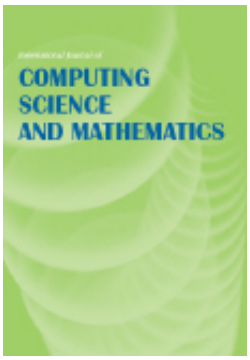

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A mathematical model and optimal control for Listeriosis disease from ready-to-eat food products

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Abstract: Ready-to-eat food (RTE) are foods that are intended by the producers for direct human consumption without the need for further preparation. In the present study, a deterministic model of Listeriosis disease transmission dynamics with control measures is analysed. Equilibrium points of the model in the absence of control measures were determined, and their local stabilities established. We formulate an optimal control problem and analytically give sufficient conditions for the optimality. The transversality conditions for the model with controls are also given. Numerical simulations of the optimal control strategies were performed to illustrate the results. The numerical findings suggest that the constant implementation of joint optimal control measures throughout the modelling time will be more efficacious in controlling or reducing the Listeriosis disease. The results of this study can be used as baseline measures in controlling Listeriosis from RTE food products.

Keywords: Listeria; contaminated food products; food contamination threshold; optimal control interventions; numerical simulations.

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1 Introduction

Human Listeriosis is a zoonotic disease a low incidence rate, but with a high mortality rate for those sickened with the infection globally (Bennion et al., 2008). The disease poses a huge public health concern, and as a result, there is a need to develop strategies to combat any outbreak of the disease. Humans are infected with Listeriosis through consumption of Listeria contaminated food products or directly from the environment by acquiring the pathogen, *Listeria monocytogenes* (*LM*), due to improper hygiene (WHO, 2020). To diagnose human Listeriosis, the active bodies responsible for foodborne diseases surveillance such as the National Institute of Communicable Disease (NICD) South Africa, Center for Disease Control (CDC) USA, and World Health Organization (WHO) obtain their results through laboratory-confirmed cases. Listeriosis is usually diagnosed when a bacterial culture grows *LM* from a body tissue or fluid, such as blood, spinal fluid, or the placental fluid (CDC, 2020). Symptoms of infection include: fever, flu-like symptoms, nausea, diarrhoea, fatigue, headache, stiff neck, convulsion, loss of balance, and muscle aches. Human Listeria infections can be treated with β-lactam antibiotic, normally ampicillin (Almudena and Payeras-Cifre, 2014). However, upon any outbreak, preventive measures such as the recall of contaminated food products, factory workers practising proper hygiene, educational campaign programs can be implemented to control the disease.

Recently, mathematical models including those with optimal control have been used to study the transmission dynamics of Listeriosis (see for instance Stout et al. (2020), Chukwu and Nyabadza (2020a), Otoo et al. (2020), Witbooi et al. (2020) and Chukwu and Nyabadza (2020b)). None of these models consider optimal control theory on the strategies to control Listeriosis in the human population by ingestion of contaminated RTE food products. The objective of this study is to develop and analyse an optimal control model of Listeriosis foodborne disease from contaminated RTE food products. The optimal control strategies to be implemented are in the form of recalls of contaminated foods products, treatment and educational campaigns.

2 Mathematical model and analysis

2.1 Model description

The model comprises of; the human population, manufactured food products, and the bacteria (*LM*) population in the environment. The total human population is divided into three compartments of susceptible $S(t)$, infected with *LM* $I(t)$, and the recovered $R(t)$, at any time t, with the total human population $N(t)$ given by $N(t) = S(t) + I(t) + R(t)$. Susceptible humans are infected by consuming contaminated food and by Listeria from the environment at a rate Λ_h defined by $\Lambda_h = \omega_1 F_c(t) + \omega_2 L_m$ where ω_1 and ω_2 are the effective contact rates (i.e the contacts that will result in infections) for susceptible humans with contaminated food and *LM* respectively. Upon infection, the susceptible humans become infectious and join the compartment $I(t)$. Once infected humans, recover at a rate α and recovery is assumed to be with temporary immunity. Thus, recovered humans become susceptible again at a rate ρ_h. The human population as a natural mortality/birth rate μ_h. We let L_m represent *LM* with a net growth rate r_l, and a carrying capacity $0 \leq \kappa_m \leq 1$. The bacteria is assumed to grow logistically so that $\frac{dL_m}{dt} = r_l L_m \left(1 - \frac{L_m}{\kappa_m}\right)$. Food products are divided into uncontaminated food products $F_u(t)$ and contaminated food products $F_c(t)$, with the total food products $F(t) = F_u(t) + F_c(t)$. We assume a constant production rate μ_f of uncontaminated food with the production assumed to produce uncontaminated food. Uncontaminated food is thus contaminated at a rate Λ_f through bacteria from the environment and contaminated food in the factory's handling and distribution processes. Here, $\Lambda_f = \omega_2 L_m + \omega_3 F_c(t)$ with ω_2 and ω_3 being the effective contact rates of the bacteria and contamination's of uncontaminated food caused contaminated food products respectively. All food products are subject to a removal rate μ_f. Figure 1 together with the model assumptions yield the following system of equations after substituting $R(t) = N(t) - S(t) - I(t)$, and setting $s = \frac{S}{N},\ i = \frac{I}{N},\ l_m = \frac{L_m}{\kappa_m},\ f_u = \frac{F_u}{F},\ \ f_c = \frac{F_c}{F}$, so that;

$$\left.\begin{array}{l} \frac{ds}{dt} = \mu_h + \rho_h(1 - s - i) - (\mu_h + \omega_1 f_c + \omega_2 l_m)s, \\ \frac{di}{dt} = (\mu_h + \omega_1 f_c + \omega_2 l_m)s - (\mu_h + \alpha)i, \ \ \frac{dl_m}{dt} = r_l l_m(1 - l_m), \\ \frac{df_u}{dt} = \mu_f - (\omega_2 l_m + \omega_3 f_c + \mu_f)f_u, \ \ \frac{df_c}{dt} = (\omega_2 l_m + \omega_3 f_c)f_u - \mu_f f_c, \end{array}\right\}, \qquad (1)$$

where the contamination rates $\omega_1 = \beta_{f_1} F,\ \omega_2 = \omega_l \kappa_m$, and $\omega_3 = \omega_{f_2} F$. System (1) is subject to non-negative initial conditions

$$s(0) > 0,\ i(0) \geq 0,\ l_m(0) \geq 0, f_u(0) \geq 0,\ f_c(0) \geq 0.$$

2.2 Feasible region and non-negativity of solutions

The positivity of the solutions is governed by the following theorem.

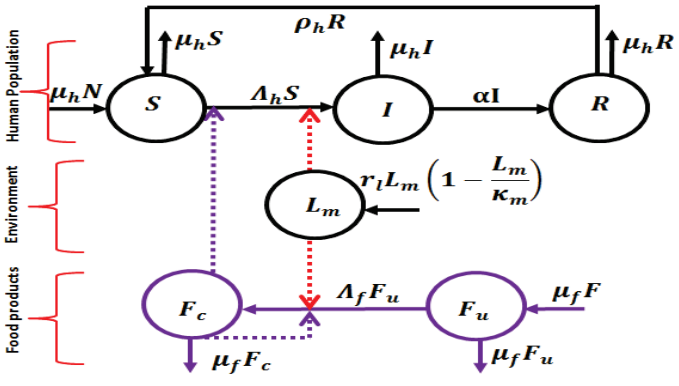
Theorem 1: *The solutions of model system (1) are contained in the region* $\Omega \in \mathbb{R}^5_+$, *which is given by* $\Omega = \left\{(s, i, l_m, f_u, f_c) \in \mathbb{R}^5_+ : 0 \leq s + i \leq 1, 0 \leq l_m \leq 1, 0 \leq f_u + f_c \leq 1\right\}$, *for the positive model initial conditions in* Ω.

Proof: Considering the total change in human population we have $\Phi_0(t) = 1 - \Phi_0(0)e^{-(\mu_h + \rho_h)t}$, where $\Phi_0 = s + i$. Here, $0 \leq \Phi_0(0) \leq 1$. The solution

of the bacterial population is given by $l_m(t) = \frac{1}{1+\Phi_1(0)e^{-r_l t}}$, where $\Phi_1(0)$ is a constant. Hence, $0 \leq l_m \leq 1$. The growth of *LM* is thus bounded. The equation of the total change in the amount of manufactured food products has a solution $f(t) = 1 - \Phi_2(0)e^{-\mu_f t}$, where $\Phi_2(0)$ is a constant. We note that $\lim_{t\to\infty} f(t) \to 1$. Hence, solutions of model equations (1) exists, and are biologically meaningful, bounded, and remain in Ω for $t > 0$. □

Theorem 2: *For the non-negative initial conditions, the solutions of model equations (1),* $(s(t),\ i(t),\ l_m(t),\ f_u(t),\ f_c(t))$ *are all non-negative for* $t \geq 0$.

Figure 1 Model diagram describing Listeriosis disease transmissions dynamics within the human population from contaminated food products and *LM* in the environment. The solid lines indicates transitions from one compartment to another, while the dotted lines represents the influences on the transitions line. Parameters are assumed to be positive except the net growth rate, r_l (see online version for colours)



Proof: Proof The first equation of the system (1) yields $s(t) \geq \frac{s(0)}{e^{\left(\mu_h+\rho_h+\int_0^t \bar{\Lambda}_h(r)dr\right)}} > 0$. Hence, the solution of $s(t)$ remains non-negative for all $t \geq 0$. Similarly, the solution to the second equation of system (1) is $i(t) \geq i(0)e^{-(\mu_h+\alpha)t} > 0$, with $i(0)$ as its initial condition. Similarly, it can be shown that the remaining equations of model system (1) are non-negative, that is; $l_m(t) > 0,\ f_u(t) > 0$, and $f_c(t) > 0$ as t tends to infinity for all time $t \geq 0$. Hence, the solutions of (1) are non-negative for all $t \geq 0$. □

2.3 *Steady states and their stability analysis*

To solve for the steady states of model equation (1), we equate the right hand side to zeros. From the bacteria equation we have that $l^*_{m0} = 0$ or $l^*_{m1} = 1$. If $l^*_{m0} = 0$, we have $f^*_{c_0} = 0$ and a nonzero solution $f^*_{c_1} = \Psi_0(\mathfrak{R}_f - 1)$, where $\Psi_0 = \frac{\mu_f}{\omega_2}$ and $\mathfrak{R}_f = \frac{\omega_3}{\mu_f} \cdot \mathfrak{R}_f$ denotes the *food contamination threshold*. It connotes the basic reproduction number, $\mathfrak{R}_0$, in infectious disease modelling as defined in Van den Driessche and Watmough (2002). Without loss of generality, $\mathfrak{R}_f$ thus represents the average amount of food products that can be contaminated, and become responsible for causing human Listeriosis. Note that f^*_{c1} exists whenever $\mathfrak{R}_f > 1$. System (1) has three steady states namely; the disease-free steady

state (DFSS) $\mathfrak{E}^* = \left(s^*,\ i^*,\ l^*_{m0},\ f^*_u,\ f^*_{c_0}\right) = \left(1, 0, 0, 1, 0\right)$, the Listeria-free steady state (LFSS) $\mathfrak{E}^{**} = \left(s^{**}, i^{**}, l^{**}_{m0}, f^{**}_u, f^{**}_{c_1}\right)$ where

$$s^{**} = \frac{(\alpha+\mu_h)(\mu_h+\rho_h)}{\mathcal{A}_1+\omega_1\Psi_1\Psi_0(\mathfrak{R}_f-1)},\ i^{**} = \frac{\Psi_0(\mathfrak{R}_f-1)(\mu_h+\rho_h)\omega_1}{\mathcal{A}_1+\omega_1\Psi_1\Psi_0(\mathfrak{R}_f-1)},$$
$$l^{**}_{m0} = 0,\ f^{**}_u = \frac{1}{1+\mathfrak{R}_e(\mathfrak{R}_f-1)},\ f^{**}_{c_1} = \Psi_0(\mathfrak{R}_f-1),$$

with $\mathcal{A}_1 = (\alpha+\mu_h)(\mu_h+\rho_h)$, $\Psi_1 = (\alpha+\mu_h+\rho_h)$, and $\mathfrak{R}_e = \frac{\omega_3}{\omega_2}$, and the Listeria endemic steady state (LESS), $\mathfrak{E}^{***} = \left(\tilde{s}^{***},\ \tilde{i}^{***},\ \tilde{l}^{***}_{m1},\ \tilde{f_u}^{***}\right)$ where

$$s^{***} = \frac{(\alpha+\mu_h)(\mu_h+\rho_h)}{\mathcal{A}_2+\mathcal{A}_3},\ i^{***} = \frac{(\mu_h+\rho_h)(\omega_2+\omega_1\tilde{f_c^*})}{\mathcal{A}_2+\mathcal{A}_3},\ l^{***}_{m1} = 1,\ f^{***}_u = \frac{1}{1+\Upsilon},$$

with $\mathcal{A}_2 = \mu_h^2 + \alpha(\omega_2+\omega_1\tilde{f_c^*})$, $\mathcal{A}_3 = \rho_h\Upsilon_0 + \mu_h\Upsilon_1$, $\Upsilon = \frac{\omega_2}{\mu_f} + \tilde{f_c^*}\mathfrak{R}_f$, $\Upsilon_0 = \left(\alpha+\omega_2+\omega_1\tilde{f_c^*}\right)$, $\Upsilon_1 = (\alpha+\rho_h+\omega_2+\omega_1\tilde{f_c^*})$ and $\tilde{f_c^*}$ being the non-negative solutions to the quadratic equation used to determine the LESS. The existence of $\mathfrak{E}^{**}$ and $\mathfrak{E}^{***}$ is subject to $\mathfrak{R}_f > 1$. Note that the DFSS is locally asymptotically stable whenever $r_l < 0$ and $\mathfrak{R}_f < 1$, and otherwise unstable. The LFSS is locally asymptotically stable whenever $r_l < 0$ and $\mathfrak{R}_f > 1$, and otherwise unstable. Also, the LESS is locally asymptotically stable if $\mathfrak{R}_f > 1$, and unstable otherwise. The proofs for the stabilities of the three steady states can easily be done by looking at the Jacobian matrices at the respective steady states and applying the Routh-Hurwitz criterion.

2.4 Parameter values and sensitivity analysis

In the absence of controls, all the model parameters were estimated except for the food product removal rate $\mu_f = 0.0076$ which is from Chukwu and Nyabadza (2020b). The model parameters used for simulations are: $\mu_h = 0.1,\ \alpha = 0.0094, \rho_h = 0.09,\ r_l = 0.02,\ \omega_1 = 0.038,\ \omega_2 = 0.002,\ \omega_3 = 0.0005$, with units per day. We carry out simulations using the following initial conditions; $s(0) = 0.6, i(0) = 0.3,\ l_m(0) = 0.1,\ f_u = 0.7,\ f_c = 0.3$ chosen hypothetically for illustrative purposes only. We use the Latin Hypercube Sampling (Blower and Dowlatabadi, 1994), to carry out the sensitivity analysis. The Tonardo plot Figure 2 depicts all the model parameters with their respective partial correlation coefficients. In Figure 2 we see that the removal rate of food products μ_f has a positive PRCC's, while the rate of contamination of uncontaminated food, ω_3, is negatively correlated. This confirms the results depicted by the food contamination threshold $\mathfrak{R}_f$ obtained in Section 2.3. Hence, increase in μ_f decreases $\mathfrak{R}_f$ and increase in ω_3 increases the $\mathfrak{R}_f$ resulting in more human Listeriosis disease.

2.4.1 Varying parameters ω_1 and ω_2 on the fractions of infected humans

In the absence of controls, the results of varying the parameters ω_1 and ω_2 on the fractions of infected humans are shown in Figure 3. We observe that an increase in the rate at which humans are infected by Listeriosis (ω_1) and the rate of food contamination by Listeria, (ω_2) results in increased infectious humans.

Figure 2 Tornado plot showing PRCC's of all the parameters of model system (1) which are responsible for Listeriosis disease epidemics (see online version for colours)

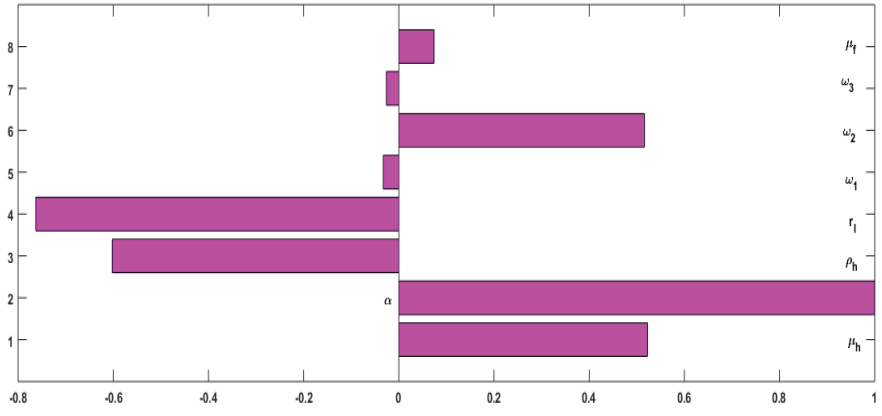
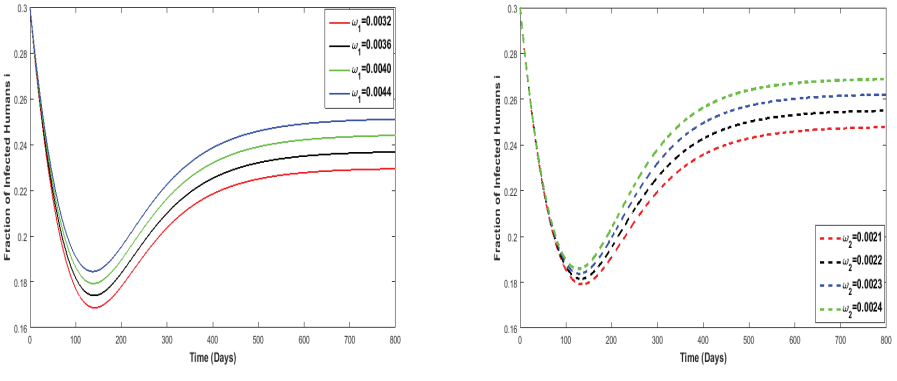


Figure 3 (a) Varying ω_1 on i and (b) varying ω_2 on i. The value for $\Re_f$ is 1.5(see online version for colours)



(a) (b)

3 Optimal control problem

We apply optimal control theory to assess the preventive measures so as to reduce the spreading of the disease in the human population. Therefore, we formulate an optimal control problem with the following permissible time dependent control variables: u_1, depicting the effects of media campaigns in reducing infection; u_2, controls the recovery of infected humans through the use of antibiotics; and u_3 controls the removal of contaminated food products through say, the recall of food products from the retail stores or factories. Introducing these permissible optimal control parameters u_1, u_2, and u_3 into model equation (1), we have:

$$\left.\begin{array}{l} \frac{ds}{dt} = \mu_h + \rho_h(1-s-i) - \mu_h s - (\omega_1 f_c + \omega_2 l_m)(1-u_1)s, \\ \frac{di}{dt} = (\omega_1 f_c + \omega_2 l_m)(1-u_1)s - \mu_h i - \alpha u_2 i, \quad \frac{dl_m}{dt} = r_l l_m(1-l_m), \\ \frac{df_u}{dt} = \mu_f - (\omega_2 l_m + \omega_3 f_c + \mu_f) f_u, \quad \frac{df_c}{dt} = (\omega_2 l_m + \omega_3 f_c) f_u - (\mu_f + u_3) f_c, \end{array}\right\}, \quad (2)$$

Note that all model parameters in Listeriosis control model equation (2) retain the same description as defined for model system (1). The objective is to reduce the number of infected humans, hence we formulate a minimisation problem with the following objective function

$$\mathfrak{J}(u_1,u_2,u_3)=\int_{t_0}^{t_f}\Big(Bi+\tfrac{1}{2}\big(C_1u_1^2+C_2u_2^2+C_3u_3^2\big)\Big)\mathrm{d}t\cdot \tag{3}$$

So $\mathfrak{J}(u_1^*,u_2^*,u_3^*)=\min_U \mathfrak{J}(u_1,u_2,u_3)$, where t_0 is the initial time, t_f is the terminal time, B is the weights associated with reducing the infected human class i, C_1,C_2,C_2 are the associated cost weights for the controls u_1,u_2 and u_3, respectively. We define the Hamiltonian function by applying the Pontryagin's Maximum Principle (Pontryagin et al., 2009) as follows

$$\begin{aligned}&\mathcal{H}=\mathfrak{L}(\tfrac{di}{dt},u_1,u_2,u_3)+\xi_1\tfrac{ds}{dt}+\xi_2\tfrac{di}{dt}+\xi_3\tfrac{dl_m}{dt}+\xi_4\tfrac{df_u}{dt}+\xi_5\tfrac{df_c}{dt}=Bi\\&+\tfrac{1}{2}(C_1u_1^2+C_2u_2^2+C_3u_3^2)+\xi_1\Big(\mu_h+\rho_h(1-s-i)-\mu_h s-(\omega_1f_c+\omega_2l_m)(1-u_1)s\Big)\\&+\xi_2\Big((\omega_1f_c+\omega_2l_m)(1-u_1)s-\mu_hi-\alpha u_2i\Big)+\xi_3\Big(r_ll_m(1-l_m)\Big)\\&+\xi_4\Big(\mu_f-(\omega_2l_m+\omega_3f_c+\mu_f)f_u\Big)+\xi_5\Big((\omega_2l_m+\omega_3f_c)f_u-(\mu_f+u_3)f_c\Big),\end{aligned}$$

where $\xi_j,\ \ j=1,\cdots,5$ are the adjoint variables. We have the following theorem on the existence of the optimal control.

Theorem 3 (Lenhart and Workman, 2007)**:** *There exist an optimal control* $u_*=(u_1^*,u_2^*,u_3^*)\in U$ *such that* $\mathfrak{J}(u_1^*,u_2^*,u_3^*)=\min_{(u_1,u_2,u_3)\in\cup}\mathfrak{J}(u_1,u_2,u_3)$ *subject to the optimal control model system (2) with its initial conditions.*

3.1 Optimality of the model system

Theorem 4: *Let* $s^*,i^*,l_m^*,f_u^*,$ *and* f_c^* *be the solutions of the optimal control model system (2) and (3) associated with the optimal control variables* $(u_1^*,\ u_2^*,\ u_3^*)$*. Then there exist an adjoint system which satisfies*

$$\left.\begin{aligned}\tfrac{d\xi_1(t)}{dt}&=\xi_1(\rho_h+\mu_h+(\omega_1f_c+\omega_2l_m)(1-u_1))+\xi_2(\omega_1f_c+\omega_2l_m(1-u_1)),\\\tfrac{d\xi_2(t)}{dt}&=\xi_1(\rho_h-B)+\xi_2(\mu_h+\alpha u_2),\\\tfrac{d\xi_3(t)}{dt}&=\xi_1\omega_2(1-u_1)s-\xi_2\omega_2(1-u_1)s-\xi_3(r_l-2r_ll_m),\\\tfrac{d\xi_4(t)}{dt}&=-\xi_4(\omega_3l_m+\omega_3f_c+\mu_f)+\xi_5(\omega_2l_m+\omega_3f_c),\\\tfrac{d\xi_5(t)}{dt}&=\xi_1\omega_1(1-u_1)s-\xi_2\omega_1(1-u_1)s+\xi_4\omega_3f_u-\xi_5\omega_3f_u+\xi_5u_3,\end{aligned}\right\},$$

with transversality boundary conditions $\xi_j(t_f)=0,\ \ j=1,2,\ldots,5,$ *denoted by*

$$u_j^*=\begin{cases}0 & \text{if } u_j\le 0,\\ u_j & \text{if } 0<u_j<1,\\ 1 & \text{if } u_j\ge 1,\end{cases}$$

where the permissible control functions $u_1^*,u_2^*,$ *and* u_3^**, are obtained by setting* $\frac{\partial H}{\partial u_i}=0,\ i=1,2,3.$ *Thus*

$$\left.\begin{aligned}&\tfrac{\partial\mathcal{H}}{\partial u_1}:=C_1u_1+s\xi_1\ (f_c\,\omega_1+l_m\,\omega_2)-s\xi_2\ (f_c\,\omega_1+l_m\,\omega_2)=0,\\&\tfrac{\partial\mathcal{H}}{\partial u_2}:=C_2u_2-i\xi_2\alpha=0,\ \ \tfrac{\partial\mathcal{H}}{\partial u_3}:=C_3u_3-f_c\xi_5=0.\end{aligned}\right\} \tag{4}$$

Solving for u_1^*, u_2^*, and u_3^* from equation (4), result in the following permissible control solutions $u_1^* := \frac{\phi_0}{C_1}$, $u_2^* := \frac{\alpha\,\xi_2 i}{C_2}$, $u_3^* := \frac{\xi_5 f_c}{C_3}$. where

$$\phi_0 = \xi_2(f_c\,\omega_1 + l_m\,\omega_2)s - \xi_1\,(f_c\,\omega_1 + l_m\,\omega_2)\,s.$$

Now, using the upper and lower constraints on the admissible controls u_1^*, u_2^*, u_3^*, it can be seen that the optimal controls can be characterised as:

$$u_1^* = \min\left\{\max\left(0, \frac{\phi_0}{C_1}\right), 1\right\},\ \ u_2^* = \min\left\{\max\left(0, \frac{\alpha\,\xi_2 i}{C_2}\right), 1\right\},$$
$$u_3^* = \min\left\{\max\left(0, \frac{f_c\,\xi_5}{C_3}\right), 1\right\}.$$

4 Numerical results and discussions

Using the Forward-Backward Sweep method (Lenhart and Workman, 2007), we note that, this optimal control problem is a two-point boundary-value problem with boundary conditions at time $t_0 = 0$ and $t_f = 100$. The simulations are carried out for a maximum of 100 days. We assumed that the estimated weight of the costs of media campaigns, treatment and removal of food products have equal values of 0.9 respectively and $B = 1$. In these numerical findings, we considered combination of the following best effective possible control strategies, which are:

- control strategy using media campaigns, treatment and removal of food products
- control strategy using treatment only
- control strategy using treatment and removal of food products only
- control strategy using media campaigns and removal of food products only
- control strategy using media campaigns and treatment only.

The control profiles of u_1 and u_3 starts at the upper and lower bound respectively and remain so during the entire simulation time. These results can be interpreted as follows: during an epidemic, media campaigns need to be implemented continuously to raise awareness in order for humans to take precautionary measures to avoid been infected. Also, the removal of contaminated food products is supposed to be implemented continuously for the effective control of the disease. In Figure 4(a) we observe that, in the presence of controls, the fractions of infected humans reduce over the modelling time. This decline in the number of infectious individuals reflects that, the controls; media campaigns and treatment have a significant impact on the infected human population and thus causes a reduction in the number of infected humans during an outbreak. Employing treatment only as a control measure, we notice that the optimal trajectories of Figure 4(b) do not achieve the targeted goal of minimising the disease in the infected humans. This implies that using treatment only is not an effective way of reducing the number of infected humans in any community induced with Listeriosis as the reduction is not significant. In Figure 4(c), we consider the dynamics of employing treatment and removal of food products only as a control measures. We notice that the combination of these two controls reduces the disease significantly. Hence in the absence of media campaign, Listerisois can also be contained in any community induced with the disease by the use of treatment and removal of food product only. Further,

we study the optimal trajectories generated by using media campaigns and treatment only. We observe that the use of these control reduces the number of infections in the infected human compartment as depicted in Figure 4(d) but does not help in eradicating the disease during the entire period of the simulation period.

Figure 4 (a) The dynamics using controls $u_1^*,\ u_2^*$ and u_3^* for infected; (b) the dynamics of using permissible control $u_2 \neq 0$ for infected; (c) the dynamics with permissible controls $u_1 = u_3 \neq 0$ for infected and (d) the dynamics with permissible controls $u_2 = u_3 \neq 0$ for infected (see online version for colours)

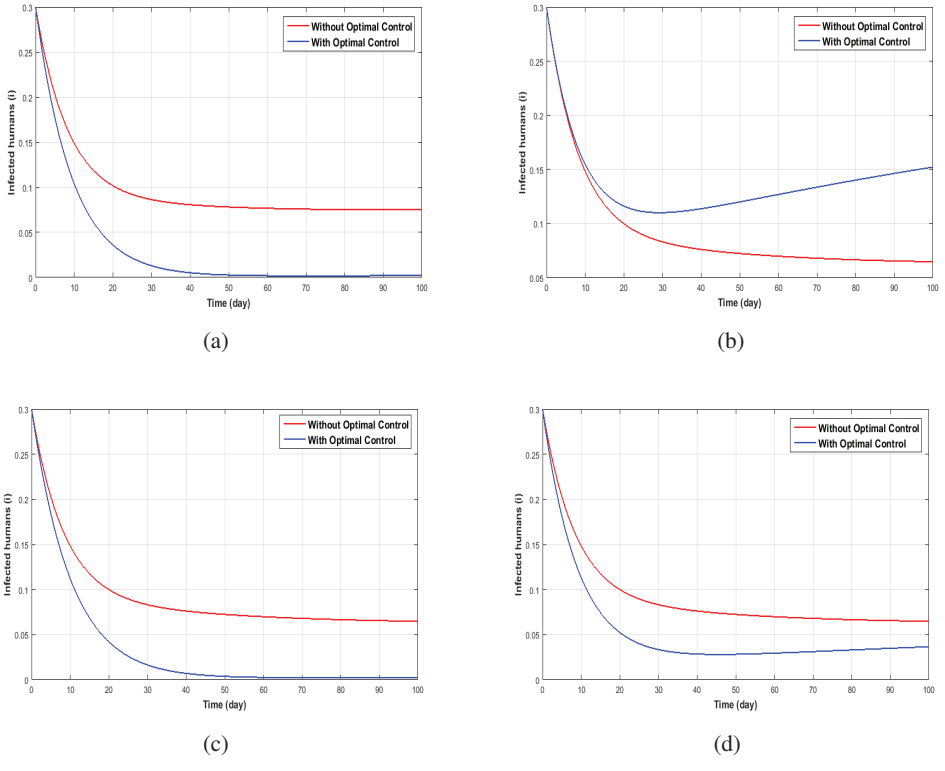
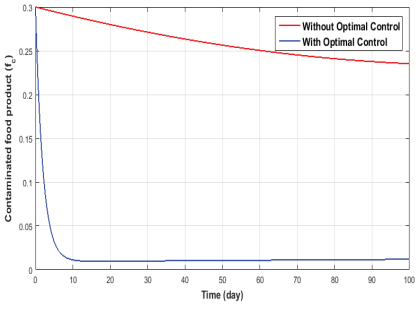
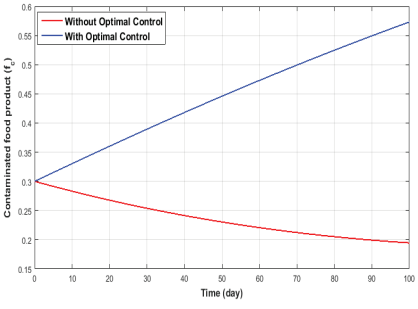


Figure 5(a) depicts that the control parameter u_3 has a positive impacts on the contaminated food products. We notice that in the presence of controls there is a reduction in the number of food products as time increases. This result reflects that controlling the removal of food products is essential in order to eradicate food-borne Listeriosis. In Figure 5(b) we observe that using media campaigns and treatment only as controls does not reduce the contaminated food products. This indicates that to reduce the total amount of contaminated food products, we require the combinations of all three controls.

Figure 5 (a) The dynamics using controls $u_1^*,\ u_2^*$ and u_3^* for contaminated food products and (b) the dynamics with permissible controls $u_2 = u_3 \neq 0$ for contaminated food products (see online version for colours)



(a)



(b)

5 Conclusion

In this paper, we studied the transmission dynamics of human Listeriosis disease resulting from the consumption of contaminated RTE food products which is described using a set of six differential equations with the inclusion of control strategies. Mathematical analysis of the model equations without controls is presented. We found that the model exhibits three steady states which were established to be locally asymptotically stable using the food contamination threshold $\mathfrak{R}_f$ under some specified conditions. An optimal control problem was formulated with the aim of reducing the number of human Listeria infections. Numerical simulations reveal that constant implementation of media campaigns, treatment and removal of food products will very effective measures in the control and management of Listeriosis in the event of an outbreak. The model presented in the paper is not without shortcomings. We considered a constant human population model, implying that the long term dynamics of the model assumes the population will be constant over the modelling time, which may not be realistic. The contribution of the bacteria from the environment in an epidemic is minute, but instrumental in the growth of an epidemic. The growth function for the *LM* is assumed to be logistic but environmental changes could impact on the growth of the bacteria and hence a periodic function could better model the growth and abundance of the bacteria. Despite the shortcomings, the results obtained from our findings could be used in food risk assessment to quantify the potential public health benefits on the most effective control strategies to reduce food contamination and control of Listeriosis.

Conflict of interest

Authors declare no competing and conflict of interests.

References

Almudena, H. and Payeras-Cifre, A. (2014) 'What is new in Listeriosis?', *BioM. Res. Int.*, Vol. 2014, Article ID 358051, 7 pages, https://doi.org/10.1155/2014/358051

Bennion, J.R., Sorvillo, F., Wise, M.E., Krishna, S. and Mascola, L. (2008) 'Decreasing Listeriosis mortality in the United States', *Clinic. Infect. Dis.*, Vol. 47, No. 7, pp.867–874.

Blower, S.M. and Dowlatabadi, H. (1994) 'Sensitivity and uncertainty analysis of complex models of disease transmission: an HIV model, as an example', *Int. Stat. Review/Revue Inter. de Stat.*, pp.229–243.

Center for Disease Control (CDC) (2020) https://www.cdc.gov/Listeria/prevention.html (Accessed 26 May, 2020).

Chukwu, C.W. and Nyabadza, F. (2020) *Mathematical Modeling of Listeriosis Incorporating Effects of Awareness Programs*, medRxiv. 2020, https://www.medrxiv.org/content/10.1101/2020.04.26.20080762v1

Chukwu, C.W. and Nyabadza, F. (2020) 'A theoretical model of Listeriosis Driven by cross-contamination of ready-to-eat food products', *Int. J. Math. and Mathc. Sci.*, Hindawi, p.14, Article-ID 2020.

Lenhart, S. and Workman, J.T. (2007) *Optimal Control Applied to Biological Models*, Chap. and Hall/CRC.

Osman, S., Otoo, D. and Sebil, C. (2020) 'Analysis of Listeriosis transmission dynamics with optimal control', *Appl. Math.*, Vol. 11, No. 7, pp.712–737.

Pontryagin, L.S., Boltyanskii, V.G., Gamkrelidze, R.V. and Mishchenko E.F. (1962) *The Mathematical Theory of Optimal Processes*, Inter. Pub., New York.

Stout, A., Van Stelten-Carlson, A., Marquis, H., Ballou, M., Reilly, B., Loneragan, G.H., Nightingale, K. and Ivanek, R. (2020) 'Public health impact of foodborne exposure to naturally occurring virulence-attenuated Listeria monocytogenes: inference from mouse and mathematical models', *Interf. Foc.*, Vol. 10, No. 1, p.20190046.

Van den Driessche, P. and Watmough, J. (2002) 'Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission', *Math. Biosc.*, Vol. 180, Nos. 1–2, pp.29–48.

World Health Organization (WHO) (2020) *World Health Organization*, International Travel and Health, https://www.who.int/ith/diseases/Listeriosis/en/ (Accessed 26 June, 2020).

Witbooi, P.J, Africa, C., Christoffels, A. and Ahmed, I.H.I. (2020) 'A population model for the 2017/18 Listeriosis outbreak in South Africa', *Plosone*, Vol. 15, No. 3, p.e0229901.