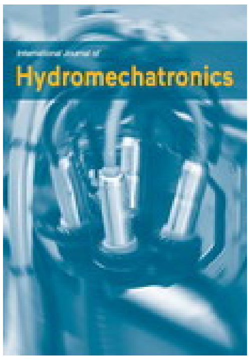

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Applications of level set method in computational fluid dynamics: a review

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Abstract: Tracking of free interfaces in two-phase and multi-phase fluids is a critical step in computational fluid dynamics. Among the many methods, because of no need for parameterisation of curves and an excellent solution to the problem of evolutionary curve topology change, the level set method (LSM) is widely used in the field and has achieved good results. The paper reviews applications of LSM in the tracking of free interfaces, including theory fundamental, solving the basic partial differential equation used to represent fluids in LSM, free interfaces tracking of two-phase fluids, interfaces evolutions of multi-phase fluids, and coupling with other methods to increase tracking performance. Based on the summaries, we confirm the level set method has achieved excellent results in fluid interface tracking either alone or coupled with other algorithms. Of course, the level set method requires further optimisation in terms of initialisation and mass conservation.

Keywords: computational fluid dynamics; CFD; curve evolution; interface capture; level set method; LSM; implicit representation.

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1 Introduction

Free interface tracking is a very important stage in fluid dynamics (Yang et al., 2006). Methods for studying interface revolution usually include theoretical methods, experimental methods, and numerical simulations (Ferziger et al., 1997). There is still much work to be done in applying the theory to practice due to the current high computational cost. In many cases, experimental studies are difficult to carry out (Olsson et al., 2007). Even if they are carried out, the problem of low precision is always unavoidable. Therefore, numerical simulation methods are gaining more and more attention and have achieved good results in calculating two or more fluid interactions (Huang et al., 2007).

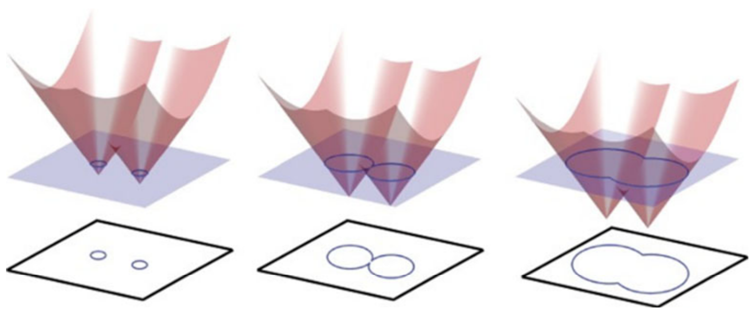
The application of numerical simulation in fluid mechanics is also known as computational fluid dynamics (CFD). To simulate the evolution of the interface between different fluids or different forms of the same fluid, the interface must be represented mathematically firstly (Cottet et al., 2008). The motion interface is usually expressed using Lagrangian or Eulerian formulas (White, 2002). Front tracking methods based on the Lagrangian formula can deal with the deformation and interaction of the two-phase interface well (Adalsteinsson and Sethian, 1995). But they have a very high computation cost, and the methods are not good at dealing with complex topology change problems (Sussman et al., 1998). The methods with an implicit representation of interface are usually based on the Eulerian formula, level set method (LSM), lattice Boltzmann method (LBM), volume-of-fluid (VOF), and phase-field method (PFM) are typical representatives (Schunk et al., 2003). They are also known as interface capturing methods which are particularly suitable for performing complex structural microfluidic calculations (Deng et al., 2018).

WF Noh et al. combined Lagrange's formula and Euler's formula and created a new calculation method: arbitrary Lagrange Euler (ALE) to study two-dimensional fluid dynamics problems (Noh, 1963). Although ALE combined many advantages of the Lagrangian and Eulerian formulas, only small boundary deformations can be tracked (Osher and Fedkiw, 2001). The performance of the fluid boundary tracking would be intensely affected when there are topological changes such as separation and folding

(Hua et al., 2013; Breen and Whitaker, 2001). As a result, the implicit methods based on the Euler formula are widely used in performing motion interface tracking.

The LSM was devised by Osher and Sethian in 1988 to research the shape change problem of flame. No curve parameterisation is required, and the method is straightforward to use (Sethian, 1996). A curve is suitable for dealing with large deformations and complex topology changes if implanted on a higher one-dimensional surface as a zero level set, which can significantly reduce the computation time (Adalsteinsson and Sethian, 2003; Coquerelle and Cottet, 2008). The LSM method has been successfully utilised in two-dimensional and three-dimensional fluid interfaces interaction calculations (Gibou et al., 2017), such as bubble formation and generation in liquids, droplet breakup, etc. (Figure 1).

Figure 1 Free interface capture based on the LSM (see online version for colours)



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In this paper, we summarise applications of the LSM in CFD focusing on two-phase fluid and multi-phase fluid. The paper is organised as follows: in Section 2, basic theory of curve evolution and LSM are listed; in Section 3, solving of level set equations are surveyed; in Section 4, we review the use of LSM to track free surface in two-phase fluid; in Section 5, utilisations of LSM in the multi-phase fluid are analysed; in section 6, coupling of LSM and other interface tracking methods is described in detail. Finally, the future development of LSM is prospected.

2 Fundamentals of LSM

The evolution process of the free moving interface based on the LSM method is still essentially the evolution of curves. Therefore, we firstly give a simple description of the curve evolution.

2.1 Evolution of the curve

The theory of curve evolution is essentially the study of the variations of curves with time under the action of geometric parameters (Sethian, 1985). The geometric parameters include unit normal vector and curvature, etc. The direction of curves change was described by the unit normal vector, while the degree of curves bend was determined by the curvature (Garcke, 2013). The process of curves evolution can be described as follows: a closed smooth curve changes along the normal direction with a specific speed to form curve clusters, and the time is treated as a variable (Duarte et al., 2004) [Figure 3(a)].

Assume $C = C(p)$ is a closed smooth curve, p are random parameterised variables (Gage, 1984). In two-dimensional Euclidean space R^2, $C(p, t) = (x(p, t), y(p, t))$ is a closed curve that varies with time *t*. Then the variation of the curve with time can be expressed using the following partial differential equation:

$$\frac{\partial C}{\partial t} = V(p,t) = \alpha\vec{T}(p,t) + \beta\vec{N}(p,t) \tag{1}$$

$\vec{T}$ means the unit tangential vector, and $\vec{N}$ denotes the unit normal vector, α and β are constants (Huisken, 1984). V is a speed vector consisting of the unit tangential vector $\vec{T}$ and the unit normal vector $\vec{N}$.

Since only the velocity in the normal direction affects shapes of the curve, while the velocity in the tangential direction does not play a role in the change of curve shape, equation (1) can be simplified as:

$$\frac{\partial C}{\partial t} = V(p,t) = \beta\vec{N}(p,t) \tag{2}$$

The equation of curve evolution can be rewritten as:

$$\frac{\partial C}{\partial t} = F\vec{N} \tag{3}$$

F is the velocity function that determines the evolutionary velocity of the points on the curve, and $\vec{N}$ is the unit normal vector that determines the direction of the evolution of points on the curve (Sethian, 1989). The curve is deformed by the combined action of F and $\vec{N}$. Driven by the velocity function F, the curve evolves with a certain velocity along the normal direction. There are many ways to express the velocity function F, and the common forms are the constant evolution and the curvature evolution.

If F is a constant, the curve evolution equation can be expressed as a constant evolution equation:

$$\frac{\partial C}{\partial t} = V_0\vec{N} \tag{4}$$

It is called constant evolution, and the velocity (V_0) at any point on the curve is the same. If $V_0 > 0$, the curve shrinks inward, and if $V_0 < 0$, curves expand outward. Since the velocity is the same at each point of the curve, smooth and closed curves driven by this equation are prone to sharp corners and cannot handle topology changes (Sethian and Smereka, 2003).

If F is a function of variations, the evolution equation is usually expressed by the following equation:

$$\frac{\partial C}{\partial t} = \alpha k\vec{N} \tag{5}$$

α is a constant, and k is the curvature of the curve. Velocity function F is proportional to the curvature k. The greater the curvature is, the faster the curve evolves.

2.2 LSM theory

Traditional curve evolution requires a parametric representation of curves, but parametric curves can hardly cope with topological changes during curve evolution. The change of curves involves the computation of the normal vector and the curvature, so researchers apply the LSM to curve evolution calculation.

Free interface tracking is usually divided into three steps:

1. Describe the location and shape of the interface using appropriate numerical methods.
2. Introduce appropriate algorithms approach to track the change of the interface.
3. Adding interface boundary conditions.

The presentation of the LSM will also start from these three processes.

2.2.1 LSM-based interface representation

The core of the LSM is a continuous function in the one-dimensional higher space containing the interface. The position of the phase interface can be computed implicitly by tracing the contour of the function in space. The moving deformed curve (surface) is embedded as a zero level set into a one-dimensional higher function, and the evolution equation of the function can be obtained from the evolution equation of the closed hypersurface (Zheng and Zhang, 2000). The embedded closed curve (surface) is always kept as the point set of the function on the zero level section. Finally, the evolution result of the moving deformed curve (surface) can be obtained as long as the points position of the evolution function on the zero level section is obtained (Sussman et al., 1994).

A plane curve is represented through the equation, $y = f(x)$. Then, an implicit function $y - f(x) = 0$ can describe the relationship between x and y. Therefore, the curve can be argued as a particular unique example of the equation, $\phi(x, y) = y - f(x)$. $\phi(x, y) = 0$ is the implicit representation of the curve. The function $\phi(x, y)$ is also known as the level set function (LSF).

Combined with the evolution time parameter, the LSF is converted to $\phi(C(t), t)$. Its zero level set $\phi(C(t), t) = 0$ represents the curve in the evolutionary process (Sharma, 2015).

According to the chain rule of derivation, the following equation can be obtained for the LSF of the curve:

$$\frac{\partial \phi}{\partial t} + \nabla \phi \cdot \frac{\partial C}{\partial t} = 0 \tag{6}$$

$\nabla \phi$ is the gradient of ϕ, $\nabla \phi = \frac{\partial \phi}{\partial c} = \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial x}$.

Assuming that the function has a negative value inside the curve C and a positive value outside, its inward unit normal vector $\vec{N} = -\frac{\nabla\phi}{|\nabla\phi|}$.

With the above equations, the following derivation can be made:

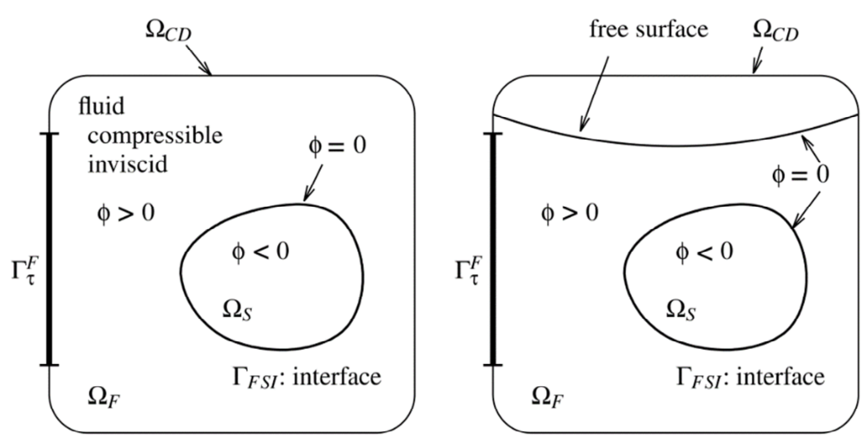
$$\frac{\partial\phi}{\partial t} = -\nabla\phi\cdot\frac{\partial C}{\partial t} = -\nabla\phi\cdot\left(\alpha k\vec{N}\right) = -\nabla\phi\cdot\left(\alpha k\left(-\frac{\nabla\phi}{|\nabla\phi|}\right)\right) = \alpha k\,|\nabla\phi| \tag{7}$$

The formula is the iteration equation for the LSF ϕ. According to the geometric meaning of curvature, $=\nabla\cdot\frac{\nabla\phi}{|\nabla\phi|}$. It is evident that curve C is no longer included in the above equation, so the curve evolution driving force is not related to the curve parameters, but only to the geometric properties of the curve (Samuel, 2014).

The evolution equation for the LSF illustrates that it is sufficient to follow the evolution of the curve C by solving the time-varying partial differential equation [formula (7)].

In the interface evolution problem, there can be many continuous functions that can be used to represent the initial interface. In solving partial differential equations, the calculation of the first-order derivatives and second-order derivatives of the LSF is inevitable (Wacławczyk, 2017). Thereby the smoothness of the LSF is crucial. Otherwise, the accuracy of the calculation will be seriously affected.

Figure 2 Fluid interaction, (a) with no free interface (b) with free interface



(a) (b)

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The symbolic distance function (SDF) is usually chosen as the initial expression of the LSF. Next, we describe the SDF. Take the evolution of the LSF in two-dimensional space as an example, $C = C(p)$ is the initial curve. Generally, the SDF is defined as:

$$\phi(x, y, t=0) = sign\left(x, y, C(t=0)\right)\cdot dist\left(x, y, C(t=0)\right) \tag{8}$$

sign$(x, y, C(t = 0))$ is a symbolic function with the value +1 or –1 and *dist*$(x, y, C(t = 0))$ is the shortest distance from a point (x, y) to the initial curve $C(t = 0)$. When a point (x, y) is in the interior of the curve $C(t = 0)$, *sign*$(x, y, C(t = 0)) = -1$. Conversely, if the point is outside the curve, *sign*$(x, y, C(t = 0)) = +1$. Obviously, if the point is on the curve, *dist*$(x, y, C(t = 0) = 0$.

In summary, the SDF can be represented as the following form:

$$\phi_0 = \begin{cases} -dist\left(x, y, C(t=0)\right) & (x, y) \text{ inside } C \\ 0 & (x, y) \text{ on } C \\ dist\left(x, y, C(t=0)\right) & (x, y) \text{ outside } C \end{cases} \tag{9}$$

An essential reason for SDF suitable for a LSF is that its gradient is 1, $|\nabla\phi| = 1$. Therefore, the size of the discrete grid can be guaranteed to be 1 in the following numerical calculation, which makes the numerical calculation with high accuracy (Legay et al., 2006) (Figure 2).

2.2.2 Computational solution for interface evolution and re-initialisation

As mentioned above, the essence of the LSM is to solve a time-varying partial differential equation, and the discrete form of the evolution equation is an essential step for numerical computation (Lervåg, 2014). Since the LSM always remains as a valid function during the iteration, the LSF $\phi(x, y, t)$ can be expressed in the form of a discrete grid.

Take two-dimensional space as an example, h is the interval of the discrete grid, Δt is the time step. At moment n, the value of the LSF of the grid point (x, y) is $\phi(ih, jh, n\Delta t)$, the curve evolution equation [formula (7)] can be discretised as:

$$\frac{\phi_{ij}^{n+1} - \phi_{ij}^{n}}{\Delta t} = \alpha k_{ij}^{n} \left|\nabla_{ij}\phi_{ij}^{n}\right| \tag{10}$$

αk_{ij}^{n} denotes the evolutionary speed at the point (x, y) at the moment n. There are many ways to solve the formula (10), and the following is a description of a frequently used upwind finite difference method (Firehammer and Desjardins, 2015).

The first-order central difference operators are defined as follow:

$$\phi_x^0 = \frac{1}{2h}\left(\phi_{i+1,j} - \phi_{i-1,j}\right) \tag{11}$$

$$\phi_y^0 = \frac{1}{2h}\left(\phi_{i,j+1} - \phi_{i,j-1}\right) \tag{12}$$

The first-order forward difference operators are defined as follow:

$$\phi_x^+ = \frac{1}{h}\left(\phi_{i+1,j} - \phi_{i,j}\right) \tag{13}$$

$$\phi_y^+ = \frac{1}{h}\left(\phi_{i,j+1} - \phi_{i,j}\right) \tag{14}$$

The first-order backward difference operators are defined as follow:

$$\phi_x^- = \frac{1}{h}\left(\phi_{i,j} - \phi_{i-1,j}\right) \tag{15}$$

$$\phi_y^- = \frac{1}{h}\left(\phi_{i,j} - \phi_{i,j-1}\right) \tag{16}$$

With the six operators from formula (11) to formula (16), discretised curve evolution equation [formula (10)] transforms into:

$$\phi_{ij}^{n+1} = \phi_{ij}^n + \Delta t\left(\max\left(\alpha k_{ij}^n, 0\right)\nabla^+ + \min\left(\alpha k_{ij}^n, 0\right)\nabla^-\right) \tag{17}$$

∇^+ and ∇^- are defined as follow separately:

$$\nabla^+ = \left[\max\left(\phi_x^-, 0\right)^2 + \min\left(\phi_x^+, 0\right)^2 + \max\left(\phi_y^-, 0\right)^2 - \min\left(\phi_y^+, 0\right)^2\right]^{1/2} \tag{18}$$

$$\nabla^- = \left[\max\left(\phi_x^+, 0\right)^2 + \min\left(\phi_x^-, 0\right)^2 + \max\left(\phi_y^+, 0\right)^2 - \min\left(\phi_y^-, 0\right)^2\right]^{1/2} \tag{19}$$

The LSF can be continuously updated through the formula (17). To keep the stability and convergence of the evolution, attention must be paid to the choice of the time step Δt. It is sufficient to satisfy the following equation during the evolutionary process with constant interval h of the discrete grid (Sethian and Adalsteinsson, 1997).

$$\alpha k \cdot \Delta t \le h \tag{20}$$

The curve evolution requires several iterations until it reaches a stable convergence state. Therefore, the LSF also requires several iterations (Nochetto and Walker, 2010) [Figure 3(b)]. The LSF is likely to create oscillations during the evolution process. The fluctuations do not guarantee the smoothness and distance function properties, resulting in error accumulation, and will cause the final result deviates from the actual situation. Therefore, during the iteration of the LSF, it is necessary to conduct the re-initialisation. Recalculate the SDF on account of the closed curve $C(x, y, t)$ represented by the zero level set to take the place of the contemporary LSF $\phi(x, y, t)$.

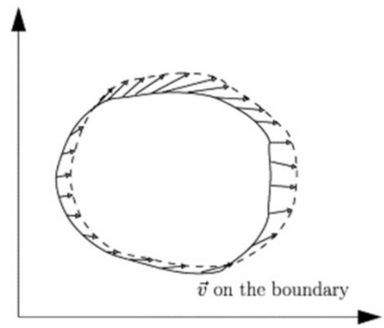
Re-initialisation of the LSF increases the computational burden of geometric curve evolution, and the iteration interval for re-initialisation of the LSF is very difficult to determine (Slavov and Dimova, 2007). A small iteration interval can lead to a more severe computational burden, which dramatically decreases the efficiency of the LSM. In contrast, a large iteration interval will lead to instability of the LSM, resulting in excessive computational errors.

A widely used re-initialisation method is realised by solving the under partial differential equation:

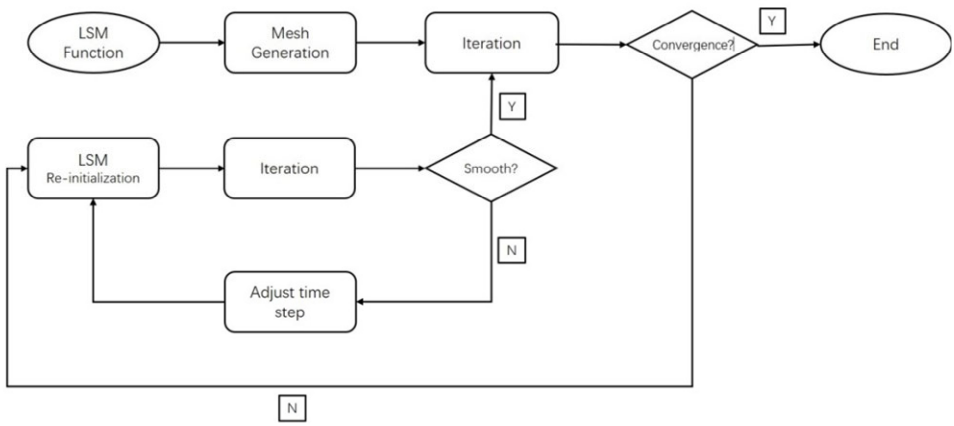
$$\frac{\partial \phi}{\partial t} = sign\left(\phi_0\right)\left(1 - |\nabla \phi|\right) \tag{21}$$

ϕ_0 is the LSF which should be re-initialised. *sign*() is a symbolic function used to label the symbol value of the distance function inside and outside the closed curve.

Figure 3 (a) The boundary velocity determines the change in the shape of the curve (b) The process of solving LSM function



(a)



(b)

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2.2.3 Conditions for evolution termination

Free interface evolution cannot proceed indefinitely, and even if it can, it will not make much sense. If the zero level set evolves to the interest target boundary, the curve stops evolving to avoid generating too many iterations. Therefore, the level set algorithm needs an iteration termination condition, and the termination conditions are required during the evolution of the iterative solution.

Currently, the literature dealing with termination conditions of the level set algorithm is relatively scarce. Wang et al. (2010) introduced a termination condition based on the evolutionary curve length change. The length difference of the contour curve is calculated twice in a row, and the iteration stops when the curve evolves to the target region with little length change. The termination condition based on the change of external forces was presented by Zhang et al. (2013). During the curve evolution, external forces attract the curve close to the target boundary. When the evolution reaches the natural border, the curve evolution slows down and finally stops at the target boundary.

Therefore, the absolute value of the difference of the two successive external forces is calculated. If the change in the external force is less than a given threshold for a set number of consecutive times, the evolution will stop.

In practical applications, the number of iterations is usually set in advance. Wait until the end of the iteration the final result is checked. If the evolutionary effect is not sufficient, the number of iterations is increased. Conversely, the number of iterations is reduced. The above presentation implies that experience plays a critical role during the evolution of the free interface through the LSM.

3 Advances on LSM solving

The LSM includes initialisation, iteration, re-initialisation, and other steps. To increase the efficiency and stability of the algorithm, we need to study its various stages. Any improvement in the detail of LSM is likely to result in better algorithm performance.

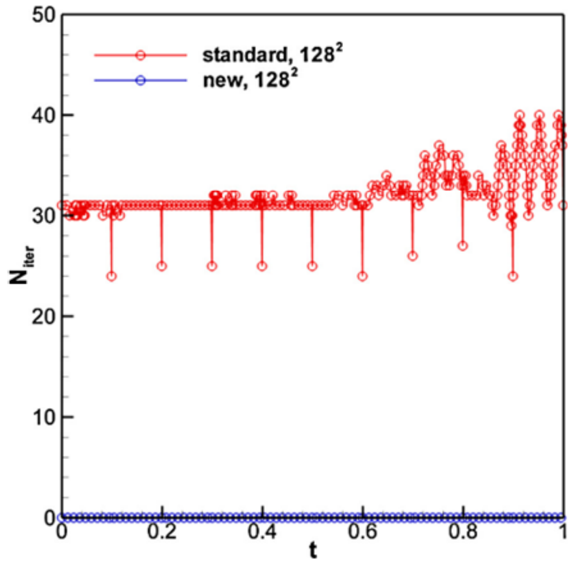
3.1 Improvements for re-initialisation issue

Re-initialisation is a crucial stage in the LSM. If the re-initialised function does not stay smooth enough, or if the deviation from the symbolic distance function is far, the re-initialisation fails. Even if the re-initialisation is successful, it makes the LSM much more computationally intensive.

Chiodi and Desjardins (2017) designed a re-initialisation method for conservative LSM. They re-initialised the convection and compression terms in the equation with the distance level set. The normals were recalculated using the fast forward method. Of course, they had experimentally verified that the performance of the method was much improved compared with the original method, and volume conservation was still guaranteed in complex fluid calculations. A source term was embedded in the LSF by Sabelnikov et al. (2014) to simplify the LSM re-initialisation (Figure 4). The equation satisfied the Eikonal equation automatically and gained a zero value on the interface. The source term improvement method was similar to the velocity expansion method, but was simpler and required re-initialisation. Compared with the traditional initialisation of the level set equation, the amount of initialisations was reduced, the calculation was more superficial, the interface was more explicit, and the number of initialisations could be controlled by changing the precision of the source term. During free interface tracking, non-ideal normals could cause false movement of the interface, so it was possible to remove them from the LSF. McCaslin and Desjardins (2014) introduced a novel local re-initialisation method of LSM. They constructed an equation equivalent to the original LSF and removed all areas that did not need to be re-initialised. Experiments such as drop impact simulations confirmed that the method was much less computationally intensive and was reliable and effective.

Li et al. (2005) devised a novel variational formulation for the active boundary. The variational formulation contained an internal energy term which made a lower deviation of the LSF from the distance function, and an external energy term that allowed the zero LSF closer to the interface. The advantage of the design was that the interface evolution process did not require re-initialisation of the LSF. The plan had already achieved perfect goals in digital image processing, and there was still a lot of experimentation and exploration to be done in the area of fluid dynamics.

Figure 4 Standard and new initialisation process requirements for the number of iterations (see online version for colours)



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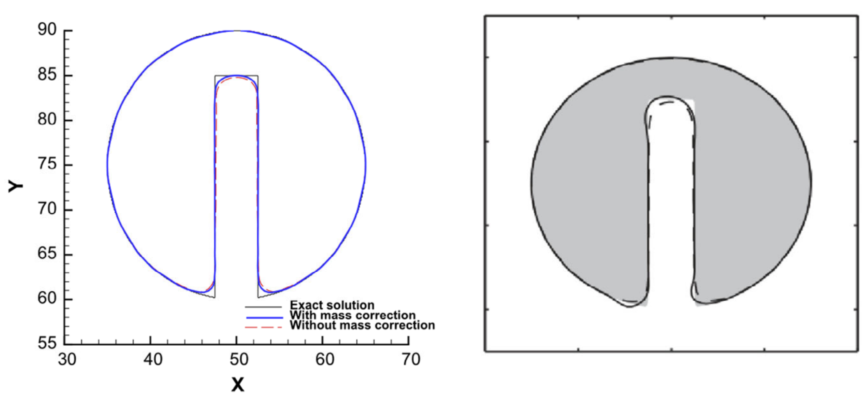
3.2 Efforts at mass conservation

The numerical calculation based on LSM is short of mass conservation due to the constant re-initialisation during the iteration. The issue significantly limits the application scope of LSM, and many researchers are working to solve the problem.

Figure 5 Interface predicted after one iteration, (a) conventional LSM; (b) LSM with mass conservation (see online version for colours)



(a) (b)

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Olsson and Kreiss (2005) employed a fuzzy heaviside function as a LSF instead of the traditional SDF. The function had a value 0 on one side and 1 on the other side, with the transition region in the middle. The transition region could implicitly represent the change of the motion interface. The entire numerical calculation was evolved using the conservation formula. Numerical tests proved that there was a second-order accuracy which also guaranteed mass conservation. Yu et al. (2016) also proposed an LSM that could ensure mass conservation during their study of bubbles rising in three-dimensional space (Figure 5). The team used a convection equation as the LSF. The iterative process re-initialised the LSF by a smoothing heaviside function to obtain a new LSF. A mass redress term was inserted into the re-initialisation process to ensure that the new LSF satisfied mass conservation. Numerical results from many experiments had shown that the LSM satisfied the requirement of mass conservation.

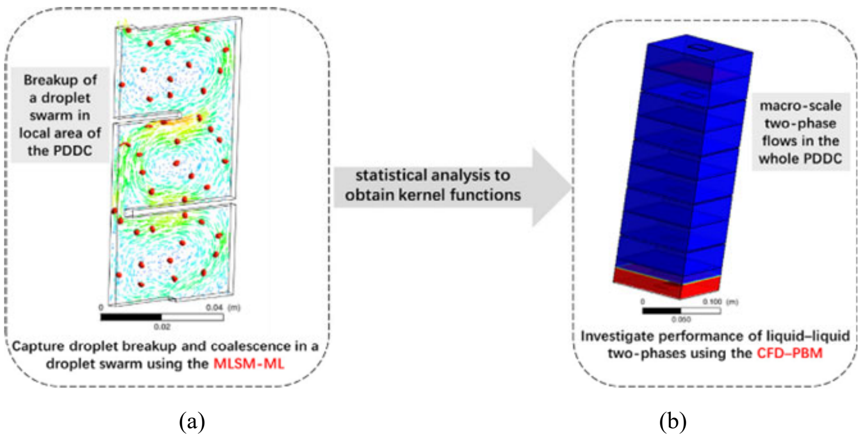
4 LSM used in two-phase flow

4.1 Liquid-liquid phase applications

Vivacqua et al. (2016) studied the aggregation behaviour of oil droplets on the water surface under the action of electric fields, and LSM was used to track the interface change between oil and water. The calculation results were sensitive to parameters such as grid size and interface thickness. There was a mapping relationship as a function between the initial volume of the oil droplet and the volume after evolving on the water surface. Dynamic mixing of plasma and blood was researched by Mehrdad and Ngok (2017) The LSM was utilised to track the mixing degree of plasma and blood. They discovered that the mixing of the two flows was enhanced if there was an obstacle between them. Guan et al. (2018) carried out work on tracking nanofluid motion using LSM, and the method was validated by nanofluid convection solution. The team performed a series of experiments about water-alumina, and the model was validated. Because driven by convection, nanoparticles were weakly mobile and could only diffuse into adjacent fluid regions. A two-phase flow numerical simulation framework was proposed by Yu et al. (2020) to research the fragmentation behaviour of droplet populations using LSM (Figure 6). After statistical analysis, they found that droplets with larger diameters were more likely to rupture and formed sub-droplets with similar radii. There was also a relationship between the droplet fragmentation frequency and its diameter. The distribution pattern of the droplets was consistent with the experimental results, indicating that the framework could replace part of the experiment.

Feppon et al. (2021) used LSM to optimise the topology of the heat exchanger. The team utilised grid evolution to track the shape and deformation of the heat exchange. All the works were performed on both two-dimensional and three-dimensional heat exchange models, and a large number of test cases had proven that the simulation method was effective. A creative framework was reported by Gao et al. (2018) to compute actions of two-phase flow. The LSM was solved using the second-order Runge-Kutta method for tracking the moving interface. The LSM function did not need to be re-initialised after each iteration, and the re-initialisation process incorporated a mass correction term, which ensured that the LSM achieved mass conservation. Several challenging experiments demonstrated the usefulness of the method. Ayuba et al. (2022) analysed the distribution of fouling in crude oil transportation pipelines based on laminar LSM. They first obtained the volumetric flow profile and the density profile inside the transport pipeline. Then they checked whether the density of a specific region was the same as that of the crude oil. If they were the same, it indicated the presence of fouling. The error analysis showed that the method was effective.

Figure 6 Frameworks used to research liquid-liquid two-phase fluidics, (a) LSM coupled with multi-levels (b) pollution balance model (see online version for colours)



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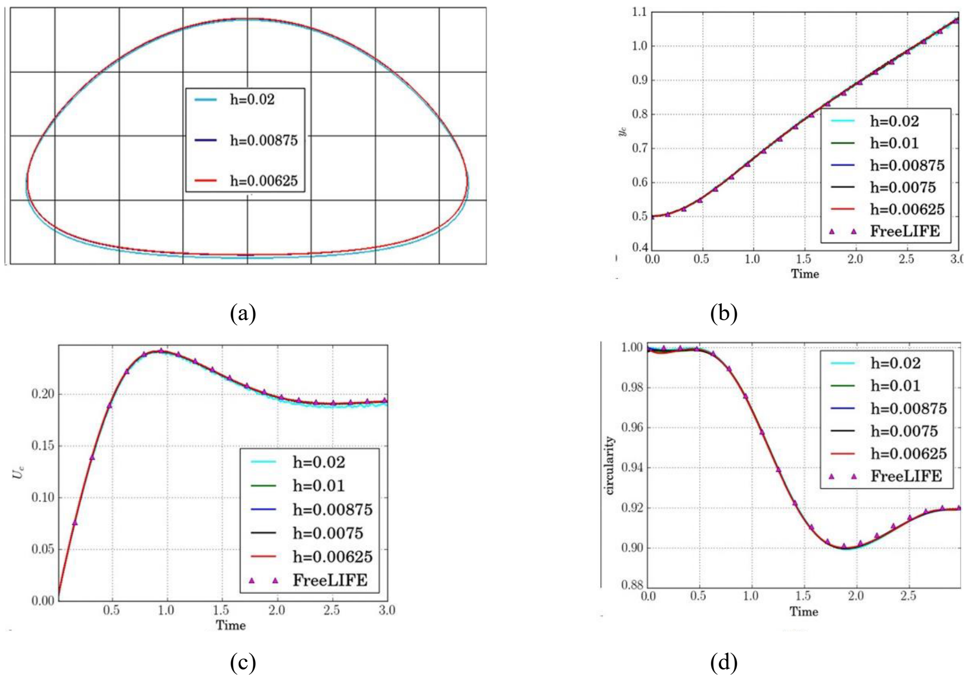
Frantzis and Grigoriadis (2019) proposed a numerical method to simulate the interaction of two incompressible fluids. The conservative LSM was used to track the evolution of the boundary, and the immersion boundary method and the fast direct solution method were used together to represent the boundary conditions. Such a combination ensured that complex shapes could be simulated without increasing the computational cost. The comparison of simulated data with experimental results proved that the method was not only practical but also computationally efficient. Mancilla et al. (2019) introduced a computational model to analyse the interaction process of two fluids with relatively significant differences in viscosity. While the immersion boundary method was used to describe the geometry of the object, the LSM distinguished and calculated the interface between fluids. The interface deformation was mainly caused by hydrodynamic stresses, and tensions and the interface would move towards the middle under the effect of gravity. The validity of the model was also confirmed by comparing the simulated data with the experimental data.

4.2 Liquid-gas phase applications

Lalanne et al. (2015) devised a new scheme to compute the viscous term of two fluids based on the hybrid LSM and ghost fluid method. The method enabled the time integration of implicit terms. Three examples of bubble-fluid interaction showed that the technique was fit for complex problems, although with minor errors. A two-fluid model based on LSM and finite element method was devised by Doyeux et al. (2013) and was used to study the process of bubbles rising in a viscous fluid (Figure 7). A Lagrange multiplier was introduced into the model to simulate the mechanical actions of red blood cells. Because the high-order polynomial was utilised in the model, the simulation accuracy had improved considerably. The simulation verified that the model was correct and valid.

Figure 7 Evolution of ellipsoidal bubbles, (a) final shape (b) vertical position change process (c) vertical speed change process (d) circularity of bubbles (see online version for colours)



Notes: (a) Shape at final shape ($t = 3$), (b) y_c vertical position, (c) vertical velocity and (d) circularity.

Source: Reproduced with permission from ref. Doyeux et al. (2013), Copyright (2013), Elsevier

Calderer et al. (2014) researched the coupling between air/water flow and rigid body with LSM. They calculated the force exerted on the object indirectly by projecting the pressure normal. This method significantly improved the ability to predict the motion of floating bodies. The simulation results showed that the scheme converged faster and reduced the error significantly, consistent with the experimental results. Flame can be seen as a special kind of fluid. Chen et al. (2018) constructed a framework to predict the spread of fire. The LSM was used to track the flame propagation front and predicted the extent to which the fire would cover. Information such as temperature field, soot, etc., could also be calculated by the framework. The validity of the framework was confirmed by comparing the simulated data with the experimental results of pine needle plates. Brissot et al. (2021) simulated the jump changes of interface and temperature during evaporation. The LSM tracked the boiling interface and calculated the speed of the jump. The calculation of temperature jump was very complex, and more work was needed to explore it. The results obtained from the analysis of the simulated data were consistent with the effects observed in the experiment. Adhikari et al. (2021) calculated the pouring process of the resin. The depth-averaged model was coupled to the LSM to track the interface change between the liquid resin and compressed air and to calculate the instantaneous position. The simulation results were consistent with the experimental data obtained using pressure sensors and cameras.

4.3 Liquid-solid phase applications

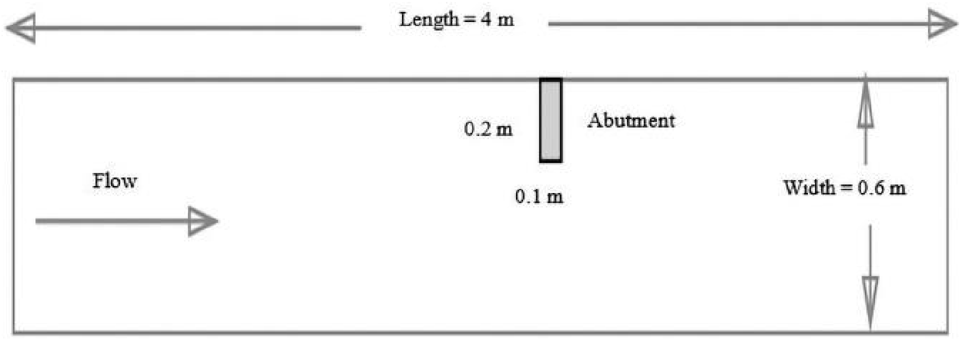
Cottet and Maitre (2006) provided an in-depth analysis of the interaction between a fluid with an immersed film and a solid by replacing the re-initialisation function of the LSM with a renormalisation process. The LSM function provided the surface stretching information to express the membrane change process. The exact discretisation operation of the partial differential equation in the solution process ensured that the method satisfied energy conservation and mass conservation. The approach was experimentally proven to be effective and could effectively control the computational cost. A new model combining LSM with immersion boundary method and deformation mesh method was devised by Jenkins and Maute (2016) to investigate the interaction between fluid and solid. The way could accommodate the moderate deformation of solids and the possible floating problem, which was of great reference value for the floating and large-scale deformation problems of rigid bodies in the future. Sun et al. (2017) calculated the issue of small-scale deformation of semi-solid materials. The interface between the solid and the fluid was implicitly represented and traced by a LSF, and the finite element calculations were performed on an anisotropic mesh. Factors such as the ratio between solid and liquid, and the physical shape of the solid were analysed for their influence during the simulation. They also found that if the fluid was fluidly restricted inside, it could trigger thermal tearing. A LSM with grid reconfiguration technology was combined to investigate the powder melting process under laser action by Zhang et al. (2018). The simulation results showed that thermodynamic surface tension had a direct effect on the area of the melt pool at the mesoscale, and at a large scale all powders including unirradiated powders must be considered as a whole. A scheme was presented by Zhang et al. (2019) to evaluate the hydrodynamic aspects of the solidification process. The team performed separate calculations for solid and fluid, tracking changes at the solid interface through LSM and predicting changes in the fluid by calculating the velocity field of the fluid. Tests on the directional solidification problem showed that the simulation data was consistent with the experimental result.

Feppon et al. (2019) researched topology optimisation between fluid and solid interaction interface using LSM and adaptive reconfigurable grid. The sensitivity of the shape was obtained by solving the derivative of the objective function, which was derived for the normal corresponding force on the interaction interface. The shape optimisation process was demonstrated through several test cases. Another optimisation framework was proposed by Li et al. (2022) for weak fluid-solid coupling. The LSM tracked changes in the interface on an adaptive grid, and the grid could be split into multiple sub-grids before finally being merged. The LSM function was re-initialised based on the reaction-diffusion function. The validity of the model was verified by engineering ions in practice. Kubo et al. (2021) optimised the two-dimensional turbulence based on LSM and immersion boundary method. The LSM traced the flow-solid interface, and the immersion boundary method added moving boundary conditions to the flow-solid interface. The standard wall function was used to estimate the velocity and pressure of the turbulent flow, etc. Numerical experiments confirmed that the method was correct. A model was proposed by Moon et al. (2020) to simulate the nonlinear movement of soft worm robots with a meshless LSM. Since the grid structure was destroyed when the robot performed nonlinear and intense movements, the tracking of the robot behaviour using the approach would be more accurate. The simulation results also proved this. Afzal et al. (2020) conducted a numerical simulation of the changes in the riverbed around the bridge platform using LSM, calculating scouring and deposition patterns in the riverbed on a crossed grid [Figure 8(a)]. The calculation results showed that the model could predict riverbed changes very well. Gautam et al. (2021) used LSM to simulate the scouring process of water flow in monopiles and calculated the changes in the submerged riverbed. The active wave absorption method was used to simulate the back and forth motion of waves, the LSM calculated the change of own surface, and the morphological model assisted in the calculation of sediment scouring and deposition. The modified model had been validated and tested in practice.

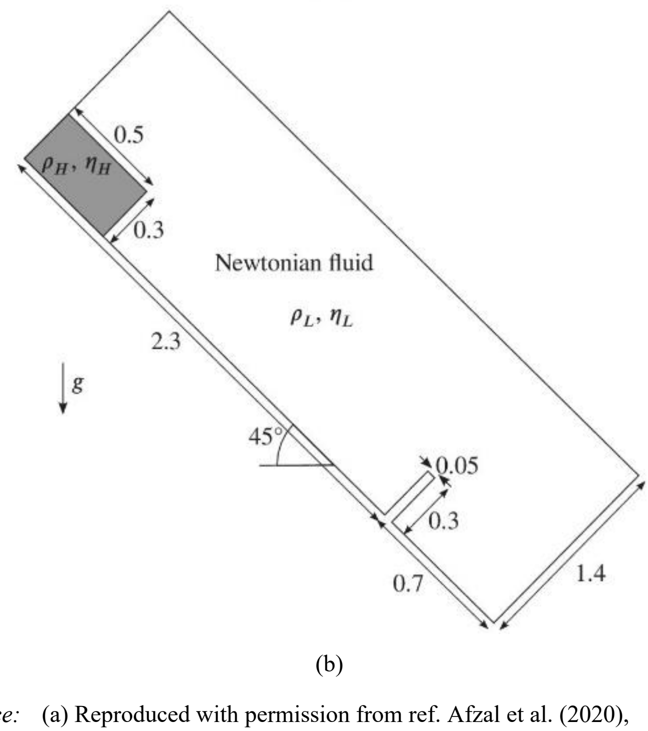
Shamsipour et al. (2021) investigated the technique for adding solid or liquid particles to liquid droplets that were very widely used in industry, and they developed a model coupling LSM and ALE. It was found that the particles stayed inside the droplet only when their initial momentum was within a specific range. Otherwise, the particles could not approach or pass directly through the droplet. Therefore, the density and speed of the particles had a decisive effect on the final fusion effect of the bureau. Larese et al. (2012) developed a program to simulate the flow of a fluid on a fixed grid over a pestle. The Navier-Stokes equations were modified because there were gaps between the stones. The structural model was constructed based on the Lagrangian method, and the fluid was simulated using the Eulerian model. The LSM traced the fluid flow on the stone surface and between the stones. The experimental results corroborated that the simulation results of the model were correct. Basting and Weismann (2013) presented a novel LSM by adding hermit expression to study fluid-solid coupling and two-phase flow interaction applications. The numerical calculation process used the finite element method, and the divided mesh was automatically aligned with the geometry. The final calculated curvature was more accurately evolved to obtain a more transparent interface. Experiments such as the interaction between blood and heart valves demonstrated the validity of the method and proved its good generality.

Cadiou et al. (2020) studied the melting and deposition processes of metal wires in a three-dimensional model. The process of melting of the metal wire and droplets separation were simulated by LSM. The model could calculate the variation of fluid velocity and temperature in the melt pool to provide theoretical support for the description of sediment growth. The experimental data verified the correctness of the simulation result. Gesenhues and Behr (2021) researched the interaction between dense particles and fluids in both time and space dimensions [Figure 8(b)]. The evolution of the interface was captured by LSM. In the time dimension, the calculation at the interface was based on an unstructured grid, which allowed for a high resolution. The location of the interface that needs to be refined was also specified by the LSM. The simulation result was in good agreement with the available experimental data. He and Qiao (2011) introduced a modified LSM to simulate the interaction of solid and fluid. They added a phase solid-liquid interface force to the momentum equation and solved it in a Cartesian network instead of a Lagrangian network. The method was experimentally proven to be effective, and it could be extended to use in three-dimensional space. Pino Muñoz et al. (2013) researched the interaction between fluids and elastic solids under Eulerian lattices considering surface tension based on finite element LSM. The interface captured between the fluid and the elastic solid was done by the LSM. The authors also proposed methods such as pocket cells, sub-grids, and surface integration for controlling the problem of possible spurious pressure oscillations at the intersection of fluids and elastic solids.

Figure 8 (a) Schematic diagram of the interaction between abutment and fluidic (b) Design of the dam in the dam-break experiment



(a)



(b)

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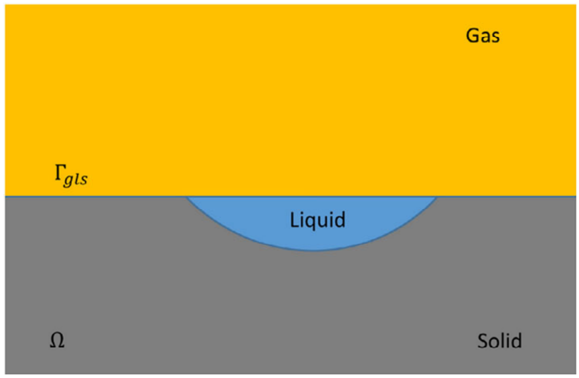
5 Multiphase flow applications of LSM

Compared to the evolution of the interface between two fluids, the multiphase flow interface tracking control equations are more complex, and the computational process is more complicated. But the LSM can still play an important role.

Figure 9 Schematic image of solid-liquid-gas triple interaction (see online version for colours)



Source: Reproduced with permission from ref. Yan et al. (2018), Copyright (2018), Elsevier

LSM was applied in the electrodynamic fluid solver to track interface by Van Poppel et al. (2010). They utilised higher-order polynomials for level-set function reconstruction, which provided an interface change process with very high accuracy. LSF transfer based on semi-Lagrangian technique eliminated the limitation on the time step during interface evolution. Analysis of the simulation results and experiments validated that the LSM maintained mass conservation as much as possible while ensuring the accuracy of the simulation. Karakus et al. (2018) introduced the Galerkin method of higher-order complete discontinuity when solving the interface of the level set of incompressible multiphase flow. The implicit solution based on Galerkin discretisation during the iteration process of the LSF could effectively reduce the need for computer memory during the simulation. A series of numerical experiments confirmed that the method was simple to model, computed fast, and guaranteed mass conservation. Yan et al. (2018) developed a thermal multiphase flow computational framework in which LSM was applied (Figure 9). The variational multi-scale formulation was involved in the LSM solution process, which was more effective when dealing with problems with significant density differences such as solid-liquid. In addition, the method allowed for mass conservation if the densities between the fluids were very close. Four sets of numerical experiments clarified the validity of the form and the consistency of the available results.

Qin and Riaz (2021) combined LSM with semi-sharpening methods to construct a computational framework for solving the evolution of multiphase flow interfaces. The framework performed the construction of the LSF by projecting the interface into a Cartesian coordinate system. The framework did not accumulate errors in the re-initialisation process during the interface iteration, and achieved good results in topological shape preservation and mass conservation. The accuracy of the model was verified by reproducing several engineering systems. Kaiser et al. (2021) proposed Lagrangian particle storage and access algorithm to simulate high-resolution multiphase flows. The property that LSM was good at handling sharp topological changes were utilised to perform differences to solve partial differential equations. The experimental results of the particle model were proved to be correct. The LSM was applied to a tool called multiUQ to simulate gas-liquid multiphase flow by Turnquist and Owkes (2021) The interface evolution of uncertain multiphase flows was done based on stochastic LSM. Many test cases demonstrated the effectiveness and robustness of the tool multiUQ, which indirectly illustrated the correctness of LSM. Barton et al. (2021) used LSM to study multiple systems of solid and fluid interactions in a fixed grid. The LSM was devoted to tracking the internal boundaries of the system. The model could separate different interface areas and had a perfect handling effect for complex shapes. Interface tracking via level set could significantly decrease the computational effort of the system, since much of the geometric information could be accessed directly, eliminating the need to reconstruct the cutting unit.

6 Coupling with other methods

There are many free interface tracking methods based on Euler's formula, and LSM is just one of them. Other widely used algorithms include VOF, PFM, LBM, etc. Each algorithm has its irreplaceable advantages, but accordingly, each algorithm inevitably has disadvantages. While the LSM method is good at tracking interfaces where topology changes dramatically and is easy to implement, it requires continuous re-initialisation during iterations. Continuous re-initialisation not only increases the computational cost, but also causes loss or gain of mass when the interface is stretched or torn. The above descriptions mean that the LSM does not satisfy the conservation of mass in fluid mechanics. Other algorithms also have more or fewer shortcomings that cannot be overcome.

Since every algorithm has both advantages and disadvantages, it is possible to create new algorithms with better performance if two or more ways can be coupled. The coupling will increase the computational effort, but if the advantages of multiple algorithms can be implemented to combine and avoid the disadvantages of each method, it is possible to find new approaches with more powerful performance. Take LSM as an example. Although it cannot guarantee mass conservation, other algorithms such as VOF and PFM can.

6.1 LSM coupled with volume-of-fluidic

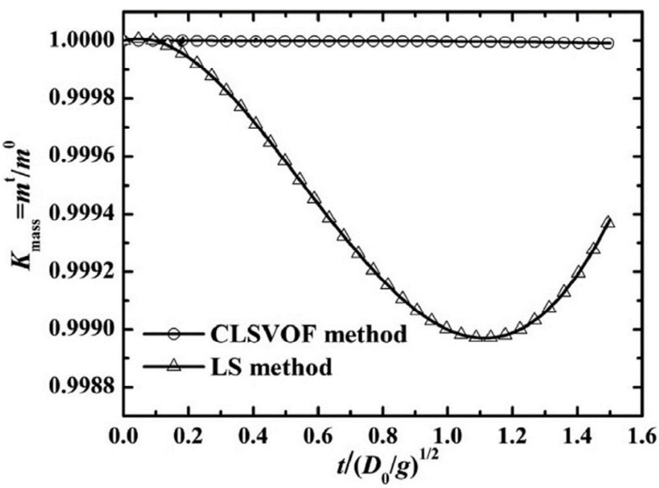
It is prevalent to use LSM and VOF coupled together in practical applications, and the coupling is also the most frequently used. Compared to LSM, the VOF maintains high precision of mass conservation and volume conservation. However, VOF is quite difficult to calculate the unit normal vector and the radius of curvature for the interface. The VOF method is coupled to the LSM to obtain the level set volume of fluid method (CLSVOF). In the new method, the VOF is used for interface reconstruction, and the LSM is used to compute the surface tension and curvature of the interface.

The solver of CLSVOF is the crucial part of free interface tracking. Skarysz et al. (2018) introduced an algorithm that allowed for fast interface reconstruction in convex meshes for CLSVOF solver. The manner divided the mesh into tetrahedrons first. The advantage of the operation was that the volume of these tetrahedrons could be calculated more quickly than current techniques such as cropping and covering. A newly designed root-finding algorithm was used when coupling volume fraction to an interface position. Many numerical tests demonstrated the algorithm had the fastest evolution rate compared to the commonly used methods. Shiratori et al. (2021) constructed a two-phase flow solver on an unstructured grid. Hyperbolic switching captured the interface, while coupling LSM and VOF evolved the interface. The iteration speed of the LSF was greatly improved due to the use of an efficient re-initialisation function. The solution of the three benchmark problems proved that the solver was adequate, and the performance was not weaker than other solvers. Bilger et al. (2017) tried to adapt the CLSVOF solver so that the coupling of the LSM method and the VOF method could be optimised [Figure 11(a)]. The team implemented these two algorithms separately in the same program, and each implementation achieved its optimal performance. Then, they were tested independently using the same test benchmarks. The test results showed that LSM and VOF were sensitive to the input parameters. When the two methods were coupled for operation, the parameters must be carefully tuned to achieve optimal performance.

A computational framework used for motion interface tracking was proposed by Yokoi (2013). The framework was suitable for complex geometric structure evolution problems because of curvature interpolation based on LSM. Droplet splash experiments demonstrated the effectiveness of the framework. Arienti et al. (2012) developed a multiphase flow dynamics program based on the CLSVOF approach to simulate the process of atomisation o in an injector. This program simulated phase change processes such as liquid cracking and bubble flow by tracking changes in the kinematic interface. In addition, the authors implemented the Lagrangian representation of droplets by performing Lagrangian discretisation on Eulerian lattices. The advantage of the design was that the droplets could be tracked over a larger area and for a longer time without loss of accuracy. Simulation results of liquid rocket motors cold started at high, moderate, and low injection velocities demonstrated the effectiveness of the procedure. Zhang et al. (2021) researched the movement of the solid/gas interface inside a solid rocket due to fuel combustion based on LSM and VOF. The porous media model divided the interior of the shelf into a solid and a gaseous calculation region and represented the respective combustion surfaces. With these two burning surfaces, it was possible to calculate the burning area at a different moment. The validity of the simulated data of the method was proved through comparing it with the experimental data.

Chakraborty et al. (2013) used the CLSVOF method to simulate the various behaviours and changes of bubbles in a stationary liquid. The resistance and final morphology of a single bubble during its ascent in a liquid, the buoyancy and deformation of two co-axial bubbles, and the deformation and rupture of bubbles at the free interface had all been studied in detail by the researchers. The calculated results were consistent with the experimental data in the relevant kinds of literature. Wang et al. (2021b) employed LSM and VOF to study the flow problem of a fully enclosed cylinder in the water. The simulation data demonstrated that the size of the wake bubble was positively correlated with the turbulence intensity. When the water contained bubbles, the turbulence intensity became smaller, and the number of bubbles decreased. The condensation behaviour of bubbles was investigated by Bahreini et al. (2021) based on CLSVOF. For individual bubbles, the size of the bubble and the flow rate of the fluid had a powerful influence on the deformation process and the condensation rate of the bubble. For multiple bubbles, the whole process was more complicated due to the interaction between bubbles. Data in the published literature demonstrated the effectiveness of the method. Tsui et al. coupled LSM and VOF in the two-phase flow calculation process (Liu et al., 2017a). The re-initialisation of the LSM was no longer performed by solving the advection equation, instead of using the interface position expansion evolved from the VOF. The re-initialisation equations were then solved based on the finite element method in a high-resolution bounded format. The simulation results of the bubble rise process based on this method were performed closer to the experimental data. Haghshenas et al. (2017) coupled LSM with VOF in the simulation of interfacial capillary motion. The volume fraction of VOF provided the interface seed position for the re-initialisation of LSM, and the advective LS length served as the initial condition for re-initialisation. The simulation results validated that the surface tension calculation was more accurate due to the addition of LSM, the parasitic currents were reduced, and the evolved interface was in better agreement with the measured interface.

Figure 10 Comparison of mass conservation performance between CLSVOF and LSM

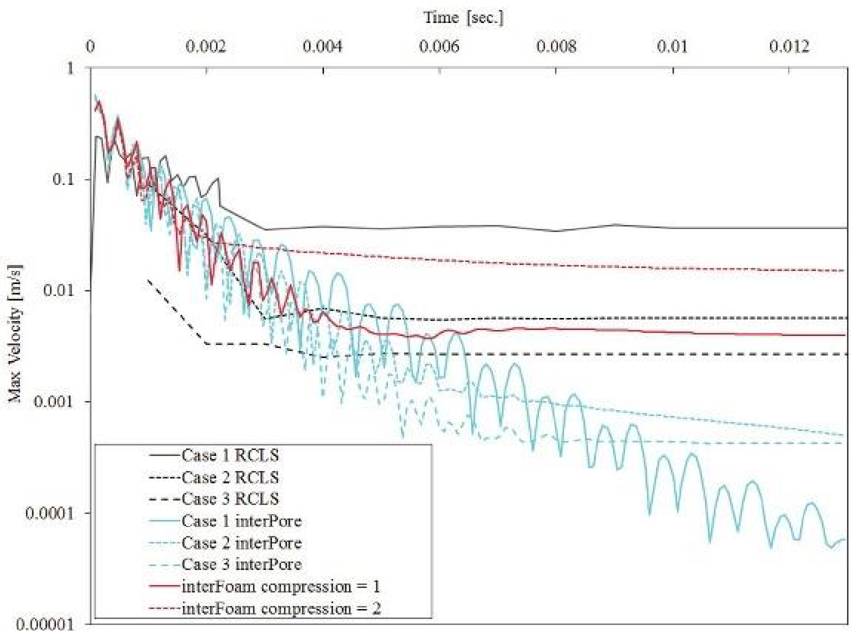


Source: Reproduced with permission from ref. Liu et al. (2017b), Copyright (2017), Elsevier

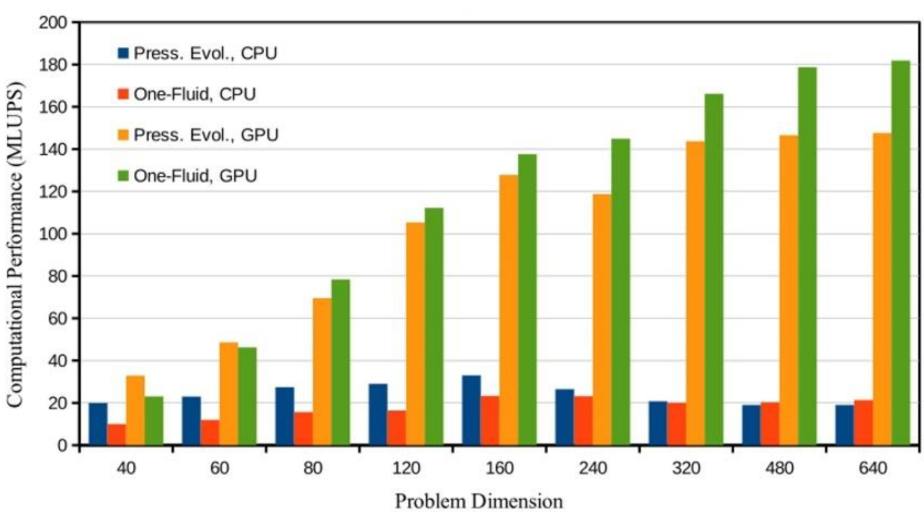
The problem of ship movement in the water was researched by Nguyen Duy and Hino (2010) through coupling LSM and VOF. The second-order discrete format is used when re-initialised the LSF with the function gradient, which dramatically improved the solution accuracy. Due to the introduction of LSM, the computation of both interface normal vector and curvature was more exact. The validity of the method was verified by three different arithmetic cases. The team also designed an enhanced scheme containing LSM and VOF simultaneously to compute two-phase flow (Duy et al., 2021). The LSF took a conservative form when performing the re-initialisation transformation, and the interface evolved by this method was smoother and more accurate. The curvature of the interface normal vector calculated by the procedure was more accurate and satisfied the requirement of mass conservation. A set of benchmark tests demonstrated the significant performance improvement of the technique over the original VOF. Michael et al. (2017) explored the evolution of the sharp interface by coupling these two algorithms. The LSM was mainly used to study the jump condition at the interface, and its initialisation function was a volume fraction function. In addition, owing to the parallel fast advance method, the interface information could be conducted quickly. Experimental data in both two-dimensional and three-dimensional space demonstrated the high performance of the manner for handling pointed interfaces. The evolution problem of axisymmetric incompressible fluids could also be studied using the coupled LSM and VOF approach. Liu et al. (2017b) coupled the two algorithms through an explicit relational equation (Figure 10). The LSM implicitly performed the interface evolution, and the VOF solved the mass non-conservation problem that may arise in the process. The simulation data revealed that the calculated results of the method were consistent with both the theoretical and experimental values.

Hong and Wang, (2017) introduced a coupling way of LSM and VOF to survey the impact effects of double droplets on spherical membranes. Study results showed that the curvature and diameter of the droplet, the distance between the droplet and the membrane, and the impact velocity all affected the final morphology of the system. It was essential to study the deformation and heat transfer of a liquid droplet flows in a fluid. Coupled LSM and VOF were combined with a solver by Talebanfard et al. (2019) to research the flow and heat transfer of liquid droplets. The simulation results demonstrated that Reynolds number and the initial shape of the droplet were vital factors in droplet deformation and heat transport. At the same time, surface tension had a more significant influence on the degree of droplet deformation. Capobianchi et al. (2019) studied the role played by viscoelasticity during the movement of droplets in stationary thermal capillaries based on the hybrid LSM and VOF method. The motion of the droplet was caused by the temperature gradient, and the effect of gravity could be largely ignored. The simulation results showed that the velocity of droplet motion was negatively correlated with the Deborah number, and the curvature of the interface was positively correlated with the number. Huang et al. (2019) investigated the aggregation and deformation of liquid droplets in a viscous fluid of a direct current electric field using coupled LSM and VOF methods. The team firstly used the technique to simulate some classical problems to determine the correctness of the way. They then simulated droplets in a viscous fluid and found that the reduction in viscosity caused a weakening of the damping effect. Gu et al. (2019) utilised both LSM and VOF in their study of bubble/droplet two-phase flow. While VOF was mainly used to calculate surface tension and performed interfacial reconstruction, LSM was employed to calculate the interfacial normal vector and the tension term in the Navier-Stokes equations. Simulations of droplet clash and bubble rupture confirmed that the results of the method corresponded with experimental results, satisfying both mass conservation and high computational accuracy. Wang et al. (2021a) applied LSM and VOF to demonstrate continuous double droplet impact on the pipe. The simulation data revealed that the larger the eccentricity distance was, the easier the droplets spread. The smaller the eccentricity for a constant impact velocity, the easier it was for the droplet to bounce back without breaking after hitting the pipe.

Figure 11 (a) Comparison of the numerical predictions of different solvers for pseudo currents (b) Computational cost for different problem dimensions (see online version for colours)



(a)



(b)

Source: (a) Reproduced with permission from ref. Bilger et al. (2017), Copyright (2017), Elsevier. (b) Reproduced with permission from ref. Safi and Turek (2015), Copyright (2015), Elsevier

Zhao and Chen (2017) proposed a new method for coupling LSM and VOF on structured mesh systems such as embedded overlapping shifts. The approach had a robust interfacial capture capability and also maintained mass conservation. A series of experiments showed that the plan was effective. Liu et al. (2021) used adaptable unstructured mesh in coupling LSM and VOF. The algorithm was coupled to a solver by a sparse matrix conjugate method. The validation of four classical problems, such as liquid jet, showed that the procedure was correct. More simulations had demonstrated that the way was not only faster but also required fewer computational resources than the structured grid method. The coupling of LSM and VOF was also researched on the unstructured grid by Cao et al. (2018) The process was mainly utilised in irregular regions to simulate the incompressible two-phase flows, with segments of the linear interface. Volume fraction was solved based on the Lagrangian-Eulerian format, and an iterative LSF was created in the unstructured grid. The simulation results were consistent with the results of many experiments. LeBlanc et al. (2018) coupled LSM and VOF on an unstructured grid. In this hybrid approach, the VOF first determined the interface location to ensure that the algorithm met the mass conservation requirement. Then, the LSM employed a re-definition distance function to characterise the thickness of the interface based on the established interface location. The performance of the method exceeded the effect of either LSM or VOF evolved alonely.

Ling et al. (2019) calculated the motion of the free interface in an arbitrary polygon mesh based on the coupling method of LSM and VOF. The polygonal interface was tracked based on the Newton iteration method, and the volume fraction was calculated based on the incremental remapping method, the second-order accuracy could be obtained by this operation. Many experiments were conducted, and their results showed the correctness of the simulation. Dianat et al. (2017) had done all the simulation work on an unstructured and non-orthogonal grid by adding LSM to perform the exterior water management of the car. The new algorithm after coupling could be iterated on arbitrarily shaped cells, and the re-initialisation function was also applicable to unstructured mesh. The simulation result compared with the experimental data showed that the algorithm was successful. Kim and Park (2021) combined LSM and VOF to investigate lubrication of flat plates in the fluid by air to provide a theoretical reference for the design of the ship with less resistance. As the air was injected between the plate and the fluid faster, the resistance of the plate became smaller. But when the air covered the entire plate, the resistance no longer changed. Compared to using only VOF, the method was more accurate in predicting the discontinuous air layer. Tayeb and Zhang (2021) devised a scheme of LSM and VOF combination to study particle deposition due to evaporation. They reported that the longer the evaporation time lasted, the faster the deposition rate would be, and then it would gradually decrease. The density of particle deposition was lower in the marginal part. The numerical results of these simulations were consistent with the results of the actual experiments. Li et al. (2021) analysed Rayleigh-Taylor instability used LSM and VOF to uncover the physical factors that influenced the evolution of its interface. After validation of the combination by a single-mode Rayleigh-Taylor instability issue, coefficient of viscosity, surface tension, and other factors created influence through the velocity gradient were investigated, and the essential physical principles were revealed.

An algorithm used to simulate the interaction between water, air, and thin walls was reported by He et al. (2022) The CLSVOF was used to capture free interface, and the equations were solved based on the finite volume method. The immersion boundary method was used to add pressure conditions at the boundary, and the fluid motion was solved by the finite difference method. Simulations of two benchmark examples demonstrated that the approach accorded well with the actual data. Martinez et al. (2021) simulated the interaction between compressible fluids based on CLSVOF. Thanks to the simultaneous coupling of LSM and VOF, the method was able to treat compressible fluids undergoing phase change while ensuring mass conservation. The effectiveness of the method on the problems of phase change, mass transfer, and large curvature deformation was experimentally demonstrated. Du et al. (2020) utilised CLSVOF to investigate the rupture behaviour of bubbles in a device with four parallel sub-microchannels. They compared the historical data and the simulation result to verify the correctness of the method. Then they analysed the effects of sub-microchannel width, bubble velocity, and fluid concentration on the bubble break-up process and gave a function for predicting the bubble variation.

6.2 LSM coupled with LBM

LBM is based on the theory of molecular motion and statistical physics, and is a method between macroscopic and microscopic. It solves the linear Boltzmann equation through a spatial grid, which is easy to implement in algebraic form and has good parallelism with reliable computational accuracy. LBM has been successfully applied to calculate the interfacial tension and computation with complex boundary conditions where the concentration ratio of the two phases is large. Coupling the LBM with the LSM helps perform a faster and more accurate evolution of the free interface while ensuring the mass conservation of the fluid.

Safi and Turek (2015) researched the behaviour of bubbles rising in different fluids with relatively disparate differences in density and viscosity, using both LBM and LSM. When LSM was used for interface evolution, the pressure solution based on the Boltzmann function gave a higher pressure resolution in the final result. The introduction of the minimum computational cost way made the re-initialisation step of the LSM simpler and more efficient [Figure 11(b)].

By coupling the LBM and LSM, Dugast et al. (2018) performed work on topological structure calculation in the thermal fluid. The LBM implemented the forward evolutionary work of the interface, and the LSM performed the geometric structure update after each iteration. In the way, the interface between different phases was obvious. The effectiveness of the coupling method was verified by comparing it with existing examples. The optimisation for thermal fluid described with different cost functions showed the robustness of the coupling method. A model was devised by Ando et al. (2021) to interpret the bypass problem using both LSM and LBM to solve fluid-solid interaction. LBM was utilised to predict the flow of fluid and movement of solid, while LSM was made use of interface tracking between fluid and solid. The wind tunnel experimental data had a tiny error compared to the simulated data.

7 Summary

As a way of curve evolution, the LSM has been widely used and achieved good results in numerical computation and image processing due to its high computational accuracy.

This paper summarises the applications of the LSM in CFD. After describing the curve evolution, a detailed explanation of the computational procedure of the LSM is given. A review of the application of the LSM in two-phase and multi-phase flow interface tracing shows that it has made a significant contribution to guiding scientific research and industrial production. Of course, the constant re-initialisation of the LSM in the iterative procedure leads to a high computational cost to ensure mass conservation and limits its further application. Therefore, a continuous improvement of the LSM solution process is also necessary. Alternatively, as summarised in the paper, it is a better solution to apply the LSM in combination with other methods such as VOF to obtain the advantages of both algorithms.

Acknowledgements

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