
**International Journal of Quality Engineering and Technology**

ISSN online: 1757-2185 - ISSN print: 1757-2177
https://www.inderscience.com/ijqet

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**DOI:** 10.1504/IJQET.2023.10054274

**Article History:**

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|---|---|
| Received: | 04 December 2021 |
| Last revised: | 06 December 2021 |
| Accepted: | 02 March 2022 |
| Published online: | 21 February 2023 |

# Pareto efficient correlated multi-response optimisation by considering customer satisfaction


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**Abstract:** The capability of a manufacturer in satisfying the customer's requirements is an important issue in the current competitive market. Since customers consider several correlated quality characteristics for selecting a product, designing the process variables to meet the required specification limits of the quality characteristics is essential. Furthermore, to attain the most satisfactory solution, a decision maker's preference information should be incorporated into the optimisation procedure. This study suggests a posterior preference articulation approach based on NSGA-II and MOPSO, which is capable to increase customer satisfaction by generating non-dominated solutions within conformance region. The proposed method takes also into account the location and dispersion effects along with the correlation of among quality characteristics as well as the relative importance of them. To demonstrate the applicability of the approach, a computational analysis on two case studies is performed. Results confirm superiority of the suggested method in comparison with the existing posterior approaches.



**Keywords:** multiple response optimisation; customer satisfaction; correlation; location effect; dispersion effect.

**Reference** to this paper should be made as follows: Salmasnia, A. and Mokhtari, H. (2023) 'Pareto efficient correlated multi-response optimisation by considering customer satisfaction', *Int. J. Quality Engineering and Technology*, Vol. 9, No. 1, pp.34–74.


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## 1 Introduction

The current competitive market has forced the manufacturers to provide customers' needs by improving the quality of their products. In this regard, process design optimisation is an essential issue, which includes selecting design variables to meet the required specification of quality characteristics in a process. For this purpose, response surface methodology (RSM) recently has attracted the most attention due to its well performance in comparison with other approximation approaches. RSM explores the relationship between design variables and a quality characteristic via a group of statistical and mathematical techniques and then optimises quality characteristic with respect to the design variables. Although most of the RSM-based techniques focus on problems with only a quality characteristic, the real-world applications often encounter more than one interested quality characteristic which is called the multiple response optimisation (MRO) problem. According to the definition of MRO problem, the multi-objective optimisation (MOO) methods can be successfully utilised for solving MRO problem. In the MOO survey, the existing approaches are classified into three major groups:

1 prior preference articulation

2 progressive preference articulation

3 posterior preference articulation (Korhenen et al, 1992; Steuer, 1986).

The majority of the existing approaches in MRO literature such as Vining (1998), Allandeh et al. (2010), Najafi et al. (2011), Salmasnia et al. (2012a, 2012b), Hejazi et al. (2013), Salmasnia and Bashiri (2015), Ouyang et al. (2013), Sharma et al. (2013), Babu et al. (2013), Bera and Mukherjee (2013), Pervez et al., (2018), Moslemi et al. (2018a), Limon-Romero et al., (2018), Köksoy and Zeybek (2019), Ganapathy et al., (2019), Chakraborty et al., (2019), Saini et al., (2019), and Tajane and Pawar (2019) are categorized into the prior preference articulation. Also, several progressive approaches have been proposed to find a compromise solution in MRO problem. Some instances are Koksalan and Plante (2003), Jeong and Kim (2003; 2005; 2009), Park and Kim (2005), Koksoy (2006a, 2006b, 2008), Lee and Kim (2012), Salmasnia et al. (2013a, 2017) and Noorossana et al. (2014). According to the best of our knowledge, only few research including Peterson (2004), Costa et al. (2011), Lee et al. (2010; 2011) and Salmasnia et al. (2013b), Costa and Lourenço (2015) have been developed posterior approaches for solving MRO problems.

The prior preference articulation approaches combine multiple responses into a single function and solve it as a single objective optimisation problem. In such approaches, a decision maker (DM) before the solving process must specify the required preference information. However, in many situations the preferred trade-off among responses can not been determined in advance because of difficulties in assessment of the DM's preference structure. The Progressive preference articulation methods give DM the opportunity to incorporate his/her preferences during the solving process. However, the progressive methods often need a considerable amount of time on the part of DM and may not be very useful for large size problems. The Posterior preference articulation approaches do not require any articulation of DM's preference information in advance or during the solving process. In Posterior approaches, after generating all (or most) of the non-dominated solutions, DM selects the most preferred solution among the obtained non-dominated solutions. However, such approaches usually generate a large number of solutions, and hence, it becomes difficult for decision maker to select the best solution among the efficient solutions. In spite of the advantages of posterior methods, they are rarely utilized to solve the MRO problems. Koksoy (2008) proposed a new approach based on the mean square error and solved the suggested model by generalised reduced gradient (GRG) method. Lee et al. (2010) presented a three-stage method to optimise location and dispersion effects of a single response variable. In this method, a set of non-dominated solutions are generated by applying the ε-constraint method. Lee et al. (2011) developed their previous posterior approach by using a modified ε-constraint method to attain the strongly non-dominated solutions. In addition, they employed an interactive selection technique to choose the most satisfactory solution. It is worth to mention that that method only considers the location effect ignoring dispersion effect of responses. Costa et al. (2011) introduced a method for optimising dual and multiple response problems via employing two approaches of the mean square error and global criteria method. Baril et al. (2011) proposed a method to generate Pareto set, which integrates the feasibility modeling technique and the interactive multi-objective algorithm under DM' preferences (IMOP) in a unified framework. Salmasnia et al. (2013b) presented a robust posterior articulation approach that uses Taguchi's signal to noise ratio for considering both location and dispersion effects. Costa and Lourenço (2015) suggested an approach with three separate methods namely, Desirability-based method, Global Criteria-based approach and Physical Programming. In these models location effect of responses along with relative importance of them are considered. Moslemi etal (2018, b) presented a new posterior method for cascade processes consisting of multiple stages. All of the above mentioned posterior methods ignore possible correlation among quality characteristics, which may lead to an unrealistic solution. They also do not guarantee that all quality characteristics fall within their corresponding specification limits.

Another important issue in optimisation problems is consideration of customer satisfaction. Two types of approaches are able to consider this property:

1 desirability function-based approaches

2 process capability index-based approaches.

Desirability function initially introduced by Harington (1965) and then modified by Derringer and Suich (1980). This function transforms an estimated response into a scale free value in the interval [0, 1]. Most of desirability function-based approaches despite considering the customer satisfaction neglect the variance-covariance structure of responses. Although there are few studies such as Salmasnia et al. (2012a, 2012c, 2013c) that take into account the correlation of among quality characteristics, they often do not pay attention to relative importance of responses.

Process capability analysis is concerned with assessing the capability of a process in satisfying the customer's requirements by producing products within conformance region. In MRO literature, only few approaches are in the basis of the process capability Indices. These approaches assume either independency among responses or equal relative importance for quality characteristics. In addition, the most of them are applicable for only nominal-the-best type responses. Some of the proposed methods in this context are as follows:

Ch'ng et al. (2004) proposed the sum of the weighted univariate $C_{pm}$ indices to aggregate mean and variance of several responses into a unique model. Plante (2001) suggested the geometric mean of the univariate process capability index ($C_{pm}$) of responses as a new unifying mathematical model. Recently, Noorossana et al. (2014) proposed a three stages interactive approach in basis of artificial neural network, genetic algorithm and the sum of the weighted univariate $C_{pm}$ indices. The mentioned approaches take into account both location and dispersion effects as well as the relative importance of quality characteristics in the optimisation process. However, these approaches assume that responses are mutually independent which especially in cases with high correlation among responses may lead to an unrealistic result. To overcome this drawback, Awad and Kovach (2011) proposed a method to maximise the multivariate process capability index ($MC_{pm}$) that was introduced by Chan et al. (1991). Amiri et al. (2012) suggested an approach for problems with several non-normal responses. In that approach $MC_{pm}$ is computed for each treatment and then the geometric mean of $MC_{pm}s$ is obtained for each factor level. Finally, the factor level with the highest geometric mean value is selected as optimal level. Bera et al. (2013) suggested an approach based on the principle component analysis and multivariate process capability index to take into account the location and dispersion effects of correlated responses. Although Awad and Kovach (2011), Amiri et al. (2012) and Bera and Mukherjee (2013) consider the variance-covariance structure of the responses, they assume that the covariance value is constant over the process region. Furthermore, they do not take into account the relative importance of responses in the optimisation process.

According to the mentioned above, this study suggests a posterior articulation method that considers customer satisfaction via employing the multivariate process capability index. Furthermore, the proposed method takes into consideration the location and dispersion effects of responses along with correlation structure and relative importance of quality characteristics. Table 1 shows the characteristics of different multi-response approaches presented in the literature.

**Table 1** A characteristic comparison of the existing approaches and the proposed method

| *Method* | *TS* | *TDMP* | *LE* | *DE* | *CE* | *SLN* | *CS* | *RI* |
|---|---|---|---|---|---|---|---|---|
| Derringer (1994) | Continuous | Prior | ✓ | | | ✓ | ✓ | ✓ |
| Kim and Lin (2000) | Continuous | Prior | ✓ | | | ✓ | ✓ | |
| Kim and Lin (2006) | Continuous | Prior | ✓ | ✓ | | ✓ | ✓ | |
| Salmasnia et al. (2012b) | Continuous | Prior | ✓ | ✓ | ✓ | ✓ | ✓ | |
| Pignatiello (1993) | Continuous | Prior | ✓ | ✓ | ✓ | | | ✓ |
| Vining (1998) | Continuous | Prior | ✓ | | ✓ | | | ✓ |
| Ko et al. (2005) | Continuous | Prior | ✓ | ✓ | ✓ | | | ✓ |
| Lin and Tu (1995) | Continuous | Prior | ✓ | ✓ | | ✓ | | |
| Kazemzadeh et al. (2008) | Continuous | Prior | ✓ | ✓ | ✓ | | ✓ | ✓ |
| Su and Tong (1997) | Discrete | Prior | | ✓ | ✓ | | | |
| Antony (2000) | Discrete | Prior | | ✓ | ✓ | | | |
| Fung and Kang (2005) | Discrete | Prior | | ✓ | ✓ | ✓ | | |
| Plante (2001) | Continuous | Prior | ✓ | ✓ | | | ✓ | ✓ |
| Awad and Kovach(2011) | Continuous | Prior | ✓ | ✓ | ✓ | | ✓ | |
| Amiri et al. (2012) | Discrete | Prior | ✓ | ✓ | ✓ | | ✓ | |
| Bera and Mukherjee (2013) | Continuous | Prior | ✓ | ✓ | ✓ | ✓ | ✓ | |
| Noorossana et al. (2014) | Continuous | Progressive | ✓ | ✓ | | | ✓ | ✓ |
| Jeong and Kim (2009) | Continuous | Progressive | ✓ | | | ✓ | ✓ | |
| Koksoy (2006a) | Continuous | Progressive | ✓ | ✓ | | | | |
| Lee and Kim (2012) | Continuous | Progressive | ✓ | ✓ | | | | ✓ |
| Park and Kim (2005) | Continuous | Progressive | ✓ | ✓ | | ✓ | | ✓ |
| Koksoy (2008) | Continuous | Posterior | ✓ | ✓ | | ✓ | | ✓ |
| Lee et al. (2010) | Continuous | Posterior | ✓ | ✓ | | ✓ | | ✓ |
| Lee et al. (2011) | Continuous | Posterior | ✓ | | | ✓ | | ✓ |
| Costa et al. (2011) | Continuous | Posterior | ✓ | ✓ | | ✓ | | ✓ |
| Baril et al. (2011) | Continuous | Posterior | ✓ | ✓ | | ✓ | | ✓ |
| Salmasnia et al. (2013b) | Continuous | Posterior | ✓ | ✓ | | ✓ | | ✓ |
| Costa and Lourenço (2015) | Continuous | Posterior | ✓ | | | ✓ | | ✓ |
| Pervez et al. (2018) | Continuous | Prior | ✓ | ✓ | | | | |

Notes: Type of methods by timing of the decision maker's preference information articulate (TDMP); type of search within the experimental design (TS); location effect (LE); dispersion effect (DE); correlation among responses (CE); usability for all three types of smaller-the-better, larger-the-better and nominal-the-best responses, (SLN); customer satisfaction (CS); and relative importance of responses (RI).

**Table 1** A characteristic comparison of the existing approaches and the proposed method (continued)

| *Method* | *TS* | *TDMP* | *LE* | *DE* | *CE* | *SLN* | *CS* | *RI* |
|---|---|---|---|---|---|---|---|---|
| Moslemi et al. (2018b) | Continuous | Posterior | ✓ | ✓ | | | | |
| Moslemi et al. (2018a) | Continuous | Prior | ✓ | ✓ | | | | |
| Limon-Romero et al. (2018) | Continuous | Prior | ✓ | ✓ | | | | |
| Köksoy and Zeybek (2019) | Continuous | Prior | ✓ | ✓ | ✓ | | | |
| Ganapathy et al. (2019) | Continuous | Prior | ✓ | ✓ | | | | |
| Chakraborty et al. (2019) | Discrete | Prior | ✓ | | | ✓ | | ✓ |
| Saini et al. (2019) | Continuous | Prior | ✓ | ✓ | | | | |
| Tajane and Pawar (2019) | Continuous | Prior | ✓ | ✓ | ✓ | | | |
| The proposed method | Continuous | Posterior | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Notes: Type of methods by timing of the decision maker's preference information articulate (TDMP); type of search within the experimental design (TS); location effect (LE); dispersion effect (DE); correlation among responses (CE); usability for all three types of smaller-the-better, larger-the-better and nominal-the-best responses, (SLN); customer satisfaction (CS); and relative importance of responses (RI).

The rest of this paper is presented in the following order: Section 2 presents the proposed method for solving the MRO problems. Section 3 demonstrates the applicability of the suggested method through two industrial case studies from the literature and provides three comparative studies for each numerical example. Finally, conclusions are reported in Section 4.

## 2 Proposed methods

In this work, a posterior preference articulation method is suggested. It uses the multivariate process capability index to consider the customer's needs via producing the products in a way that the quality characteristics meet their corresponding conformance regions. It also employs the weighted statistical distance to take into account the relative importance and variance-covariance structure of quality characteristics in the optimisation procedure. Therefore, the proposed method reduces the MRO problem to a bi-objective optimisation problem with the weighted statistical distance and the multivariate process capability index as objectives. NSGA-II and MOPSO are conducted on the mentioned objectives to generate the non-dominated solutions. Finally, three performance measures are utilised to assess the generated Pareto sets. To develop the method, we first define the variables and parameters of the method. Then, the methodology of the suggested approach is described in detail.

### *2.1 Parameters and variables*

The used parameters in the paper are defined as follows:

*Problem parameters*

| | |
|---|---|
| $x$ | Design vector (is a $n \times 1$ vector, $n$ = the number of control variables). |
| $y$ | Response vector (is a $r \times 1$ vector, $r$ = the number of response variables). |
| $y_{ijt}$ | The observed value of the $j^{th}$ response under the $i^{th}$ experimental run in the $t$+ replication. |
| $\bar{y}_{ij}$ | The sample mean of the $j^{th}$ response under the $i^{th}$ experimental run. |
| $\sigma_{ij}^2$ | The sample variance of the $j^{th}$ response under the $i^{th}$ experimental run. |
| $\sigma_{ijk}$ | The sample covariance between the $j^{th}$ and $k^{th}$ responses under the $i^{th}$ experimental run. |
| $LSL_j$ | Lower specification limit of the $j^{th}$ response. |
| $USL_j$ | Upper specification limit of the $j^{th}$ response. |
| $LPL_j$ | Lower process limit of the $j^{th}$ response. |
| $UPL_j$: | Upper process limit of the $j^{th}$ response. |
| $T_j$ : | Target value of the $j^{th}$ response. |
| $Ln(W_{SDi})$ | The natural logarithm of weighted statistical distance for the $i^{th}$ experimental run. |
| $Ln(CpM_i)$ | The natural logarithm of process capability index for the $i^{th}$ experimental run. |
| $\hat{y}_{Ln}(WSD)$ | The fitted response surface for the natural weighted statistical distance. |
| $\hat{y}_{Ln}(CpM)$ | The fitted responses surface for the natural logarithm of process capability index. |
| $W$ | The weight matrix for y (is a $r \times r$ diagonal matrix). |
| $w_j$ | The weight of the $j^{th}$ response. |
| $\sum_i$: | The variance-covariance matrix of responses in the $i^{th}$ iteration (is a $r \times r$ matrix). |
| $m$ | The number of replications of each response variable in the experimental design |
| $\Delta$ | Spacing metric. |
| $\bar{C}$ | Set coverage metric. |
| $NPS$ | Number of Pareto solution. |
| $\Omega$ | Experimental region. |

*Method parameters*

| | |
|---|---|
| $Q_t$ | Offspring at iteration t in NSGA II |
| $N$ | population size in NSGA II |

| | |
|---|---|
| $P_t$ | parent population in NSGA II |
| $R_t$ | entire population in NSGA II |
| $F_i$: | fronts in NSGA II |
| $V_t^i$ | velocity of the $i^{th}$ particle at iteration t in MOPSO |
| $p_t^i$ | position of the $i^{th}$ particle at iteration t in MOPSO |
| $\bar{C}$ | Normalized set coverage metric |
| $\Delta$ | Spacing metric |
| *NPS* | Number of Pareto solutions. |

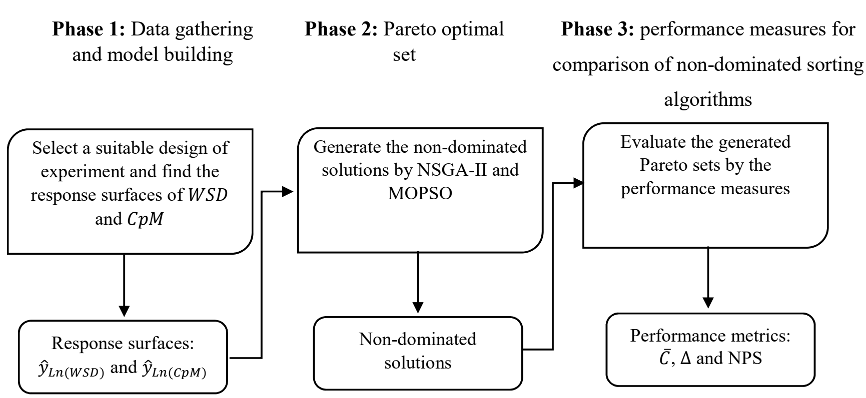
## 2.2 Model development

The framework of the proposed method consists of three phases:

1. data gathering and model building
2. pareto optimal set
3. performance measures for comparison of non-dominated sorting algorithms.

In the first phase, the used weighted statistical distance and multivariate process capability index are introduced and then are estimated in terms of design variables. Next, in the second phase, the Pareto optimal sets are generated by conducting the non-dominated sorting algorithms on the mentioned objectives. Finally, in the third phase, the generated Pareto sets are statistically compared with respect to three performance metrics. Figure 1 depicts the conceptual framework of the suggested method.

**Figure 1** Conceptual framework of the proposed method




### 2.2.1 Phase 1: data gathering and model building

The weighted statistical distance (WSD) is a multivariate function, which shows the weighted deviation of the mean quality characteristics from their corresponding targets by considering the variance-covariance structure of quality characteristics.

$$WSD = \left(\overline{y}_j - T\right)^T W\Sigma^{-1}\left(\overline{y}_j - T\right) \quad (1)$$

where $\Sigma^{-1}$ is the inverse sample variance-covariance matrix of responses.

Process capability analysis is a known method used to relate product/ process performance to customer specifications. It quantifies a process's ability to meet customer requirements. As a result, process capability indices can be used to provide a measure of customer satisfaction. For calculating these indices firstly for each quality characteristic three parameters of:

1 target value

2 upper specification limit (USL)

3 lower specification limit (LSL) are estimated by marketing department based on customer s' point-of-view.

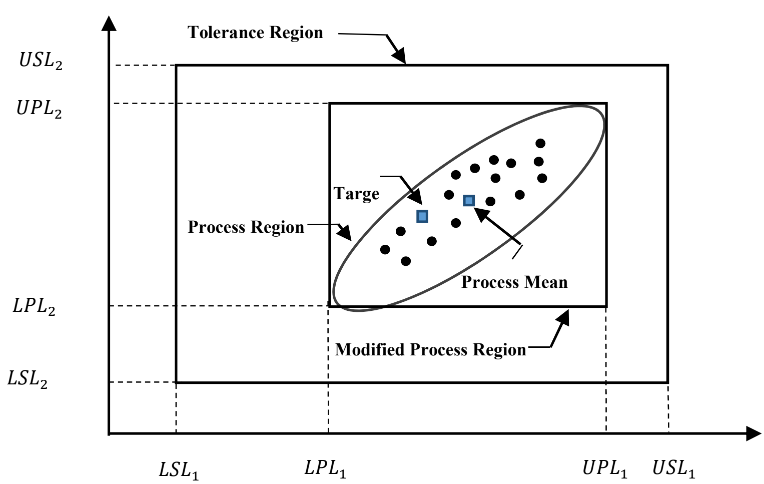
The difference between USL and LSL, called the customer tolerance interval, provides a measure of allowable process spread (i.e. customer requirements). In other words, an item produced outside the customer tolerance interval is called a defective product.

According to the mentioned explanations, customer satisfaction increases as the process has greater ability to be near the target. Larger values of the process capability indices indicate higher customer satisfaction while lower values show poorer customer satisfaction.

The employed multivariate process capability index (*CpM*) is based on the ratio of the engineering tolerance region volume to the modified process region volume. The modified process region is defined the smallest region similar in shape to the engineering tolerance region. Figure 2 illustrates the elliptical actual process region, the rectangular modified process region, and the engineering tolerance region for $r$ = 2. In this Figure, $UPL_j$ and $LPL_j$ are the edges of the modified process region.

$$CpM = \left(\frac{Vol.\,engineering\ tolreance\ region}{Vol.\,modified\ \ process\ region}\right)^{\frac{3}{r}} \quad (2)$$

**Figure 2** Illustration of multivariate process capability index when r = 2



Since the multivariate process capability index is a ratio of the tolerance region to the modified process region, the values greater than or equal to one indicate that all responses are within the specification limits that predetermined by customers. Thus, the customers are satisfied by results that fulfil their needs. On the other hand, values less than one cannot satisfy customers because at least one quality characteristic is outside the tolerance region. The *CpM* ratio is also capable of considering the correlation among quality characteristics, but it cannot take into account the location effect.

The combination of weighted statistical distance function and multivariate process capability index makes a suitable model that is able to find solutions which satisfies the customer's needs and incorporates the variance-covariance structure of responses and the deviation of the mean responses from their corresponding targets in a unifying model. It also relaxes the assumption of equal relative importance of interested quality characteristics, which is neglected in many MRO approaches. In another word, these two criteria complete each other's tasks; one considers the location effect and relative importance of responses but cannot guarantee that the non-dominated solutions fall within the specification region, whereas the other one can consider this property. The first step in MRO-solving approaches is often identification of the most important control variables. With no exception, also in this study, the design factors that may have significant effects on the weighted statistical distance and the process capability index must be identified. After finding the significant design variables, design of experiment (DOE) and RSM are applied to estimate the objectives under consideration (i.e., *WSD*, *CpM*). Therefore, a suitable experimental design, as a test or series of tests in which some changes deliberately are made on the control variable values, is utilised to detect the reasons of changing that may observe in the weighted statistical distance and the process capability index. Experimental designs such as Central composite, Box-Behnken, full factorial and fractional factorial designs are common in the data collection phase.

Since variance-covariance matrix is required for computing *WSD* and *CpM*, the sample mean and variance of the $j^{\text{th}}$ response as well as the sample covariance among the $j^{\text{th}}$ and the $k^{\text{th}}$ quality characteristics in the $i^{\text{th}}$ experimental run can be computed by following formulas:

$$\bar{y}_{ij} = \sum_{t=1}^{m} \frac{y_{ijt}}{m} \tag{3}$$

$$\sigma_{ij}^2 = \frac{1}{m-1} \sum_{t=1}^{m} \left( y_{ijt} - \bar{y}_{ij} \right)^2 \tag{4}$$

$$\sigma_{ijk} = \frac{1}{m-1} \sum_{t=1}^{m} \left( y_{ijt} - \bar{y}_{ij} \right)\left( y_{ikt} - \bar{y}_{ik} \right), \qquad \forall j \neq k \tag{5}$$

After calculating the sample mean and constructing the sample variance-covariance matrix ($\sum_i$) of the $i^{\text{th}}$ iteration, the values of the weighted statistical distance function and multivariate process capability index for the $i^{\text{th}}$ experimental run can be evaluated via equations (6) and (7), respectively.

$$\sum_i = \begin{pmatrix} \sigma_{i1}^2 & \sigma_{i12} & \cdots & \sigma_{i1r} \\ \sigma_{i2r} & \sigma_{i2}^2 & \cdots & \sigma_{i2r} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{ir1} & \sigma_{ir2} & \cdots & \sigma_{ir}^2 \end{pmatrix} \tag{6}$$

$$WSD_i = \left(\bar{y}_{ij} - T\right)^T W \sum_i^{-1} \left(\bar{y}_{ij} - T\right)$$

where W is a diagonal matrix, which the sum of its diagonal elements is equal to 1. The $\Sigma_i^{-1}$ denotes the inverse variance-covariance matrix in the $i^{\text{th}}$ iteration.

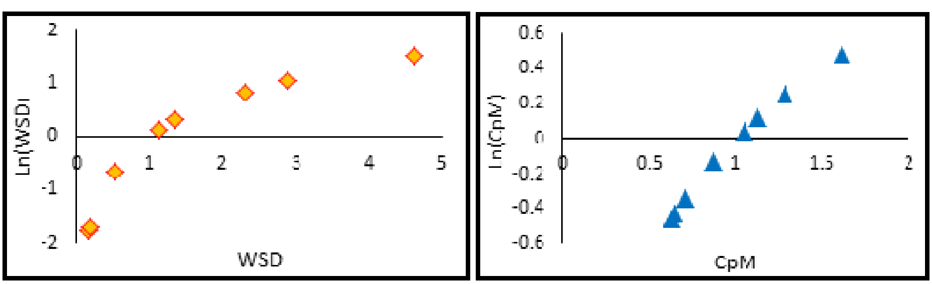
$$CpM_i = \left( \frac{\prod_{j=1}^{r} \left(USL_j - LSL_j\right)}{\prod_{j=1}^{r} \left(USL_j - LSL_j\right)} \right)^{\frac{1}{r}} \tag{7}$$

As mentioned before, $UPL_j$ and $LPL_j$ are the upper and lower process limits of the $j^{\text{th}}$ response and are obtained as follows:

$$UPL_j = \bar{y}_{ij} + \sqrt{\frac{\chi_{\alpha,r}^2 \det\left(\Sigma_{ij}^{-1}\right)}{\det\left(\Sigma_i^{-1}\right)}}, \qquad LPL_j = \bar{y}_{ij} - \sqrt{\frac{\chi_{\alpha,r}^2 \det\left(\Sigma_{ij}^{-1}\right)}{\det\left(\Sigma_i^{-1}\right)}} \tag{8}$$

where det $\left(\Sigma_i^{-1}\right)$ denotes the determinant of the inverse variance-covariance matrix of responses in the $i^{\text{th}}$ iteration and det $\left(\Sigma_{ij}^{-1}\right)$ is determinant of $\Sigma_{ij}^{-1}$ which is obtained via deleting the $j^{\text{th}}$ row and column from $\sum_i^{-1}$. $\chi_{\alpha,r}^2$ denotes the upper 100(α)% of a chi-square distribution with $r$ degree of freedoms associated with the probability contour.

**Figure 3** Relationship of the original criteria values and their natural logarithm values (see online version for colours)




After the computation of the weighted statistical distance and the multivariate process capability index in all experimental runs, the response surfaces of *Ln* ($CpM_i$) and *Ln* ($WSD_i$) can be fitted in terms of significant control variables. Since the weighted statistical distance and the multivariate process capability index are inherently positive, their natural logarithm transformations are estimated. It is worth to remark that even though $\hat{y}_{Ln(CpM)}$ and $\hat{y}_{Ln(WSD)}$ take negative values, their corresponding *CpM* and *WSD* would always be positive. Figure 3 illustrates relationship of the original values of the equations (6) and (7) with their natural logarithm values by a small instance. It is important to note that the relationship between design factors and the mentioned criteria must be well modelled. Otherwise, obtained solutions from the model by any MRO approach may not be reliable.

### 2.2.2 Phase 2: Pareto optimal set

This phase generates a set of non-dominated solutions as a Pareto optimal set by solving the following mathematical problem:

$$\begin{aligned} &Minimize\ Z = \left[-\hat{y}_{Ln(CpM)}, \hat{y}_{Ln(WSD)}\right] \\ &s.t: X \in \Omega \end{aligned} \tag{9}$$

*Definition*: A point $x^*$ is considered as a non-dominated solution, if there does not exist another point x in the experimental region such that one of the three below modes occurs:

1 $\hat{y}_{Ln(CpM)}(x) < -\hat{y}_{Ln(CpM)}(x^*)$ and $\hat{y}_{Ln(WSD)}(x) \le \hat{y}_{Ln(WSD)}(x^*)$

2 $\hat{y}_{Ln(CpM)}(x) \le -\hat{y}_{Ln(CpM)}(x^*)$ and $\hat{y}_{Ln(WSD)}(x) < \hat{y}_{Ln(WSD)}(x^*)$

3 $\hat{y}_{Ln(CpM)}(x) < -\hat{y}_{Ln(CpM)}(x^*)$ and $\hat{y}_{Ln(WSD)}(x) < \hat{y}_{Ln(WSD)}(x^*)$

This phase aims to generate a Pareto optimal set to provide sufficient insight into trade-offs between two criteria under consideration. Carlyle (2003) introduced three desirable properties for evaluating the quality of a Pareto optimal set namely, diversity (a wide range of non-dominated solutions), uniformity (a uniform distribution of the non-dominated solutions) and cardinality (a large number of non-dominated solutions). According to the mentioned properties, two evolutionary multi-objective optimisation algorithms NSGA-II and MOPSO are selected to generate the efficient Pareto optimal set that will be explained in below.

#### 2.2.2.1 NSGA-II

NSGA-II is one of the most popular methods among the evolutionary multi-objective optimisation (EMO) algorithms that was introduced by Deb et al. (2000) and recently has been employed widely by some researchers such as Frotus and Tohme (2013), Yadav et al. (2014), Safarsadeh and Matahhar (2014), Rajabi-Bahmani et al. (2015), Liu et al. (2015) and Arora et al. (2016). It has the ability to find much spread solutions over the Pareto optimal set-in contrast to most of the conventional techniques that employ one elite-preservation strategy. In the first step, NSGA-II creates the offspring $Q_t$ from the parent population $P_t$ by utilising tournament selection, recombination (crossover) and mutation operators. Then, these two populations are combined to form the entire population $R_t$ of size of $2N$, where N is population size. Next, a non-dominated sorting is conducted to classify $R_t$. Finally, the new population is filled by solutions of different fronts $F_i$ in the basis of their ranks. When the last front is being considered, there may exist more solutions than the remaining slots in the new population. In this situation, a crowding sort of procedure is done to choose the members of the last front in a way that a diverse set of solutions is selected from this front set.

The schematic of NSGA-II algorithm is depicted in Figure 4 and pseudo code of algorithm also illustrated in Figure 5.

**Figure 4** Schematic of the NSGA-II procedure

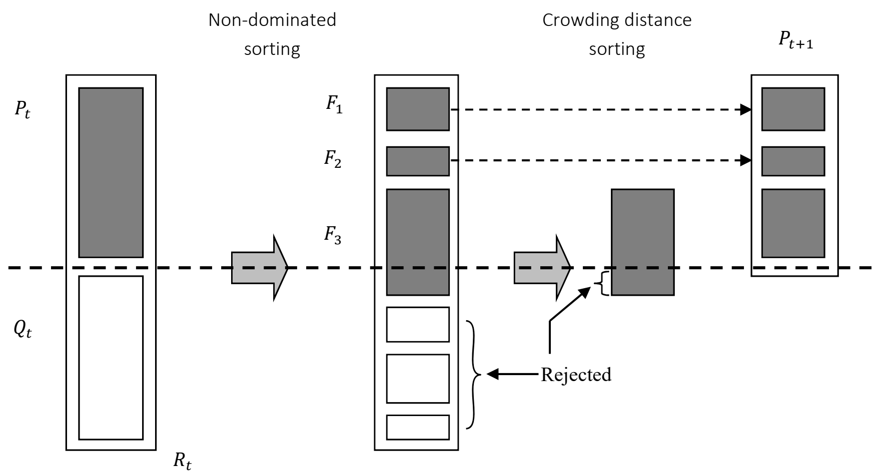



**Figure 5** NSGA-II algorithm (see online version for colours)

1. While iteration number is less than the maximum iteration do
2. Create $R_t$ by incorporating offspring and parent populations
3. Carry out a non-dominated sorting to $R_t$ and identify different fronts $F_i$
4. Set new population $P_{t+1} = \emptyset$ and counter $i = 1$.
5. While $|P_{t+1}| + |F_i| < N$ do
6. Carry out $P_{t+1} = P_{t+1} \cup F_i$ and $i = i + 1$
7. End while
8. Carry out the crowding sort process and involve the most widely spread $(N - |P_{t+1}|)$ solutions.
9. Create offspring population $Q_{t+1}$ from $P_{t+1}$ using the crowded tournament selection, crossover and mutation operators.
10. End while

### *2.2.2.2 MOPSO*

Coello et al. (2004) proposed MOPSO algorithm as one of the fastest algorithms among the EMO algorithms. Zhu et al. (2015), Meza et al. (2015) and Zhang and Chen (2016) are some researchers who employed the MOPSO algorithm in their studies. In the first step, MOPSO initialises a swarm of particles. Then, the non-dominated solutions are determined and stored in an external archive called repository. Next, the hypercubes are constructed via dividing the search space in order to determine a leader for each particle of the swarm. The classical roulette wheel selection is employed to select a hypercube in which selection probability of each hypercube has inverse relationship with the number of repository members in the given hypercube. Later on, a leader is selected randomly, and the position of particle is updated. Finally, the mutation operator is performed for better search and the personal best position is updated. Figures 6 and 7 illustrate the representation of a searching point and the pseudo code of MOPSO algorithm, respectively.

**Figure 6** Schematic of the MOPSO procedure (see online version for colours)

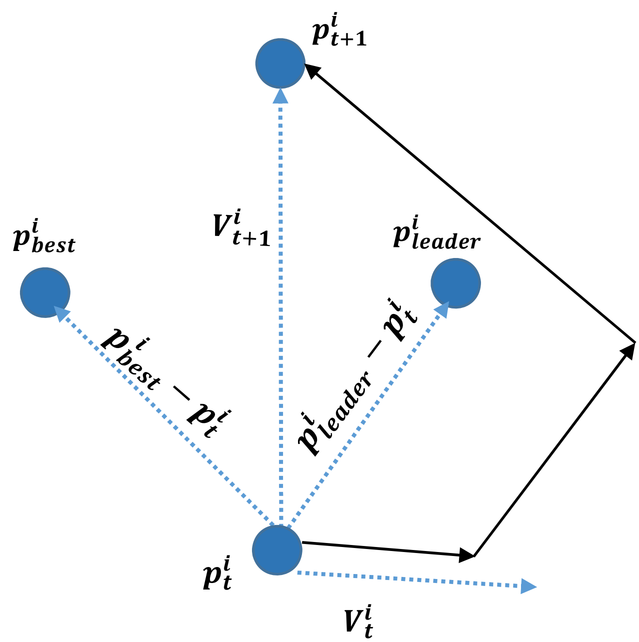



**Figure 7** MOPSO algorithm

1. Initialize a swarm of particles.
2. Initialize velocity and personal best of the $i^{th}$particle as $\boldsymbol{V}_0^i = \boldsymbol{0}$ and $\boldsymbol{P}^i_{best} = \boldsymbol{P}^i$, respectively
3. Construct the repository by identifying the non-dominated solutions
4. While iteration number is less than the Maximum iteration do
5. For each particle do
6. Construct the hypercube via dividing the search space
7. Select a leader for the $i^{th}$particle from the repository
8. Update velocity and position of the $i^{th}$particle
9. Conduct mutation operator on particles and update the personal best of the $i^{th}$particle.
10. End for
11. Update the repository.
12. End while.

### 2.2.3 Phase 3: performance measures for comparison of non-dominated sorting algorithms

The purpose of this phase is introducing three performance measures for comparison of the obtained Pareto optimal sets from non-dominated sorting algorithms. As mentioned in the previous phase unlike the single objective optimisation methods, which aim to find the best or near the best solution, the multi-objective optimisation algorithms such as NSGA-II and MOPSO look for three features simultaneously:

1 diversity

2 uniformity

3 cardinality.

In this regard, three famous metrics are utilised as follows:

#### 2.2.3.1 Normalised set coverage metric $(\bar{C})$

Set coverage metric was proposed by Zitzler and Thiele (1998) for comparing two different Pareto sets. Assume in a problem, there are two sets of non-dominated solution *A* and *B*. set coverage metric *C* (*A*, *B*) computes the fraction of *B*, which is weakly dominated by *A* as equation (10):

$$C(A, B) = \frac{\left|\{b \in B \middle| \mathrm{E}\alpha \in A : a \prec b\}\right|}{|B|} \tag{10}$$

where $a \preccurlyeq b$ means that solution a weakly dominates solution *b*. *C* (*A*, *B*) = 1 demonstrates that all the non-dominated solutions in B are weakly dominated by *A*. On the other hand, *C* (*A*, *B*) = 0 means that none of the points in set B can be weakly dominated by A. It is worth to notice that since the *C* (*A*, *B*) is not necessary equals to 1 –*C* (*B*, *A*) both *C* (*A*, *B*) and *C* (*B*, *A*) must be considered. For simplifying the comparison of the two sets, the normalized set coverage metric $(\bar{C})$ is proposed as equation (11). This equation assures $\bar{C}(A, B) = 1 - \bar{C}(B, A)$ which makes comparison of algorithms easier than set coverage metric. With respect to the mentioned definition, $\bar{C}(A, B) \geq \bar{C}(B, A)$ indicates that set *A* has better coverage than B.

$$\bar{C}(A, B) = \frac{C(A, B)}{C(A, B) + C(B, A)} \tag{11}$$

#### 2.2.3.2 Spacing metric (Δ)

Spacing metric was introduced by Deb et al. (2001). It measures the spread of solutions of a Pareto set in the entire region through computing variance of distances of the neighbouring solutions in the given Pareto set.

$$\Delta = \sum_{i=1}^{|n|} \frac{|d_i - \bar{d}|}{|n|} \tag{12}$$

where

$$d_i = \min k \in n,$$

$$k \neq i \left\{ \sqrt{\left[ \hat{y}_{Ln(CpM)}(x^i) - \hat{y}_{Ln(CpM)}(x^k) \right]^2 + \left[ \hat{y}_{Ln(WSD)}(x^i) - \hat{y}_{Ln(WSD)}(x^k) \right]^2} \right\}, \bar{d}$$

is mean of the Euclidian distances ($d_i$) and n denotes the number of non-dominated solutions in the Pareto set. According to the mentioned definition for spacing metric, the smaller value of Δ is more desirable.

#### *2.2.3.3 Number of Pareto solutions*

This metric is used to measure the cardinality of algorithm and the higher value of NPS shows the better performance of algorithm.

## 3 Numerical example

In order to illustrate the applicability of the proposed method, two industrial case studies from Pignatiello (1993) and Costa et al. (2011) are employed. In each numerical example three comparative studies are performed. Since posterior approaches have been rarely investigated in the MRO literature, In the first comparative study the performance of the most popular methods for generating Pareto set consisting of NSGAII, MOPSO and $\varepsilon$-constraint are compared to each other. The second comparative study includes the comparison of the most well-known Posterior approaches in the MRO literature. Finally, for more clarification of joint optimisation importance of multivariate process capability index and weighted statistical distance, the proposed method is compared to state-of-the-art methods in the literature as well as individual optimisation of multivariate process capability index and weighted statistical distance, separately.

### *3.1 Example 1*

The case study given in Pignatiello (1993), which is used in this section, consists of two correlated quality characteristics ($y_1$,$y_2$) and three process variables ($x_1$, $x_2$, $x_3$). It is assumed that the target values of the quality characteristics are 103 and 73 and the specification limits are (97,109) and (70, 76) for $y_1$ and $y_2$, respectively.

The procedure of the proposed method on this example is described as follows:

#### *3.1.1 Phase 1: data gathering and model building*

The experiment is conducted in a full factorial design with four replications and the results are displayed in Table 2.

The $\bar{y}_{ij}$, $\sigma_{ij}^2$ and $\sigma_{ijk}$ for these two responses in eight experimental runs are calculated via equations (3) to (5), and the obtained results are shown in Table 3. Then, the weight matrix should be selected based on standards of organisation or decision maker's subjective judgments. For further analysis in the cases with more than two responses, the weight matrix can be determined through one of the weight generation methods in literature such as Moeini et al. (2011) and Ahmadi-Javid and Moeini (2015).

**Table 2** The Experimental results of numerical example

| *Experimental run* | $x_1$ | $x_2$ | $x_3$ | $y_1$ | | | | $y_2$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | *1* | *2* | *3* | *4* | *1* | *2* | *3* | *4* |
| 1 | –1 | –1 | –1 | 109.895 | 109.759 | 110.704 | 109.773 | 67.6974 | 67.2374 | 67.962 | 66.9268 |
| 2 | 1 | –1 | 1 | 100.192 | 99.634 | 100.269 | 100.6 | 67.0264 | 66.1779 | 66.5758 | 67.9431 |
| 3 | –1 | 1 | –1 | 106.078 | 105.642 | 105.671 | 105.393 | 72.9353 | 72.8508 | 72.5768 | 72.3754 |
| 4 | 1 | 1 | –1 | 104.12 | 104.802 | 104.203 | 104.335 | 72.9878 | 74.2487 | 73.9371 | 73.2824 |
| 5 | –1 | –1 | 1 | 113.515 | 111.121 | 112.854 | 106.666 | 68.2934 | 68.4693 | 68.9576 | 64.7051 |
| 6 | 1 | 1 | 1 | 98.732 | 99.357 | 102.842 | 94.235 | 67.0955 | 63.6112 | 68.647 | 62.4118 |
| 7 | –1 | 1 | 1 | 103.145 | 106.959 | 107.62 | 103.44 | 71.6818 | 75.2657 | 77.4958 | 76.3739 |
| 8 | 1 | 1 | 1 | 104.454 | 105.029 | 99.786 | 104.923 | 76.9003 | 77.0322 | 68.989 | 75.7691 |

**Table 3** The results of sample mean, variance and covariance

| *Experimental run* | $x_1$ | $x_2$ | $x_3$ | $\overline{y}_1$ | $\overline{y}_2$ | $\sigma_1^2$ | $\sigma_2^2$ | $\sigma_{12}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | –1 | –1 | –1 | 100.033 | 65.4559 | 0.0204 | 0.214 | 0.1679 |
| 2 | 1 | –1 | –1 | 100.174 | 66.9308 | 0.1608 | 0.5756 | 0.2686 |
| 3 | –1 | 1 | –1 | 105.696 | 72.6846 | 0.0804 | 0.0659 | 0.0611 |
| 4 | 1 | 1 | –1 | 104.365 | 73.614 | 0.0927 | 0.3364 | 0.1295 |
| 5 | –1 | –1 | 1 | 111.039 | 67.6064 | 9.5182 | 3.8199 | 5.6372 |
| 6 | 1 | –1 | 1 | 98.791 | 65.4431 | 12.4972 | 8.4992 | 8.5411 |
| 7 | –1 | 1 | 1 | 105.291 | 75.4543 | 5.4127 | 6.6345 | 4.1672 |
| 8 | 1 | 1 | 1 | 103.548 | 74.4227 | 6.3525 | 18.7178 | 10.7214 |

$$\hat{y}_{Ln(CpM)} = 0.1389 - 0.1538x_1 + 0.1007x_2 - 0.9597x_3 - 0.9991x_2x_3 \quad (13)$$

$$W = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$\hat{y}_{Ln(WSD)} = 2.509 - 0.803x_1 - 1.214x_2 - 1.312x_3 - 0.791x_2x_3 + 0.562x_1x_2x_3 \quad (14)$$

$$W = \begin{bmatrix} 0.34 & 0 \\ 0 & 0.66 \end{bmatrix}$$

$$\hat{y}_{Ln(WSD)} = 2.509 - 0.941x_1 - 1.038x_2 - 1.462x_3 - 1.055x_2x_3 + 0.468x_1x_2x_3 \quad (15)$$

$$W = \begin{bmatrix} 0.57 & 0 \\ 0 & 0.43 \end{bmatrix}$$

$$\hat{y}_{Ln(WSD)} = 2.539 - 1.071x_1 - 1.029x_2 - 1.594x_3 - 0.227x_1x_3 - 1.203x_2x_3 + 0.468x_1x_2x_3 \quad (16)$$

$$W = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.8 \end{bmatrix} \quad (17)$$

$$\hat{y}_{Ln(WSD)} = 2.613 - 0.836x_1 - 1.411x_2 - 1.088x_3$$

In this example, four weight matrices are considered. The natural logarithm of the process capability index and the weighted statistical distance are calculated in each experimental run based on the response values in Table 2 and the weight matrices. Table 4 summarises the obtained results for the interested criteria. Finally, the response surfaces of *Ln*(*CpM*) as well as *Ln*(*WSD*) for different weight matrices are estimated as follows:

**Table 4** The results of constructing the new responses

| *Experimental run* | $x_1$ | $x_2$ | $x_3$ | *Ln(CpM)* | *Ln(WSD)* | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | $W=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix}$ | $W=\begin{bmatrix}0.34 & 0\\ 0 & 0.66\end{bmatrix}$ | $W=\begin{bmatrix}0.57 & 0\\ 0 & 0.43\end{bmatrix}$ | $W=\begin{bmatrix}0.2 & 0\\ 0 & 0.8\end{bmatrix}$ |
| 1 | –1 | –1 | –1 | 0.99277 | 4.4432 | 4.8708 | 4.2465 | 5.1366 |
| 2 | 1 | –1 | –1 | 0.80491 | 3.4657 | 3.947 | 3.3123 | 3.9414 |
| 3 | –1 | 1 | –1 | 1.52161 | 5.2405 | 4.9461 | 5.3463 | 4.5899 |
| 4 | 1 | 1 | –1 | 1.07688 | 2.736 | 2.181 | 2.9068 | 1.1336 |
| 5 | –1 | –1 | 1 | –0.64817 | 4.7053 | 4.7148 | 4.7012 | 4.723 |
| 6 | 1 | –1 | 1 | –0.95581 | 1.5735 | 2.019 | 1.2916 | 2.2927 |
| 7 | –1 | 1 | 1 | –0.68558 | –0.58983 | –0.625 | –0.5748 | –0.6567 |
| 8 | 1 | 1 | 1 | –0.98457 | –1.5024 | –0.6631 | –2.3586 | –0.2596 |

### *3.1.2 Phase 2: Pareto optimal set*

In this section, two suggested meta-heuristic algorithms namely, NSGA-II and MOPSO as well as ε-constraint method, which is proposed in Lee et al. (2011), are conducted on the response surfaces of $\hat{y}_{Ln(CpM)}$ and $\hat{y}_{Ln(WSD)}$ to generate the non-dominated solutions. The conventional ε-Constraint method considers one of the objectives under consideration as main objective and other objectives as constraints. The conventional $\varepsilon$-Constraint method generates the weakly non-dominated solutions (Hassannayebi et al., 2011). To overcome this drawback, the modified $\varepsilon$-Constraint method is introduced which considers a sufficient small coefficient ($\rho$) to generate the strongly non-dominated solutions. The optimisation scheme of the modified ε-Constraint method which adapted in this study is given:

$$\begin{aligned} &Minimize\left\{-\hat{y}_{Ln(CpM)} - \rho\left(\hat{y}_{Ln(WSD)}\right)\right\} \\ &s.t.: \hat{y}_{Ln(WSD)} < \varepsilon \\ &X \in \Omega \end{aligned} \tag{18}$$

To generate the efficient solutions, the mentioned algorithms were coded in the MATLAB software. The used parameters are given in Table 5. To compare the non-dominated sorting algorithms, each of them is executed six times for each weight matrix. Consequently, 24 Pareto sets are generated, which are used for comparative parametric and non-parametric statistical tests.

**Table 5** The used parameters in Pareto set generation algorithms

| *Algorithm* | *Parameter* | *Value* |
|---|---|---|
| NSGA-II | Population size | 80 |
| | Maximum iteration | 120 |
| | Crossover rate | 0.7 |
| | Mutation rate | 0.4 |
| MOPSO | Population Size | 80 |
| | Repository size | 80 |
| | Maximum Iteration | 120 |
| | Number of divisions | 30 |
| ε-Constraint | ρ | 0.001 |
| | ε | 0.03 |

### *3.1.3 Phase 3: comparison of NSGA-II, MOPSO and ε-constraint*

To measure the performance of the non-dominated sorting algorithms, three metric criteria are conducted on the 24 generated Pareto sets. The obtained metric criteria values are shown in Table 6 and the average of the metric criteria are given in the last row. In Table 6 *A*, *B* and *C* denote the Pareto sets generated by NSGA-II, MOPSO and *ε-constraint* algorithms, respectively. Figure 8 shows the results of the performance metrics of the three algorithms graphically. The average values of measures imply that the NSGA-II algorithm generates the best Pareto sets. It has the smallest Spacing metric values and the greatest NPS and normalised set coverage values among the employed algorithms. Furthermore, the results in the last row of Table 6 confirm that the MOPSO algorithm generates the more appropriate Pareto sets compared to *ε-constraint* algorithm. The results also are evaluated by both parametric and non-parametric tests. The parametric t-test is used to study the difference between mean values of NPS, Δ and $\overline{C}$ in three algorithms. On the other hand, the nonparametric Mann-Whitney test is employed for evaluation of difference in median values of criteria under consideration except for the comparison of median NPS of NSGA-II algorithm with the other algorithms, which the sign test is used due to the constant NPS values of NSGA-II algorithm. The hypothesises are given in Table 7 in which μ and m are mean and median, respectively. The given *P*-values in Table 7 confirm that the NSGA-II generates significantly the better Pareto sets and also show the better performance of MOPSO algorithm compared to ε-Constraint method.

**Figure 8** Comparison of NSGA-II, MOPSO and ε-constraint based on metric measures (see online version for colours)

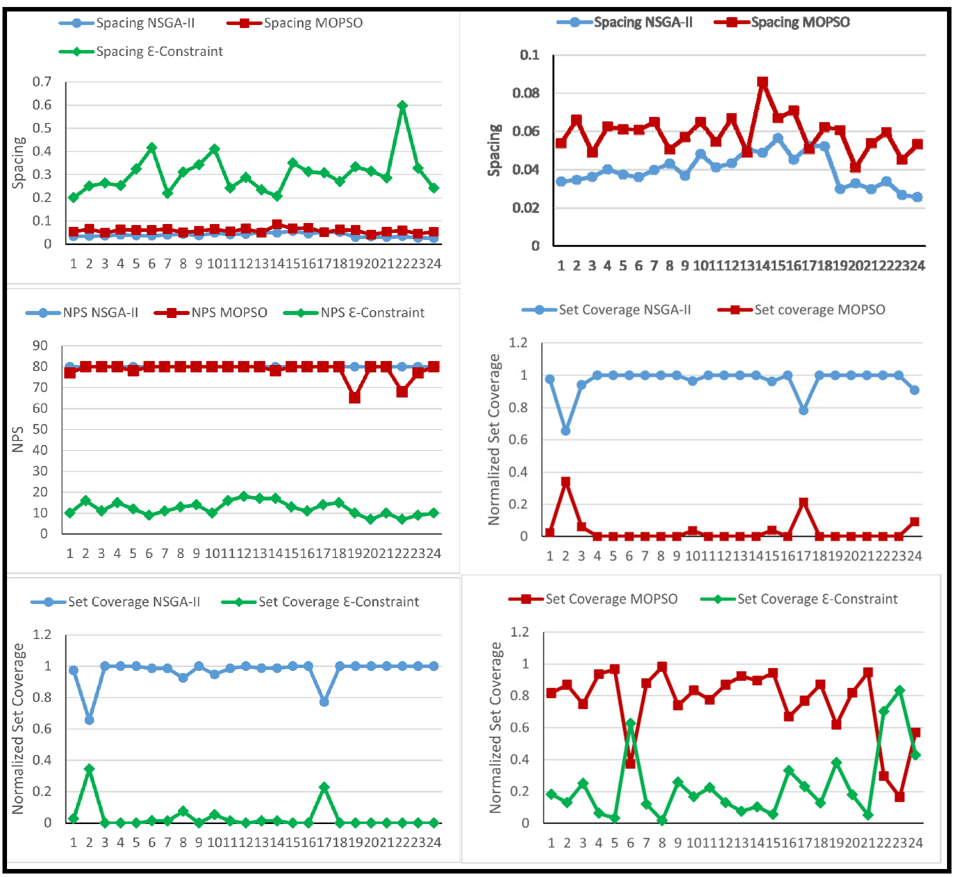


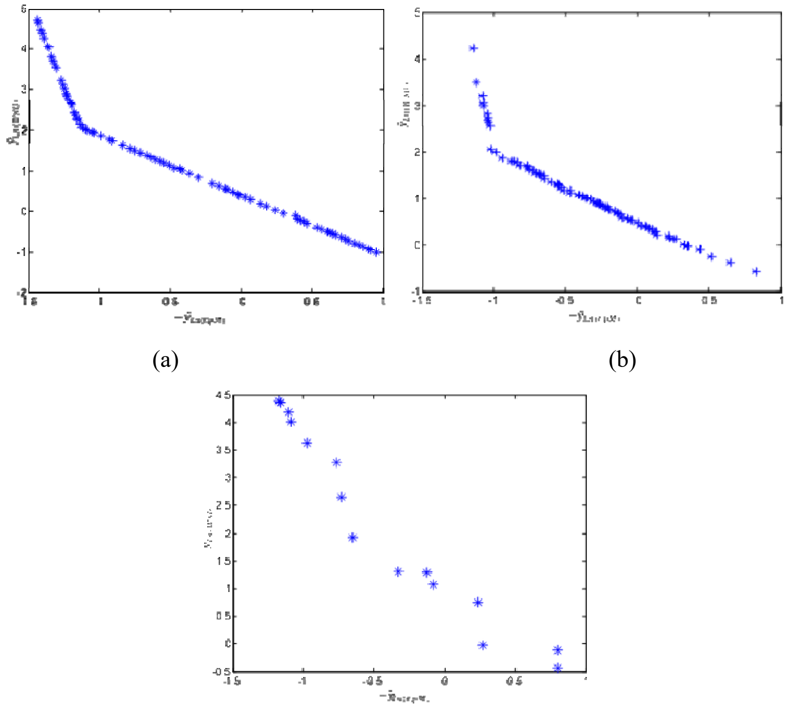
**Table 6** The results of performance metrics

| *Weight matrix* | *NSGA-II* | | | | *MOPSO* | | | | *ε-constraint* | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Delta$ | $\bar{C}(A, B)$ | $\bar{C}(A, C)$ | *NPS* | $\Delta$ | $\bar{C}(B, A)$ | $\bar{C}(B, C)$ | *NSP* | $\Delta$ | $\bar{C}(C, A)$ | $\bar{C}(C, B)$ | *NSP* |
| $\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ | 0.0338 | 0.9756 | 0.9730 | 80 | 0.0538 | 0.0244 | 0.8179 | 77 | 0.2007 | 0.0270 | 0.1821 | 10 |
| | 0.0347 | 0.6557 | 0.6557 | 80 | 0.0661 | 0.3443 | 0.8696 | 80 | 0.2512 | 0.3443 | 0.1304 | 16 |
| | 0.0362 | 0.9398 | 1.0000 | 80 | 0.0490 | 0.0602 | 0.7484 | 80 | 0.2636 | 0.0000 | 0.2516 | 11 |
| | 0.0402 | 1.0000 | 1.0000 | 80 | 0.0625 | 0.0000 | 0.9362 | 80 | 0.2528 | 0.0000 | 0.0638 | 15 |
| | 0.0374 | 1.0000 | 1.0000 | 80 | 0.0611 | 0.0000 | 0.9670 | 78 | 0.3243 | 0.0000 | 0.0330 | 12 |
| | 0.0361 | 1.0000 | 0.9861 | 80 | 0.0608 | 0.0000 | 0.3721 | 80 | 0.4165 | 0.0139 | 0.6279 | 9 |
| $\begin{bmatrix} 0.34 & 0 \\ 0 & 0.66 \end{bmatrix}$ | 0.0399 | 1.0000 | 0.9864 | 80 | 0.0649 | 0.0000 | 0.8791 | 80 | 0.2192 | 0.0136 | 0.1209 | 11 |
| | 0.0431 | 1.0000 | 0.9249 | 80 | 0.0506 | 0.0000 | 0.9823 | 80 | 0.3108 | 0.0751 | 0.0177 | 13 |
| | 0.0368 | 1.0000 | 1.0000 | 80 | 0.0571 | 0.0000 | 0.7407 | 80 | 0.3424 | 0.0000 | 0.2593 | 14 |
| | 0.0482 | 0.9639 | 0.9474 | 80 | 0.0649 | 0.0361 | 0.8333 | 80 | 0.4103 | 0.0526 | 0.1667 | 10 |
| | 0.0412 | 1.0000 | 0.9868 | 80 | 0.0545 | 0.0000 | 0.7759 | 80 | 0.2421 | 0.0132 | 0.2241 | 16 |
| | 0.0434 | 1.0000 | 1.0000 | 80 | 0.0669 | 0.0000 | 0.8696 | 80 | 0.2873 | 0.0000 | 0.1304 | 18 |
| $\begin{bmatrix} 0.57 & 0 \\ 0 & 0.43 \end{bmatrix}$ | 0.0506 | 1.0000 | 0.9869 | 80 | 0.0489 | 0.0000 | 0.9244 | 80 | 0.2344 | 0.0131 | 0.0756 | 17 |
| | 0.0489 | 1.0000 | 0.9869 | 80 | 0.0860 | 0.0000 | 0.8958 | 78 | 0.2069 | 0.0131 | 0.1042 | 17 |
| | 0.0565 | 0.9615 | 1.0000 | 80 | 0.0670 | 0.0385 | 0.9442 | 80 | 0.3506 | 0.0000 | 0.0558 | 13 |
| | 0.0453 | 1.0000 | 1.0000 | 80 | 0.0709 | 0.0000 | 0.6689 | 80 | 0.3129 | 0.0000 | 0.3311 | 11 |
| | 0.0525 | 0.7843 | 0.7715 | 80 | 0.0508 | 0.2157 | 0.7692 | 80 | 0.3078 | 0.2285 | 0.2308 | 14 |
| | 0.0523 | 1.0000 | 1.0000 | 80 | 0.0622 | 0.0000 | 0.8727 | 80 | 0.2700 | 0.0000 | 0.1273 | 15 |
| $\begin{bmatrix} 0.2 & 0 \\ 0 & 0.8 \end{bmatrix}$ | 0.0299 | 1.0000 | 1.0000 | 80 | 0.0607 | 0.0000 | 0.6191 | 65 | 0.3337 | 0.0000 | 0.3809 | 10 |
| | 0.0328 | 1.0000 | 1.0000 | 80 | 0.0410 | 0.0000 | 0.8205 | 80 | 0.3151 | 0.0000 | 0.1795 | 7 |
| | 0.0298 | 1.0000 | 1.0000 | 80 | 0.0538 | 0.0000 | 0.9474 | 80 | 0.2861 | 0.0000 | 0.0526 | 10 |
| | 0.0339 | 1.0000 | 1.0000 | 80 | 0.0595 | 0.0000 | 0.2970 | 68 | 0.5975 | 0.0000 | 0.7030 | 7 |
| | 0.0268 | 1.0000 | 1.0000 | 80 | 0.0453 | 0.0000 | 0.1659 | 77 | 0.3276 | 0.0000 | 0.8341 | 9 |
| | 0.0256 | 0.9080 | 1.0000 | 80 | 0.0533 | 0.0920 | 0.5714 | 80 | 0.2421 | 0.0000 | 0.4286 | 10 |
| Average | 0.0398 | 0.9662 | 0.9669 | 80 | 0.0588 | 0.0338 | 0.7620 | 78.458 | 0.3044 | 0.0331 | 0.2380 | 12.2917 |

**Table 7** Statistical test results

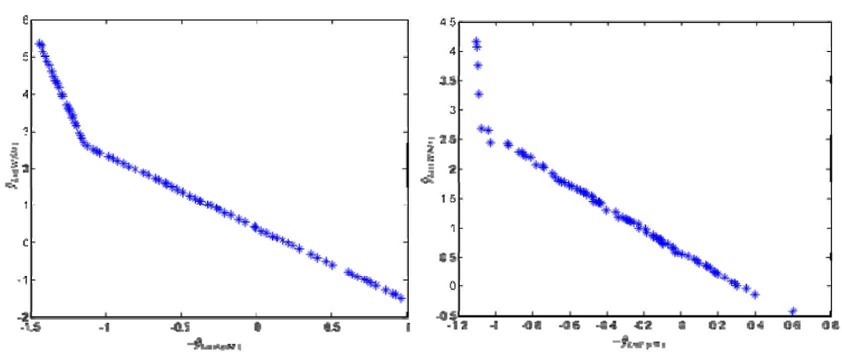
| *Criteria* | *Parametric test* | | | | *Nonparametric test* | | | |
|---|---|---|---|---|---|---|---|---|
| | *Hypothesis* | *P-value* | *Hypothesis* | *P-value* | *Hypothesis* | *P-value* | *Hypothesis* | *P-value* |
| NPS | $\begin{cases} H_0: \mu_A = \mu_B \\ H_1: \mu_A > \mu_B \end{cases}$ | 0.031 | $\begin{cases} H_0: \mu_A = \mu_C \\ H_1: \mu_A > \mu_C \end{cases}$ | 0.00 | $\begin{cases} H_0: m_A = m_B \\ H_1: m_A > m_B \end{cases}$ | 0.0156 | $\begin{cases} H_0: m_A = m_C \\ H_1: m_A > m_C \end{cases}$ | 0.00 |
| | - | | $\begin{cases} H_0: \mu_B = \mu_C \\ H_1: \mu_B > \mu_C \end{cases}$ | 0.00 | | | $\begin{cases} H_0: m_B = m_C \\ H_1: m_B > m_C \end{cases}$ | 0.00 |
| $\Delta$ | $\begin{cases} H_0: \mu_A = \mu_B \\ H_1: \mu_A < \mu_B \end{cases}$ | 0.00 | $\begin{cases} H_0: \mu_A = \mu_C \\ H_1: \mu_A < \mu_C \end{cases}$ | 0.00 | $\begin{cases} H_0: m_A = m_B \\ H_1: m_A < m_B \end{cases}$ | 0.00 | $\begin{cases} H_0: m_A = m_C \\ H_1: m_A < m_C \end{cases}$ | 0.00 |
| | - | | $\begin{cases} H_0: \mu_B = \mu_C \\ H_1: \mu_B < \mu_C \end{cases}$ | 0.00 | | | $\begin{cases} H_0: m_B = m_C \\ H_1: m_B < m_C \end{cases}$ | 0.00 |
| $\bar{C}$ | $\begin{cases} H_0: \mu_A = \mu_B \\ H_1: \mu_A > \mu_B \end{cases}$ | 0.00 | $\begin{cases} H_0: \mu_A = \mu_C \\ H_1: \mu_A > \mu_C \end{cases}$ | 0.00 | $\begin{cases} H_0: m_A = m_B \\ H_1: m_A > m_B \end{cases}$ | 0.00 | $\begin{cases} H_0: m_A = m_C \\ H_1: m_A > m_C \end{cases}$ | 0.00 |
| | - | | $\begin{cases} H_0: \mu_B = \mu_C \\ H_1: \mu_B > \mu_C \end{cases}$ | 0.00 | | | $\begin{cases} H_0: m_B = m_C \\ H_1: m_B > m_C \end{cases}$ | 0.00 |

**Figure 9** The Pareto sets when $W = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ (a) NSGA-II Pareto optima 1 set (b) the MOPSO Pareto optima 1 set and (c) the $\varepsilon$-constraint Pareto optima 1 set (see online version for colours)



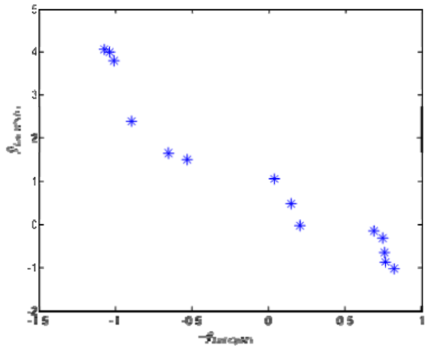

(c)

**Figure 10** The Pareto sets when $W = \begin{bmatrix} 0.34 & 0 \\ 0 & 0.66 \end{bmatrix}$ (a) NSGA-II Pareto optima 1 set (b) the MOPSO Pareto optima 1 set and (c) the $\varepsilon$-constraint Pareto optima 1 set (see online version for colours)



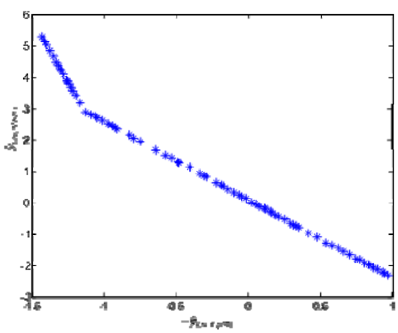
(a) (b)

**Figure 10** The Pareto sets when $W = \begin{bmatrix} 0.34 & 0 \\ 0 & 0.66 \end{bmatrix}$ (a) NSGA-II Pareto optima 1 set (b) the MOPSO Pareto optima 1 set and (c) the $\varepsilon$-constraint Pareto optima 1 set (see online version for colours) (continued)

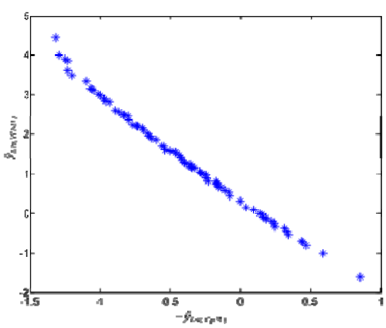


(c)

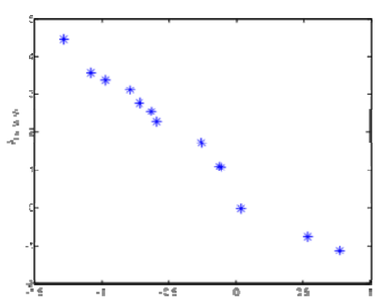
**Figure 11** The Pareto sets when $W = \begin{bmatrix} 0.57 & 0 \\ 0 & 0.43 \end{bmatrix}$ (a) NSGA-II Pareto optima 1 set (b) the MOPSO Pareto optima 1 set and (c) the $\varepsilon$-constraint Pareto optima 1 set (see online version for colours)



(a)

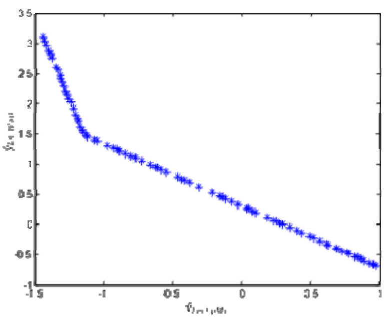


(b)

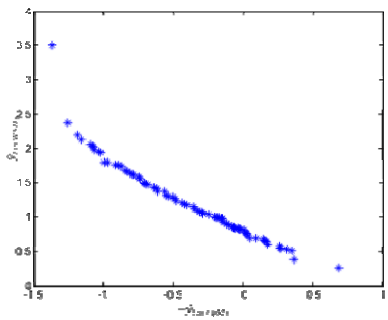


(c)

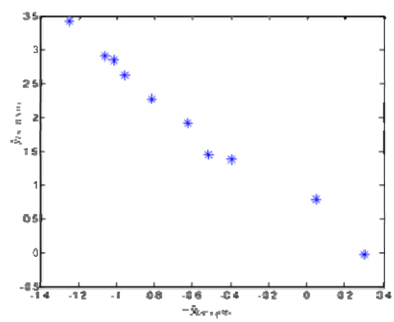
**Figure 12** The Pareto sets when $W = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.8 \end{bmatrix}$ (a) NSGA-II Pareto optima 1 set (b) the MOPSO Pareto optima 1 set and (c) the $\varepsilon$-constraint Pareto optima 1 set (see online version for colours)



(a)



(b)



(c)

## 3.2 *Comparison of the proposed method with the posterior methods in example 1*

In this section, the suggested approach is compared with the existing posterior preference articulation methods. In this regard, the presented approaches in Lee et al. (2011) and Salmasnia et al. (2013b) and two presented models in Costa and Lourenço (2015) are conducted on the numerical example. Then, one of the generated non-dominated solutions is determined as the best solution via their proposed best solution selection phase. Table 8 depicts the achieved results from the mentioned methods along with the results obtained from employing VIKOR method, which is described briefly in appendix A, on the generated non-dominated solutions in the previous Section. Moreover, the last four columns in Table 8 give the obtained results for the natural logarithm of the process capability index and the weighted statistical distance in addition to performance measures of NPS and Δ. For the better comparison, the response surfaces of standard deviation of original responses are estimated and their results are given in the columns 4 and 5 for $y_1$ and $y_2$, respectively. The estimated surfaces of these responses can be seen in the appendix C.

**Table 8** Comparison of the proposed method with some posterior approaches in example 1

| *Weigh matrix* | *Model* | *X* | $\hat{y}_{1\sigma}$ | $\hat{y}_{2\sigma}$ | $\hat{y}_{Ln(CpM)}$ | $\hat{y}_{Ln(WSD)}$ | *NPS* | *Δ* |
|---|---|---|---|---|---|---|---|---|
| $\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ | Lee et al. (2011) | (1,0.6856,1) | 2.6889 | 3.6223 | –0.9735 | –0.5953 | 4 | 0.2073 |
| | Salmasnia et al. (2013b) | (–0.9970,0.9976, –0.9998) | 0.1099 | 0.3610 | 1.4510 | 4.7580 | 7 | 0.0012 |
| | Costa and Lourenço (2015) model 1 | (1,0.8037,0.2346) | 1.7021 | 2.4306 | –0.1778 | 0.3793 | 10 | 0.1275 |
| | Costa and Lourenço (2015) moddel2 | (1,0.7998, 0.5760) * | 2.1299 | 2.8451 | –0.5328 | –0.1262 | 7 | 0.2358 |
| | The proposed method (NSGA–II) | (0.9802,0.9872, –0.9613) | 0.1606 | 0.7237 | 1.1041 | 2.0124 | 80 | 0.0402 |
| | The proposed method (MOPSO) | (0.9790,0.9579, –0.4032) | 0.8657 | 1.5439 | 0.5100 | 1.1820 | 80 | 0.0625 |
| $\begin{bmatrix} 0.34 & 0 \\ 0 & 0.66 \end{bmatrix}$ | Lee et al. (2011) | (1,0.6856, 1) | 2.6889 | 3.6223 | –0.9735 | –1.0008 | 4 | 0.2073 |
| | Salmasnia et al. (2013b) | (–0.9989,0.9977, –0.9995) | 0.1102 | 0.3609 | 1.4510 | 5.3928 | 7 | 0.0012 |
| | Costa and Lourenço (2015) model 1 | (1,0.7956,1) | 2.6609 | 3.3598 | –0.9733 | –1.1868 | 10 | 0.1275 |
| | Costa and Lourenço (2015) moddel2 | (0.1064,0.7199, –0.0005) * | 1.4295 | 1.7731 | 0.1955 | 1.6627 | 7 | 0.2358 |
| | The proposed method (NSGA-II) | (0.9739,0.9879, –0.8991) | 0.2382 | 0.8136 | 1.0395 | 2.4139 | 80 | 0.0368 |
| | The proposed method (MOPSO) | (0.9699,0.8710, –0.3852) | 0.9103 | 1.5675 | 0.4804 | 1.4571 | 80 | 0.0571 |
| $\begin{bmatrix} 0.57 & 0 \\ 0 & 0.43 \end{bmatrix}$ | Lee et al. (2011) | (0.4839,0.6004,0.9373) | 2.6321 | 3.1875 | –0.8304 | –0.9451 | 4 | 0.2073 |
| | Salmasnia et .al (2013b) | (–0.9966,0.9979, –0.9997) | 0.1099 | 0.3611 | 1.4509 | 5.3903 | 7 | 0.0012 |
| | Costa and Lourenço (2015) moddel1 | (1,0.7955,1) | 2.6609 | 3.3598 | –0.9733 | –1.9975 | 10 | 0.1275 |
| | Costa and Lourenço (2015) moddel2 | (1,0.7964, 0.9066) * | 2.5440 | 3.2464 | –0.8763 | –1.7687 | 7 | 0.2358 |
| | The proposed method (NSGA-II) | (0.9791,0.9857, –0.3733) | 0.8960 | 1.5880 | 0.4823 | 1.2760 | 80 | 0.0565 |
| | The proposed method (MOPSO) | (0.9012,0.8852, –0.4333) | 0.8465 | 1.4788 | 0.5433 | 1.5887 | 80 | 0.0670 |
| $\begin{bmatrix} 0.2 & 0 \\ 0 & 0.43 \end{bmatrix}$ | Lee et al. (2011) | (1,0.6856,1) | 2.6889 | 3.6223 | –0.9735 | –0.2783 | 4 | 0.2073 |
| | Salmasnia et al. (2013b) | (–0.9989,0.9977, –0.9995) | 0.1102 | 0.3609 | 1.4510 | 3.1278 | 7 | 0.0012 |
| | Costa and Lourenço (2015) moddel1* | (1,0.7952,1) | 2.6610 | 3.3598 | –0.9733 | –0.4330 | 10 | 0.1275 |
| | Costa and Lourenço (2015) moddel2* | (0.6754,0.7745,0.0121) | 1.4314 | 2.0204 | 0.1005 | 0.9425 | 7 | 0.2358 |
| | The proposed method (NSGA-II) | (0.9897,0.9924, –0.9719) | 0.1461 | 0.7091 | 1.1150 | 1.4427 | 80 | 0.0298 |
| | The proposed method (MOPSO) | (0.5014,0.9065, –0.5290) | 0.7215 | 1.2261 | 0.7083 | 1.4903 | 80 | 0.0538 |

Notes: *desirability function–based model and global criterion–based model are considered as models 1 and 2, respectively.

The results of Table 8 demonstrate that Lee et al. (2011), Salmasnia et al. (2013b) and Costa and Lourenço (2015) have poor performance in the NPS criterion with only four, seven and ten generated non-dominated solutions. The results in the last column denote that the generated Pareto sets by Lee et al. (2011) and Costa and Lourenço (2015) models do not cover the entire feasible region in comparison with the other approaches. Salmasnia et al. (2013b) method has a very small spacing metric value that means an appropriate uniformity but generating only 7 non-dominated solutions shows that the method only focuses on a small part of the acceptable region and ignores the other parts of the experimental region. In other words, it is only capable of generating a few efficient solutions in a small neighbourhood. In contrast, both suggested sorting non-dominated algorithms generated 80 non-dominated solutions with small uniformity. It confirms that the suggested approach generates better Pareto optimal sets in comparison with the other existing posterior MRO approaches.

Lee et al. (2011) and Costa and Lourenço (2015) have the negative values for $\hat{y}_{Ln(CpM)}$, which is equivalent with values of less than 1 for the multivariate process capability index. It supports the claim that both of these approaches ignore customer's needs in process optimisation. Furthermore, the great values for $\hat{y}_{j\sigma}$ demonstrate that the mentioned methods do not consider dispersion effect of responses. The obtained values for the design variables by Salmasnia et al. (2013b) lead to great values for $\hat{y}_{Ln(WSD)}$ because of ignoring of this method from covariance among quality characteristics. This comparison confirms the efficiency of the proposed method against the existing posterior articulation approaches in the MRO literature.

**Table 9** Comparison of the proposed method with some prior approaches in example 1

| *Model* | | $X$ | $\hat{y}_{1\sigma}$ | $\hat{y}_{2\sigma}$ | $\hat{y}_{Ln(CpM)}$ | $\hat{y}_{Ln(WSD)}$ |
|---|---|---|---|---|---|---|
| Awad and Kovach (2011) | | (1.0000, –1.0000, –1.0000) | 0.6170 | 0.9319 | 0.7450 | 4.0030 |
| Babu et al. (2013) | | (1, 0.796, 1) | 2.6608 | 3.3598 | –0.9733 | –0.7546 |
| Hejazi et al. (2011) | | (0.9530, 0.7090, 0.4070) | 1.9417 | 2.5971 | –0.3553 | 0.2753 |
| Díaz-García and Bashiri (2014) | | (1.0000,0.7070,0.4520) | 1.9984 | 2.6945 | –0.4092 | 0.1815 |
| Individual CpM | | (-1.0000, 1.0000, –1.0000) | 0.1090 | 0.3603 | 1.4522 | 4.7630 |
| Individual WSD | | (1.0000, 1.0000, 1.0000) | 2.6090 | 3.3598 | –0.9730 | –1.0490 |
| The proposed method | NSGA-II | (0.9802, 0.9872, –0.9613) | 0.1606 | 0.7237 | 1.1041 | 2.0124 |
| | MOPSO | (0.9790, 0.9579, –0.4032) | 0.8657 | 1.5439 | 0.5100 | 1.1820 |

### 3.2.1 *Comparison of the proposed method with existing prior methods in example 1*

In this section, to illustrate the advantages of simultaneous optimisation of multivariate process capability index and weighted statistical distance, the proposed method is compared to state-of-the-art methods in the literature as well as individual optimisation of multivariate process capability index and weighted statistical distance, separately. The reported results in the last column of Table 9 reveal that Awad and Kovach (2011) and Individual *CpM* do not consider location effects, properly. Furthermore, the obtained negative values for natural logarithm of process capability index confirm the claim that

Babu et al. (2013), Hejazi et al. (2011) and Díaz-García and Bashiri (2014) and Individual WSD set the controllable variable values without considering the customer's requirements, which is not acceptable in the current competitive market. Eventually, the obtained results denote the outstanding performance the suggested method.

$\overline{y}_{ij}$, $\sigma_{ij}^2$ and $\sigma_{ijk}$ of two interested responses for each experimental run are calculated via equations (3) to (5), which are given in Table 11. Then, three weight matrices are randomly determined. Table 12 summarises the obtained results for process capability index and weighted statistical distance. The response surfaces of *Ln* (*CpM*) and *Ln* (*WSD*) are estimated, subsequently.

### *3.3.2 Phase 2: Pareto optimal set*

In this section, similar to the first numerical example NSGA-II and MOPSO algorithms as well as ε-constraint method are conducted on the response surfaces of $\hat{y}_{Ln(CpM)}$ and $\hat{y}_{Ln(WSD)}$ to generate the non-dominated solutions. For this purpose, the used parameters are given in Table 13. To compare the non-dominated sorting algorithms, each of them is executed eight times for each weight matrix. Consequently, 24 Pareto sets are generated, which are used for comparative parametric and non-parametric statistical tests.

### *3.3.3 Phase 3: Comparison of NSGA-II, MOPSO and ε-constraint*

Three metric criteria are conducted on the 24 generated Pareto sets to measure the performance of the non-dominated sorting algorithms. The obtained results are shown in Table 14 while the average values of the metric criteria are given in the last row. Similar to the case study 1, the results imply that the NSGA-II algorithm generates the best Pareto sets. It has the smallest spacing metric values and the greatest NPS and normalised set coverage values among the employed algorithms. Furthermore, the results in the last row of Table 14 confirm that the MOPSO algorithm generates the more appropriate Pareto sets compared to ε-constraint algorithm. The results also are statistically compared by both parametric and non-parametric tests. The considered hypothesises and their corresponding P-values are given in Table 15. The obtained P-values confirm that the NSGA-II generates significantly the better Pareto sets and also show the better performance of MOPSO algorithm in comparison with ε-Constraint method.

### *3.2.1 Comparison of the proposed method with existing posterior methods in example 2*

In this section, the suggested approach is compared with some posterior preference articulation methods. The control variable vectors given in third column of Table 16 are obtained through conducting VIKOR method on the generated Pareto sets. The obtained results in Table 16 confirm the mentioned discussion in example 1.

**Table 10** The Experimental results of numerical example 2

| $x_1$ | $x_2$ | $x_3$ | $y_1$ | | | | | $y_2$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | *1* | *2* | *3* | *4* | *5* | *1* | *2* | *3* | *4* | *5* |
| –1.000 | 1.000 | –1.000 | 74.432 | 73.016 | 74.5 | 74.009 | 74.125 | 51.404 | 52.508 | 54.188 | 53.485 | 53.303 |
| 1.000 | 1.000 | –1.000 | 50.159 | 50.321 | 51.49 | 51.69 | 51.203 | 63.203 | 62.397 | 63.678 | 63.173 | 62.654 |
| –1.000 | –1.000 | 1.000 | 87.587 | 88.761 | 88.961 | 87.718 | 88.613 | 53.961 | 52.847 | 53.608 | 53.311 | 54.174 |
| 1.000 | –1.000 | 1.000 | 70.083 | 70.643 | 69.448 | 69.52 | 69.542 | 62.779 | 62.694 | 62.873 | 63.312 | 61.789 |
| –1.000 | 1.000 | 1.000 | 70.094 | 69.522 | 71.063 | 71.564 | 70.456 | 56.401 | 56.899 | 57.09 | 58.295 | 56.746 |
| 1.000 | 1.000 | 1.000 | 90.366 | 90.689 | 89.818 | 90.387 | 89.714 | 68.258 | 67.559 | 67.927 | 68.373 | 68.327 |
| –1.682 | 0.000 | 0.000 | 66.539 | 65.991 | 66.505 | 65.967 | 66.475 | 60.092 | 59.885 | 60.622 | 59.248 | 59.721 |
| 1.682 | 0.000 | 0.000 | 96.318 | 96.95 | 96.819 | 97.51 | 96.119 | 67.844 | 68.886 | 67.333 | 67.591 | 67.419 |
| 0.000 | –1.682 | 0.000 | 76.951 | 74.43 | 76.57 | 78.137 | 77.632 | 59.064 | 58.809 | 58.867 | 59.087 | 59.055 |
| 0.000 | 1.682 | 0.000 | 78.619 | 78.017 | 78.309 | 76.373 | 78.549 | 66.04 | 65.688 | 66.064 | 65.928 | 66.128 |
| 0.000 | 0.000 | –1.682 | 85.768 | 84.901 | 86.418 | 84.369 | 85.65 | 58.592 | 60.563 | 62.376 | 59.375 | 62.055 |
| 0.000 | 0.000 | 1.682 | 93.825 | 102.277 | 94.842 | 97.683 | 100.255 | 56.776 | 62.938 | 60.095 | 63.333 | 58.975 |
| 0.000 | 0.000 | 0.000 | 54.936 | 55.102 | 55.02 | 54.895 | 55.136 | 57.457 | 58.017 | 57.368 | 57.532 | 57.114 |
| 0.000 | 0.000 | 0.000 | 81.09 | 80.916 | 80.983 | 80.989 | 81.008 | 63.284 | 63.59 | 63.315 | 62.834 | 63.455 |
| 0.000 | 0.000 | 0.000 | 90.903 | 85.439 | 70.546 | 78.373 | 86.978 | 61.607 | 58.899 | 56.19 | 58.608 | 62.974 |
| 0.000 | 0.000 | 0.000 | 79.464 | 72.3 | 78.765 | 84.358 | 82.01 | 63.279 | 61.181 | 57.124 | 56.218 | 61.435 |
| 0.000 | 0.000 | 0.000 | 77.671 | 81.747 | 77.608 | 73.247 | 74.151 | 61.142 | 63.578 | 60.81 | 64.414 | 63.677 |
| 0.000 | 0.000 | 0.000 | 86.053 | 80.349 | 83.103 | 80.442 | 91.657 | 62.486 | 60.453 | 61.927 | 61.968 | 61.235 |
| –1.000 | 1.000 | –1.000 | 82.035 | 78.625 | 80.23 | 81.935 | 72.136 | 62.038 | 59.658 | 60.793 | 60.127 | 63.178 |
| 1.000 | 1.000 | –1.000 | 90.213 | 85.786 | 86.64 | 90.861 | 88.702 | 62.406 | 58.563 | 55.698 | 56.693 | 61.782 |

**Table 11** The results of sample mean, variance and covariance of example 2

| $\overline{y}_1$ | $\overline{y}_2$ | $\sigma_1^2$ | $\sigma_2^2$ | $\sigma_{12}$ |
|---|---|---|---|---|
| 74.0164 | 0.1082 | 1.13129 | 0.3548 | 52.9776 |
| 50.9726 | 0.1557 | 0.25299 | 0.4805 | 63.021 |
| 88.328 | –0.0622 | 0.27711 | 0.3976 | 53.5802 |
| 69.8472 | 0.0056 | 0.31003 | 0.2621 | 62.6894 |
| 70.5398 | 0.4411 | 0.52037 | 0.6411 | 57.0862 |
| 90.1948 | –0.0579 | 0.11825 | 0.1709 | 68.0888 |
| 66.2954 | 0.0962 | 0.25365 | 0.084 | 59.9136 |
| 96.7432 | 0.062 | 0.3968 | 0.3017 | 67.8146 |
| 76.744 | 0.1621 | 0.01652 | 2.0392 | 58.9764 |
| 77.9734 | 0.0557 | 0.03 | 0.856 | 65.9696 |
| 85.4212 | 0.6788 | 2.70124 | 0.636 | 60.5922 |
| 97.7764 | 5.7079 | 7.57456 | 12.658 | 60.4234 |
| 55.0178 | –0.0007 | 0.10914 | 0.0107 | 57.4976 |
| 80.9972 | –0.0049 | 0.08142 | 0.0039 | 63.2956 |
| 82.4478 | 18.6962 | 7.12501 | 64.8043 | 59.6556 |
| 79.3794 | –5.3426 | 9.16607 | 20.5523 | 59.8474 |
| 76.8848 | –1.8072 | 2.66494 | 11.3723 | 62.7242 |
| 84.3208 | 0.3968 | 0.61881 | 22.2809 | 61.6138 |
| 78.9922 | –3.5267 | 2.07525 | 16.6483 | 61.1588 |
| 88.4404 | 2.0715 | 8.93819 | 4.8393 | 59.0284 |

**Table 12** The results of constructing the new responses of example 2

| *Ln(CpM)* | *Ln(WSD)* | | |
|---|---|---|---|
| | $W = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ | $W = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.84 \end{bmatrix}$ | $W = \begin{bmatrix} 0.79 & 0 \\ 0 & 0.21 \end{bmatrix}$ |
| 0.6024 | 6.8635 | 5.6925 | 7.3263 |
| 0.901 | 8.1977 | 7.352 | 8.4351 |
| 0.9256 | 5.4513 | 4.8817 | 5.7664 |
| 1.0017 | 7.4896 | 6.4599 | 7.9268 |
| 0.6486 | 7.3689 | 6.1773 | 7.835 |
| 1.3496 | 6.3081 | 6.5598 | 6.028 |
| 1.3363 | 9.4433 | 8.4138 | 9.7603 |
| 0.9048 | 5.1628 | 5.5471 | 4.6525 |
| 1.2218 | 7.4107 | 7.2777 | 7.5117 |
| 1.2897 | 7.6697 | 7.9544 | 7.34 |
| 0.2389 | 5.5416 | 4.6198 | 5.9561 |

**Table 12** The results of constructing the new responses of example 2 (continued)

| *Ln(CpM)* | *Ln(WSD)* | | |
|---|---|---|---|
| | $W = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ | $W = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.84 \end{bmatrix}$ | $W = \begin{bmatrix} 0.79 & 0 \\ 0 & 0.21 \end{bmatrix}$ |
| -0.7666 | 0.5519 | 0.7497 | 0.3453 |
| 2.0623 | 11.4573 | 10.3179 | 11.9147 |
| 2.3879 | 10.7886 | 9.5862 | 11.2566 |
| -1.1596 | 2.8583 | 2.4588 | 3.1058 |
| -0.9355 | 2.3913 | 1.0573 | 2.8789 |
| -0.4787 | 3.178 | 2.3022 | 3.5818 |
| -0.2818 | 3.0583 | 3.2922 | 3.8036 |
| -0.5114 | 2.602 | 1.0418 | 3.117 |
| -0.5676 | 2.798 | 1.8062 | 3.2275 |

$$\hat{y}_{Ln(CpM)} = 0.64976 + 0.62968_1^2 + 0.1007x_2 + 0.97231x_3^2 \tag{19}$$

$$W = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$
$$\hat{y}_{Ln(WSD)} = 0.70169 - 1.57337x_1^2 + 2.83973x_3^2 - 1.08923x_1x_3 \tag{20}$$

$$W = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.84 \end{bmatrix}$$
$$\hat{y}_{Ln(WSD)} = 2.10149 - 0.51168x_2 + 1.83921x_1^2 + 2.6649x_3^2 - 0.71524x_1x_3 \tag{21}$$

$$W = \begin{bmatrix} 0.79 & 0 \\ 0 & 0.21 \end{bmatrix} \tag{22}$$

$$\hat{y}_{Ln(WSD)} = 3.1878 - 0.8021x_2 + 1.3886x_1^2 + 2.8589x_3^2 - 1.273x_1x_3 \tag{23}$$

**Table 13** The used parameters in Pareto set generation algorithms of example 2.

| *Algorithm* | *Parameter* | *Value* |
|---|---|---|
| NSGA-II | Population size | 80 |
| | Maximum Iteration | 120 |
| | Crossover rate | 0.8 |
| | Mutation rate | 0.3 |
| MOPSO | Population Size | 80 |
| | Repository size | 80 |
| | Maximum Iteration | 120 |
| | Number of division | 30 |
| ε-Constraint | ρ | 0.001 |
| | ε | 0.05 |

**Table 14** The results of performance metrics of example 2

| *Weight matrix* | *NSGA-II* | | | | *MOPSO* | | | | *ε-Constraint* | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Δ* | $\bar{C}(A,B)$ | $\bar{C}(A,C)$ | *NPS* | *Δ* | $\bar{C}(B,A)$ | $\bar{C}(B,C)$ | *NSP* | *Δ* | $\bar{C}(C,A)$ | $\bar{C}(C,B)$ | *NSP* |
| $W=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix}$ | 0.0590 | 1.0000 | 1.0000 | 80 | 0.0847 | 0.0000 | 1.0000 | 80 | 0.6092 | 0.0000 | 0.0000 | 20 |
| | 0.0703 | 1.0000 | 1.0000 | 80 | 0.1062 | 0.0000 | 0.9540 | 80 | 0.8221 | 0.0000 | 0.0460 | 18 |
| | 0.0614 | 1.0000 | 0.9868 | 80 | 0.0966 | 0.0000 | 0.9552 | 80 | 0.6023 | 0.0132 | 0.0448 | 15 |
| | 0.0641 | 1.0000 | 1.0000 | 80 | 0.1046 | 0.0000 | 1.0000 | 80 | 0.5549 | 0.0000 | 0.0000 | 17 |
| | 0.0558 | 1.0000 | 1.0000 | 80 | 0.1032 | 0.0000 | 0.8000 | 80 | 0.6041 | 0.0000 | 0.2000 | 16 |
| | 0.0647 | 1.0000 | 1.0000 | 80 | 0.0913 | 0.0000 | 1.0000 | 80 | 0.8990 | 0.0000 | 0.0000 | 18 |
| | 0.0541 | 1.0000 | 1.0000 | 80 | 0.1004 | 0.0000 | 0.6452 | 80 | 0.3917 | 0.0000 | 0.3548 | 16 |
| | 0.0628 | 1.0000 | 0.866 | 80 | 0.0918 | 0.0000 | 0.9773 | 80 | 0.8052 | 0.0134 | 0.0227 | 13 |
| $W=\begin{bmatrix}0.16 & 0\\ 0 & 0.84\end{bmatrix}$ | 0.0720 | 1.0000 | 1.0000 | 80 | 0.0936 | 0.0000 | 0.8983 | 80 | 0.3692 | 0.0000 | 0.1017 | 22 |
| | 0.0625 | 1.0000 | 0.9870 | 80 | 0.1128 | 0.0000 | 0.8672 | 80 | 0.5495 | 0.0130 | 0.1328 | 21 |
| | 0.0638 | 0.9821 | 1.0000 | 80 | 0.0835 | 0.0179 | 1.0000 | 80 | 0.3333 | 0.0000 | 0.0000 | 22 |
| | 0.0780 | 1.0000 | 1.0000 | 80 | 0.1274 | 0.0000 | 0.8791 | 80 | 0.4013 | 0.0000 | 0.1209 | 19 |
| | 0.0735 | 0.9848 | 1.0000 | 80 | 0.0964 | 0.0152 | 0.9586 | 80 | 0.3955 | 0.0000 | 0.0414 | 24 |
| | 0.0624 | 1.0000 | 1.0000 | 80 | 0.0990 | 0.0000 | 0.9845 | 80 | 0.3627 | 0.0000 | 0.0155 | 23 |
| | 0.0645 | 1.0000 | 1.0000 | 80 | 0.0827 | 0.0000 | 0.8743 | 80 | 0.4207 | 0.0000 | 0.1257 | 24 |
| | 0.0813 | 1.0000 | 1.0000 | 80 | 0.1074 | 0.0000 | 0.9043 | 80 | 0.3360 | 0.0000 | 0.0957 | 14 |
| $W=\begin{bmatrix}0.79 & 0\\ 0 & 0.21\end{bmatrix}$ | 0.0558 | 1.0000 | 0.9867 | 80 | 0.0969 | 0.0000 | 0.6349 | 80 | 0.5498 | 0.0133 | 0.3651 | 12 |
| | 0.0621 | 1.0000 | 0.9852 | 80 | 0.0855 | 0.0000 | 0.9211 | 80 | 0.6667 | 0.0148 | 0.0789 | 13 |
| | 0.0656 | 1.0000 | 0.9736 | 80 | 0.0651 | 0.0000 | 0.9610 | 80 | 0.5634 | 0.0264 | 0.0390 | 14 |
| | 0.0681 | 1.0000 | 0.9867 | 80 | 0.0759 | 0.0000 | 0.9809 | 80 | 0.6590 | 0.0133 | 0.0191 | 17 |
| | 0.0602 | 1.0000 | 0.9869 | 80 | 0.0934 | 0.0000 | 0.9839 | 80 | 0.4231 | 0.0131 | 0.0161 | 13 |
| | 0.0581 | 1.0000 | 0.9866 | 80 | 0.0990 | 0.0000 | 0.9326 | 80 | 1.1072 | 0.0134 | 0.0674 | 17 |
| | 0.0539 | 1.0000 | 1.0000 | 80 | 0.1007 | 0.0000 | 1.0000 | 80 | 0.3941 | 0.0000 | 0.0000 | 12 |
| | 0.0614 | 1.0000 | 0.9852 | 80 | 0.0928 | 0.0000 | 1.0000 | 80 | 0.6063 | 0.0148 | 0.0000 | 20 |
| Average | 0.0640 | 0.9921 | 0.9931 | 80 | 0.0955 | 0.0014 | 0.9214 | 80 | 0.5594 | 0.0069 | 0.0786 | 17.5 |

**Table 15** Statistical test results of example 2

| *Criteria* | *Parametric test* | | | | *Nonparametric test* | | | |
|---|---|---|---|---|---|---|---|---|
| | *Hypothesis* | *P–value* | *Hypothesis* | *P–value* | *Hypothesis* | *P–value* | *Hypothesis* | *P–value* |
| NPS | – | | $\begin{cases} H_0 : \mu_A = \mu_C \\ H_1 : \mu_A > \mu_C \end{cases}$ | 0.00 | – | | $\begin{cases} H_0 : m_A = m_C \\ H_1 : m_A < m_C \end{cases}$ | 0.00 |
| $\Delta$ | $\begin{cases} H_0 : \mu_A = \mu_B \\ H_1 : \mu_A > \mu_B \end{cases}$ | 0.00 | $\begin{cases} H_0 : \mu_A = \mu_C \\ H_1 : \mu_A > \mu_C \end{cases}$ | 0.00 | $\begin{cases} H_0 : m_A = m_B \\ H_1 : m_A < m_B \end{cases}$ | 0.00 | $\begin{cases} H_0 : m_A = m_C \\ H_1 : m_A < m_C \end{cases}$ | 0.00 |
| | – | | $\begin{cases} H_0 : \mu_B = \mu_C \\ H_1 : \mu_B > \mu_C \end{cases}$ | 0.00 | – | | $\begin{cases} H_0 : m_B = m_C \\ H_1 : m_B < m_C \end{cases}$ | 0.00 |
| $\bar{C}$ | $\begin{cases} H_0 : \mu_A = \mu_B \\ H_1 : \mu_A > \mu_B \end{cases}$ | 0.00 | $\begin{cases} H_0 : \mu_A = \mu_C \\ H_1 : \mu_A > \mu_C \end{cases}$ | 0.00 | $\begin{cases} H_0 : m_A = m_B \\ H_1 : m_A < m_B \end{cases}$ | 0.00 | $\begin{cases} H_0 : m_A = m_C \\ H_1 : m_A < m_C \end{cases}$ | 0.00 |
| | – | | $\begin{cases} H_0 : \mu_B = \mu_C \\ H_1 : \mu_B > \mu_C \end{cases}$ | 0.00 | – | | $\begin{cases} H_0 : m_B = m_C \\ H_1 : m_B < m_C \end{cases}$ | 0.00 |

**Table 16** Comparison of the proposed method with some posterior approaches in example 2

| *Weigh matrix* | *Model* | *X* | $\hat{y}_{\ln(\sigma 1)}$ | $\hat{y}_{\ln(\sigma 2)}$ | $\hat{y}_{Ln(CpM)}$ | $\hat{y}_{Ln(WSD)}$ | NPS | Δ |
|---|---|---|---|---|---|---|---|---|
| $W=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix}$ | Lee et al. (2011) | (–0.6461,1.6820, –0.4298) | 0.1594 | 0.1703 | –0.2074 | 2.5145 | 9 | 9.5769 |
| | Salmasnia et al. (2013b) | (1.6646, –1.6096,1.6760) | –4.3729 | –3.2943 | 3.8197 | 13.2428 | 13 | 0.1589 |
| | Costa and Lourenço (2015) moddel1 | (–0.6212,1.682, –0.4143) | 0.1909 | 0.2040 | –0.2399 | 2.4498 | 12 | 0.2330 |
| | Costa and Lourenço (2015) moddel2 | (–0.6576,0.0468, –0.3547) | 1.0642 | 0.1917 | –0.2553 | 3.5666 | 6 | 0.4698 |
| | The proposed method (NSGA–II) | (1.5572,1.6590,0.9546) | –1.6736 | –1.9149 | 1.7577 | 6.4357 | 80 | 0.0631 |
| | The Proposed Method (MOPSO) | (1.3403,1.3659,0.8782) | –0.9519 | –1.3124 | 1.2274 | 5.5918 | 80 | 0.0918 |
| $W=\begin{bmatrix}0.16 & 0\\ 0 & 0.84\end{bmatrix}$ | Lee et al. (2011) | (–0.6461,1.6820, –0.4298) | 0.1594 | 0.1703 | –0.2074 | 2.3023 | 9 | 9.5769 |
| | Salmasnia et al. (2013b) | (–1.6812,1.6700, –1.6731) | –4.4416 | –3.3349 | 3.8451 | 11.8933 | 13 | 0.1589 |
| | Costa and Lourenço (2015) moddel1 | (–0.6215,1.682, –0.4139) | 0.1912 | 0.2039 | –0.2400 | 2.2238 | 12 | 0.2330 |
| | Costa and Lourenço (2015) moddel2 | (–0.5988,1.682, –0.4511) | 0.1590 | 0.2084 | –0.2261 | 2.2494 | 6 | 0.4698 |
| | The proposed method (NSGA–II) | (1.1510,1.6462,1.1853) | –1.8654 | –1.2854 | 1.5479 | 6.4640 | 80 | 0.0625 |
| | The proposed method (MOPSO) | (–1.1690,1.0444, –1.1882) | –1.4170 | –1.3241 | 1.5808 | 6.8494 | 80 | 0.1128 |
| $W=\begin{bmatrix}0.79 & 0\\ 0 & 0.21\end{bmatrix}$ | Lee et al. (2011) | (–0.6461,1.6820, –0.4298) | 0.1594 | 0.1703 | –0.2074 | 2.5929 | 9 | 9.5769 |
| | Salmasnia et .al (2013b) | (1.6637, –1.6767,1.6757) | –4.4350 | 3.2912 | 3.8169 | 12.8549 | 13 | 0.1589 |
| | Costa and Lourenço (2015) moddel1 | (–0.7586,1.682, –0.5015) | 0.0003 | 0.0006 | –0.0434 | 2.8725 | 12 | 0.2330 |
| | Costa and Lourenço (2015) moddel2 | (–0.7755,1.0616, –0.1614) | 0.7979 | 0.1088 | –0.2464 | 3.0865 | 6 | 0.4698 |
| | The proposed method (NSGA–0II) | (–1.6171,1.6487, –0.8678) | –1.5386 | –1.9818 | 1.7230 | 5.8631 | 80 | 0.0625 |
| | The proposed method (MOPSO) | (1.4551,1.4140,0.8623) | –1.0909 | –1.5630 | 1.4017 | 5.5222 | 80 | 0.0855 |

### 3.2.2 Comparison of the proposed method with existing posterior methods in example 1

As mentioned earlier to illustrate the main advantages of the suggested method including consideration of customer satisfaction and variance-covariance structure of quality characteristics, the suggested method is compared to some popular methods in the literature. The obtained results from Costa and Pereiria (2010) and Sharma et al. (2013) approaches imply that these models only focus on the location effects of responses, ignoring the dispersion effects of the quality characteristics and customer satisfaction. Awad and Kovach (2011), Yadav et al. (2014a) and the suggested process capability index mostly pay attention to customer satisfaction. However, these methods do not consider the weighted statistical distance of mean responses from their corresponding target values. Eventually, the obtained results of the suggested approach confirm the superiority of this model against the other considered approaches.

**Table 17** Comparison of the proposed method with some prior approaches in example 2

| *Model* | | $X$ | $\hat{y}_{1\sigma}$ | $\hat{y}_{2\sigma}$ | $\hat{y}_{Ln(CpM)}$ | $\hat{y}_{Ln(WSD)}$ |
|---|---|---|---|---|---|---|
| Sharma et al. (2013) | | (–0.5440, –0.0920, –0.4800) | 0.9777 | 0.2449 | –0.2392 | 3.7158 |
| Awad and Kovach (2011) | | (1.6820, –0.5320, –0.6697) | –0.4880 | –1.9844 | 1.5611 | 10.1409 |
| Yadav et al. (2014a) | | (1.6820, –1.6820,1.6820) | –4.4955 | –3.3543 | 3.8758 | 13.3977 |
| Costa and Pereira (2010) | | (–0.6428,1.6820, –0.3182) | 0.2756 | 0.2219 | –0.2912 | 2.3504 |
| Individual CpM | | (–1.6820, 0, –1.6820) | –3.6643 | –3.3543 | 3.8758 | 12.2194 |
| Individual WSD | | (0,1.6820, 0) | –0.5914 | 0.6233 | –0.6488 | 1.6356 |
| The proposed method | NSGA-II | 0.9777 | 0.2449 | –0.2392 | 3.7158 | 2.0124 |
| | MOPSO | –0.4880 | –1.9844 | 1.5611 | 10.1409 | 1.1820 |

## 4 Conclusions

With respect to the current competitive world, this study aims to find the controllable variable values that maximise the customer satisfaction. For this purpose, a multivariate process capability index was suggested. This index considers dispersion effect of quality characteristics as well as covariance of among responses. However, it cannot guarantee falling all quality characteristics within their corresponding specification limits. On one hand, to overcome this shortcoming, and on the other hand, since relative importance of quality characteristics in most of the real-world problems is different, the weighted statistical distance as another index was considered. This index is able to consider the deviation of the mean responses from their corresponding target values. Therefore, by combination of these two criteria, a multi-responses problem was converted to a bi-objective problem. Moreover, to reduce the cognitive effort on DM, the NSGA-II and MOPSO algorithms were suggested as two posterior preference articulation approaches. Then, these two approaches were statistically compared with the ε-constraint method, which was presented in Lee et al. (2011), based on the three quality performance measures for generating Pareto optimal sets. The comparison results indicated that the NSGA-II algorithm has the better performance than MOPSO and ε-constraint methods. Finally, the proposed approach was compared with the most well-known methods in the

MRO literature. The obtained results demonstrated the superiority of the suggested method compared to the existing approaches.

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