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Yingli Shu, Quande Yuan, Jian Zhang, Huazhong Li, Yuzhen Pi, Wen-de Ke

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## **A method of designing swinging-leg walking trajectory for biped robot on plat ground**

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**Yingli Shu**

School of Electrical Engineering and Information Technology,  
Changchun Institute of Technology,  
Changchun, 130012, China  
Email: shuyl@scitimes.com

**Quande Yuan**

School of Computer Technology and Engineering,  
Changchun Institute of Technology,  
Changchun, 130012, China  
Email: yuanquande@qq.com

**Jian Zhang**

Department of Oil Saving Engineering,  
Yantai Longyuan Power Technology Co., Ltd.,  
Yantai, 264000, China  
Email: riefu1012@163.com

**Huazhong Li\***

Sino-German School,  
Shenzhen Institute of Information Technology,  
Shenzhen, 518172, China  
Email: chinawwwsl@163.com  
\*Corresponding author

**Yuzhen Pi**

School of Electrical Engineering and Information Technology,  
Changchun Institute of Technology,  
Changchun, 130012, China  
Email: piyz@airlab.ac.cn

**Wen-de Ke**

Department of Mechanical and Energy Engineering,  
Southern University of Science and Technology,  
Shenzhen, 518055, China  
Email: wendeke@163.com

**Abstract:** The periodic walking of biped robot involves the alternate movement of supporting leg and swinging leg. In order to quickly plan the gait, it is necessary to select the key posture of biped walking on the premise of maintaining the stability of the robot. Based on the known information, the spline curve is designed and solved to construct the ankle trajectory of the swinging leg of the robot. Simulation results showed the feasibility of the method.

**Keywords:** biped robot; trajectory; walking; ZMP; zero moment point.

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**Biographical notes:** Yingli Shu received his MPhil from Changchun University of Technology in 2013. Now, he is a Senior Experimentalist at Changchun Institute of Technology. His current research focuses on robotics and machine-learning.

Quande Yuan received his PhD in Computer Application Technology from Harbin Institute of Technology in 2016. He is an Associate Professor at Changchun Institute of Technology. His research interests include robotics, machine learning and smart grid.

Jian Zhang received his Bachelor degree in Computer Science and Technology from Beihua University in 2004. Now, he is an Engineer at Yantai Longyuan Power Technology Co., Ltd. His research interests include robotics and smart grid.

Huazhong Li received his MS and PhD degree from Harbin Institute of Technology, China in 1963 and 1999, respectively. Now, he is the Professor of Shenzhen Institute of Information Technology. His research interests include artificial intelligence, intelligent robots and complex network technology.

Yuzhen Pi received her MPhil degree in Computer Application Technology from Northeast Electric Power University in 2007. She is a Lecturer at Changchun Institute of Technology. Her research interests include machine learning and smart grid.

Wen-de Ke received his PhD degree from Harbin Institute of Technology in 2013. Now, he is the Professor in Southern University of Science and Technology. His research interests focus on robotics and intelligent algorithms.

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## 1 Introduction

Biped robot shows the human like walking style, and its walking, running, jumping, stretching and other forms of motion are hot research issues in the field of robot. Motion planning method firstly needs to solve the problem of joint trajectory generation, that is, how to design the joint trajectory and joint torque that can meet the mechanical characteristics of the robot, and the change of its motion parameters to reflect the motion characteristics of the humanoid body of the biped robot; secondly, it is needed to consider

how to generate the whole body motion mode of the robot to meet the specific target environment, analyse the characteristics of the environment structure, and use it to adjust the trajectory of the robot joints that make its motion stable. At present, the method based on motion analytic equation is regarded as the traditional motion planning one for biped robot. The trajectory of robot joints is obtained by establishing the motion analytic equation and solving the motion trajectory.

Taking the biped motion planning of biped robot as an example, the advantage of this traditional trajectory design method is that the robot motion shape reflects the characteristics of mathematical model, which is easy to describe and analyse, and it is easy to meet the mechanical constraints, kinematic and dynamic stability conditions of the robot, and make the trajectory smooth. Many scholars have carried out relevant research, for examples, Li et al. (2014) proposed a central pattern generator based motion control strategy for active/passive walking for a biped robot using CPG with sensory interaction. Li et al. (2021) proposed the sequential sensor fusion-based real-time LSTM gait pattern controller for biped robot. Catalano et al. (2021) presents the results of testing two such soft feet on the humanoid robot HRP-4, and compares them to what obtained with the original flat feet of the robot. Yoo et al. (2018) proposed Biped robot walking on uneven terrain using impedance control and terrain recognition algorithm and tested its validation. Znegui et al. (2020) illustrates a stabilisation approach of the passive bipedal locomotion of the compass-gait biped model based on an exclusively developed enhanced design of the closed form of the Controlled Poincaré Map (CPM). Added and Gritli (2020) proposed the model strategy for locomotion mechanism of the passive-dynamic compass-type biped robot by an impulsive hybrid nonlinear system and believed it was the best model for mimicking the human walking. Azimi et al. (2021) presented and experimentally implemented three different adaptive and robust adaptive controllers as the first steps toward using model-based controllers for transfemoral prostheses. Bzhikhatlov et al. (2020) proposed the estimation approach that was used to estimate model parameters using the foot trajectory, in which the method used only foot coordinates of captured data and shows fast estimation, etc. These studies have achieved good results.

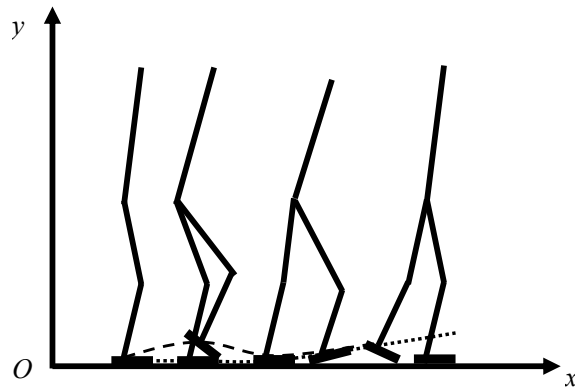
On the other hand, how to reduce the trajectory design process and make the bipedal joint trajectory more quickly is what we need to consider in this paper. We have studied the walking for biped robot on stepping downstair (Ke et al., 2020), herein we mainly study how to plan the trajectory for walking on the plat ground. The organisation of this paper is as follows: Section 1 describes the trajectory characteristics of the biped robot, Section 2 describes the key posture constraints, Section 3 describes the motion constraints of biped walking, and in the experiments, three groups of different leg raising height and speed are analysed.

## 2 Trajectory analysis of biped robot

The biped robot is an open chain structure with high centre of gravity and weak motion stability. The walking process is realised by alternating swinging of two feet. It is set on the  $y$ - $z$  vertical plane. As shown in Figure 1, the two dashed lines respectively represent the movement track of the ankle joint of both feet. It can be seen that in a walking cycle, the gait of supporting foot and swinging foot alternate and are completely symmetrical; the supporting phase of both feet shows the static stable closed-loop kinematics, and the

supporting phase of one foot shows the characteristics of dynamic unstable open-loop kinematics.

**Figure 1** The period of biped walking on ground



When planning the robot's gait, firstly, the walking behaviour is analysed, that is, the parameters such as walking cycle and step distance are set; according to the parameters and the robot's connecting rod data, the key postures in the biped walking cycle are determined, and the joint angle information corresponding to these key postures is calculated; based on the parameters of the key postures, the cubic spline interpolation function is constructed to realise the key control of the robot's walking cycle. The smooth motion curve is obtained by point interpolation.

In the motion design of biped robot with kinematic constraints, there may be the jumping problem of connection points between sub phases, that is, the key pose at the end point of the current sub phase cannot smoothly connect to the key pose at the start point of the next sub phase, and when the steering gear amplitude suddenly increases, the motion trajectory will jump as well.

In order to solve this problem, it is necessary to carry out linear interpolation on the junction points to ensure the smooth transition of sub phases. After interpolation, the sub phase connection becomes smooth and natural.

In Ke et al. (2020), we have mentioned the Spline interpolation of key postures for stepping downstairs. In the walking process of biped robot, trigonometric function interpolation method is used to calculate the angle and angular velocity of the joint of adjacent sub phases.

The walking motion of biped robot is periodic, and polynomial trajectory is used to design. Specify the start point and end point of the swing leg (similar to the support leg), determine the middle point of the trajectory (swing leg raise point and drop point), and match the position, velocity and acceleration of each point on the two motion segments to plan a continuous trajectory. According to the boundary conditions of the start point and the end point and the determined intermediate points, the polynomial is used to plan the trajectory and make it pass through all the specified points. Mathematically, the derivative of cubic spline interpolation function has second-order continuity, and its adjacent attitude points are described by polynomials. The trajectory can be connected by three segments, including the starting point, the first middle point, two adjacent middle points, the last middle point and the end point

$$\begin{cases} \varphi_{\text{swinging-}d_1}(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 \\ \varphi_{\text{swinging-}d_2}(t) = b_0 + b_1t + b_2t^2 + b_3t^3 \\ \varphi_{\text{swinging-}d_3}(t) = c_0 + c_1t + c_2t^2 + c_3t^3 + c_4t^4 \end{cases} \quad (1)$$

The corresponding angle, velocity and angular velocity are

$$\begin{cases} \varphi_{\text{swinging-}d_1}(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 \\ \dot{\varphi}_{\text{swinging-}d_1}(t) = a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 \\ \ddot{\varphi}_{\text{swinging-}d_1}(t) = 2a_2 + 6a_3t + 12a_4t^2 \end{cases}$$

$$\begin{cases} \varphi_{\text{swinging-}d_2}(t) = b_0 + b_1t + b_2t^2 + b_3t^3 + b_4t^4 \\ \dot{\varphi}_{\text{swinging-}d_2}(t) = b_1 + 2b_2t + 3b_3t^2 + 4b_4t^3 \\ \ddot{\varphi}_{\text{swinging-}d_2}(t) = 2b_2 + 6b_3t + 12b_4t^2 \end{cases}$$

$$\begin{cases} \varphi_{\text{swinging-}d_3}(t) = c_0 + c_1t + c_2t^2 + c_3t^3 + c_4t^4 \\ \dot{\varphi}_{\text{swinging-}d_3}(t) = c_1 + 2c_2t + 3c_3t^2 + 4c_4t^3 \\ \ddot{\varphi}_{\text{swinging-}d_3}(t) = 2c_2 + 6c_3t + 12c_4t^2 \end{cases}$$

Because the neighbourhood of the joint point can be derived everywhere, the joint curve formed by the interpolation point is continuous and smooth everywhere. The trigonometric function interpolation method can be used to calculate the angle and angular velocity of the joint point at the joint of adjacent sub phases.

### 3 Key pose constraints

The key pose refers to the posture when the human movement pauses at a certain time, which can well reflect the movement characteristics of the human body. In the analysis and selection of key poses, it is necessary to learn from the process of bipedal walking. Although biped robots have human shape and limbs, there are morphological differences in size and limb length between robots. In the aspect of form differentiation control, the joint trajectory is adjusted to adapt to different forms of biped robot. In the decomposition and reconstruction of basic actions, the periodic motion is decomposed into basic actions or key postures, and relevant kinematic constraints are applied to achieve similarity control. In the design of complex movements, the research is mainly carried out from the base segment segmentation, movement rhythm control and optimisation. In the design of learning ability, it mainly focuses on complex behaviour learning and trajectory imitation learning.

Key poses are usually handled in three ways:

- 1 *Extraction.* For example, with the help of a single mobile camera, combined with motion detection and trajectory tracking, Fossati et al. (2010) proposed the key postures extraction from motion sequences of continuous images, and robust 3D human motion recovery was realised.

- 2 *Recognition and matching.* For example, Gu et al. (2010) proposed a two-dimensional behaviour recognition algorithm based on the three-dimensional model of human behaviour. In the key pose set, the best matching three-dimensional key pose frame was found for each two-dimensional observation sample, and the three-dimensional key pose sequence corresponding to the two-dimensional observation sample sequence was recognised by the behaviour classifier.
- 3 *Classification.* For example, Hsieh et al. (2010) proposed a new grouping scheme to construct the model space for classification and recognition of human key postures, and a model driven method was used to divide the closed region into multiple single objects to solve the segmentation problem of closed human motion trajectory ; etc.

In the periodic walking process of biped robot, the key poses of the swinging leg are as follows:

- 1 *Initial state:* the distance between the robot's feet is constant, the soles of the robot's feet are in full contact with the ground, and the speed is 0, then

$$C^{\text{initial}} \equiv \begin{cases} \mathbf{K}_{\text{initial-ankle}} = [x \ y \ z]^T \\ \mathbf{K}_{\text{initial-knee}} = [x \ y + h^{\text{knee}} \ 0]^T \\ \mathbf{K}_{\text{initial-hip}} = [x \ y + h^{\text{hip}} \ 0]^T \\ \mathbf{K}_{\text{initial-velocity}} [x \ y \ z]^T \equiv \mathbf{0} \end{cases} \quad (2)$$

among which,  $[x \ y \ z]^T$  describes the coordinate values of the related points of the swing leg, including the ankle joint, knee joint and hip joint.

- 2 *Key pose 1:* When the rotation angle of the robot's swinging leg around the toe is constant, the sole of the supporting leg is in full contact with the ground, the distance between the two feet is constant, and the speed is non-zero, then the robot's swinging leg is stable

$$C^{\text{key1}} \equiv \begin{cases} \mathbf{K}_{\text{key1-ankle}} = [x_{\text{key1-ankle}} \ y_{\text{key1-ankle}} \ z_{\text{key1-ankle}}]^T \\ \mathbf{K}_{\text{key1-knee}} = [x_{\text{key1-knee}} \ y_{\text{key1-knee}} + h^{\text{knee}} \cos(\alpha_{\text{key1-knee}}) \ 0]^T \\ \mathbf{K}_{\text{key1-hip}} = [x_{\text{key1-hip}} \ y_{\text{key1-hip}} + h^{\text{hip}} \cos(\beta_{\text{key1-hip}}) \ 0]^T \\ \mathbf{K}_{\text{key1-velocity}} [x \ y \ z]^T \neq \mathbf{0} \end{cases} \quad (3)$$

among which,  $\alpha_{\text{key1-knee}}$  and  $\beta_{\text{key1-hip}}$  are the angle between the knee joint and hip joint of swing leg and the ground, respectively.

- 3 *Key pose 2:* In order to ensure that the robot can cross a certain height of obstacles when walking, make the robot's foot at the level when the swing leg is lifted to the highest point, and the speed is non-zero, then

$$C^{\text{key2}} \equiv \begin{cases} \mathbf{K}_{\text{key2-ankle}} = [x_{\text{key2-ankle}} \ y_{\text{key2-ankle}} \ z_{\text{key2-ankle}}]^T \\ \mathbf{K}_{\text{key2-ankle}} = [x_{\text{key2-ankle}} \ y_{\text{key2-ankle}} \ z_{\text{key2-ankle}}]^T \\ \mathbf{K}_{\text{key2-hip}} = [x_{\text{key2-hip}} \ y_{\text{key2-hip}} + h^{\text{hip}} \cos(\beta_{\text{key2-hip}}) \ 0]^T \\ \mathbf{K}_{\text{key2-velocity}} [x \ y \ z]^T \neq \mathbf{0} \\ z_{\text{key2-ankle}} \neq h \end{cases} \quad (4)$$

among which,  $h$  is the height from the bottom of the palm of the swinging leg to the ground.

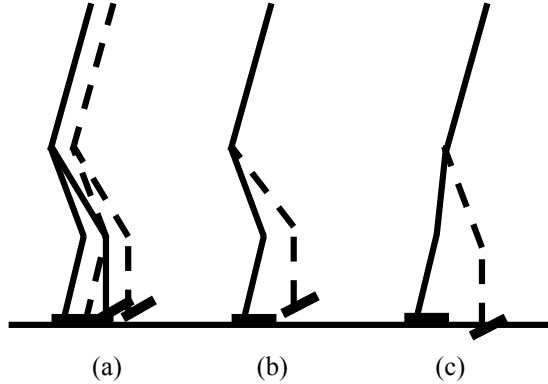
- 4 *Key pose 3*: The distance between the two feet is constant, the heel of the swinging leg touches the ground, and the speed is non-zero, then

$$C^{\text{key3}} \equiv \begin{cases} \mathbf{K}_{\text{key3-ankle}} = [x_{\text{key3-ankle}} \ y_{\text{key3-ankle}} \ z_{\text{key3-ankle}}]^T \\ \mathbf{K}_{\text{key3-ankle}} = [x_{\text{key3-ankle}} \ y_{\text{key3-ankle}} \ z_{\text{key3-ankle}}]^T \\ \mathbf{K}_{\text{key3-hip}} = [x_{\text{key3-hip}} \ y_{\text{key3-hip}} + h^{\text{hip}} \cos(\beta_{\text{key3-hip}}) \ 0]^T \\ \mathbf{K}_{\text{key3-velocity}} [x \ y \ z]^T \neq \mathbf{0} \\ z_{\text{key3-ankle}} \equiv 0 \end{cases} \quad (5)$$

#### 4 Kinematic constraints

The trajectory obtained by spline function is based on the ankle joint. When the actual robot's foot falls to the ground, the length of the foot is not considered, and there may be interference between the foot and the ground. The constraint condition of the contact between the foot and the ground is the key factor of the stability. Although the expected and actual zero moment points are in the stable area of the support polygon formed by the contact with the ground, there are still three situations of the swinging leg landing, such as sliding, suspension and foot penetrating the ground theoretically, as shown in Figure 2. Both sliding and foot penetrating the ground are in contact with the ground.

**Figure 2** Contact states between swinging leg and ground: (a) sliding; (b) suspension and (c) penetrating



Sliding is caused by the movement track noise or the smooth ground, resulting in the displacement of the contact position between the robot's foot and the ground. Suspension and foot penetrating the ground are caused by the abnormal joint torque due to the difference of joint proportion, which makes the landing foot suspend on the ground or excessively compress the ground. When the swinging foot falls to the ground, it will



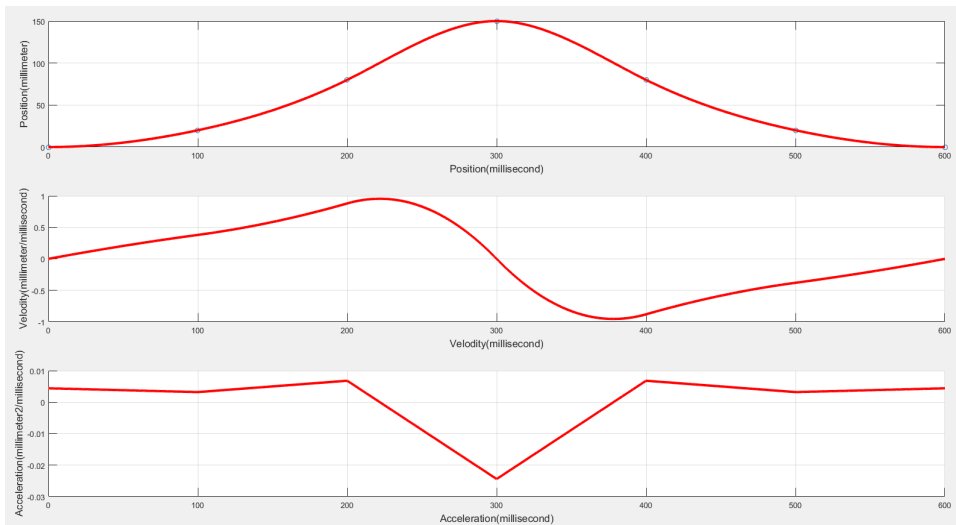
cause impact vibration, which directly affects and destroys the movement stability. Qualitative, and even damage the robot mechanical link components.

In order to solve the problem of landing collision of swinging leg sole, the elastic buffer components can be added to the mechanism, such as springs, cushions, etc. This method is passive and related to the mechanical characteristics of the robot. It can alleviate the collision to a certain extent, but it cannot really solve the compliance requirements of rigid body and collision. In the actual design, the active compensation mechanism can be used to adjust the landing height and position of the swing leg, and the compliant collision can be achieved by reducing the contact error.

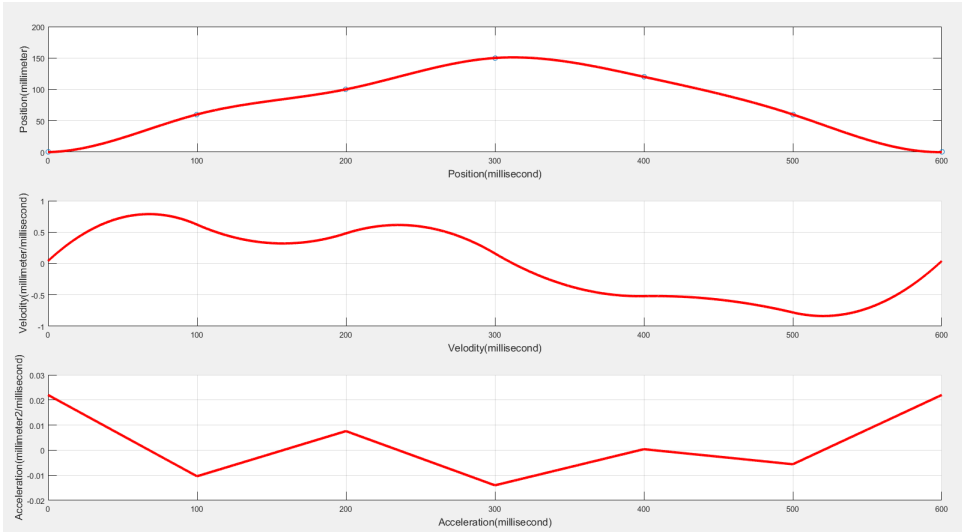
## 5 Experiments

A biped walking cycle of biped robot is extracted. Because the key poses are set, the y-axis coordinate position in the space plane, the corresponding position time, the interpolation period, the spline function value, the velocity, the acceleration value, the initial velocity  $v_0$  and the final velocity  $v_n$  of the ankle joint of the swing leg are set, and the maximum leg lifting height is a fixed value  $h$ . When all these values are given as well as every time point and the related velocity boundary, the periodic cubic spline trajectory effect is solved as shown in the figure. The initial setting of position and speed within 600 ms is carried out respectively, and the corresponding spline curve is solved by MATLAB programming. Three different groups of data are planned, and the spline curves are shown in Figures 3–5.

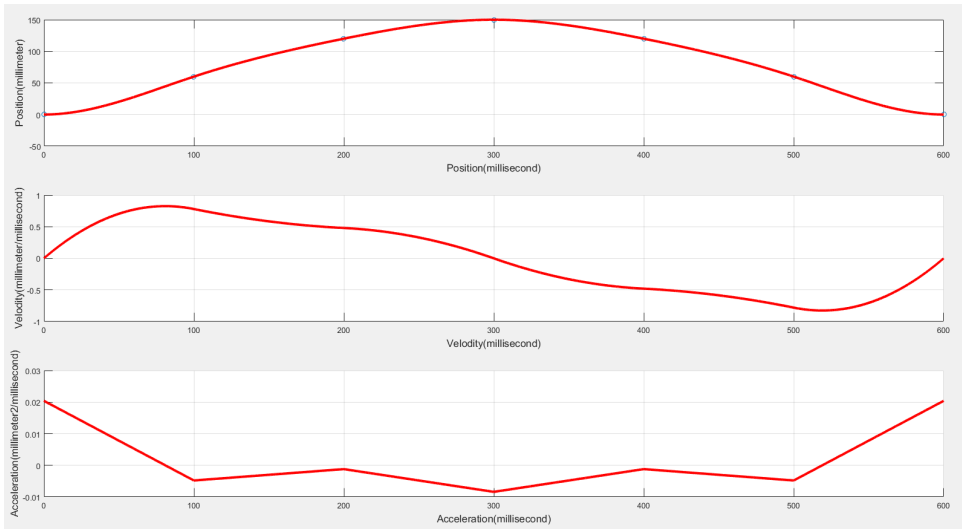
**Figure 3** Group 1 (position-velocity-acceleration) for swinging leg (see online version for colours)



**Figure 4** Group 2 (position-velocity-acceleration) for swinging leg (see online version for colours)



**Figure 5** Group 3 (position-velocity-acceleration) for swinging leg (see online version for colours)



From the three different sets of data, it can be seen that in order to quickly lift the ankle joint of swinging leg to a fixed height, its position change process can be realised by increasing the torque of hip joint and knee joint in a short time. From the whole ascending and descending cycle, the change of velocity and acceleration is obvious, which will reduce the walking stability of biped robot.

## 6 Conclusion

When the biped robot maintains its balance, it can walk effectively through the periodic interaction between the supporting leg and the swinging leg. Therefore, the trajectory change of the ankle joint is the primary consideration of gait planning. In order to improve the walking effect of the robot, the related trajectory can be obtained through the spline curve. Under the constraints of stability, the continuous walking effect can be achieved by solving the joint angle change of the robot through inverse kinematics.

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