aligned}\tag{3}$$

Now, the disturbance term $\varphi(x, u) \Delta_\theta$ must satisfy certain conditions, for example $\|\varphi(x, u) \Delta_\theta\| < \delta_1$ and/or $\|d(\varphi(x, u) \Delta_\theta)/dt\| < \delta_2$, for a bounded input u , where δ_1 and δ_2 are known constants.

The least squares algorithm is a very simple and useful tool to estimate the vector of parameter uncertainties Δ_θ based on the knowledge of $\varphi(x, u)$ and $\varphi(x, u) \Delta_\theta$. However, in many practical situations, some of the signals included in $\varphi(x, u)$ and the term $\varphi(x, u) \Delta_\theta$ are not measured, which prevents the implementation of the method. Also, in an experimental context, there is no strategy to validate the results.

Therefore, the problem addressed in this paper is to propose a methodology to estimate the uncertain parameter vector Δ_θ in the class of dynamical systems defined by equation (1), using the least squares algorithm, taking into account the case that some variables in $\varphi(x, u)$ and the term $\varphi(x, u) \Delta_\theta$ are not measured.

3 Disturbance estimation using discontinuous state observers

We can use state observers and differentiators to estimate all non-measured signals included in $\varphi(x, u)$ and the disturbance term $\varphi(x, u) \Delta_\theta$. Several state observers can solve these problems, see for example Bu et al. (2015), Chen (2004) and Ren et al. (2018). However, in this work, we use high-order sliding mode observers because they present the phenomenon of equivalent output injection, which will be useful for evaluating the quality of the parameter estimation. The concept of equivalent output injection in discontinuous state observers exhibiting the sliding mode phenomenon is similar to equivalent control in sliding mode control systems. The difference is that,

in general, the discontinuity surface in control systems is defined as a function of the state variables. On the other hand, in state observers, the discontinuity surface is defined as a function of the error between the plant output and the observer output; an output injection is made.

At this point we make the following assumption: There is a robust discontinuous state observer for system (3) which estimates the disturbance term $\varphi(x, u) \Delta_\theta$, through the equivalent output injection principle, and all non-measured signals needed to implement the term $\varphi(x, u)$.

In this work, the main objective is not the proposal of new state observers for systems with the form (3), but to use the observers that have already been developed. Therefore, in this section, we present two previously published state observers for the estimation of disturbances. In later sections, we use them for the parameter identification.

3.1 A nonlinear disturbance observer for a class of first-order systems

This observer is based on Bu et al. (2015) and Wang et al. (2015). Consider a first-order nonlinear system

$$\begin{aligned}\dot{x} &= f(x) + g(x)u + \gamma(x, t), \\ y &= x,\end{aligned}\quad (4)$$

where $x \in \mathfrak{R}$ is the state, $f(x)$ and $g(x)$ are well known linear or nonlinear functions, and $\gamma(x, t)$ is a disturbance term which satisfies $|\dot{\gamma}(x, t)| < \delta$, where δ is a known constant. In this case the problem is the estimation of the disturbance term $\gamma(x, t)$. The observer is given by

$$\begin{aligned}\dot{\hat{x}} &= f(x) + g(x)u + \omega_1 |y - \hat{y}|^{\frac{1}{2}} \text{sign}(y - \hat{y}) + \omega_2 p, \\ \dot{p} &= -T_s p + \text{sign}(y - \hat{y}), \\ \hat{y} &= \hat{x}.\end{aligned}\quad (5)$$

The dynamics of the error variable $e = y - \hat{y}$ are given by

$$\begin{aligned}\dot{e} &= \gamma(x, t) - \omega_1 |e|^{\frac{1}{2}} \text{sign}(e) - \omega_2 p, \\ \dot{p} &= -T_s p + \text{sign}(e).\end{aligned}\quad (6)$$

If $\omega_1 = 1.5\sqrt{\delta}$, $\omega_2 = 1.1\delta$ and $T_s > 0$, where $|\dot{\gamma}(x, t)| < \delta$, e and \dot{e} converge to zero in finite time (Bu et al., 2015; Wang et al., 2015).

Therefore, the equivalent output injection $u_{eq}(\cdot)$ is given by

$$u_{eq}(\cdot) = \omega_1 |e|^{\frac{1}{2}} \text{sign}(e) + \omega_2 p = \gamma(x, t). \quad (7)$$

In conclusion, the observer (5) estimates the disturbance $\gamma(x, t)$ in equation (4) in finite time and it does not need a low-pass filter.

3.2 A nonlinear disturbance observer for a class of second-order systems

Consider the second-order system

$$\begin{aligned}\dot{x}_1 &= x_2, \\ \dot{x}_2 &= f(x_1, x_2) + g(x_1)u + \gamma(x_1, x_2, t), \\ y &= x_1,\end{aligned}\quad (8)$$

where $f(\cdot)$ and $g(\cdot)$ are well known functions and $\gamma(\cdot)$ is a bounded disturbance $|\gamma(\cdot)| \leq \delta$, where δ is a known constant.

The state observer for system (8) is given by Almeida et al. (2007)

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + c_1(y - \hat{y}), \\ \dot{\hat{x}}_2 &= f(x_1, \hat{x}_2) + g(x_1)u + c_2(y - \hat{y}) \\ &\quad + c_3 \text{sign}(y - \hat{y}), \\ \hat{y} &= \hat{x}_1.\end{aligned}\quad (9)$$

To prove the convergence of the observer state to the plant state we define the error variables $e_1 = x_1 - \hat{x}_1$, $e_2 = x_2 - \hat{x}_2$, whose dynamics are given by

$$\begin{aligned}\dot{e}_1 &= e_2 - c_1 e_1 \\ \dot{e}_2 &= -c_2 e_1 - c_3 \text{sign}(e_1) + \rho(\cdot)\end{aligned}$$

where $\rho(\cdot) = f(x_1, x_2) - f(x_1, \hat{x}_2) + \gamma(x_1, x_2, t)$. Now, making a change of variables $z_1 = e_1$ and $z_2 = e_2 - c_1 e_1$, whose dynamics are

$$\begin{aligned}\dot{z}_1 &= z_2, \\ \dot{z}_2 &= -c_2 z_1 - c_1 z_2 - c_3 \text{sign}(z_1) + \rho(\cdot).\end{aligned}\quad (10)$$

With a suitable selection of c_1 , c_2 and c_3 the state variables converge asymptotically to zero, see Almeida et al. (2007). Then, \hat{x}_1 and \hat{x}_2 converge to x_1 and x_2 , respectively.

System (10) has a discontinuity surface in $z_1 = 0$ and the term $c_3 \text{sign}(z_1)$ produces a second-order sliding mode, because the discontinuous output injection $u_{eq}(\cdot)$ appears until the second time derivative of the function defining the discontinuity surface

$$\ddot{z}_1 = -c_2 z_1 - c_1 z_2 - u_{eq}(\cdot) + \rho(\cdot) = 0.$$

Then, the equivalent output injection is present at $z_1 = z_2 = 0$, and so

$$u_{eq}(\cdot) = \rho(\cdot) = f(x_1, x_2) - f(x_1, \hat{x}_2) + \gamma(x_1, x_2, t).$$

Because \hat{x}_2 converges to x_2 we have

$$u_{eq}(\cdot) = \gamma(x_1, x_2, t).$$

We can see that the equivalent output injection produces a disturbance term and, as we know, it is the average of the term $c_3 \text{sign}(z_1)$ when the trajectories stay at the origin. Therefore, we have to use a low-pass filter to recover the equivalent output injection (Almeida et al., 2007).

3.3 Parameter estimation through the equivalent output injection

The disturbance term $\varphi(x, u) \Delta_\theta = u_{eq}(\cdot)$, where $u_{eq}(\cdot)$ is the equivalent output injection, has the form of a regressor, so we can apply the least squares algorithm proposed in Almeida et al. (2007) and Davila et al. (2006), and used in Imine et al. (2015) to estimate the uncertain parameter vector Δ_θ .

We want to find a vector $\hat{\Delta}_\theta$ that minimises

$$J = \frac{1}{t} \int_0^t (u_{eq}(\cdot) - \varphi(\cdot) \hat{\Delta}_\theta)^T (u_{eq}(\cdot) - \varphi(\cdot) \hat{\Delta}_\theta) d\tau, \quad (11)$$

The optimal solution is

$$\Delta_\theta = \left[\int_0^t \varphi^T(\cdot) \varphi(\cdot) d\tau \right]^{-1} \int_0^t \varphi^T(\cdot) u_{eq}(\cdot) d\tau,$$

where the matrix

$$\int_0^t \varphi^T(\cdot) \varphi(\cdot) d\tau,$$

must be non-singular. Define a new variable

$$\Gamma_t = \left[\int_0^t \varphi^T(\cdot) \varphi(\cdot) d\tau \right]^{-1},$$

using the following identities

$$\begin{aligned} \Gamma_t^{-1} \dot{\Gamma}_t &= I \\ \Gamma_t^{-1} \dot{\Gamma}_t + \dot{\Gamma}_t^{-1} \Gamma_t &= 0. \end{aligned}$$

Then we have

$$\dot{\Gamma}_t = -\Gamma_t \varphi^T(\cdot) \varphi(\cdot) \Gamma_t. \quad (12)$$

Now, a parameter identification algorithm, based on the equivalent output injection, is given by

$$\dot{\hat{\Delta}}_\theta = \Gamma_t \varphi^T(\cdot) \left(u_{eq}(\cdot) - \varphi^T(\cdot) \hat{\Delta}_\theta \right). \quad (13)$$

Considering that the matrix $\Gamma_t \varphi^T(\cdot) \varphi(\cdot)$ is Hurwitz, we can conclude that equations (12) and (13) provide the actual values of the uncertain parameter vector Δ_θ ; $\hat{\Delta}_\theta = \Delta_\theta$.

4 Parameter estimation methodology

In an ideal context, where we have all necessary signal measurements to implement the regressor $\varphi(\cdot)$, as well as the equivalent output injection $u_{eq}(\cdot)$, and assuming that these signals are noise-free, equations (12) and (13) estimate in a precise form the vector of parameter uncertainties Δ_θ . However, in a practical context, some state variables are not measured and must be estimated. In general, the estimated signals have errors with respect to the real ones and also contain noise and small delays, which produces an imprecise estimation of the parameter uncertainties Δ_θ .

To resolve these problems in an experimental context we propose in this section a recursive methodology, offline, that uses discontinuous state observers to estimate the non-measured signals to implement the term $\varphi(\cdot)$, and equivalent output injection $u_{eq}(\cdot)$ and, at the same time, implements a signal processing step to reduce the noise in the estimated signals.

The methodology to identify Δ_θ begins with the definition of the mathematical model of the plant, and we have to write it in the form (3). Also, we must guarantee a bounded behaviour of the state variables, outputs, and disturbances if we apply a bounded input signal u , which satisfies the persistence condition when the plant is in open-loop. Then, using some methodology or by empirical knowledge, we propose a vector of nominal parameters of the plant to define the nominal part of the system (3). Now, using the nominal part of the model, design and implement a discontinuous state observer to estimate the non-measured signals needed to implement the regressor $\varphi(\cdot)$, as well as the equivalent output injection $u_{eq}(\cdot)$, which corresponds to an estimation of the term $\varphi(x, u) \Delta_\theta$. Here it is assumed that the observer already exists in the literature; otherwise, we have to design one.

Now, implement an offline process to obtain an estimation of the actual values of the parameters. It is essential to mention that if the signals have noise, it is necessary to filter them to reduce the high-frequency components but, at the same time, avoiding introducing long time delays.

To have a measure of the magnitude of the equivalent output injection, we define the index $P(u_{eq})$, which is the power of $u_{eq}(\cdot)$ in the discrete domain. It is given by

$$P(u_{eq}) = \frac{1}{N+1} \sum_{n=1}^N |u_{eq}(\cdot)_n|^2, \quad (14)$$

where N is the number of samples of $u_{eq}(\cdot)$. It is important to notice that

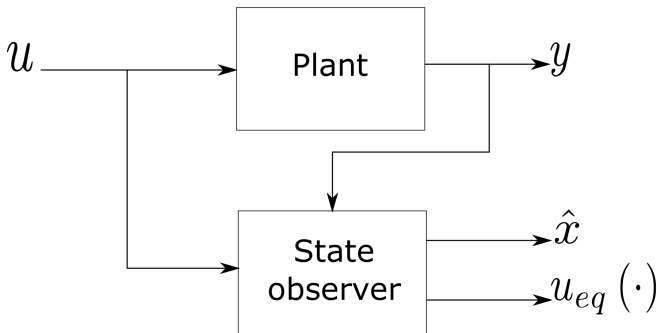
$$P(u_{eq}) = \frac{1}{N+1} \sum_{n=1}^N |\varphi(\cdot)_n \Delta_\theta|^2.$$

Hence, for the same sequence $\varphi(\cdot)$, the magnitude of the index $P(u_{eq})$ is directly proportional to the magnitude of the vector of parameter uncertainties Δ_θ . Now, we carry out the following iterations.

- 1 Connect the plant and the state observer as shown in Figure 1. The parameters of the observer, related to the plant, are equal to θ_0 . Apply the input signal u and store all signals, both measured and estimated, to implement the regressor $\varphi(\cdot)$, and the equivalent output injection $u_{eq}(\cdot)$. If these signals have noise, filter them. In an offline process to reduce high-frequency components, while avoiding the introduction of large time delays. Obtain the index $P_1(u_{eq})$, where the subscript 1 indicates that it corresponds to the first iteration. Finally, implement equations (12) and (13) to obtain the first estimation of Δ_θ , which we call $\hat{\Delta}_{\theta(1)}$.
- 2 Upgrade the parameter vector in the state observer which corresponds to the plant to $\theta_0 + \hat{\Delta}_{\theta(1)}$. Apply the input signal u and store all signals, both measured and estimated, to implement the regressor $\varphi(\cdot)$, and the equivalent output injection $u_{eq}(\cdot)$. If these signals have noise, filter them to reduce high-frequency components, but avoid introducing large time delays. Obtain the index $P_2(u_{eq})$. Finally, implement equations (12) and (13) to obtain the second estimation of $\hat{\Delta}_{\theta(2)}$.
- 3 Upgrade the parameter vector in the state observer, which corresponds to the plant, to $\theta_0 + \hat{\Delta}_{\theta(1)} + \dots + \hat{\Delta}_{\theta(i-1)}$. Apply the input signal u and store all signals, both measured and estimated, to implement the regressor $\varphi(\cdot)$, and the equivalent output injection $u_{eq}(\cdot)$. If these signals have noise, filter them to reduce high-frequency components, but avoid introducing large time delays. Obtain the index $P_i(u_{eq})$. Finally, implement equations (12) and (13) to obtain the i estimation of $\hat{\Delta}_{\theta(i)}$.

Make as many iterations as necessary until one sees only small variations in the index $P(u_{eq})$.

Figure 1 A block diagram of the plant and the state observer to estimate the terms needed for the estimation of the parameter uncertainties



Among all the iterations made, find the k^{th} iteration where the index $P_k(u_{eq})$ has the minimum value. Then, the estimation of the vector of parameter uncertainties $\hat{\Delta}_\theta$ is given by

$$\hat{\Delta}_\theta = \sum_{j=1}^{k-1} \hat{\Delta}_{\theta(j)},$$

and the value of the actual parameter vector is

$$\theta = \theta_0 + \sum_{j=1}^{k-1} \hat{\Delta}_{\theta(j)}.$$

It is important to note that the values of $P(u_{eq})$, for all the iterations performed, form a sequence. In this case, this sequence doesn't need to converge: we only require a minimum.

5 Performance of parameter estimation strategy

5.1 Parameter estimation of a level system

Consider the level system shown in Figure 2. A simplified model of this system is given by

$$\dot{x} = \frac{1}{A} (f_{in}(t) - f_{out}(t)),$$

where x is the level of the liquid, A is the cross-sectional area of the tank, $f_{in}(t)$ is the inflow rate, and $f_{out}(t)$ is the outflow rate. The inflow rate is given by

$$f_{in}(t) = kV(t),$$

where k is the pump constant and $V(t)$ is the voltage applied to the pump. Now, using Bernoulli's law for flows through small orifices, the outflow velocity is

$$v_{out}(t) = \sqrt{2gx},$$

where g is the gravitational acceleration constant. Then, the outflow is

$$f_{out}(t) = a\sqrt{2gx},$$

where a is the cross-sectional area of the outflow orifice. Therefore, the model is given by

$$\dot{x} = -\frac{a}{A}\sqrt{2gx} + \frac{k}{A}V(t), \quad (15)$$

System (15) has three not exactly known parameters: A , a and k . Therefore, it takes the form

$$\dot{x} = -(\alpha + \Delta_\alpha)\sqrt{2gx} + (\beta + \Delta_\beta)V(t), \quad (16)$$

where α and β are the nominal values of a/A and k/A , respectively, while Δ_α and Δ_β are the uncertainties of those parameters.

Because g is a known constant, system (16) satisfies the property of linearity with respect to the parameters, therefore

$$\dot{x} = -\alpha\sqrt{2gx} + \beta V(t) + \varphi^T(x, V)\theta,$$

where

$$\varphi^T(x, V) = [-\sqrt{2gx} V(t)],$$

and

$$\theta^T = [\Delta_\alpha \Delta_\beta].$$

In this case, the state variable x is available. Then we need a state observer to estimate only the disturbance term $\varphi^T(x, V)\theta$. Based on the observer (5) we propose the state observer given by

$$\begin{aligned} \dot{\hat{x}} &= -\alpha\sqrt{2gx} + \beta V(t) + \omega_1 |x - \hat{x}|^{\frac{1}{2}} \text{sign}(x - \hat{x}) \\ &\quad + \omega_2 p, \\ \dot{p} &= -T_s p + \text{sign}(x - \hat{x}). \end{aligned} \quad (17)$$

To prove the stability of the observer, define the error $e = x - \hat{x}$, whose dynamics are given by

$$\begin{aligned} \dot{e} &= -\omega_1 |e|^{\frac{1}{2}} \text{sign}(e) - \omega_2 p + \varphi^T(x, V)\theta, \\ \dot{p} &= -T_s p + \text{sign}(e), \end{aligned}$$

where

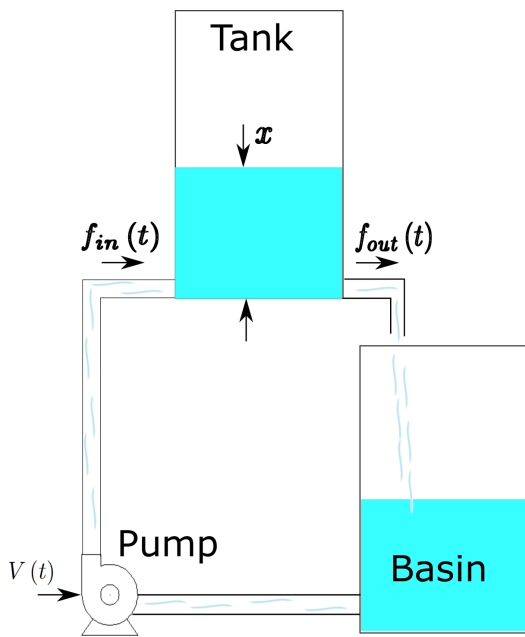
$$\left| \frac{d\varphi^T(x, V)\theta}{dt} \right| \leq \delta.$$

If $\omega_1 = 1.5\sqrt{\delta}$ and $\omega_2 = 1.1\delta$, we can guarantee that the error e and its derivative converge to zero in finite time. Hence

$$v = \omega_1 |e|^{\frac{1}{2}} \text{sign}(e) + \omega_2 p = \varphi^T(x, V)\theta$$

in finite time (Bu et al., 2015; Wang et al., 2015).

Figure 2 Scheme of the level system (see online version for colours)

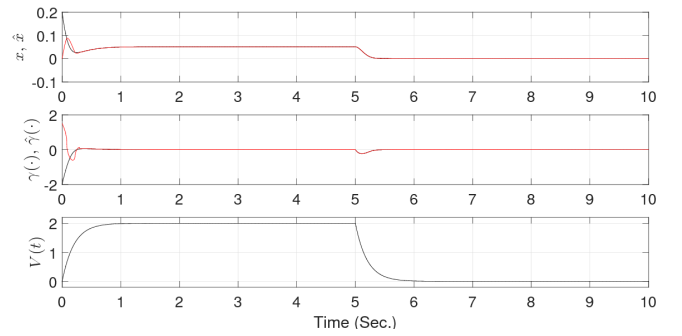


5.1.1 Numerical results of the parameter estimation of a level system

To illustrate the performance of the methodology, as a first stage, we made a numerical experiment. For system (16), we propose the nominal values for the parameters to be $\alpha = 0.6047$ and $\beta = 3.0075 \times 10^{-3}$, and the parameter uncertainties as $\Delta_\alpha = -1$ and $\Delta_\beta = 0.5$. The parameters for observer (17) are $\delta = 5$ and $T_s = 1$.

The performance of the state observer is shown in Figure 3. We can see that the convergence of \hat{x} to x and the estimation of the disturbances $\hat{\gamma}(\cdot)$ are in finite time. It is important to note that signals x , $\hat{\gamma}(\cdot)$ and $V(t)$ do not have noise, therefore a filtering process is not necessary.

Figure 3 Performance of the state observer in estimating the disturbance (see online version for colours)



Notes: Black lines correspond to actual signals and red lines to estimated signals.

The results of the methodology for parameter estimation are in Table 1. Because the signals are noise-free, we have a good estimation in the first iteration.

Table 1 Numerical results for parameter estimation of a level system

$It. i$	$P_i(u_{eq_i})$	$\tilde{\Delta}_{\alpha_i}$	$\tilde{\Delta}_{\beta_i}$
1	0.0013	0.9999	0.5
2	4.0293e-07	-	-

5.1.2 Experimental results of the parameter estimation of a level system

The experiment was made with a level system, manufactured by FESTO, shown in Figure 4, which is composed of a basin, tank, pump, and a level sensor. The experiments were made in the real-time platform MicrolabBox 1102 from dSPACE, using a sample time of 0.0001 seconds.

For the experiments, the nominal values for the plant are $\alpha = 0.0060469$ and $\beta = 0.0030075$, and the parameters for the state observer are $T_s = 1$ and $\delta = 10$.

Figure 5 shows the performance of the observer. Here we can see that \hat{x} converges to x , but all signals, x , $\hat{\gamma}(\cdot)$ and v , have noise. Therefore, we apply a filtering processes to reduce it. Figure 6 shows the equivalent output injection

for the first iteration, which corresponds to the black line and the red line's second iteration. Because the nominal parameters were close to the real ones, the decrease in amplitude was small.

Figure 4 Level system used in the experiments (see online version for colours)

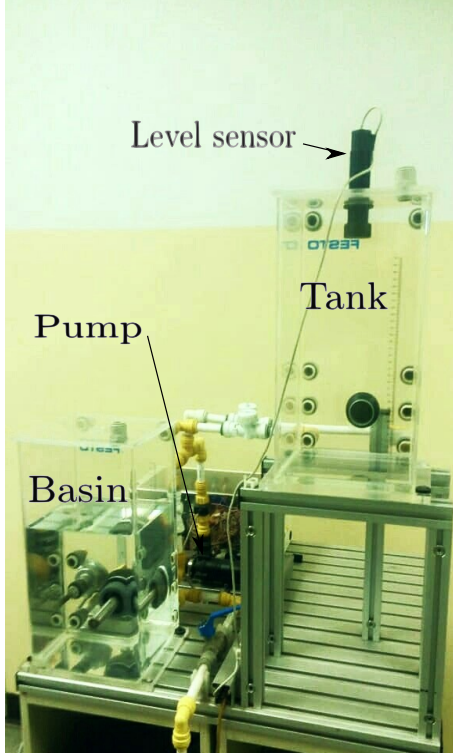
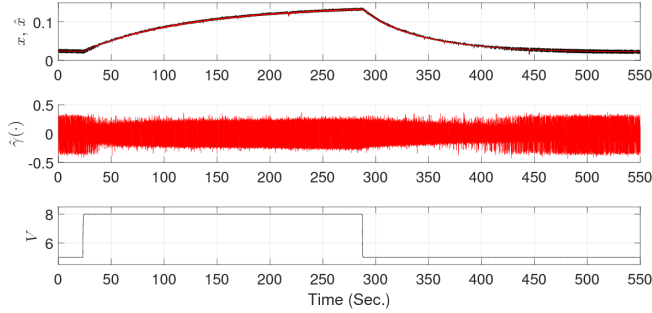


Figure 5 Experimental results (see online version for colours)



Notes: Performance of the state observer in estimating the disturbance. Black lines correspond to actual signals and red lines to estimated signals.

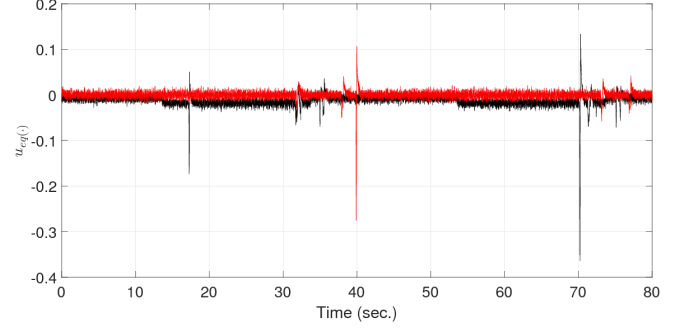
Table 2 Experimental results for parameter estimation of a level system

It. i	$P_i(u_{eq_i})$	$\tilde{\Delta}_{\alpha_i}$	$\tilde{\Delta}_{\beta_i}$
1	3.2863e-04	-0.004271	-0.00273
2	8.1318e-05	-6.795×10^{-5}	-7.592×10^{-6}
3	1.7447e-04	3.946×10^{-5}	-7.691×10^{-7}
4	1.1578e-04	-	-

Table 2 shows the results of the parameter estimations for four iterations. As we can see, the minimum value of the

index $P(u_{eq})$ is obtained after the first iteration. Therefore, the final values of the parameters are $\alpha = 0.0017759$ and $\beta = 0.0047834$.

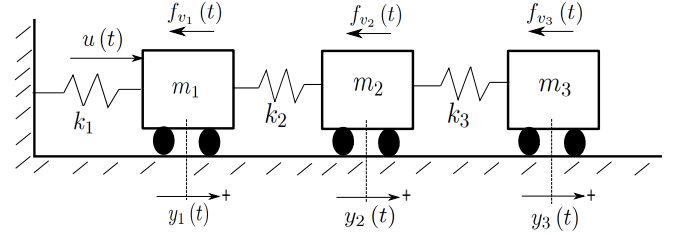
Figure 6 Comparison of the equivalent output injection of the first and second iterations, black line y red line respectively (see online version for colours)



5.2 Parameter estimation for a three-DOF mass spring damper

In this section we apply the proposed methodology to a linear system with order six, with one input and three outputs. Consider the mass-spring-damper mechanical system shown in Figure 7, where $f_{v_i}(t)$, for $i = 1, 2, 3$, are the viscous friction forces in each mass.

Figure 7 Diagram of a mass-spring-damper mechanical system with three DOFs



The model of this mechanical system is given by

$$\begin{aligned}
 \dot{x}_{11} &= x_{21}, \\
 \dot{x}_{12} &= x_{22}, \\
 \dot{x}_{13} &= x_{23}, \\
 \dot{x}_{21} &= -a_2 x_{11} + a_3 x_{12} - a_4 x_{21} + a_1 u + \gamma_1(\cdot), \\
 \dot{x}_{22} &= b_1 x_{11} - b_2 x_{12} + b_3 x_{13} - b_4 x_{22} + \gamma_2(\cdot), \\
 \dot{x}_{23} &= c_1 x_{12} - c_2 x_{13} - c_3 x_{23} + \gamma_3(\cdot), \\
 y_1 &= x_{11}, \\
 y_2 &= x_{12}, \\
 y_3 &= x_{13}
 \end{aligned} \tag{18}$$

where for $i = 1, 2, 3$, x_{1_i} are the positions, x_{2_i} are the velocities, a_i , b_i and c_i are the nominal parameters, and the terms $\gamma_i(\cdot)$ are given by

$$\gamma_1(\cdot) = \begin{bmatrix} u \\ -x_{11} \\ x_{12} \\ -x_{21} \end{bmatrix}^T \begin{bmatrix} \Delta_{a_1} \\ \Delta_{a_2} \\ \Delta_{a_3} \\ \Delta_{a_4} \end{bmatrix}$$

$$\gamma_2(\cdot) = \begin{bmatrix} x_{11} \\ -x_{12} \\ x_{13} \\ -x_{22} \end{bmatrix}^T \begin{bmatrix} \Delta_{b_1} \\ \Delta_{b_2} \\ \Delta_{b_3} \\ \Delta_{b_4} \end{bmatrix}$$

$$\gamma_3(\cdot) = \begin{bmatrix} x_{12} \\ -x_{13} \\ -x_{23} \end{bmatrix}^T \begin{bmatrix} \Delta_{c_1} \\ \Delta_{c_2} \\ \Delta_{c_3} \end{bmatrix}$$

For system (18) we propose a state observer based on the observer (9), which is given by

$$\begin{aligned} \dot{\hat{x}}_{11} &= \hat{x}_{21} + c_{11}(y_1 - \hat{y}_1), \\ \dot{\hat{x}}_{12} &= x_{22} + c_{12}(y_2 - \hat{y}_2), \\ \dot{\hat{x}}_{13} &= x_{23} + c_{13}(y_3 - \hat{y}_3), \\ \dot{\hat{x}}_{21} &= -a_2x_{11} + a_3x_{12} - a_4\hat{x}_{21} - a_5\text{sign}(\hat{x}_{21}) + a_1u \\ &\quad + c_{21}(y_1 - \hat{y}_1) + c_{31}\text{sign}(y_1 - \hat{y}_1) \\ \dot{\hat{x}}_{22} &= b_1x_{11} - b_2x_{12} + b_3x_{13} - b_4\hat{x}_{22} - b_5\text{sign}(\hat{x}_{22}) \\ &\quad + c_{22}(y_2 - \hat{y}_2) + c_{32}\text{sign}(y_2 - \hat{y}_2) \\ \dot{\hat{x}}_{23} &= c_1x_{12} - c_2x_{13} - c_3\hat{x}_{23} - c_4\text{sign}(\hat{x}_{23}) \\ &\quad + c_{23}(y_3 - \hat{y}_3) + c_{33}\text{sign}(y_3 - \hat{y}_3) \\ \hat{y}_1 &= \hat{x}_{11} \\ \hat{y}_2 &= \hat{x}_{12} \\ \hat{y}_3 &= \hat{x}_{13} \end{aligned}$$

For this observer it is necessary to use low-pass filters to recover the equivalent control u_{eq} . We use the second-order low-pass filters given by

$$\ddot{y}_f + 1.414\omega_c\dot{y}_f + \omega_c^2y_f = \omega_c^2u_f, \quad (19)$$

where ω_c is the cut-off frequency, y_f is the output and u_f is the input. We use a filter for each discontinuous term in the observer and to estimate each disturbance term.

5.2.1 Numerical results of the parameter estimation for a three-DOF mass spring damper

In the numerical simulation, we take the nominal parameters to be $a_1 = 5$, $a_2 = 1,100$, $a_3 = 600$, $a_4 = 1.5$, $b_1 = 400$, $b_2 = 666.67$, $b_3 = 266.67$, $b_4 = 1$, $c_1 = 533.33$, $c_2 = 533.33$, and $c_3 = 3.0$. The parameter uncertainties are $\Delta_{a_1} = 0.05$, $\Delta_{a_2} = 10$, $\Delta_{a_3} = -3$, $\Delta_{a_4} = 1$, $\Delta_{b_1} = -5$, $\Delta_{b_2} = 12$, $\Delta_{b_3} = 1$, $\Delta_{b_4} = 0.4$, $\Delta_{c_1} = -5$, $\Delta_{c_2} = 5$, and $\Delta_{c_3} = 1$.

The parameters of the state observer are $c_{11} = c_{12} = c_{13} = 19$, $c_{21} = c_{22} = c_{23} = 94$, $c_{31} = 2$, $c_{32} = 1.3$ and $c_{33} = 2$. The cut-off frequency of the low-pass filters is $\omega_c = 90$ rad/seg.

It is important to note that the signals generated by the observer have noise. Therefore, it was necessary apply a filter.

The results for four iterations are given in Tables 3, 4 and 5, where the minimum values for $P_1(u_{eq})$ and $P_3(u_{eq})$ are found in iteration four, and for $P_2(u_{eq})$, in iteration three. Therefore, the estimated parameter uncertainties are

$$\begin{aligned} \hat{\Delta}_{a_1} &= 0.049492, \hat{\Delta}_{a_2} = 9.8749, \hat{\Delta}_{a_3} = -3.0748, \hat{\Delta}_{a_4} = \\ &1.000279, \hat{\Delta}_{b_1} = -5.0304, \hat{\Delta}_{b_2} = 11.9155, \hat{\Delta}_{b_3} = 0.946800, \\ &\hat{\Delta}_{b_4} = 0.400305, \hat{\Delta}_{c_1} = -5.017468, \hat{\Delta}_{c_2} = 4.982555, \text{ and} \\ &\hat{\Delta}_{c_3} = 0.999919. \end{aligned}$$

Table 3 Simulation results for the estimation of parameter uncertainties Δ_{a_1} , Δ_{a_2} , Δ_{a_3} and Δ_{a_4}

<i>It. i</i>	$P_{1_i}(u_{eq_i})$	$\tilde{\Delta}_{a_1}$	$\tilde{\Delta}_{a_2}$	$\tilde{\Delta}_{a_3}$	$\tilde{\Delta}_{a_4}$
1	0.0444	0.04495	8.845	-3.644	1.031
2	0.000069645	0.003827	0.8502	0.4613	-0.03058
3	0.000065718	0.000715	0.1797	0.1079	-0.000141
4	0.000060197	-	-	-	-

Table 4 Simulation results for the estimation of parameter uncertainties Δ_{b_1} , Δ_{b_2} , Δ_{b_3} and Δ_{b_4}

<i>It. i</i>	$P_{2_i}(u_{eq_i})$	$\tilde{\Delta}_{b_1}$	$\tilde{\Delta}_{b_2}$	$\tilde{\Delta}_{b_3}$	$\tilde{\Delta}_{b_4}$
1	0.1783	-5.613	10.43	0.1254	0.4617
2	0.000050329	0.4739	1.218	0.6655	-0.06018
3	0.000034503	0.1087	0.2675	0.1559	-0.001215
4	0.000038463	-	-	-	-

Table 5 Simulation results for the estimation of parameter uncertainties Δ_{c_1} , Δ_{c_2} , and Δ_{c_3}

<i>It. i</i>	$P_{3_i}(u_{eq_i})$	$\tilde{\Delta}_{c_1}$	$\tilde{\Delta}_{c_2}$	$\tilde{\Delta}_{c_3}$
1	0.0726	-5.199	4.817	1.031
2	0.000038947	0.1817	0.1657	-0.03108
3	0.00005942	-0.000168	-0.000146	-0.000007832
4	0.000021241	-	-	-

5.2.2 Experimental results of the parameter estimation for a three-DOF mass spring damper

For the experiments we used the mechanical system shown in Figure 8 with the same nominal parameters as were used in the numerical simulations. The parameters of the state observer are $c_{11} = c_{12} = c_{13} = c_{21} = c_{22} = c_{23} = 94$, $c_{31} = 2$, $c_{32} = 1.3$ and $c_{33} = 2$. The cut-off frequency of the low-pass filters is $\omega_c = 90$ rad/seg.

The results for five iterations are given in Tables 6, 7 and 8, where the minimum value for $P_1(u_{eq})$ and $P_2(u_{eq})$ are found in iteration four, and for $P_3(u_{eq})$, in iteration three. Then, the final values for the parameters of the system are $a_1 + \hat{\Delta}_{a_1} = 2.1671$, $a_2 + \hat{\Delta}_{a_2} = 653.85$, $a_3 + \hat{\Delta}_{a_3} = 517.58$, $a_4 + \hat{\Delta}_{a_4} = 6.8732$, $b_1 + \hat{\Delta}_{b_1} = 541.43$, $b_2 + \hat{\Delta}_{b_2} = 795.4$, $b_3 + \hat{\Delta}_{b_3} = 227.63$, $b_4 + \hat{\Delta}_{b_4} = 3.2338$, $c_1 + \hat{\Delta}_{c_1} = 216.65$, $c_2 + \hat{\Delta}_{c_2} = 219.49$ and $c_3 + \hat{\Delta}_{c_3} = 2.3237$.

Finally, Figure 9 shows a comparison between the equivalent output injection for the first iteration, black line, and for the iteration which corresponds to the best parameter estimation, red line. Here we can notice a significant attenuation of the equivalent output injection for each degree of freedom.

Table 6 Experimental results for the estimation of parameter uncertainties Δ_{a_1} , Δ_{a_2} , Δ_{a_3} and Δ_{a_4}

It. i	$P_{1_i}(u_{eq_i})$	$\tilde{\Delta}_{a_1}$	$\tilde{\Delta}_{a_2}$	$\tilde{\Delta}_{a_3}$	$\tilde{\Delta}_{a_4}$
1	3.7413	-1.628	-218.6	22.98	-1.257
2	0.548	-1.035	-157.1	-46.34	6.636
3	0.0132	-0.1699	-70.45	-59.06	-0.005794
4	0.0111	-45.46	-45.46	-38.69	-0.2615
5	0.0505	-	-	-	-

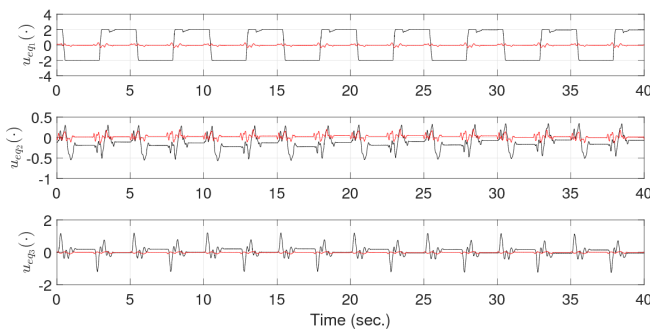
Table 7 Experimental results for the estimation of parameter uncertainties Δ_{b_1} , Δ_{b_2} , Δ_{b_3} and Δ_{b_4}

It. i	$P_{2_i}(u_{eq_i})$	$\tilde{\Delta}_{b_1}$	$\tilde{\Delta}_{b_2}$	$\tilde{\Delta}_{b_3}$	$\tilde{\Delta}_{b_4}$
1	0.0445	26.2	-20.15	-41.02	4.089
2	0.051	68.66	79.89	-20.06	-1.964
3	0.0035	46.57	68.99	22.04	0.1088
4	0.003	27.43	38.88	11.69	0.1193
5	0.0056	-	-	-	-

Table 8 Experimental results for the estimation of parameter uncertainties Δ_{c_1} , Δ_{c_2} , and Δ_{c_3}

It. i	$P_{3_i}(u_{eq_i})$	$\tilde{\Delta}_{c_1}$	$\tilde{\Delta}_{c_2}$	$\tilde{\Delta}_{c_3}$
1	0.1055	-283.9	-281.5	-0.4647
2	0.003	-32.78	-32.34	-0.2116
3	0.0011	-1.03	-0.9549	-0.007009
4	0.0015	-1.698	-1.717	-0.01018
5	0.0045	-	-	-

Figure 8 Mass-spring-damper mechanical system with three degrees of freedom used in experiments (see online version for colours)

Figure 9 Comparison between the equivalent output injection for the first iteration, black line, and for the iteration which corresponds to the best parameter estimation, red line (see online version for colours)


5.3 Parameter estimation of a simple pendulum

In this section we present the parameter estimation of a simple pendulum, with Coulomb friction, shown in Figure 10. The model of this system is given by

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -a \sin(x_1) - bx_2 - c \operatorname{sign}(x_2) + du + \varphi^T(\cdot)\theta, \\ y &= x_1, \end{aligned} \quad (20)$$

where x_1 is the angular position, x_2 is the angular velocity, a , b , c and d are the nominal parameter values, and

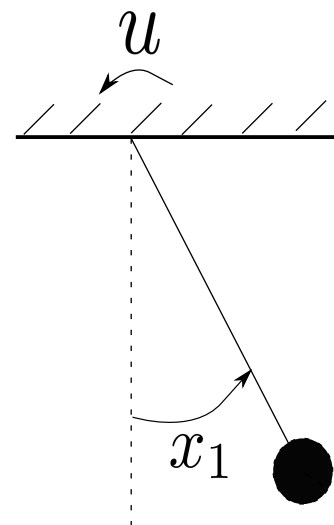
$$\begin{aligned} \varphi(\cdot) &= \begin{bmatrix} -\sin(x_1) \\ -x_2 \\ -\operatorname{sign}(x_2) \\ u \end{bmatrix}, \\ \theta &= [\Delta_a \Delta_b \Delta_c \Delta_d]^T, \end{aligned}$$

where Δ_a , Δ_b , Δ_c and Δ_d are the parameter uncertainties. If the state variables x_1 and x_2 are bounded, the term $\varphi^T(\cdot)\theta$ is bounded too.

As in the previous subsection, we use the state observer (9) to estimate the velocity and the disturbance term in system (20). The state observer is given by

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + c_1(y - \hat{y}), \\ \dot{\hat{x}}_2 &= -a \sin(x_1) - b\hat{x}_2 - c \operatorname{sign}(\hat{x}_2) + du \\ &\quad + c_2(y - \hat{y}) + c_3 \operatorname{sign}(y - \hat{y}), \\ y &= x_1, \end{aligned}$$

and we use the low-pass filter given by equation (19) to recover the equivalent control.

Figure 10 A simple pendulum


5.3.1 Numerical results of the parameter estimation of a simple pendulum

For the numerical simulations, the nominal values of the parameters of the pendulum are $a = 76$, $b = 2.5$, $c = 0.5$ and the parameter uncertainties are $\Delta_a = 5$, $\Delta_b = -0.8$, $\Delta_c = -0.2$, $\Delta_d = -4$. For the state observer the gains are $c_1 = 17.5$, $c_2 = 57.25$ and $c_3 = 30$. The input u is a square wave with amplitude 0.2 and frequency 0.1 Hz.

In this case, the estimated velocity and disturbance terms have noise, and so we filter those signals.

The results of the estimation process are shown in Table 9 where we can see that the fifth iteration corresponds to the minimum value of the $P(u_{eq})$, with the estimated parameter uncertainties $\hat{\Delta}_a = 4.9257$, $\hat{\Delta}_b = -0.8009$, $\hat{\Delta}_c = -0.18915$ and $\hat{\Delta}_d = -4.1018$, which are very near to the actual parameter uncertainties.

Table 9 Simulation results for the parameter estimation of a simple pendulum with Coulomb friction

It. i	$P(u_{eq})$	$\hat{\Delta}_a$	$\hat{\Delta}_b$	$\hat{\Delta}_c$	$\hat{\Delta}_d$
1	11.3516	1.946	-1.051	-0.2541	-7.64
2	0.3507	3.283	0.1274	0.04252	3.847
3	0.0595	0.2678	0.1334	0.01207	0.3875
4	0.0223	-0.4244	0.008904	0.005385	-0.5084
5	0.0171	-0.1467	-0.0196	0.004979	-0.1879
6	0.0159				

5.3.2 Experimental results of the parameter estimation of a simple pendulum

For the experiments, we used the simple pendulum shown in Figure 11, and the same nominal parameter values as well as the same state observer and low-pass filter as were used in the previous simulation.

Figure 11 Simple pendulum used in experiments



The results of the estimation process are shown in Table 10 where we can see that the fourth iteration corresponds

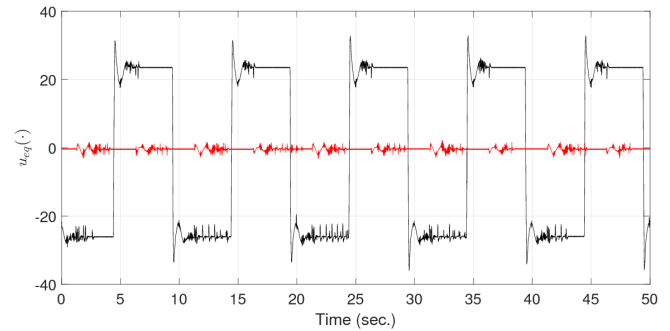
to the minimum value of the $P(u_{eq})$, with the estimated parameter uncertainties $\hat{\Delta}_a = -49.467$, $\hat{\Delta}_b = -2.31417$, $\hat{\Delta}_c = -0.029669$ and $\hat{\Delta}_d = -96.0454$. Therefore, the actual values of the parameters of the simple pendulum are $a + \hat{\Delta}_a = 24.709$, $b + \hat{\Delta}_b = 0.1657$, $c + \hat{\Delta}_c = 0.479985$ and $d + \hat{\Delta}_d = 6.657$.

Table 10 Experimental results for the parameter estimation of a simple pendulum with Coulomb friction

It. i	$P_{1_i}(u_{eq_i})$	$\hat{\Delta}_a$	$\hat{\Delta}_b$	$\hat{\Delta}_c$	$\hat{\Delta}_d$
1	608.1348	-23.14	-0.4726	-0.009184	-87.7
2	1.2931	-24.78	-1.319	-0.0129	-7.614
3	0.3781	-3.371	-0.5427	0.002069	-1.029
4	0.3543	1.824	0.02013	-0.009654	0.2976
5	0.4494				

Finally, Figure 12 shows a comparison between the equivalent output injection for the first iteration, black line, and for the iteration which corresponds to the best parameter estimation, red line. A significant attenuation of the equivalent output injection can be seen observed.

Figure 12 Comparison between the equivalent output injection in the simple pendulum for the first iteration, black line, and for the iteration which corresponds to the best parameter estimation, red line (see online version for colours)



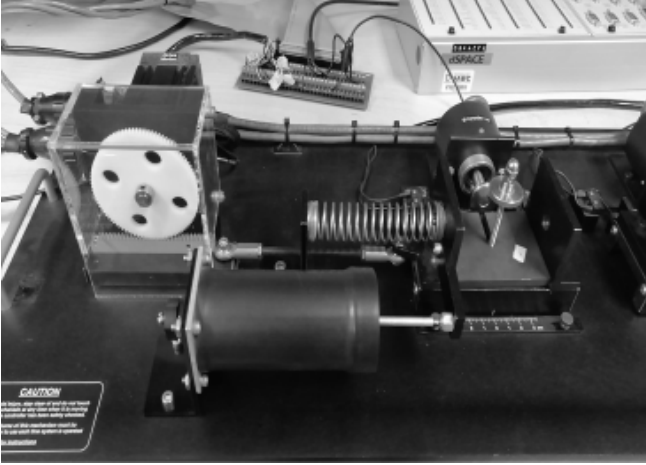
6 Performance comparison with other parameter identification strategies

In this section, we compare the performance of the proposed parameter identification methodology with different approaches published previously, in particular, with the tools in the system identification toolbox from MATLAB, and the VSS-based direct identification method proposed in Xu and Hashimoto (1993). In this comparison, we use a mass-spring-damper system of one degree of freedom, shown in Figure 13, whose model is given by

$$\begin{aligned}
 \dot{x}_1 &= x_2, \\
 \dot{x}_2 &= -\alpha x_1 - \beta x_2 + \rho u, \\
 y &= x_1,
 \end{aligned} \tag{21}$$

where x_1 and x_2 are the position and velocity, respectively, α , β , and ρ are the parameters to identify.

Figure 13 Mass-spring-damper system of one degree of freedom



For all strategies, we applied a square wave with a period of six seconds, 50% duty cycle, and an amplitude of 1 volt. For the system identification toolbox from MATLAB, we carried out six experiments: we created the objects with the function `iddata()` and merged all objects with `merge()`. Finally, we obtained the transfer function of the system using `tffest()`, getting the parameters $\alpha = 174.2$, $\beta = 12.570$, and $\rho = 2.567$. The index $P(u_{eq})$ for these parameters is $P(u_{eq}) = 0.0207$.

For the VSS-based direct identification method proposed in Xu and Hashimoto (1993) we assume that the parameters satisfy the following inequalities

$$\begin{aligned} -500 < -\alpha < -100, \\ -20 < -\beta < -5, \\ 1 < \rho < 5. \end{aligned}$$

System (21) has the form

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \rho(x, p) + \lambda(x, p)u, \end{aligned}$$

where $\rho(x, p) = -\alpha x_1 - \beta x_2$ and $\lambda(x, p) = \rho$. Then, the identifier has the form

$$\begin{aligned} \hat{x}_1 &= \hat{x}_2 \\ \dot{\hat{x}}_2 &= \hat{\alpha}_0(t)^T \begin{bmatrix} x_1 \\ x_2 \\ u \end{bmatrix} + v \end{aligned}$$

where $\hat{\alpha}_0(t) = [\hat{\alpha}_{0_1}(t) \hat{\alpha}_{0_2}(t) \hat{\alpha}_{0_3}(t)]^T$ and each element has the form defined in Xu and Hashimoto (1993). $v = v_c + v_v$, where $v_c = h_1(x_2 - \hat{x}_2)$, $h_1 = 20$, $v_v = d^T |\zeta|_1 \text{sign}(\sigma)$, $|\zeta|_1 = [|x_1| |x_2| |v|]^T$, $d = [400 \ 15 \ 4]^T$ and $\sigma = h_1(x_1 - \hat{x}_1) + (x_2 - \hat{x}_2)$. The identification algorithm is

$$\dot{\hat{\alpha}}(t) = \Gamma \zeta \omega, \quad (22)$$

where $\Gamma = \Gamma^T > 0$. The equivalent control is given by $\omega = u_{eq} - v_c + (\hat{\alpha}_0(t) - \hat{\alpha}(t))^T \zeta$. To obtain it we use a first-order low-pass filter given by $\dot{y}_f = -y_f/\tau + u_f/\tau$ with $\tau = 0.02$. With this method we obtain the parameters $\alpha = 161.2$, $\beta = 8.095$ and $\rho = 2.281$, and the index $P(u_{eq}) = 0.0245$.

Lastly, we applied the methodology proposed in this paper. In this case, we took the parameters obtained with the system identification toolbox functions as the nominal values of the system and used the observer proposed in Subsection 3.2 to estimate the velocity and the equivalent output injection. After five iterations, we obtained the parameters $\alpha = 140.903$, $\beta = 10.0086$ and $\rho = 2.249$, and the index $P(u_{eq}) = 0.0028$.

As we can see, the methodology proposed in this paper obtained the best estimation according to the index $P(u_{eq})$.

7 Conclusions

We have proposed a methodology to estimate the parameters of a broad class of dynamical systems: linear, nonlinear, and discontinuous, that satisfy the property of parameter linearity. The equivalent output injection phenomenon, present in discontinuous state observers, is the basis for implementing the terms needed by the least squares algorithm used in the methodology. The main advantages of this proposal are the following: the existence of an index that allows the evaluation of the parameter estimation in each iteration, its capacity to estimate parameters in discontinuous systems, like the coefficient of the signum function in the simple pendulum, and the facility to incorporate new observers to improve its performance.

Because the methodology has to process and analyse signals offline, it takes much time to develop all experiments. Also, if the signals involved in the identification process have noise, the noise must be reduced using some appropriate technique because the least squares algorithm is sensitive to noise. However, this additional effort is compensated by obtaining a model that will later facilitate the design and implementation of controllers.

It is important to note that a factor that affects the parameter estimation strategy is the exactitude of estimating disturbance term, which depends on the performance of the state observer. Therefore, in an experimental context, we have to use a state observer with adequate performance. Finally, as future work, we propose the design of discontinuous observers with better experimental performance to improve the quality of estimation of the disturbance terms; in this way, the methodology performance will improve.

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