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A survey on network design problems: main variants and resolution approaches

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Abstract: Over the last decades, network design problems (NDPs) have been one of the most investigated combinatorial optimisation problems that are still catching the interest of both practitioners and researchers. In fact, *NDPs* pose significant algorithmic challenges, as they are notoriously *NP*-hard, and arise in several applications, mainly in logistics, telecommunication, and production systems. Based on the literature published mainly between 1962 and 2021, this paper provides a comprehensive taxonomy of *NDPs* and also identifies the most investigated variants as well as their main fields of application. This taxonomy highlights the diversity as well as the assets of this core class of operations research problems. Moreover, the main mathematical formulations and solution methods are reported. Finally, directions for future research on *NDPs* are derived. [Submitted: 14 March 2021; Accepted: 23 January 2022]

Keywords: network design problems; NDPs; literature review; survey; combinatorial optimisation.

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1 Introduction

Network flow problems are well-known as NP-hard operations research problems that have been widely investigated since the early 20th century and are closely related to computer science as well as to combinatorial optimisation and many fields of engineering and management (Johnson et al., 1978). In general, network flow problems could be classified into two principal categories: network routing and network design problems (NDPs). Network routing problems require determining routes on networks and arise in several contexts (Carré, 1971). For example, in telecommunications, including message routing through congested logic networks (Barnhart et al., 1995), routing assignments on virtual circuit data networks (Lin and Yee, 1992), or routing on a ring private network (Shepherd and Zhang, 2001). Again, path finding problems of different natures also appear in logistics and scheduling areas requiring the identification of the shortest paths between specific source nodes and sink nodes on a time-space network where a flow to be shipped can be a tanker fleet (Bellmore et al., 1971), a type of aircraft that must fly on each flight segment (Hane et al., 1995), freight car traffic on rail networks (Kwon et al., 1998), or seasonal products in warehousing and distributing context (Jewell, 1957). On the other hand, *NDPs* involve designing a network at the lowest cost allowing the total or partial circulation of material or data flows without violating the installed capacities. In this context, Chopra et al. (1996) have mentioned that the *NDPs* are still NP-hard and constitute a crucial class of difficult combinatorial optimisation problems. Accordingly, most of these problems could be solved in polynomial time using linear programming models (Ouorou et al., 2000). The extensive relevant literature on *NDPs* was inspired by the works of Ford and Fulkerson (2015) and Hu (1963). Excellent reviews such as in (Kennington, 1978) were addressed in the late 1970s.

The literature on *NDPs* is extensive, as these combinatorial problems are extremely interesting, but yet complicated. How to make optimal design decisions is among the main challenges of *NDPs* as it involves a difficult trade-off between different constraints with varying levels of complexity. Several problems' features and assumptions give rise to different variants and models, with many degrees of difficulty and size. The reason why studies on *NDPs* have received substantial attention from researchers and practitioners is that this class of problems is considered a powerful methodology appropriate to capture realistic assumptions and effectively model various real-life situations that correspond to several issues of interest (Hewitt et al., 2021).

Thus, numerous scientific researches of the description, modelling strategies, and applications of *NDPs* are available. Actually, *NDPs* are prominent in several practical situations such as telecommunications, logistics and supply chain, manufacturing and production planning, localisation, aircraft assignments, economic settings, electric systems, energy transportation, to quote just a few. Therefore, covering all real-life applications of *NDPs* poses extremely difficult challenges and is probably impossible.

In this paper, we do not attempt to review all the applications of *NDPs* which is an endeavour in itself, we have restricted our survey to most relevant applications which are practical of interest and rise a large set of issues related to sustainable policy for modern human life. In addition to other relevant applications, we particularly focused on smart telecommunication systems (e.g., smart grid transmission systems, survivable NDPs with relays), green logistics and sustainable supply chain networks (e.g., liner shipping network design with emission control areas, sustainable multimodal coastal container transport problem), as well as intelligent energy networks (e.g., energy hubs optimisation problem).

1.1 Related work

Over the last four decades and with the advanced evolution of the telecommunications sector, *NDPs* have naturally captured the interest of scientists and have been the subject of numerous researches that arise in a great variety of applications. These studies address real-world problems such as designing local access networks with one or more types of technologies or designing fibre optic networks with different bandwidths. In addition to the communication and computer networks, *NDPs* have been considerably recognised in several fields such as logistics and transportation, traffic engineering, economic systems, and energy where they have a crucial impact on companies' profitability. A growing body of literature exists regarding the theoretical and practical perspectives of the *NDPs*. Nevertheless, despite the multiple level studies on these topics, the lack of a complete, comprehensive, and detailed survey has attracted the attention of certain researchers. Previous reviews on *NDPs* were published almost four decades ago, such as the survey initiated by Assad (1978). Assad not only provided an interesting analysis of solution methods for both linear and nonlinear *NDPs* but also presented a comprehensive survey by including recent advances on specific processing techniques which have a significant potential impact on the computational experience for both kinds of problems. Afterward, Minoux (1989) published a survey for optimum NDPs with concave differentiable cost functions. He briefly reviewed the diversity of the underlying solution algorithms in this area. Accordingly, following in the wake of earlier papers, it is notable that the theoretical aspect of the *NDPs* has received much interest from the operations research community. More precisely, the focus was on finding pertinent models and effective techniques as optimal resolution approaches. In this regard, many articles have already established an excellent background in the bibliography (Scott, 1969; Karp, 1975; Wong, 1976; Johnson et al., 1978; Dionne and Florian, 1979; Okamura and Seymour, 1981; Okamura, 1983; Erdös and Székely, 1992; Ouorou et al., 2000).

In addition to earlier reviews, more surveys have recently been proposed by focusing on both theoretical and practical sides of *NDPs*, particularly many new methods and relevant applications in the field have been addressed and discussed. In this context, Yaghini and Akhavan (2012) presented a review of multicommodity NDPs modelling, their possible applications in rail freight planning, and the techniques that have been developed to solve them. Other studies focus on network design applications such as freight routing (Crainic, 2000), railroad blocking (Barnhart et al., 2000), and microelectronic routing (Hu and Sapatnekar, 2001). Minoux (2001) proposed an overview of available exact solution methods for discrete cost multicommodity network optimisation, as well as many variants related to the telecommunications network. Later, Costa (2005) proposed a survey on several solution methodologies for NDPs and underlined the economic interest of the related problems.

During the last ten years, additional review papers have been published to explore the *NDPs* in several engineering applications from an algorithmic and experimental point of view. This highlights the growing interest in these problems over the years. Accordingly, impressive surveys have recently been conducted by Wang (2018a, 2018b). In fact, in the first part of his survey, the author summarised most applications and different mathematical formulations for modelling the main variants of multicommodity network flow problems (*MCNFs*), particularly network design and network routing problems. Then, as a follow-up survey paper to the previous Part I (Wang, 2018a), Part II gave a great summary of the *MCNF* solution methods that have been reported in the literature including two categories: conventional primal or dual frameworks, as well as the approximation methods, interior-point methods, and convex programming methods.

Likewise, the most recent survey paper on MCNFs was published by Salimifard and Bigharaz (2020). The authors classified the *MCNF* problems into three major categories: the max *MCNF* problem which aims at maximising the totality of flows associated to all commodities crossing the network from the source nodes to the sink nodes; the max-concurrent flow problem that aims at maximising the fractional satisfied demands corresponding to all commodities; and the min-cost *MCNF* problem, whose purpose is to determine the optimal flow configuration that satisfies the demand for all commodities at the lowest cost and with respect to the installed arc capacities. In this context, the authors were initially looked at the most popular applications of the *MCNF* categories in different frameworks. Then, they analysed the associated articles according to applied solution algorithms.

Recently NDPs have been tackled from another perspective, based on the goal of reducing the environmental impacts of engineering activities, commonly referred to as 'green NDPs'. For an extensive bibliography regarding green NDPs, the reader is referred to the original recent survey published by Dukkanci et al. (2019). The authors presented an extremely exciting review on green logistics problems within operations research, including those relevant to supply chain management. Major definitions of these problems as well as mathematical models, and practical applications were provided.

1.2 Aims and framework of the survey

This paper is intended as a survey in the context of NDPs. The survey covers papers on *NDPs* published since 1962. Precisely, we have reviewed over hundreds of publications and summarised the relevant references. The range of publications considered in our bibliographic search was constrained to a multitude of articles reported by the most prominent publishers in the network design topics such as Elsevier, Science Direct, Springer, Scopus, and IEEE. These journals are regarded as dominant publishers who produce many of the most interesting researches in the *NDPs* publishing landscape. To extend the pile of references, we have also considered related relevant conference proceedings.

After reviewing the general literature on *NDPs*, we realised that there is a need to classify this class of combinatorial problems according to specific features particularly, the nature of input data. We have thus been led to consider two main categories around which the overall document is structured: deterministic and stochastic *NDPs*. Then, we have considered characteristics including capacity and cost criteria, flow policies, among many others that will be discussed in more detail later, to classify the main variants for each category. In the following, we present firstly this classification and we provide the various corresponding contexts of applications as well as related mathematical formulations. Then, we classify publications from another perspective based on the most studied solution methods in the literature for solving these variants with a focus on those approaches that currently are considered to be the most effective or prominent. To the best of our knowledge, there is no complete and comprehensive survey study devoted to classify the main variants associated with both deterministic and stochastic *NDPs* in various fields of applications, at the same time. The goal is to give the lecturer a reasonably rich coverage based on pertinent classifications in the field that allow him to clearly define the context as well as the characteristics of the problem he has to address and to compare it with the related published works.

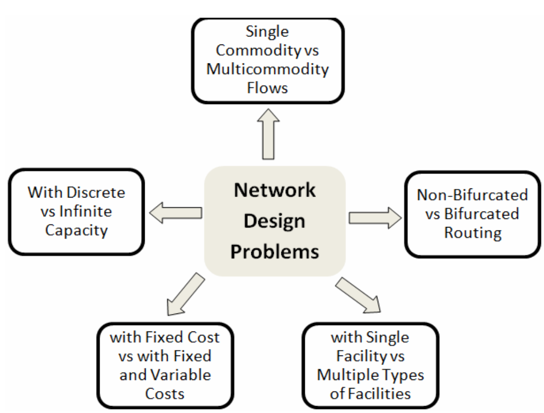
1.3 Paper structure

The road map for this paper is as follows. Sections 2, 3, 4 and 5 cover the first part of our review. In Sections 2 and 3, we formally define the *NDPs* properties and we present an illustrative example to highlight their complexity from a practical point of view. Then in Section 4, we classify the main related variants for both deterministic and stochastic categories. To provide global coverage of their great interest from a practical perspective, an overview of the landscape of their contexts of applications is then presented. In addition to the classical real-world situations, Section 5 is particularly dedicated to some specific applications which are practical of interest and which rise a large set of issues related to sustainable policy for modern human life. Section 6 presents the second part of our survey that reviews the main computational methods for solving both deterministic and stochastic *NDPs* to give as many fine details as possible. The limits of underlying approaches are also discussed. In Section 7, we provide some observations and we discuss the most challenging future researches. Finally, Section 8 draws the conclusion of this paper.

2 Characteristics of NDPs

NDPs are well-investigated problems in graph theory as fundamental optimisation problems in their own right. To successfully resolve them, it is necessary to have a good comprehension of their properties and to define the interactions between them. The different characteristics of *NDPs* are detailed in Figure 1. In general, the purpose is to design a network of links (or facilities) with minimum cost that allows the total or partial circulation of material or data flows of commodities in a way that some demand characteristics can be satisfied without violating capacities.

Figure 1 Different characteristics of NDPs (see online version for colours)



In general, *NDPs* can be defined as follows. Let a graph $G = (V, E)$ be characterised by:

- A set of nodes $V = \{1, 2, \ldots, n\}$ that can correspond, depending on the specific context, to distribution points in a telecommunication network, towns in a transportation system, electric production stations in an energetic structure, to quote a few.
- A set of links $E = \{1, 2, \ldots, m\}$ between pairs of nodes in the graph. A link may be restricted to be oriented (we call it an arc) or non-oriented (called an edge).
- A set of facilities that can be installed on each link. According to the context under study, these facilities may represent a transmission technology such as fibre-optic or copper cables, motorways, electrical line types, etc. These facilities allow the passage of a material or data flow such as data transmission packets, people, car or truck traffic, electricity transport, etc.
- Capacity: A discrete value that represents the maximum amount of material and/or data transfer across a given facility. It may correspond to network bandwidth measuring the maximum throughput in a digital communication system, number of passenger or vehicle traffic in the transit network, electrical power for energy systems, etc.
- Cost: This is the cost associated with a facility on each link of the graph. It includes a fixed cost that is paid as soon as a route is exploited for the first time (also called opening cost) or additional facilities are installed. According to the specific areas, these costs may represent for example the cost of installing a type of fibre optic cable, constructing a road, or line electric installation. In addition to a fixed cost, a per-unit cost (variable cost) can be also imposed. The latter, also known as the flow routing cost, depends on the volume of each commodity crossing the link.
- Type of facility on a link: On each link, one or more types of facilities may be available, such as fibre optic cables with different bandwidths, a fleet of heterogeneous vehicles, power lines with various voltage levels, etc.
- Demand: This is the amount of materials/objects such as products, machinery, vehicles, and even people, or the amount of data (also called the flow) to be routed between two particular nodes in the graph, called source/origin and destination/sink. Several flows policies may be envisaged: the case of the single commodity flows where there is only one demand to be routed between a specific source and a specific destination, for example, data transmission between a concentrator and a switching centre in centrally managed system infrastructure. Also, there is a case of multicommodity flows where demands have a single source but multiple destinations such as delivering products from a central factory to multiple distribution points. Another routing policy for multicommodity flows where there are different source-sink pairs, and for each pair of nodes a certain demand amount has to be shipped from a source node to a sink node. The case of the planning and scheduling problem is an example where commodities are products to be shipped from multiple manufacturing firms to different customers. An important aspect of multicommodity flows is related to other different routing schemes: at one extreme, we have a non-bifurcated flow which means that flow must be shipped exactly on one path

from origin to destination. In the other extreme, we find bifurcated routing flow that enables the flow bifurcations.

Thus, *NDPs* require deciding on which links to install facilities with a discrete or infinite capacity to determine the optimal values of flow demands to be delivered with a minimum total cost (Atamtürk and Günlük, 2018). Not surprisingly, these problems are NP-hard (Johnson et al., 1978; Chopra et al., 1996) and their resolution poses significant research challenges.

3 Illustrative example

NDPs are particularly interesting as, on one side, they are notoriously difficult (given their combinatorial aspects plus the difficulties of dealing with the complex nature of their constraints, variables, and objective functions, most of *NDPs* are known to be strongly NP-hard), and on the other side, these problems are highly relevant since they have wide applicability in many real-world situations.

Usually, several problems' features give rise to different variants and problem formulations, with different levels of difficulty and size. Significant features of *NDPs* can often be considered such as the edge capacity restrictions that constitute a challenging decision in realistic contexts and greatly increase the computational challenges of making optimal decisions. Typically, the capacity may be restricted to be directed, bi-directed, or undirected, according to the specific application area and the required technology used for setting the capacity. In the first configuration, the amount of materials/data through a directed arc is restricted to the capacity of this arc. In the bi-directed configurations, if a given material or data flow is installed across an arc, so the corresponding value of capacity should equally be set on the opposite arc. While in undirected configurations, the total flow on an arc plus its opposite arc is restricted to the capacity of the non-oriented edge corresponding to both arcs.

Additional assumptions can be envisaged such as the cost structure where the compromises between fixed installation costs and variable flow routing costs when constructing a solution are so complicated, as well as the crucial interaction between assigning capacities on the links and routing simultaneous flows on possibly multiple paths between different source-destination pairs.

NDPs' models can be differentiated according to these various features. In order to illustrate what we mean by an *NDP*, how decisions can be made to design a network, then to highlight the complexity of *NDPs* from a practical point of view, consider the illustrative example of Figure 2 that represents a telecommunication network. The nodes of this network refer to customer nodes that must be interconnected via communication sections. These sections correspond to the edges of the network, a set of possible physical connections where communication facilities (transmission equipment such as fibre optic cables) may be installed to enable the circulation of the network's traffic flow. Now, in the network example, we want to illustrate, we have five nodes, eight edges, and there are two distinct possible facilities to be selected, and then at most one of them can be installed on each edge. These facilities can transmit data at various speed rates (discrete capacity expressed in Mb/sec) and require different fixed installation costs. As expected, the solution of the discrete link capacity problem is difficult to find when there is a large number of integer variables. In our case, we neglect the variable transmission costs, which are dominated by the fixed investment costs.

In addition, there are a set of different traffic demands between all pairs of nodes to be routed simultaneously along several different paths on the network, and these are assumed to share the common amounts of capacity on the facilities. In such a situation, we consider the case of the capacitated multifacility multicommodity flow with different source-sink node pairs. Table 1 describes the characteristics of the demand for each commodity k, $k = 1, \ldots, K$, by defining the source-destination nodes (s_k, t_k), and the quantity d_k that corresponds to the total amount of communication required between s_k and t_k. For example, the quantity of flow to be shipped from node 1 to node 4 through the graph G is $d_3 = 7$. Precisely, let $G = (V, E)$ be the graph of the network described in Figure 2. The goal is to find which facility to install on each edge in order to route simultaneously all the point-to-point demands with a minimum total cost.

Table 1 Flow requirements

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
s_k	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
t_k	2	3	4	5	1	3	4	5	1	2	4	5	1	2	3	5	1	2	3	4
d_k	5	3	7	2	2	1	3	4	6	2	1	2	4	1	2	3	1	2	1	3

Figure 2 Graph of the illustrative example (G) (see online version for colours)

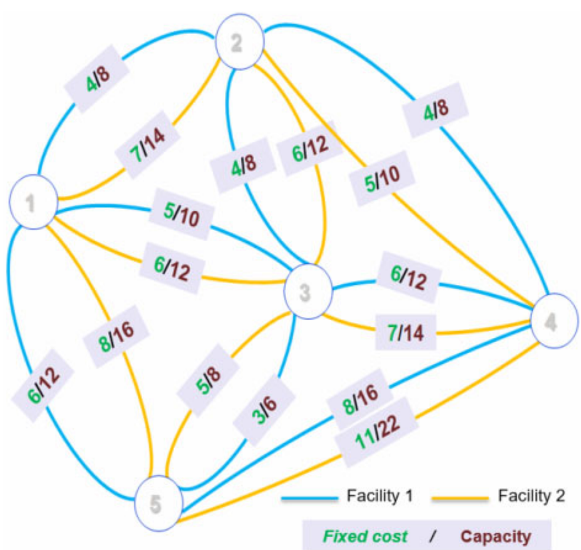


Figure 3 shows the optimal solution given after the exact resolution of this example. We can see that the minimum total installation cost is 39. The optimal network is given by installing one facility on edges (1, 2), (1, 3), (1, 5), (3, 5), (5, 4), and (2, 4) respectively without installing facilities on edges (2, 3) and (3, 4). Thus it is possible to route all the traffic demands between all source-destination pairs simultaneously.

Now, let us consider solution G2, shown in Figure 4, which is obtained by installing the facility of the maximum capacity amount on each edge of the network. The total cost of this configuration is 55 which correspond to an upper bound for the optimal solution.

We can see that we have sufficient capacity to carry all the required demands between the users of the network. Thus, the current solution is feasible but incurs additional costs. Therefore, it is reasonable to envisage decreasing the cost of fibre cables deployment (facilities) by reducing their capacity values.

Figure 3 Optimal solution/network (G1) (see online version for colours)

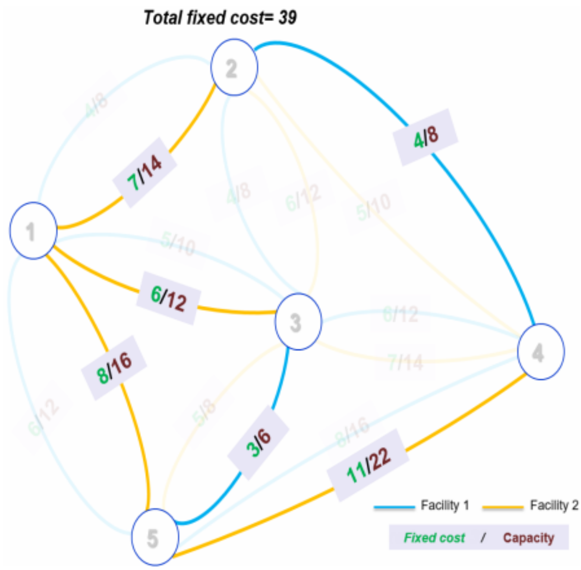
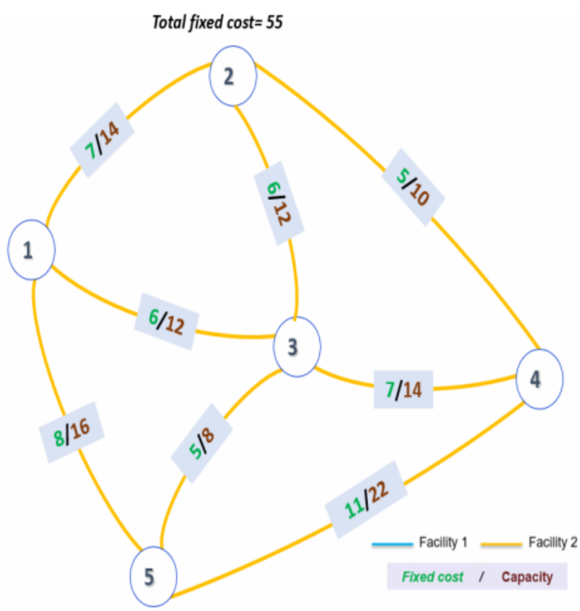
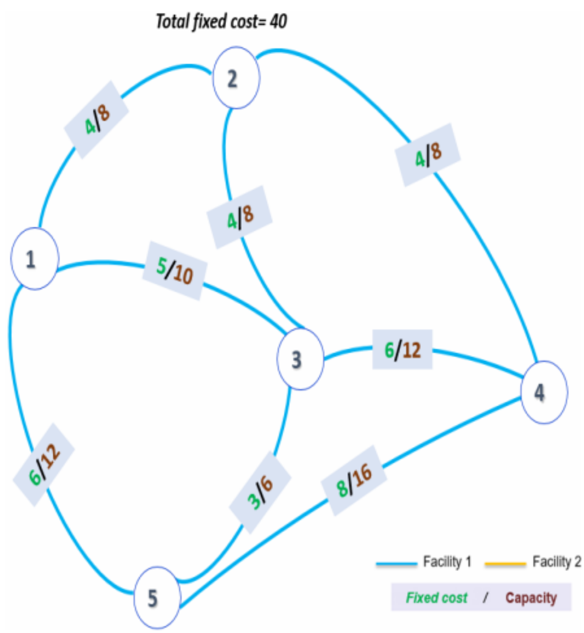


Figure 4 Network configuration with maximum capacities (G2) (see online version for colours)



On the other hand, assume that the problem seeks the least-cost set of fibre cables to be deployed on the network. Thus, a significant cost reduction can be achieved if we install the facility of the minimum capacity amount on each edge. The total cost of the corresponding network (G3), shown in Figure 5, is 40 which is more costly than the obtained optimal configuration. Thus, it should be noted that installing the minimum capacity values does not lead necessarily to a valid lower bound as it may be a feasible solution depending on the commodity demand amounts. In addition, in other cases, considering minimum amounts of capacities can yield an infeasible solution as the associated network configuration has insufficient capacity to carry all traffic demands. Hence, this poses the question of how to get a better configuration that optimises assigning a minimum cost set of communication facilities and routing simultaneous flows between different source-destination nodes, which is the scope of several advanced methods in the literature.

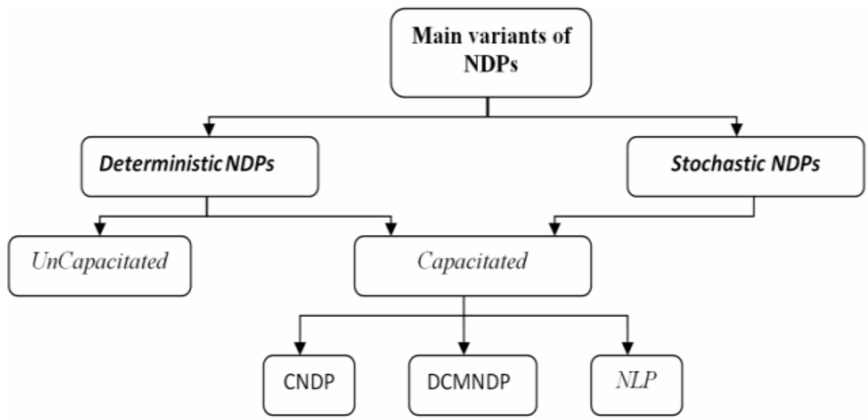
Figure 5 Network configuration with minimum capacities (G3) (see online version for colours)



4 Main variants of NDPs

As aforementioned, *NDPs* can be classified according to the nature of input data. In this regard, the problem parameters (traffic demands, capacities, etc.) can be deterministic and known in advance, or stochastic, and therefore uncertain. The two main variants are therefore deterministic and stochastic *NDPs*. Next, we looked at characteristics including capacity and cost criteria, flow schemes, as well as other properties, to classify the main variants in each category. In this section, we describe each variant by focusing on the corresponding contexts of application. Figure 6 summarises different main variants of the NDPs.

Figure 6 Main variants of NDPs



4.1 Deterministic NDPs

Deterministic variants of *NDPs* can be categorised into uncapacitated and capacitated NDPs. Clearly, considering a capacity on the arcs of the network makes this problem even more complex (Balakrishnan et al., 1997). As a result, different capacitated *NDPs* (CNDPs) have been investigated in the past, each with its own properties and computational issues. Some of the research works on each of these categories and their specific characteristics are summarised in Tables 2 and 3.

4.1.1 Uncapacitated NDPs

The first studies on network design considered the installation of links within finite capacity. These are the uncapacitated NDPs (*UCNDP*), where on each edge of the network, a decision about installing or not an infinite capacity link should be taken. Initial researches have studied this variant with multicommodity and fixed installation costs. From these works, we mention those that considered:

- The case of multiple types of facilities, as in Minoux (1974, 1976). In these works, Minoux studied the problem of medium- and long-term planning of telecommunications networks considering a set of demands between distinct source-destination pairs.
- The case of the single facility to be installed on each link, as in Hoc (1982). In this work, Hoc studied the problem of choosing investments in network transport infrastructure by considering that the capacities of the links are unlimited.

Later on, this variant *UCNDP* was explored by considering two types of costs: a fixed installation cost and a variable cost depending on the routed amount of flow. Several works have studied this variant in connection with telecommunications and transportation networks by considering:

- The case of the single commodity flow, for instance, we cite one of the most important problems in the transportation context proposed by Jakob and Pruzan (1983), which consists in transporting goods from a production centre to its direct customer, while looking at the fixed costs of transport and the variable costs of production.
- Another line of research for addressing the uncapacitated variant is to consider the case of multicommodity flows with demands between multiple source-destination pairs. This case has been addressed in logistics to determine the best location of warehouses (Magnanti and Wong, 1984; Magnanti et al., 1986), production planning (Balakrishnan, 1987; Balakrishnan et al., 1989), as well as in telecommunications to solve the classical uncapacitated hub location problem with multiple assignments (Contreras et al., 2011).
- Herein, we focus on the multicommodity flows where demands have multiple destinations, but only one source. This case has been approached extensively in telecommunications to design a local access network allowing the routing of a set of demands between a distribution point and different witching centres while considering that there are no capacity constraints on the links. We refer to the works of Randazzo and Luna (2001) and Randazzo et al. (2001).

4.1.2 Capacitated NDPs

In this section, we review *CNDPs* where the amount of traffic to be routed over a link is subject to certain discrete capacity restrictions. *CNDPs* can be easily stated but they are very difficult to solve (Crainic et al., 2000). There are effective theoretical findings as well as empirical evidence to prove that the *CNDPs* are NP-hard (Balakrishnan et al., 1997). Here, we present a selection of available CNDPs with emphasis on three variants which are the most common among existing problems. These problems have been classified according to their cost functions. Table 3 summarises research works on different variants of CNDPs and highlights the difference between each of them.

4.1.2.1 NDPs with fixed and variable costs

The first studies, which focused on *CNDPs*, have considered the installation of a single facility on each edge while minimising two types of costs: a fixed-charge cost that can be the cost of constructing arcs and a variable cost associated with the operating cost of routing flows on arcs [this problem is known as the capacitated fixed-charge NDP (*CFNDP*)]. These problems cover a wide range of applications in telecommunications, transportation, logistics, localisation, and production. We refer the reader to the work of Balakrishnan et al. (1997) for a detailed review.

The main variants of the *CFNDP* consider:

1 The capacitated fixed-charge network flow problem (*CFNFP*). Hirsch and Dantzig (1955) were the first to tackle this variant. Then, it was formulated during the 1968s by Murty (1968) in a general context. The author has shown that this problem appears in planning situations where the activities are interdependent. Then, CFNFP was applied in many application areas, particularly in logistics and production planning (Hirsch and Dantzig, 1968). Herein, the authors studied a practical situation

in which this problem may occur. They looked at a factory in which distinctive operations are executed by different machines. The problem consisted of minimising the operating costs and the fixed costs of setting up equipments under special restrictions. Then, many other works were published until 2003 treating applications in the fields of telecommunications and logistics (Rardin and Choe, 1979; Khang and Fujiwara, 1991; Kim and Pardalos, 1999; Kim and Hooker, 2002; Atamtürk et al., 2016). Ekşioğlu et al. (2003) studied the problem of delivering products from a factory to a distribution centre to satisfy its demand. They focused on minimising the fixed costs of preparing a delivery and the variable costs associated with fuel consumption, driver's salary, etc.

2 The case of the multicommodity capacitated fixed-charge NDP (*MCFNDP*). The compromises between the variable and fixed costs when constructing a solution, plus the interaction between assigning capacities on the links and routing of multiple flows simultaneously, make it more difficult to efficiently address and solve this variant of the problem (Agarwal et al., 2022; Kazemi et al., 2021). In addition to its various applications in telecommunications (Crainic et al., 2000; Holmberg and Yuan, 2000; Frangioni and Gendron, 2009; Chouman et al., 2009; Katayama et al., 2009; Hewitt et al., 2013; Gendron and Larose, 2014; Yaghini et al., 2014; Momeni and Sarmadi, 2016; Atamturk et al., 2017; Munguía et al., 2017; Chouman et al., 2018; Gendron et al., 2018), product distribution, logistics, urban traffic, oil industry, as well as aircraft assignment are among the principal areas of application for this class of problem. Accordingly, Geoffrion and Graves (1974) have addressed a common problem in multicommodity distribution network design. The objective is to identify the optimal way for locating intermediate distribution systems between plants and customers. Costa (2005) was particularly interested in the service NDP arising in airline and trucking companies. The author applied the *MCFNDP* to decide the frequency off lights according to aircraft availability to maximise profits. In power systems, this variant of the fixed-charge NDP was formulated to obtain the optimal energy transmission system from the production centres to the end-users. Recently, Guimarães et al. (2020) investigated the same variant in a transport traffic context by considering multiple lines and a heterogeneous fleet for transporting freight in buses from multiple origins to multiple destinations (bus stations). The objective is to reduce the travel operating costs. Hellsten et al. (2021) considered the transit time constraints to tackle the MCFNDP variant in the context of transporting perishable goods. Li et al. (2021) investigated a new service NDP with heterogeneous resource constraints in the context of freight carrier's transportation. Considering such application, services referred to the transportation of products (commodities) between origin-destination terminals. While the vehicles involved in transport services were heterogeneous and associated with specific node terminals. The problem consists of identifying the services to be scheduled, constructing routes for the considered heterogeneous resources, and routing the commodities into the service network configuration while minimising the sum of variable flow routing costs and fixed costs for identifying resources.

Table 2 Characteristics of deterministic *UCNDPs*

Reference	*Capacity*		*Cost*		*Facility*		*Flow*		
							Single	*Multicommodity*	
	With	*Without*	*Fixed*	*Variable*	*Single*	*Multiple*	*Single source and single destination*	*Single source multiple destinations*	*Multiple sources and destinations*
Minoux (1974)		X	X			X			X
Minoux (1976)		X	X			X			X
Hoc (1982)		X	X		X				X
Jakob and Pruzan (1983)		X	X	X		X	X		
Magnanti and Wong (1984)		X	X	X	X				X
Magnanti et al. (1986)		X	X	X		X			X
Balakrishnan (1987)		X	X	X		X			X
Balakrishnan et al. (1989)		X	X	X		X			X
Cruz et al. (1998)		X	X	X	X		X		
Frangioni and Gallo (1999)		X	X		X				X
Randazzo and Luna (2001)		X	X	X		X		X	
Randazzo et al. (2001)		X	X	X		X		X	
Contreras et al. (2011)		X	X	X	X				X
Tadayon and Smith (2014)		X		X	X				X

Table 3 Characteristics of deterministic *CNDPs*

Reference	*Capacity*		*Cost*		*Facility*		*Flow*		
							Single	*Multicommodity*	
	With	*Without*	*Fixed*	*Variable*	*Single*	*Multiple*	*Single source and single destination*	*Single source and multiple destinations*	*Multiple sources and destinations*
CFNDP									
Hirsch and Dantzig (1968)	X		X	X	X		X		
Murty (1968)	X		X	X	X		X		
Rardin and Choe (1979)	X		X	X	X		X		
Khang and Fujiwara (1991)	X		X	X	X		X		
Kim and Pardalos (1999)	X		X	X	X		X		
Crainic et al. (2000)	X		X	X	X				X
Holmberg and Yuan (2000)	X		X	X	X				X
Kim and Hooker (2002)	X		X	X	X		X		
Crainic and Gendreau (2002)	X		X	X	X				X
Ghamlouche et al. (2003)	X		X	X	X				X
Ekşioğlu et al. (2003)	X		X	X	X		X		
Crainic et al. (2004)	X		X	X	X				X
Costa (2005)	X		X	X	X				X
Alvarez et al. (2005)	X		X	X	X				X
Crainic et al. (2006)	X		X	X	X				X
Pedersen et al. (2006)	X		X	X	X				X
Crainic and Gendreau (2007)	X		X	X	X				X
Chouman et al. (2009)	X		X	X	X				X
Frangioni and Gendron (2009)	X		X	X	X				X

Table 3 Characteristics of deterministic *CNDPs* (continued)

Reference	*Capacity*		*Cost*		*Facility*		*Flow*		
							Single	*Multicommodity*	
	With	*Without*	*Fixed*	*Variable*	*Single*	*Multiple*	*Single source and single destination*	*Single source and multiple destinations*	*Multiple sources and destinations*
CFNDP									
Katayama (2009)	X		X	X	X				X
Chouman and Crainic (2010)	X		X	X	X				X
Rodríguez-Martín and Salazar-González (2010)	X		X	X	X				X
Hewitt et al. (2013)	X		X	X	X				X
Gendron and Larose (2014)	X		X	X	X				X
Yaghini et al. (2014)	X		X	X	X				X
Atamtürk et al. (2016)	X		X	X	X		X		
Momeni and Sarmadi (2016)	X		X	X	X				X
Atamturk et al. (2017)	X		X	X	X				X
Munguía et al. (2017)	X		X	X	X				X
Chouman et al. (2018)	X		X	X	X				X
Gendron et al. (2018)	X		X	X	X				X
Kazemzadeh et al. (2022)	X		X	X	X				X
Katayama (2020)	X		X	X	X				X
Guimarães et al. (2020)	X			X	X				X
Shibasaki et al. (2021)	X		X	X	X				X
Agarwal et al. (2022)	X		X	X	X				X

Table 3 Characteristics of deterministic *CNDPs* (continued)

Reference	*Capacity*		*Cost*		*Facility*		*Flow*		
							Single	*Multicommodity*	
	With	*Without*	*Fixed*	*Variable*	*Single*	*Multiple*	*Single source and single destination*	*Single source and multiple destinations*	*Multiple sources and destinations*
CFNDP									
Kazemi et al. (2021)	X		X	X	X				X
Hellsten et al. (2021)	X		X	X	X				X
Li et al. (2021)	X		X	X	X				X
Avella et al. (2021)	X		X	X	X				X
DCMNDP									
Minoux (1989)	X		X			X	X		
Stoer and Dahl (1994)	X		X			X			X
Gabrel et al. (1999)	X		X			X			X
Minoux (2001)	X		X			X			X
Gabriel et al. (2003)	X		X			X			X
Roussel et al. (2004)	X		X			X			X
Mrad and Haouari (2008)	X		X			X			X
Aloise and Ribeiro (2011)	X		X			X			X
Layeb et al. (2017)	X		X			X			X
Mejri et al. (2019a)	X		X			X			X
Mejri et al. (2019c)	X		X			X			X

Table 3 Characteristics of deterministic *CNDPs* (continued)

Reference	*Capacity*		*Cost*		*Facility*		*Flow*		
							Single	*Multicommodity*	
	With	*Without*	*Fixed*	*Variable*	*Single*	*Multiple*	*Single source and single destination*	*Single source and multiple destinations*	*Multiple sources and destinations*
NLP									
Lee et al. (1989)	X		X		X				X
Magnanti et al. (1993)	X		X		X				X
Magnanti et al. (1995)	X		X			X			X
Barahona (1996)	X		X		X				X
Bienstock and Günlük (1996)	X		X			X			X
Bienstock et al. (1998)	X		X		X				X
Chopra et al. (1998)	X		X		X		X		
Dahl and Stoer (1998)	X		X			X			X
Günlük (1999)	X		X		X				X
Berger et al. (2000)	X		X			X			X
Sridhar et al. (2000)	X		X		X				X
Mirchandani (2000)	X		X			X	X		
Agarwal (2002)	X		X			X			X
Gendron et al. (2002)	X		X			X			X
Avella et al. (2007)	X		X		X				X
Ljubić et al. (2012)	X		X			X		X	
Mattia (2012)	X		X		X				X
Atamtürk and Günlük (2021)	X		X			X			X

4.1.2.2 NDPs with fixed costs

This variant of problem considers a fixed installation costs on a given network with various types of discrete link capacities, to optimise the assignment of single or multicommodity flows:

1. The case of the single flow known as, discrete cost single commodity flow problem (*DCSFP*). This class of problem was initially used by Minoux (1989) to design a telecommunication network. The author found out that the single flow problem is NP-complete.
2. The case of multicommodity flow known as, discrete cost multicommodity NDP (*DCMNDP*). Several researches have been developed to deal with the *DCMNDs* particularly to design a telecommunication network by considering multiple facilities with discrete capacities and fixed installation costs (Stoer and Dahl, 1994; Gabrel et al., 1999; Minoux, 2001; Gabriel et al., 2003; Aloise and Ribeiro, 2011; Mrad and Haouari, 2008; Mejri et al., 2019a, 2019c). These costs are characterised by general discontinuous step-increasing functions. Other applications of this variant of problem can also be found in a transportation planning context (Roussel et al., 2004).

4.1.2.3 Network loading problems

In general terms, network loading problems (*NLPs*) aim to decide the number of facilities to be loaded on each link of the graph to allow simultaneous routing of traffic at the lowest cost. The particularity herein is that the loaded link-capacities can be multiples of integers and the associated cost corresponds to installing one unit of capacity on the arc. In this case, we neglect the variable flow costs, which are considered zero. Applications of *NLP* are widely found in various areas, particularly for designing telecommunication and distribution networks (e.g., Mejri et al., 2019d).

Many variants of this problem can be seen in the literature, such as:

1. The NLPs considering a single type of facility (known as, the single facility *NLP*). On each link, we can load an integer number of a single type of facility which is characterised by a fixed cost of installing a capacity unit. Closely related problems have been addressed in Magnanti et al. (1993), Barahona (1996), Bienstock et al. (1998) and Chopra et al. (1998). Accordingly, Sridhar et al. (2000) tackled this variant in a telecommunication context to configure a local area network (LAN) which offers high quality at low configuration cost.
2. A generalisation of these problems, which considers many types of facilities with a discrete capacity and fixed cost, arises in a variety of application settings. In the transportation sector, facilities can model fixed-size lorries, so that the problem is to determine the optimal plan for assigning lorries to itineraries or loading cargo on lorries (Magnanti et al., 1993). In the telecommunications sector, it can describe the configuration of private networks which consider digital communication facilities (digital circuit channels) capable of transmitting voice, data, or videos between locations. We apply a fixed leasing cost for each digital facility type (for example DSO circuit, DS1 circuit, etc). The goal is to identify the number of digital facilities

to be installed on the backbone links that will enable the routing of the expected traffic at the lowest possible cost (Magnanti et al., 1995). In the same context, Günlük (1999) was interested in the application of this variant for a long-distance service provider as well as for their users. For each issue, digital communication facilities are loaded in multiple integers of different transmission rates (for example OC1, OC3, etc). Another important feature that characterises the different variants of *NLPs* is related to routing single or multiple commodities on the network.

3. To our knowledge, the only researches that have addressed the NLPs with the single-commodity flow, were respectively the work of Chopra et al. (1998) where the author studied the one-facility, one-commodity (*OFOC*) NLP for both telecommunications and transportation networks, and the paper published by Mirchandani (2000) which explored the problem for one-commodity flow with two facility types.
4. The multicommodity flow version of *NLP* is still hard. Closely related problems are very well-investigated for a large number of telecommunications applications in Lee et al. (1989), Magnanti et al. (1995), Barahona (1996), Bienstock and Günlük (1996), Dahl and Stoer (1998), Berger et al. (2000), Agarwal (2002), Gendron et al. (2002) and Atamtürk and Günlük (2021).

4.2 Stochastic NDPs

It is worthy to note that there is tremendous literature on *NDPs* assuming that input data are predefined and fully known, i.e., deterministic *NDPs*. But in most real-world applications, these data are uncertain and exhibit significant variation, typically they are not accessible at the time we configure the network (stochastic). Accordingly, evaluating the network performance in a stochastic environment is becoming one of the most important issues for practitioners as well as for network designers, who realised that it is very challenging in reality to cope with this uncertainty so providing an optimal solution in this framework is significantly hard (e.g., Hong et al., 2015; Shen et al., 2017). Therefore, the literature addressing NDP's variants under uncertainty is relatively scant.

A review on the state of research in stochastic *NDPs* enables to detect the main works published for the different *NDPs* variants. Table 4 summarises these research works and highlights the difference between each of them.

4.2.1 NDPs with fixed and variable cost

Lium et al. (2009) proposed a detailed review in which they discuss the importance of considering data uncertainty explicitly in designing networks, and they focus particularly on demand uncertainty. Accordingly, an interesting body of literature on stochastic NDPs considering both fixed and variable costs has been proposed for the multicommodity case. Atamtürk and Zhang (2007) described a two-stage MCFNDP under only uncertain demand. The authors demonstrated that the given problem is NP-hard even on a bipartite graph and provided specific applications to the problems of lot-sizing transport and localisation. Under both uncertain transportation cost and random demand, Mudchanatongsuk et al. (2008) have shown that providing an approximate solution to the robust NDP can be performed efficiently for the case of a single source and destination per commodity. Hoff et al. (2010) investigated a multicommodity network with only uncertain demand to find an efficient service scheduling scheme provided by a fleet of homogeneous vehicles while minimising the expected total system costs including vehicle operating costs, freight transportation costs as well as investment costs. Later, Robust versions of MCFNDPs with demand uncertainty have been studied in telecommunications and transportation contexts by several authors, namely Crainic et al. (2011, 2014), Boland et al. (2016), Crainic et al. (2016), Atamturk et al. (2017), Rahmaniani et al. (2018) and Hewitt et al. (2021). In addition, distribution networks for water, oil, and gas were confronted with challenges of what constitutes a robust network design considering the disruptions in gas supply. In this context, handling uncertainties in capacity has been investigated by Thapalia et al. (2012) for the single-commodity variant under random edge capacity. Very recently, Crainic et al. (2021) addressed a two-stage stochastic multicommodity network design model under both uncertain demands and random edge capacities.

4.2.2 NDPs with fixed cost

Compared to *CFNDP*, studies on the stochastic versions of *DCMNDP* are still quite limited. Featuring polyhedral demand uncertainty, Minoux (2010) has proved that the nondeterministic versions of such problems are strongly NP-hard. In addition, the author has shown that this variant of problems is key to many real applications such as telecommunication networks under linear costs and uncertain customer requirements (see e.g., Mejri et al., 2019b; Ouorou and Vial, 2007) and power networks (see e.g., Lee et al., 2013).

4.2.3 Network loading problems

As mentioned earlier, when reviewing the available body of literature, one shows that researches addressing uncertainty in *NDP's* variants and in particular *NLP* are still very scant. We can identify the NLP under random requirements that we refer to as the robust NLP. Interested by this variant of problem in telecommunications networks, Altın et al. (2011) considered a polyhedral definition of feasible traffic demands to design a network that allows routing multiple non-simultaneous flows. In the same context, Koster et al. (2013) mainly concentrated on aspects of telecommunications networks and addressed a similar study taking into account two budgeted uncertainty. Since then, the robust NLP has become one of the most challenging problems in the telecommunications industry (Mattia, 2013; Claßen et al., 2015). Recently, Mattia and Poss (2018) proposed a comprehensive investigation on robust *NLP* in which the authors studied the impact of routing strategy changes on the complexity of the problem from both a theoretical and computational perspectives. Accordingly, they investigated the advantages and drawbacks of static, affine, and dynamic routing schemes under demand uncertainties.

Table 4 Characteristics of stochastic *NDPs*

Reference	*Capacity*		*Cost*		*Facility*		*Flow*			*Stochastic data*
							Single	*Multicommodity*		
	With capacity	*Without capacity*	*Fixed cost*	*Variable cost*	*Single facility*	*Multiple facility*		*Single source and multiple destinations*	*Multiple sources and destinations*	
CFNDP										
Atamtürk and Zhang (2007)	X		X	X	X				X	Demand
Mudchanatongsuk et al. (2008)	X		X	X	X				X	Demand cost
Hoff et al. (2010)	X		X	X	X				X	Demand
Crainic et al. (2011)	X		X	X	X				X	Demand
Thapalia et al. (2012)	X		X	X	X		X			Capacity
Crainic et al. (2014)	X		X	X	X				X	Demand
Boland et al. (2016)	X		X	X	X				X	Demand
Crainic et al. (2016)	X		X	X	X				X	Demand
Rahmaniani et al. (2018)	X		X	X	X				X	Demand
Sarayloo et al. (2020)	X		X	X	X				X	Demand
Sarayloo et al. (2021)	X		X	X	X				X	Demand
Hewitt et al. (2021)	X		X	X	X				X	Demand
Crainic et al. (2021)	X		X	X	X				X	Demand capacity
DCMNDP										
Ouorou and Vial (2007)	X		X		X				X	Demand
Minoux (2010)	X		X			X			X	Demand
Lee et al. (2013)	X		X			X			X	Demand
Mejri et al. (2019b)	X		X			X			X	Demand

Table 4 Characteristics of stochastic *NDPs* (continued)

Reference	*Capacity*		*Cost*		*Facility*		*Flow*			*Stochastic data*
								Multicommodity		
	With capacity	*Without capacity*	*Fixed cost*	*Variable cost*	*Single facility*	*Multiple facility*	*Single*	*Single source and multiple destinations*	*Multiple sources and destinations*	
NLP										
Altin et al. (2011)	X		X			X			X	Demand
Koster et al. (2013)	X		X		X		X			Demand
Mattia (2013)	X		X		X				X	Demand
Claßen et al. (2015)	X		X		X				X	Demand
Mattia and Poss (2018)	X		X		X				X	Demand

4.3 Synthesis

4.3.1 Main variants of NDPs

Interesting results have been published for the *UCNDP* between 1974 and 2014, and no studies of the stochastic version have been published to date. Probably this is due to the fact that in most real cases, real facilities are characterised by capacities. Therefore, more attention has been devoted to the *CNDP* highlighting the need for models that are closer to more realistic applications but this variant is more challenging and poses significant modelling and algorithmic difficulties (see Figure 7).

Figure 7 Trends of papers on *uncapacitated* and *capacitated NDPs* (see online version for colours)

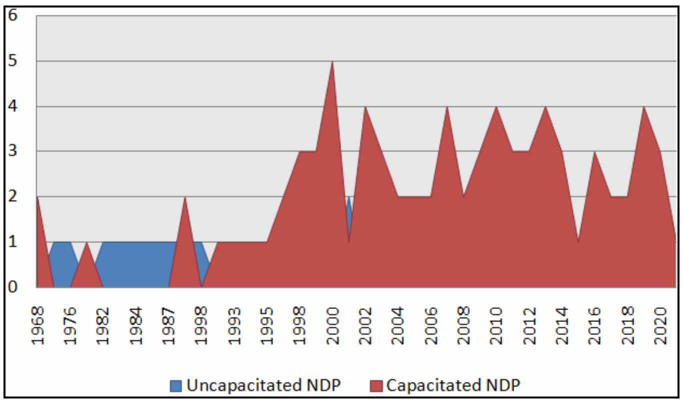
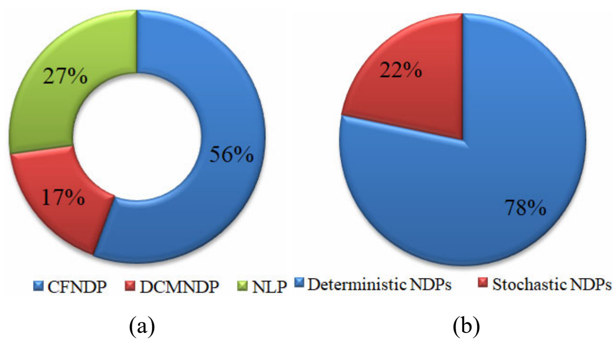


Figure 8(a) indicates that the majority of the reviewed studies have tackled the *CFNDP* and its variants. There is quite a large body of academic literature on the topic and researchers also continue to publish to this day. We believe that this is due to the flexibility of the model which could easily be adapted to cope with practical and real situations. In these works, the objective function is a minimisation of a fixed-charge cost that can be the cost of constructing arcs and a variable cost associated to the operating cost of routing flows on arcs. The last papers dealing with the *CFNDP* were recently published over the 2020–2021 period. Several studies focused on the deterministic variant (e.g., Agarwal et al., 2022; Kazemi et al., 2021; Hellsten et al., 2021; Li et al., 2021), while Sarayloo et al. (2020, 2021) and Hewitt et al. (2021) addressed the stochastic version of the same problem.

We can also notice that the *NLPs* and *DCMNDPs* address the current situation where multiple types of facilities with different fixed installation costs and discrete capacities are available instead of the case where only one type of facility is adopted as in the *CFNDP* variant. However, among the more than 100 papers mentioned in this literature survey, only 14 addressed *DCMNDPs* and only 23 focused on the *NLP* problems. Therefore, it seems clear that, despite the importance of these two variants' applications in diverse settings, the available researches have still been addressed in a quite scant way instead of the *CFNDPs*. Accordingly, we mention that the papers on deterministic *NLPs* were previously published until 2013, and works on the stochastic variant continue to be published until today. On the other hand, the last references that have considered the deterministic *DCMNDP* variant date from 2019, and very limited studies on its stochastic version have been conducted to date.

Finally, it should be noted that most of the reviewed studies have tackled the deterministic *NDPs* instead of the stochastic variants [see Figure 8(b)]. Not surprisingly, the difficulty of *NDPs* increases when the input data are uncertain. Therefore, the stochastic variants of *NDP* are more challenging and complex.

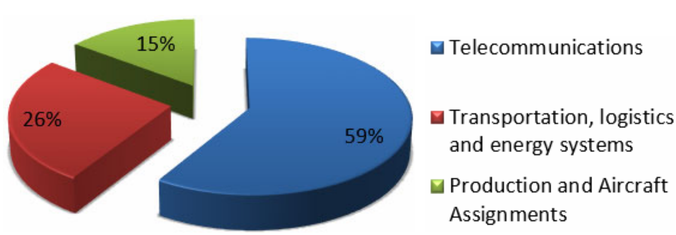
Figure 8 (a) Total number of papers on *CFNDP*, *DCMNDP*, and *NLP* variants (b) Total number of papers on deterministic and stochastic variants (see online version for colours)



4.3.2 *Areas of applications for NDPs*

The NDPs have originally motivated many interesting progresses in optimisation research because these problems have wide applicability in many real-world situations. Based on the studies we have surveyed in both deterministic and stochastic environments, we can conclude that the most relevant applications of *NDPs*, particularly *DCMNDP*, *NLP* and *CFNDP* variants, arise in the telecommunications industry (59%) that represents an active research area. Then, we find that transportation, logistics, and energy systems represent the second important areas investigated mainly by studies on *CFNDP* variant (26%). Production systems and airline industry have used also the *NDPs* for product planning and aircraft assignments (15%). Trends in applications of *NDPs* are summarised in Figure 9.

Figure 9 Main applications of *NDPs* (see online version for colours)



5 Additional relevant applications of NDPs

As mentioned earlier, NDPs are prominent in several practical situations. Since covering all real-life applications of *NDPs* is a very difficult task almost impossible, we purposely focused on some relevant applications which are practical of interest and which rise a large set of issues. Therefore, this section is particularly dedicated to specific applications in the context of smart telecommunication systems, green logistics and sustainable supply chain networks, as well as intelligent energy networks, although the other sections of the paper address other practical contexts also in telecommunications, logistics and transportation, manufacturing and production planning, aircraft assignments, economic settings, and electric systems.

5.1 Smart telecommunication systems

Influenced by the rapid rate of technological innovation and progress, the functioning of our modern human life and needs has been transformed to become increasingly dependant on advanced telecommunication networks (Houankpo and Kozyrev, 2019). Due to the transition of complicated equipment and crucial systems to novel communication networks, it has become essential to guarantee the high reliability and availability of networks-technologies-solutions, now commonly known as the smart grid transmission systems. In addition, survivability is an increasingly prominent feature in telecommunications networks, as when introducing fibre facilities of high-capacities the communications infrastructures become less dense. Thus, survivability is useful to adequately keep connectivity in the case of link failures and attacks, besides by integrating redundant paths into the configuration structure, survivability can improve the reliability of the network (Crainic et al., 2020).

A large number of publications on *NDPs* were recently interested in investigating the survivability and reliability of several complicated telecommunication networks since almost all these can be modelled as NDPs. Particularly, Kabadurmus and Smith (2016) presented an exact approach and a practical heuristic procedure to solve a specific variant of the survivable multicommodity NDP which has not been tackled before because of its computational complexity. The authors addressed the multicommodity survivable *NDP* in two ways. At the first stage, the *k*-splittable flow that enables the flow bifurcation was applied by considering relays for every split path and every survivable path. Next, the capacity edges restrictions were introduced to ensure a more realistic aspect. The purpose was to regenerator placement problem in the context of translucent optical networks by minimising the total relay and edge costs.

According to Schauer and Raidl (2016), the simplest manner to ensure survivability is to employ a ring topology as the network remains connected in the face of a single node or edge failure. In this context, the authors investigated the multi-layer hierarchical ring network design (*MLHRND*) problem, which is widely applied in hierarchically structured configurations with a high survivability requirement. The problem was modelled as a mixed-integer linear program using a multicommodity flow-based procedure. Also, a single-commodity flow formulation was adapted to achieve the connectivity of the overall network.

Another challenging issue that has been tackled in the literature is designing and measuring the reliability of wireless mesh networks to enhance the performance of their services. Recently, Samani and Wang (2018) proposed a flow MaxStream framework using two integer multicommodity flow problem formulations intending to optimise the quantity as well as the quality of video streaming services.

5.2 Green logistics and sustainable supply chain networks

As mentioned earlier, the NDPs in traditional logistic contexts have been widely addressed, but their effects on the environment has been investigated only in recent years, and are still scarce while global warming caused by the enhanced greenhouse gas (GHG) emissions, poses a major environmental issue. Accordingly, given the growing preoccupation about the transportation impacts on the global warming phenomenon, Cariou et al. (2018) presented a MILP model using a multicommodity pickup and delivery arc-flow (AF) formulation to design an optimal line service network so that the total operating benefit is maximised taking into account CO_2 and SO_x emissions. Each commodity is characterised by source-destination ports, container volume, and freight transportation revenue. Zhang et al. (2018) developed an optimisation model that simultaneously integrates corresponding logistics infrastructure investments and green transport mode subsidies with specific carbon emission reduction goals in a freight transport context. The experimental results based on real-world case studies have shown that expected carbon policies reduction can have a meaningful impact on logistics infrastructure investments in terms of the number of logistics nodes and their total cost.

A recent review on green supply chain network design was proposed by Waltho et al. (2019). This comprehensive survey aimed to understand how emissions are considered in the design of supply chains, the carbon policies that are employed to significantly reduce emissions, and their eventual effect on the supply chain operations from both financial and structural perspectives. The authors showed that all surveyed articles considered emissions from transportation/shipping and most of them were interested in the green logistics network design with a cap-and-trade scheme. They also pointed out that very scarce studies have considered the impact of gas emissions on the demand. Mehranfar et al. (2019) addressed the production-distribution problem by considering the carbon emissions in designing the supply chain network under uncertain customer demands. The authors explored different carbon emission strategies namely strict carbon cap and trade, carbon taxation, and cap and trade. The objective of this study was to design an optimal supply chain network that optimises the economic system costs and the environmental revenues. They showed that carbon cap-and-trade may be more beneficial for the considered production-distribution systems.

Recently, Jiang et al. (2020) addressed a regional multimodal logistics NDP in the context of urban cluster development. The authors considered four relevant characteristics associated with their study context: the uncertainty of future logistic demands, the environmental concerns of CO_2 emission reduction, and the presence of subsidies and multiple stakeholders. In another recent study, Yang et al. (2021) presented a local optimisation algorithm to enhance the sustainability of the coastal container multimodal transportation strategy. The proposed approach seeks to reduce carbon dioxide emissions by simultaneously optimising two different policies (the shipping network design policy and toll policy.

Many researchers have devoted their attention to attempting to redesign supply chain networks to be more sustainable. In this context, Fragoso and Figueira (2021) developed a balanced procedure that combines three aspects of sustainability into the supply chain network design, namely economic, social and environmental impacts. The authors were interested in the wine industry as a real case study. This work aimed to optimise sustainability multi-objective decisions including the minimisation of GHG emissions, water consumption, and supply chain costs and the maximisation of supply chain revenue and employment.

5.3 Intelligent energy networks

The relevance of a smart energy system is prompted by the reality that the usage of fossil fuel leads substantially to the 'greenhouse effect', hence the need to use renewable energy sources. These smart energy systems which include various energy segments (such as electricity, heat, and gas) are identified as a valuable concept for delivering an optimised alternative ensuring a practicable and sustainable energy system in the short-term horizon. Although there are many efforts to identify, develop, and optimise these schemes, the challenge of designing a smart energy system is still a top concern. Shi et al. (2016) developed a mathematical model for the multicommodity smart energy system. They established a self-sufficient hybrid energy scheme for reducing the need to exchange electricity with the external grid connexion. Ghorab (2019) proposed a complex optimisation model for the smart energy network design in a Canadian community. The objective was to reduce energy cost consumption and GHG emissions.

A successful energy strategy has to be strongly oriented toward two main objectives: minimising the consumption of energy and optimising the exploitation of the existing energy sources. In this regard, district heating (DH) is considered to be one of the relevant sources for achieving a sustainable environment and effective energy policies in modern societies. Bordin et al. (2016) developed a mathematical model for a DH NDP. The network included a set of nodes that presented potential users, characterised by their heating requirements (demands). The links in the network referred to the pipes that may be connected to other pipes to connect potential users. The goal was to identify the optimal configuration by deciding which users to select in the network in order to minimise infrastructure and operational costs.

6 Solving NDPs

In this section, we present the most commonly used formulations for modelling the *NDPs* as well as the main methods deployed to solve them. We also reviewed the related published papers for both deterministic and stochastic variants.

6.1 Mathematical formulations

To model the different variants of *NDPs*, mixed integer linear programming (MILP) is generally used. In particular, two types of formulations are deployed, namely the AF-based formulation and the arc-path-based formulation. First, let us introduce a unified notational framework for these formulations:

- $G = (V, A)$: Direct connected graph.
- V: Set of n nodes, indexed by i and j.
- A: Set of m edges connecting the nodes.

- D: Set of K traffic demands (commodities). For each commodity k, an amount of flow with value d_k must be routed between the source node s_k and the sink node t_k.
- f_{ij}: Fixed cost of installing a facility on arc (i, j). This cost is paid if arc (i, j) is considered in the network design.
- c_{ij}^{k} : Cost of routing one unit of commodity k on arc (i, j).
- u_{ij}: Capacity of the facility to be installed on arc (i, j).

6.1.1 AF-based-formulation

The AF-based formulation generally consider the following two decision variables: x_{ij}^{k} a continuous variable that represents the quantity of flow circulating on arc $(i, j) \in A$ for each commodity k, $k = 1, \ldots, K$, and y_{ij} a binary variable that models the decision to install a facility on the arc $(i, j) \in A$, such that y_{ij} is equal to 1 if a facility is installed, and 0 otherwise.

Using these notations, a general formulation of AF type is then given as follows:

$$AF : Minimise \sum_{(i,j)\in A} f_{ij} y_{ij} + \sum_{k=1}^{K} \sum_{(i,j)\in A} c_{ij}^{k} x_{ij}^{k} \tag{1}$$

Subject to:

$$\sum_{j:(i,j)\in A} x_{ij}^{k} - \sum_{j:(j,i)\in A} x_{ji}^{k} = \begin{cases} d_k & si \quad i = s_k, \\ 0 & si \quad i \in V \setminus \{s_k, t_k\}, \\ -d_k & si \quad i = t_k, \end{cases} \qquad \forall k = 1, \ldots, K, \tag{2}$$

$$\sum_{k=1}^{K} x_{ij}^{k} \le u_{ij} y_{ij}, \qquad \forall (i, j) \in A, \tag{3}$$

$$x_{ij}^{k} \ge 0, \qquad \forall (i, j) \in A, k = 1, \ldots, K, \tag{4}$$

$$y_{ij} \in \{0, 1\}, \qquad \forall (i, j) \in A. \tag{5}$$

The objective (1) is to minimise the sum of the fixed installation costs plus the variable costs of routing flows through the network. Constraint (2) express the network flow conservation constraints and guarantee that each demand k, $k = 1, \ldots, K$, is routed from its origin to its destination nodes. Constraint (3) state that the total flow crossing an open arc, $y_{ij} = 1$, does not exceed the capacity of the facility installed on this arc. If the arc is not used, $y_{ij} = 0$, the total flow has to be zero. Constraints (4) and (5) are respectively the non-negativity constraints of variables x and the integrality of variables y.

By nature, the AF formulation is a polynomial compact MILP formulation. If the underlying graph is complete then it comprises $O(n^4)$ continuous demand routing variables and $O(n^2)$ binary design variables. Furthermore, the AF formulation presents a bloc-diagonal structure and contains $O(n^3)$ constraints.

6.1.2 Arc-path-based-formulation

Generally, the commonly-known arc-path formulation (PF) defines for each commodity k, a set P_k, $k = 1, \ldots, K$, of all possible paths from the source node s_k to the sink node t_k. Let a_{ijrk} be a binary constant which is equal to 1 if arc (i, j) is part of the path $r \in P_k$ of commodity k, and 0 otherwise. The cost of routing the flow through each path $r \in P_k$ for each commodity k, $k = 1, \ldots, K$, is noted by c_r^k. Accordingly, two decision variables are considered: z_r^k a continuous variable that represents the amount of flow on the path $r \in P_k$ for each commodity k, $k = 1, \ldots, K$, from s_k to t_k, and y_{ij} a binary variable that models the decision to install a facility on arc $(i, j) \in A$, such that y_{ij} is equal to 1 if a facility is installed, and 0 otherwise. Thus, an arc-path-based formulation reads as follows:

$$PF: Minimise \sum_{(i,j)\in A} f_{ij} y_{ij} + \sum_{k=1}^{K} \sum_{r=1}^{|P_k|} c_r^k z_r^k \tag{6}$$

Subject to:

$$\sum_{r=1}^{|P^k|} z_r^k \geq d_k, \qquad k = 1, \ldots, K, \tag{7}$$

$$\sum_{k=1}^{K} \sum_{r=1}^{|P^k|} a_{ijrk} z_r^k \leq u_{ij} y_{ij}, \qquad \forall (i, j) \in A, \tag{8}$$

$$z_r^k \geq 0, \qquad \forall r \in P^k, k = 1, \ldots, K, \tag{9}$$

$$y_{ij} \in \{0, 1\}, \qquad \forall (i, j) \in A. \tag{10}$$

The objective (6) is to minimise the total cost of installing facilities on the arcs and routing the flow through the network. Constraint (7) ensures the routing of all traffic demands from the source nodes to the sink nodes. Constraint (8) guarantees that the amount of traffic routing on each arc does not exceed the capacity of the facility installed on that arc. These constraints also connect routing and design variables by interdicting any flow to cross an arc unless the associated facility is installed and therefore its fixed cost is paid. Constraints (9) and (10) are respectively the non-negativity constraints of the variables z and the integrality of the variables y.

Remark: Models AF and PF may be extended to the situation where multiple facilities of a similar nature can be installed on the same link. In fact, in these situation constraints (5) and (10) should be substituted with

$$y_{ij} \in N, \qquad \forall (i, j) \in A. \tag{11}$$

Compared to the AF formulation, the PF formulation contains fewer constraints [$O(n^2)$ vs. $O(n^3)$], the same number of binary variables y, but an exponential number of continuous variables z. It is worthy of mention that models AF and PF provide identical linear programming relaxations. In addition, we notice that from a computational point of view, commercial optimisation solvers such as CPLEX can be unlikely used to solve the

AF formulation instead of the path-based formulation which presents a certain structure that can be susceptible to being tackled by column generation resolution technique.

6.2 Solution methods

As already mentioned, the *NDPs* are NP-hard problems that are characterised by an important number of variables and constraints (Magnanti and Wong, 1984). Usually, different parameters can often influence considerably the level of *NDPs* difficulty such as capacity restrictions, simultaneous flows to be routed on multiple paths between different source-destination pairs, plus the difficulties related to data uncertainty. This has naturally inspired researchers to develop efficient algorithms to solve these problems. As a result, significant attention has been paid to the development of exact methods capable to provide optimal solutions. Heuristics and metaheuristics were also used to provide high-quality approximate solutions (Khatrouch et al., 2019).

Table 5 Solution methods for solving deterministic *NDPs*

Reference	*Solution methods*	
	Exact solution methods	*Approximate solution methods*
UCNDP		
Minoux (1974)	Branch and bound	
Minoux (1976)	Branch and bound	Heuristics based on successive approximations
Hoc (1982)	Benders decomposition and cut generation	
Jakob and Pruzan (1983)		Heuristics based on Lagrangian relaxation
Magnanti and Wong (1984)	Benders decomposition	Neighbourhood search heuristic
Magnanti et al. (1986)	Benders decomposition	
Balakrishnan (1987)	Cut generation	
Balakrishnan et al. (1989)	Cut generation and column generation	
Cruz et al. (1998)	Branch and bound	
Frangioni and Gallo (1999)	Column generation and Lagrangian relaxation decomposition method	
Randazzo and Luna (2001)	Benders decomposition and branch and cut	
Randazzo et al. (2001)	Benders decomposition	
Contreras et al. (2011)	Benders decomposition and column generation	Heuristics based on cut generation
Tadayon and Smith (2014)	Cut generation	

Table 5 Solution methods for solving deterministic *NDPs* (continued)

Reference	*Solution methods*	
	Exact solution methods	*Approximate solution methods*
CFNDP		
Hirsch and Dantzig (1968)	Benders decomposition	
Murty (1968)	Benders decomposition	
Rardin and Choe (1979)	Column generation combined with Lagrangian relaxation	
Khang and Fujiwara (1991)		Scaling heuristic and Lagrangian relaxation
Kim and Pardalos (1999)		Heuristics based on dynamic slope scaling procedure
Crainic et al. (2000)	Column generation	Tabu search
Holmberg and Yuan (2000)	Branch and bound	Lagrangian heuristic based on subgradient method
Kim and Hooker (2002)	Branch and cut	Metaheuristics based on a hybrid approach
Crainic and Gendreau (2002)		Tabu search
Ghamlouche et al. (2003)		Tabu search
Ekşioğlu et al. (2003)	Branch and bound	Heuristics based on dynamic slope scaling procedure
Crainic et al. (2004)		Slope scaling heuristic
Costa (2005)	Benders decomposition	
Alvarez et al. (2005)		Heuristics based on the scatter search approach
Crainic et al. (2006)		Tabu search
Pedersen et al. (2006)		Tabu search
Crainic and Gendreau (2007)		Scatter search
Chouman et al. (2009)	Cut generation	
Frangioni and Gendron (2009)	Cut generation combined with Lagrangian relaxation	
Katayama et al. (2009)	Column generation and cut generation	Capacity scaling heuristics and local branching
Chouman and Crainic (2010)		Tabu search
Rodríguez-Martín and Salazar-González (2010)		A local branching heuristic
Hewitt et al. (2013)	Branch and price	
Gendron and Larose (2014)	Branch, price and cut	
Yaghini et al. (2014)	Cutting plane	Tabu search
Momeni and Sarmadi (2016)		Metaheuristics based on genetic algorithms
Atamturk et al. (2017)	Branch and cut	

Table 5 Solution methods for solving deterministic *NDPs* (continued)

Reference	*Solution methods*	
	Exact solution methods	*Approximate solution methods*
CFNDP		
Munguía et al. (2017)		Local search heuristics based on solving restricted MIP subproblems
Chouman et al. (2018)	Branch and cut	
Gendron et al. (2018)		Heuristics based on dynamic slope scaling procedure
Kazemzadeh et al. (2022)		Lagrangian metaheuristic
Katayama (2020)		Neighbourhood search heuristics
Guimarães et al. (2020)		Variable fixing heuristics
Shibasaki et al. (2021)		Heuristic schemes based on Lagrangian information
Agarwal et al. (2022)	Cutting plane	
Kazemi et al. (2021)	Aggregation	Heuristic to construct partial aggregations
	LP relaxation-based approach	
	Cutting plane	
Hellsten et al. (2021)	Branch and price	
Li et al. (2021)	Branch and price	
Avella et al. (2021)	Branch and cut	Heuristic separation for flow cover inequalities
DCMNDP		
Minoux (1989)	Branch and cut	
Stoer and Dahl (1994)	Benders decomposition and cut generation	
Gabrel et al. (1999)	Branch and cut	
Minoux (2001)	Benders decomposition and cutting plane generation	
Gabriel et al. (2003)	Benders decomposition and cutting plane generation	Link-rerouting and flow-rerouting greedy heuristics
Roussel et al. (2004)	Benders decomposition and cut generation	
Mrad and Haouari (2008)		Heuristics based on the shortest path algorithms combined with the adaptive memory search method
Aloise and Ribeiro (2011)		Heuristics based on the shortest path algorithms combined with the adaptive memory search

Table 5 Solution methods for solving deterministic *NDPs* (continued)

Reference	*Solution methods*	
	Exact solution methods	*Approximate solution methods*
DCMNDP		
Mejri et al. (2019a)	Benders decomposition, column generation, and cut generation	
Mejri et al. (2019c)	Benders decomposition and column generation	
NLP		
Lee et al. (1989)	Cut generation	Heuristics based on Lagrangian relaxation
Magnanti et al. (1993)	Cut generation combined with Lagrangian relaxation	
Magnanti et al. (1995)	Branch and cut	
Barahona (1996)	Branch and cut	
Bienstock and Günlük (1996)	Cut generation	Heuristics based on a separation model
Bienstock et al. (1998)		Heuristics based on the shortest path algorithms
Chopra et al. (1998)	Column generation and cut generation	Heuristics based on a separation model
Dahl and Stoer (1998)	Cutting plane algorithm	Primal heuristics
Günlük (1999)	Branch and cut	
Berger et al. (2000)		Tabu search
Sridhar et al. (2000)		Local search heuristics
Agarwal (2002)		Local search heuristics
Gendron et al. (2002)		Local search heuristics
Avella et al. (2007)	Benders decomposition	
Ljubić et al. (2012)	Cut generation	
Mattia (2012)	Cut generation	

Recently, Salimifard and Bigharaz (2020) have demonstrated that exact resolution approaches are less and less applied for NDP since 2004. However, the development of metaheuristics has been observed over the last two decades and is predicted to further increase in the future, since the size of the investigated problems continues to rise.

Furthermore, over the last two decades, stochastic and robust programming methods have been developed to account for some parameters uncertainties when building networks.

The present survey is concerned with the main solution methods that have been used in the literature to solve different variants of *NDPs*. These include exact and approximate solution methods for deterministic cases as well as stochastic programming and robust optimisation approaches for stochastic variants. We briefly present their underlying basis and highlight their effectiveness on real problem applications. Tables 5 and 6 compiled the published papers dealing with each specific solution method for deterministic and stochastic variants, respectively.

Table 6 Solution methods for stochastic *NDPs*

Reference	*Solution methods*			
	Exact methods	*Approximate solution methods*	*Stochastic simulation-optimisation*	*Robust optimisation*
CFNDP				
Atamtürk and Zhang (2007)		Budget-of-uncertainty approach		X
Mudchanatongsuk et al. (2008)	Column generation approach to solve the LP relaxation of the path formulation			X
Crainic et al. (2011)		Lagrangian relaxation combined with metaheuristics, based on the progressive hedging algorithm of Rockafellar and Wets	X	
Thapalia et al. (2012)		Heuristic based on 'capacity scaling'		X
Crainic et al. (2014)		Lagrangian relaxation combined with metaheuristics, based on the progressive hedging algorithm of Rockafellar and Wets	X	
Boland et al. (2016)	Benders decomposition	Heuristics based on 'large-neighbourhood search (LNS)'	X	
Crainic et al. (2016)	Partial decomposition	Hybrid strategies	X	
Rahmaniani et al. (2018)	Partial decomposition and cutting planes generation	Heuristics	X	
Sarayloo et al. (2020)		Integrated learning and progressive hedging metaheuristic	X	
Sarayloo et al. (2021)		Metaheuristic approach based on reduced cost information	X	
Crainic et al. (2021)	Partial decomposition procedure		X	

Table 6 Solution methods for stochastic *NDPs* (continued)

Reference	*Solution methods*			
	Exact methods	*Approximate solution methods*	*Stochastic simulation-optimisation*	*Robust optimisation*
DCMNDP				
Minoux (2010)				X
Lee et al. (2013)	Benders decomposition and cut generation			X
Mejri et al. (2019b)	Column generation procedure embedded in a Benders decomposition schema and enhanced by set of strong inequalities	Hedging flow duality-based heuristic	X	
NLP				
Altın et al. (2011)	Branch-and-cut algorithm	Approximation rounding heuristics		X
Koster et al. (2013)	Cut generation	Heuristic separation algorithm		X
Mattia (2013)	Branch-and-cut	Heuristics based on a separation and cut generation model		X
Claßen et al. (2015)	Branch-and-cut	Heuristics based on a separation and cut generation model		X
Mattia and Poss (2018)	Benders decomposition	Heuristics based on a separation and cut generation model		X

6.2.1 Exact methods

6.2.1.1 Benders decomposition

The Benders decomposition method is a classical approach for combinatorial optimisation problems. It was published by Benders (1962), to solve linear programs with mixed variables (mixed integer programs). It consists in projecting the problem on the space of binary variables and decomposing it into two problems known as master problem and auxiliary problem. The master problem contains the binary variables and the corresponding constraints and the auxiliary problem contains the continuous variables and the associated constraints (Costa, 2005; Rahmaniani et al., 2016). The success of the Benders decomposition method started with the work of Geoffrion and Graves (1974) for the modelling of a distribution NDP with both fixed and variable costs. However, the resolution of both master and auxiliary problems requires iterative processes using excessive memory and computational time (Fragkogios and Saharidis, 2019). Accordingly, the main weaknesses of this method that have been highlighted in the literature include the iterations time consumed, the ineffectiveness of initial iterations, the low feasibility and optimality of constraints or cuts, and the slow convergence at the end of the algorithm (MirHassani et al., 2000; Fontaine and Minner, 2018). A lot of effort has been dedicated to increase the efficiency of the algorithm and accelerate its convergence (e.g., Mejri et al., 2018).

The structure of the reported *NDPs* variants provides a logical decomposition procedure for the Benders method: the design variables are given by solving the master problem while the flow variables are provided by solving the auxiliary problem. In this context, Magnanti et al. (1986) used the Benders decomposition method to solve AF formulation modelling the *UCNDP*. The authors developed a cut generation procedure to accelerate the convergence of the Benders decomposition algorithm and reduce the number of iterations. Randazzo and Luna (2001) applied the exact Benders decomposition algorithm for the same variant (*UCNDP*). Before the Benders decomposition, the authors used linear relaxation to find a feasible solution. This solution will be used to generate the initial values of the dual variables associated with the auxiliary problem. Their approach allows finding optimal solutions for instances with up to 30 nodes and 130 edges. In the same year, Randazzo et al. (2001) applied the Benders decomposition algorithm in a very similar way to the one presented in Randazzo and Luna (2001), but without the use of linear relaxation. The initial feasible solution is obtained using the shortest path algorithm and by considering only variable costs. They were able to solve instances with up to 41 nodes and 417 edges and they showed that the Benders decomposition algorithm applied to the AF formulation was more rapid than the MIP solver CPLEX (version 3.0). Gabrel et al. (1999) solved the *DCMNDP* by proposing an approach based on Benders decomposition and applied to the AF formulation. They generated constraints iteratively until obtaining the optimum. This technique allowed solving instances with up to 20 nodes and 37 edges. Mrad and Haouari (2008) proposed enhancements to the constraint generation algorithm presented in Gabrel et al. (1999) in order to accelerate its convergence. These improvements are based on the generation of efficient metric inequalities. Their approach allowed to find optimal solutions for instances with up to 50 nodes and 100 edges. Recently, Mejri et al. (2019c) proposed Benders decomposition scheme to solve the DCMNDP with a step cost function. The problem was formulated using both AF and PFs. The authors introduced a set of common valid inequalities to accelerate the basic Benders procedure. Their approach allowed solving real-world networks with up to 41 nodes and 154 edges. Based on the promising computational results found in Mejri et al. (2019c), the authors investigated in Mejri et al. (2019a) new accelerating techniques for solving large-scale instances of the same *DCMNDP* variant. They developed an original exact separation model to derive efficient cut-set inequalities. Their enhanced Benders decomposition procedure was capable to solve large-scale networks with up to 100 nodes, 600 edges, and 4,950 commodities while previously developed approaches were not able to solve *DCMNDP* problems with more than 50 nodes, 100 edges, and 1,225 commodities. Very recently, Crainic et al. (2021) proposed an enhanced partial decomposition procedure that consists of embedding a specific number of scenario subproblems to explicitly keep certain information related to the subproblems in the master problem. This strategy enhanced the master problem by providing information that can lead to better solutions. Kazemi et al. (2021) pointed out that the Benders decomposition approach is more efficient than partial formulations in the case of several multicommodity flows.

6.2.1.2 Cutting plane

The cutting planes method is commonly used to provide integer solutions for linear optimisation problems. The method consists in solving a linear relaxation of the given integer programs. If the obtained solution is optimal, but not feasible for the original problem, the method consists of generating violated constraints and adding them to the relaxed program to eliminate the resulting fractional solution. These constraints are known as cut-set inequalities. The algorithm stops when there are no more valid violated constraints to generate.

Chouman et al. (2009) considered an AF formulation to model the *CFNDP* variant. They presented five types of valid inequalities that they integrated into a cut generation algorithm. Avella et al. (2007) presented a cut generation procedure to solve the *NLP* variant. Later, Mattia (2012) enhanced this procedure. The author showed that the cut constraints are very useful to improve the performance of the proposed exact resolution approach, however, their generation requires a lot of computational time. Cut constraints for AF formulation were also applied in Magnanti et al. (1993, 1995) and Frangioni and Gendron (2009). Tadayon and Smith (2014) proposed two linearisation approaches for solving the min-cost multicommodity NDP, using cutting plane methods.

Recently, cutting plane methods have been investigated for problems specific to railroads applications (Davarnia et al., 2019). The authors have shown that cutting plane techniques have proven useful in tackling unsplittable NDPs. Particularly, they developed a cut-generating linear program to solve a multicommodity network model with NSNM requirements (no-split no-merge). The NSNM constraints arise when, for particular nodes in a network, flows are not allowed to be split or merged. The performed experiments on this configuration based on both node and path formulations suggested the computational potential of cutting plane approaches when compared to column generation techniques and main commercial branch-and-cut software. Thus, in this work, NSNM cuts were found to be very effective in gap reduction, especially for large networks.

Agarwal et al. (2022) developed an effective cutting-plane algorithm to tackle the directed fixed charge multicommodity NDP. The originality of this work consists in proposing a new procedure based on polar duality to generate valid inequalities to the model. The proposed approach enabled to exactly solve the problem instances in less than 400 seconds, on average, instead of four hours when using CPLEX MIP solver.

6.2.1.3 Column generation

The column generation method is often used for solving linear programs with a large number of decision variables. The overarching idea is that at the optimum, only some of these variables (columns) are used to solve the problem. The algorithm starts by considering only a subset of variables that can improve the objective function and then new columns are generated iteratively as the algorithm progresses.

This technique has been widely explored in the literature to solve the PFs due to the exponential number of their variables. Rardin and Choe (1979) worked on the PFs and developed a column generation algorithm combined with a Lagrangian relaxation to solve the *CFNDP* variant.

6.2.1.4 Branch and bound

Branch and bound is a method based on an enumeration algorithm that is generally used to solve combinatorial optimisation problems in an adequate way. The branch and bound algorithm consists of branching and then searching a tree whose root is the feasible domain of the initial problem and the nodes are the feasible domains of the sub-problems. It has two main phases:

1. the branching phase consists of a recursive subdivision of the initial problem into smaller sub-problems, which are easier to solve, each of them corresponds to a node of the search tree
2. the bounding phase consists in evaluating each node of the search tree to determine the subsets of solutions that may contain an optimal solution and eliminating the others, in order to reduce the search space.

The algorithm stops when there are no more nodes to evaluate.

Minoux (1974) focused on the telecommunications domain. He proposed an exact branch and bound algorithm for the UCNDP with concave cost functions. He showed that the branch and bound method is adapted to this problem and provides good solutions for small instances of up to 12 nodes. Cruz et al. (1998) proposed also a branch and bound algorithm to provide optimal solutions for the un-CFNFP. Additionally, the combination of the branch and bound method and other resolution techniques has been extensively used for solving the *NDPs* variants. These works will be described in the following paragraphs.

6.2.1.5 Branch and cut

The branch and cut is a method of combinatorial optimisation that involves combining the branch and bound algorithm and the cut generation technique for solving integer linear programs. The objective is to reduce the search space of feasible solutions.

Stoer and Dahl (1994) studied the variant of *DCMNDP*. They generated valid inequalities, which they integrated into a branch and cut procedure. This approach was able to solve instances with up to 27 nodes and 51 edges. Later, Günlük (1999) applied an exact procedure based on branch and cut algorithm to solve the *NLP* variant. The proposed approach applied for an AF formulation was capable to solve instances with up to 30 nodes and 55 edges. More recently, Chouman et al. (2018) considered an AF formulation to model the NDP with fixed and variable costs (*CFNDP*) and proposed a branch and bound procedure associated with a cut generation technique to solve the problem. They showed that the branch and cut algorithm is very competitive with the available optimisation solvers. The computation results conducted on randomly generated instances were promising, and the proposed exact procedure appears to perform efficaciously by solving instances with up to 30 nodes and 350 edges. Also, an AF formulation was proposed by Ozbaygin et al. (2018) to solve optimally the split delivery vehicle routing problem (SDVRP). The authors presented an exact solution approach enhanced by introducing framed capacity cuts and cut-set inequalities to the branch and bound root node of the search tree.

6.2.1.6 Branch and price

The branch and price is a combinatorial optimisation technique used to solve large integer linear programs and mixed-integer linear programs. This method presents a hybrid of branch and bound and column generation procedures. The method is based on the idea that at each node of the search tree, columns can be added to the relaxation of the linear program. The algorithm begins by using a reduced set of columns to identify a restricted master problem (RMP). Then, the LP relaxation of the RMP is solved and a subproblem (*pricing problem*) is generated based on the obtained dual solutions. The pricing problem is solved to find columns with negative *reduced costs*. In such a case, new columns are generated and added to the RMP, then the LP relaxation of the RMP is resolved. Branching arises when no columns with negative reduced costs are identified, and the LP relaxation solution does not respect the integrality criteria. This procedure is repeated at each iteration of the branch and bound search tree and it is stopped when the optimal integer solution is obtained.

Hewitt et al. (2013) solved the *CFNDP* problem using the PF. The authors investigated an exact procedure based on branch and price algorithm. They compared the performance of the proposed approach on benchmark instances with up to 30 nodes against an exact optimisation solver, CPLEX. They showed that the branch and price algorithm outperforms CPLEX by producing high-quality solutions more quickly. Very recently, Hellsten et al. (2021) proposed a Path formulation to model the transit time-constrained fixed charge multicommodity NDP. The authors developed a branch-and-price procedure enhanced by deep dual-optimal inequalities to solve this variant of the problem. In the proposed sensitivity analysis conducted to evaluate the branch-and-price algorithm, the authors showed that the computational problem complexity grows when considering longer transit times and a higher ratio between the fixed charge and the flow routing cost in comparison to the edge capacity, whose impact is not very significant. Li et al. (2021) proposed two mathematical formulations to model the service design problem with heterogeneous resource constraints. Firstly, based on the AF formulation, they solved small problem instances without computational difficulties using a commercial MIP solver. Then, to be able to solve larger problem instances, a PF was considered based on a branch-and-price resolution approach. The proposed strategy was enhanced by new branching techniques (based on generating the branching on slack variables and fractional design-cycle variables), as well as an acceleration procedure to identify the integer solution more quickly. The results demonstrated that the proposed branch and price procedure outperformed the column generation algorithm and the CPLEX MIP solver.

6.2.1.7 Branch, price and cut

By combining column and cut generations and integrating them into a branch and bound procedure, we obtain a branch, price and cut algorithm. The method consists in applying column and cut generation algorithms to solve, for each node of the search tree, the relaxation of the linear optimisation problem.

Gendron and Larose (2014) proposed an AF formulation to solve the MCFNDP. In this work, the authors developed a column generation algorithm that was enhanced by a cut generation process based on effective valid inequalities and then embedded within a branch and bound procedure. The resulting branch, price and cut schema was proven to be more efficient than the MIP solver CPLEX and a B&C approach that does not integrate column generation technique. The computational results showed that the proposed approach was able to provide high-quality solutions for randomly generated instances with up to 30 nodes and 350 edges with an optimality GAP of 1%.

6.2.2 Approximate solutions methods

Besides the exact techniques, a large number of approximate solution methods have been proposed in the literature to deal with the several variants of *NDPs*. The majority of the reviewed works solved the *NDPs* by heuristics and metaheuristics techniques for providing good feasible solutions. The need to use approximate solution methods has arisen due to the reasons that computing optimal solutions for difficult models is very time-consuming and may not always be feasible. Many variants of *NDPs* were investigated by heuristic approaches such as neighbourhood search heuristics (Katayama, 2020), scatter search (Crainic and Gendreau, 2007), slope scaling (e.g., Khang and Fujiwara, 1991; Kim and Pardalos, 1999; Ekşioğlu et al., 2003; Crainic et al., 2004), local branching heuristics (Rodríguez-Martín and Salazar-González, 2010), variable fixing heuristics (Guimarães et al., 2020), greedy and Lagrangian heuristics (Shibasaki et al., 2021; Ennaifer et al., 2016). A specific application of heuristic techniques built according to specifications of a metaheuristic-based optimisation framework is known as a metaheuristic. These techniques include Tabu search, genetic algorithms, simulated annealing, and ant colony optimisation, among many others.

Notable examples of numerous researches on the MCFNDPs have tackled heuristic and metaheuristic algorithms. Holmberg and Yuan (2000) proposed a Lagrangian heuristic embedded in a branch and bound procedure to solve the AF formulation modelling the *CFNDP* variant. They used a subgradient optimisation to solve the dual Lagrangian. Their approach was able to generate feasible solutions for instances with up to 100 nodes and 500 edges. Crainic et al. (2000) investigated the same problem. They developed a Tabu search method within a column generation algorithm for solving a PF. Their results showed good upper bounds for well-known benchmark instances with up to 25 nodes and 300 arcs. Alvarez et al. (2005) addressed a scatter search procedure for computing high-quality solutions to a set of large-scale problems. Katayama et al. (2009) proposed a new procedure based on capacity scaling and local branching heuristics using a column generation enhanced by a cut generation step. The idea was based on modifying capacities according to the quantity of flow circulating on the arcs. Results showed that the proposed schema was able to provide good upper bounds when solving networks with up to 100 nodes and 200 arcs. More recently, Katayama (2020) developed a capacity scaling heuristic algorithm but combined with a MIP neighbourhood search that has successfully enhanced the initial solutions and found good results in a reasonable time frame. Munguía et al. (2017) also investigated the *CFNDP* variant using an AF formulation. They developed local search heuristics based on solving restricted MIP subproblems in an iterative process. An experimental study, conducted on instances of the literature with up to 30 nodes and 700 arcs, showed the efficiency of the resulting approach being able to construct solutions on the order of 0.58% of the lower bound on average. Gendron et al. (2018) developed a heuristic procedure based on slope scaling methods. They found good upper bounds for benchmark instances with up to 100 nodes and 700 edges, within one hour of computation time. Crainic et al. (2004) presented a slope scaling/Lagrangean perturbation (SS/LP) heuristic to explore more feasible and good solutions. Rodríguez-Martín and Salazar-González (2010) applied a local branching heuristic algorithm to the AF formulation. They used a state-of-the-art MIP solver for the exploration of neighbourhoods. Crainic and Gendreau (2002), Ghamlouche et al. (2003), Crainic et al. (2006), Pedersen et al. (2006), Crainic and Gendreau (2007) and Chouman and Crainic (2010) solved the *CFNDP* problem by applying a Tabu search metaheuristics. Momeni and Sarmadi (2016) proposed other metaheuristics to solve the same variant of problem. They used the genetic algorithm to search in space solutions and introduced a relaxation induced neighbourhood search-based algorithm in a local branching procedure for mapping from one selected solution to a neighbour one. A very recent paper published by Shibasaki et al. (2021) addressed the same variant of the problem. The authors explored Lagrangian information to develop and enhance heuristic models.

The NLPs were also tackled by approximate solution techniques. Dahl and Stoer (1998) investigated a PF to model this variant of problems. They proposed primal heuristics for a cutting plane algorithm. The authors tested the resulting procedure on real-world graphs used in Norwegian Telecom and having up to 118 nodes and 134 edges. The computational experiments provided near-optimal solutions. Berger et al. (2000) were the first to develop a Tabu search metaheuristics to the NLP variant. Their approach allowed finding good results for instances with up to 200 nodes and 100 commodities. A local search heuristic was developed by Agarwal (2002) to solve the *NLP* problem. Instances with up to 20 nodes and 169 edges were solved by constructing solutions on the order of 5% of the lower bound on average. Other published works on *NLP* problems have proposed local search heuristics using AF formulations (e.g., Gendron et al., 2002; Sridhar et al., 2000). Gabrel et al. (2003) explored several link-rerouting and flow-rerouting greedy heuristics to solve the *DCMNDP* variant using the AF formulation. The developed algorithms were able to find approximate solutions for a large set of graphs having up to 50 nodes and 90 edges. In the same context, Aloise and Ribeiro (2011) proposed various heuristics based on the shortest path algorithms combined with the adaptive memory search method for solving AF formulation applied to the *DCMNDP* variant. They have shown that their approach was highly efficient when solving instances generated by Gabrel et al. (2003).

Besides, the survey on papers using approximate methods for solving the UCNDPs showed that heuristics were commonly applied as a starting strategy for the Benders decomposition algorithm and then for improving the solutions found after solving the corresponding master problem. Accordingly, Magnanti and Wong (1984) focused on solving approximately the UCNDP variant. They developed acceleration techniques to Benders decomposition algorithm in order to reduce the number of iterations as well as the computational time. They used heuristics based on neighbourhood search to solve approximately the RMP in the first iteration. Contreras et al. (2011) proposed heuristics based on cut generation procedure that integrated into a Benders decomposition schema. They observed that their heuristic methods not only improved the overall convergence of the algorithm but also found better upper bounds for instances with up to 200 nodes.

6.2.3 Solving methods for stochastic NDPs

Recently, a variety of methods were developed to deal with uncertainties characterising many variants of *NDPs*. Different parameters, in particular, the demand amount of each commodity usually induced certain variations when constructing and designing the network. The purpose was thus to install a feasible network at a minimum cost when many demand realisations are covered. Models considering explicitly these uncertainties were then suggested. These include generally robust optimisation and stochastic programs with recourse.

6.2.3.1 Two-stage stochastic programming with recourse

In recent years, two-stage stochastic programs with recourse rapidly became the method of choice to handle uncertain model parameters characterising stochastic NDPs (Crainic et al., 2016). In the first stage, the decider has to make a decision, without knowing the exact result of the uncertain parameters. In the second stage, called the recourse decision, once the parameters are known, the decider can take recourse measures or actions to accordingly adapt or adjust his plan. Stochastic programming with recourse provides a set of strategies for specifying how to react when a particular random value is observed. Therefore, the goal is to achieve a solution that, on average, produces good results over the long-term. Many solution approaches have been developed to solve these stochastic programs with recourse. Most of these algorithms were based on simulation, optimisation, and their combinations. Herein the stochastic model is decomposed into parts allowing solving it more efficiently. Thus, the simulation-optimisation schema based on the idea of associating simulation with optimisation approaches was used on a large-scale. One challenge of this procedure lies in taking advantage of the strengths of each approach to find results that would otherwise be very difficult or impossible to achieve. Therefore, all possible realisations of random variables must be considered. Unfortunately, in practice, it is very challenging to compute all these random variables. An alternative way is to consider a finite set of discrete realisations, called scenarios (Crainic et al., 2011). The stochastic optimisation (SO) considers thus the scenario-based procedure using Monte Carlo simulation techniques. Stochastic *NDPs* are generally NP-hard due to their challenging combinatorial nature plus the additional complexity resulting from data uncertainty (Rahmaniani et al., 2018). Therefore, they seem very complex to solve with exact methods. SO methods with recourse were based mostly on approximations and decomposition techniques that are embedded in a simulation-optimisation framework. In addition, to the best of our knowledge, all the works that have addressed the stochastic variants of *NDPs* have applied the commonly used AF formulations for modelling these problems. Crainic et al. (2011) developed metaheuristics, inspired by the progressive hedging algorithm of Rockafellar and Wets (1991), to solve the stochastic problem with recourse modelling *CFNDP* variant. These metaheuristics integrated into a simulation-optimisation procedure were then improved in their work that was published in 2014 (Crainic et al., 2014). Crainic et al. (2016) were the first to propose an exact 'partial Benders decomposition' algorithm embedded in a simulation-optimisation procedure to solve the same problem. While, Boland et al. (2016) developed a large-neighbourhood search (LNS) heuristic for the Benders decomposition algorithm, known as 'proximity Benders'. They have shown that their simulation-optimisation procedure, when tested on instances of Crainic et al. (2016), outperformed the state-of-the-art MIP solvers such as CPLEX. Recently, Rahmaniani et al. (2018) enhanced the existing procedures by proposing several acceleration techniques including generating valid cuts methods as well as a partial decomposition to solve the widely investigated stochastic *CFNDP*. A simulation-optimisation procedure for solving the stochastic *DCMNDP* under demand uncertainty was presented by Mejri et al. (2019b). The first-stage solution approach was based on a column generation procedure embedded in a Benders decomposition schema and enhanced by a set of strong inequalities. A Monte Carlo simulation procedure associated with a hedging flow duality-based heuristic has been presented in the second-stage problem. Very recently, an efficient learn and optimisation procedure was proposed by Sarayloo et al. (2020) to solve the stochastic *CFNDP*. They developed a two-stage integrated learning and progressive hedging metaheuristic to deal with a large number of scenarios.

6.2.3.2 Robust optimisation methods

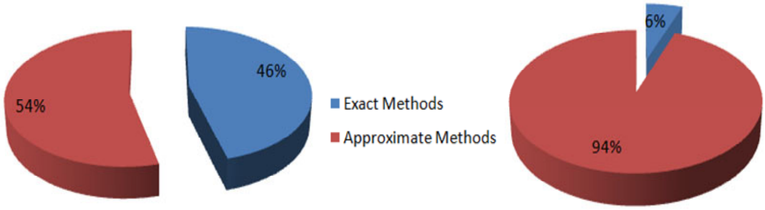
Robust optimisation approaches do not need any specific knowledge of probability distribution related to uncertain data (Lee et al., 2013), while SO considers uncertainty characterised by a known probability distribution such as uniform distribution, a normal distribution, discrete or continuous probability distribution, a binomial distribution, exponential distribution, and a Poisson distribution. The robust optimisation approach for solving an optimisation problem with uncertain data consists in providing a robust solution that can resist data variability in its worst-case realisation. Yanıkoğlu et al. (2019) presented a comprehensive survey on applications of robust optimisation methods, their advantages, and their limitations. Moreover, the concept of adjustable robust optimisation was addressed.

Atamturk et al. (2017) developed a cut generation algorithm in a robust optimisation framework to solve the *CFNDP* variant. By considering random demand realisations, they have shown that the routing of all traffic demands was not guaranteed, but the robustness of the solution was controlled. The difficulty of robust optimisation multicommodity networks was discussed in Minoux (2010). Lee et al. (2013), presented an exact resolution approach based on Benders decomposition method and embedded in a robust optimisation schema. The computational results showed that their algorithm was able to produce robust *DCMNDP* networks in a reasonable computation time. Mattia (2013) developed a robust optimisation approach for the *NLP* variant. The author presented a heuristic to build an initial feasible solution, which will be used in a branch and cut algorithm. Other heuristics for robust optimisation have been proposed in the literature to cope with the stochastic *NLP* variant (e.g., Claßen et al., 2015; Mattia and Poss, 2018). Mudchanatongsuk et al. (2008) developed a robust optimisation procedure for solving the *CFNDP* problem under both transportation cost and demand uncertainty. They proposed a column generation algorithm to solve the LP relaxation of a path-flow-based formulation.

6.3 Synthesis

The resulting survey paper could allow researchers to identify which solution approaches are the most addressed in the literature for solving the NDPs. Figures 10(a)–10(b) indicates trends in solution methods for solving the deterministic and stochastic NDPs.

Figure 10 (a) Total number of papers on solution methods for deterministic *NDPs* (b) Total number of papers on solution methods for stochastic *NDPs* (see online version for colours)



(a) (b)

We can notice that the surveyed studies have tackled the different variants of deterministic *NDPs* using both exact and approximate methods. The majority of the investigated exact methods (46%) include particularly the Benders decomposition algorithm. We believe that this is due to the structure of *NDPs* models particularly the nature of the design and flow variables which could easily provide a logical decomposition scheme for the Benders algorithm. Therefore, this method presents one of the most promising and competitive approaches for solving this class of problems. Several studies have shown that this technique may be more competitive than other popular methods such as the branch and bound algorithm, especially when enhanced by strong cut-set inequalities as well as other acceleration techniques (Mejri et al., 2019a; Costa, 2005). Additionally, the column generation method is the most commonly used technique for solving the arc-path-based formulations. The proposed approaches have been able to solve medium to large size problems having up to 100 nodes and 600 edges in a reasonable computation time that does not exceed three hours. While approximate solution methods that represent the other proportion (54%) were also frequently used for solving this class of problems. This is due to the difficulty of solving these problems to optimality. Therefore, the classical metaheuristic methods namely the Tabu search algorithm as well as other simple heuristics based on scatter search, capacity scaling, and local branching heuristics were the most explored approximation methods.

On the other hand, Figure 10(b) indicates that the approximation methods cover the largest proportion (94%) of the total number of papers dealing with the stochastic version of *NDPs*. Not surprisingly, the advances in theoretical as well as experimental investigations of approximate solution methods have been fascinating over the last two decades. As mentioned earlier, these problems are generally NP-hard due to their challenging combinatorial nature plus the additional complexity resulting from data uncertainty. Therefore, they seem very complex to solve with exact methods.

In addition, notice that to ensure resilient network designs in the stochastic context, several resolution techniques can be applied, such as two-stage stochastic, multistage stochastic, and robust optimisation. The two-stage and multistage SOs have been successfully addressed in many real-life problems. Nevertheless, one of their limitations is that, in reality, one may have insufficient data to assess the probability distribution related to uncertainty. Also, these SO approaches are sensitive to the curse of dimensionality, particularly for large-size problems. On the other hand, robust optimisation provides solutions that are immune to data perturbations. But these decision results have been proven to be too conservative and, then, costly on a day-to-day basis (Cadarso et al., 2018).

7 NDPs: recent advances and future researches

This survey on NDPs naturally opens up many interesting perspectives for future research directions:

- One such research direction concerns the development of multi-objective models where the objective function includes both cost and time minimisation. A compromise will then naturally be managed to provide more efficient and realistic solutions, i.e., networks.

  Another important research avenue would be to investigate additional complex constraints to model networks in more practical conditions. Indeed the future directions will be oriented to enhancing the *NDPs* models by introducing new features that may be interesting in realistic situations. Accordingly, one main issue emerges in the design of networks: survivability which means the maintaining of connectivity in the case of arc (or node) failure. For instance in telecommunication networks, the goal is to minimise the total installation cost of fibres cables while ensuring the connectivity of the overall network. Nonetheless, if a single node or arc fails, then the network may be unable to adequately connect all nodes. This involves certain survivability assumptions to be considered when designing the network. It is noteworthy that substantial financial losses for network providers can be generated by this service failure. Thus, designing network configurations with a suitable amount of survivability allowing restoration of services in the case of attacks or failures has recently become a top concern. Recently, many studies are focused on the survivability issue in the context of telecommunications NDPs.

  While the survivability of the networks as a crucial issue has received substantial attention, but much work remains to be done. Particularly, addressing these features in another context of applications such as logistics and supply chain networks and considering the survivability of the network in the stochastic context can be a challenging avenue for future research directions.

- We also suggest exploring the case of non-bifurcated flow which means that flow must be shipped exactly on one path from origin to destination. In such a situation, an additional binary variable can be introduced. This variable takes value 1 if a path is selected to route all the traffic demands and 0 otherwise. In this context, we notice that most of the reviewed studies have tackled the bifurcated *NDPs* variants instead of the non-bifurcated version.

- It is also interesting to mention that a fascinating creative direction to explore is the use of partially-aggregated formulations to implement specialised resolution methods for efficiently solving large-scale NDPs. Very recently, Kazemi et al. (2021) proposed novel commodity specifications enabling partial aggregation of commodities to effectively solve the multicommodity fixed-charge NDP. This creative idea allowed aggregating commodities for a subset of the network structure rather than the classical aggregation for the whole network configuration. The authors have shown that the proposed procedure that they defined as ‘dispersion of commodities’ was able to significantly ensure the compromise between the mathematical aggregated program size and the performance of the obtained LP bound. In this context, we intend to outline promising research directions in the development of acceleration techniques such as efficient valid cuts to cope with the difficulties of solving large-scale problems using partially aggregated formulations. Furthermore, following this interesting direction, we encourage researchers to investigate and extend this partial aggregation trend instead of the usual full aggregation techniques for solving other variants of *NDPs* particularly with regard to stochastic models.

- In addition, it might be highly worthwhile to develop general enhanced solution approaches that could naturally be adapted to become readily applicable for any variants of *NDPs*. We believe that this issue seems an extremely interesting avenue for solving efficiently a large variety of NDPs in a wide range of applications.
- As mentioned earlier, the advances in theoretical as well as experimental investigations of approximate solution methods have been fascinating over the last two decades. Accordingly, other promising research avenues require developing innovative metaheuristic-based algorithms in order to effectively solve very large-scale networks, particularly in a stochastic framework when considering data uncertainty. In this context, we point out that it will also be interesting to investigate the performance of the network facing the variability of certain parameters other than the demand, such as the link capacity, the operating time uncertainty, and the large variation of costs in the scenarios.
- In addition, we can expect alternative methods based on recent advances in artificial intelligence and machine learning algorithms. Adopting mathematics and computer science to integrate machine learning components in combinatorial optimisation appears to be a promising perspective for solving NP-hard NDPs in a practical time. Very recently, Bengio et al. (2021) investigated applying machine learning algorithms to tackle combinatorial problems. The authors have shown that is pertinent to enhance the resolution methods with machine learning and in particular deep learning algorithms to deal with the difficulties of solving these problems due to their high dimensionality structures. Accordingly, a comprehensive survey on the possible various approaches to incorporate learned algorithmic components into combinatorial optimisation problems was presented. In this context, Gasse et al. (2019) proposed a novel procedure to address the branching process within branch and bound framework for solving combinatorial optimisation problems such as the capacitated facility location problem. The authors applied imitation learning and a convolutional neural network model to cope with the branch and bound variable selection. The proposed approach offered a strong branching policy over traditional rules. Lombardi and Milano (2018) investigated different possible ways to apply machine learning tools for modelling the special case of combinatorial optimisation problems considering discrete decision variables, namely discrete optimisation models. The authors pointed that learning is an effective tool to model problems in the stochastic context. This can pose challenging lines of research. From this point of view, we firmly believe that the hybridisation of artificial intelligence and traditional network design optimisation techniques is a broad area for research and a fruitful perspective for future directions.
- On the other hand, in recent years, researchers have devoted particular attention to address NDPs from a different perspective, which is known as 'green NDPs'. Given the growing interest in reducing the environmental impact of industrial activities in

logistics, production, telecommunications, and transportation (such as fuel consumption and the emission of GHGs). Studies on this topic continue to be increasingly expanded. When reviewing the available body of literature, one shows that environmental effects within green network design were addressed based on different types of mathematical formulations and various solution techniques including exact and approximate methods. In general, the objective function is a minimisation of two measures, the first one is the traditional total cost including fixed installation costs plus operating flow costs, and the second one is the cost relevant to fuel consumption and emissions. Starting from a theoretical perspective, it thus seems clear that such upgrades can result in extended problems that are closer to realistic and relevant objectives but more complicated and pose different methodological challenges (Dukkanci et al., 2019). Accordingly, in recent years, green NDPs have become an exciting research area that represents many original progresses in various real-world situations. Dekker et al. (2012) propose a detailed survey of green NDPs by focusing on applications in transportation, production, and localisation. Tang et al. (2012) were the first to investigate the multicommodity flow allocation problem where the objective was to minimise the total network power consumption in order to allow routing all the traffic demands. Herein the link is characterised by a bidirectional capacity and a power consumption cost that is a discrete step increasing function. The authors developed an approximation procedure based on a greedy algorithm to find close-to-optimal solutions. For more details, we recommend the recent comprehensive review of Dukkanci et al. (2019) on green NDPs. This research area opens up several future effective opportunities.

- In addition, new aspects related to the concept of sustainability have been recently developed. Integrating economic, social, and environmental impacts into sustainable development has become a great challenge for future NDPs. Recently, numerous publications have tried to incorporate the sustainability concept into several NDPs contexts namely the supply chain networks. Nevertheless, the majority of them only address the economic and environmental aspects of sustainability, often neglecting the social perspective (Fragoso and Figueira, 2021). Taking these three dimensions of sustainability into account by considering sustainable consumption, optimised exploitation of natural resources and social sustainability issues such as poverty reduction tends to add yet further complexity to network design decisions and opens up a challenging avenue for future research.

8 Conclusions

In short, this paper has presented a survey on NDPs. First, we have formally defined *NDPs* properties that allow identifying the most studied *NDPs* variants as well as their related real-world applications. This highlighted the great interest of exploring *NDPs* from both theoretical and practical directions. Next, we have proposed a general review of models and methods developed for solving this class of problems. Finally, we conclude by suggesting some challenging future research avenues.

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