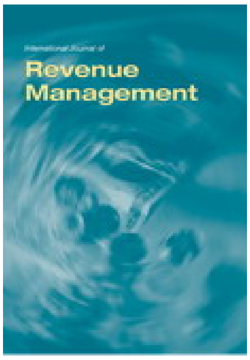

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Optimal finite horizon bargaining mechanisms with refusal cost

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Abstract: This paper considers a bargaining problem under asymmetric information between a seller and multiple buyers for selling given perishable items over a finite period. It is assumed that the seller faces a refusal cost if the item does not get sold. The problem is modelled as a Markov decision process that endogenises the marginal inventory valuation of the seller. This paper compares four bilateral bargaining mechanisms namely seller posting price, buyer posting price, difference splitting between seller's and buyer's valuation, and negotiation. For low refusal cost, the seller prefers posting price and splitting the difference between both valuations when he is in strong and weak positions, respectively. For high cost, the seller is indifferent between negotiation and buyer's posting. Also, this paper compares bargaining mechanisms and dynamic pricing with and without the refusal cost. This paper conducts simulation experiments to validate the findings of the model.

Keywords: Markov decision process; bargaining mechanisms; refusal cost; finite horizon.

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Biographical notes: K.R. Ramkishore obtained his PhD from the Department of Management Studies at the Indian Institute of Technology Madras, Chennai. His research interest includes scheduling, revenue management, and operations research.

1 Introduction

Industries having a finite stock of a product need to sell in a finite time period. There are numerous examples to highlight in day-to-day life. Examples include airlines, the hotel industry, the automobile industry, and retailers selling multi-generation products. A seller who has perishable items are constrained by deadlines. Those products have two key characteristics: limited stock availability and a deadline for sale (Gallego and Ryzin, 1994). In airlines, empty seats after the flight depart have no value. Departure time is the deadline by which seats are to be sold. Unbooked hotel rooms after midnight

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have no value. Though laptops and automobiles do not fall under perishable items, their values diminish as new generation laptops and new model automobiles enter the market. In multi-generation products, when a new version of a product arrives in the market, the retailer promotes the new version to sell. As a result, retailers tend to reduce the price to clear the older version (Kuo and Huang, 2012). To be realistic, analysing the allocation over a finite horizon is inevitable. To mention some of the literature those study over the finite horizon are Gallego and Ryzin (1994), Vulcano et al. (2002), Cao et al. (2012), and Sato (2021).

The seller has the option to decide the mechanism to allocate the item. During the last few decades, researchers' interest in bargaining has been increased as an allocation mechanism. As a consequence, analysing from managerial/seller and researcher perspectives has gained importance. In the revenue management setting, this paper focuses on bilateral bargaining mechanisms, where the seller and the buyers' valuations are private information. The seller has a fixed stock of inventory to sell in a stipulated time period. This paper assumes the buyer does not know the number of items remaining and the time period remaining to sell the inventory. This assumption makes the arrival of the buyer in any time period the same. A buyer is strategic when he postpones the purchase anticipating a lower price in the future. This paper assumes the buyers are non-strategic in nature. In each time period, a buyer arrives with a probability for a unit demand. Earlier literature those analyse static bilateral bargaining under asymmetric information are Chatterjee and Samuelson (1983), Myerson and Satterthwaite (1983), and Myerson (1985). They assume that the seller valuation and the buyer valuation are private information to each other. Based on their valuations, the mediator will determine whether the trade happens or not and payment to be made by the buyer to the seller. If the buyer's and seller's valuations are independent random variables and uniformly distributed between [0, 1], this type of bargaining problem is called symmetric uniform trading problem (SUTP) (Myerson, 1985). In the SUTP setting, Myerson (1985) analyses different bargaining mechanisms namely neutral bargaining solution (NBM), buyer posted price (BPM), seller posted price (SPM), and split the difference (STM). To describe these mechanisms briefly: the seller decides the price for his/her inventory item at which arriving buyer can either accept to buy or reject it (SPM); the arriving buyer decides the price to buy for an inventory item at which the seller can either accept to sell or reject it (BPM); the seller and the buyer negotiate each other (NBM); and the buyer's price and the seller's price would be averaged (STM).

Bhandari and Secomandi (2011) initiate the bilateral bargaining problem in the dynamic context and analyse it as a Markov decision process (MDP) that endogenously determines the marginal valuation of the seller as a function of inventory remaining in an infinite horizon period. In the SUTP setting, they compare four bilateral bargaining mechanisms analytically as well as numerically from the seller's perspective and find that SPM dominates other mechanisms. Dufalla (2014) modifies the Bhandari and Secomandi (2011) model as the finite horizon problem. She finds that the seller prefers STM, while the other three mechanisms behave similarly when the inventory levels are sufficiently high and sufficiently less time remaining. Seller prefers SPM, which outperforms STM, followed by BPM when the inventory levels are sufficiently low and sufficiently more time remaining. NBM behaves similar to SPM when the seller has less inventory and more time remaining and similar to BPM when the seller has more inventory and less time remaining.

In the traditional inventory problems, when the supplier faces stock out, either backorder or lost sale is incurred according to the customer's reaction to stock out. Seller may refuse to meet the demand of buyers even in case of no stock out, anticipating future benefits. Recently, supply chain management focuses on customer selection models that choose customers without the knowledge of the customer's arrival in the future. In revenue management problems, the demand from low-type customers is often denied in the hope of satisfying high-type customers in the future. The seller incurs refusal or rejection cost when refusing to satisfy the demand of the customer. This refusal cost includes loss of goodwill, lost sale penalty, or price paid to a third party. Bhaskaran et al. (2010), Elmachtoub and Levi (2015), Elmachtoub and Levi (2016), and Ramkishore and Amit (2019) consider rejection cost in their work.

Ramkishore and Amit (2019) extend the Bhandari and Secomandi (2011) work with refusal cost. For lower refusal cost, the seller prefers to post the price. When the refusal cost increases, STM dominates SPM. For the high refusal cost, BPM and NBM behave similarly and they outperform STM and SPM. From the perspective of the seller, this paper analyses the four bilateral bargaining mechanisms in a finite time horizon. This paper considers the seller faces a cost of refusal if the item does not get sold. For lower refusal cost, the seller prefers SPM, which dominates STM, followed by BPM when there is sufficiently low inventory and more time remaining. NBM behaves similar to SPM. Seller prefers STM, which dominates BPM and NBM together, followed by SPM when there is sufficiently more inventory and less time remaining. As the refusal cost increases, STM is the preferred mechanism when the inventory levels are sufficiently low and more time remaining. NBM and BPM together dominate other mechanisms when the inventory levels are sufficiently more and less time remaining. When the refusal cost is high, BPM and NBM together dominate STM, followed by SPM, irrespective of the inventory level and time remaining.

The researchers have intensified their attention on dynamic pricing (posted price). Some of the recent dynamic pricing studies include Kim et al. (2016), Lu et al. (2018), and Sato (2021). Varma and Vettas (2001) model infinite time horizon dynamic pricing problem in which the seller has a finite amount of a non-perishable good and posts the price as a function of inventory remaining. They assume each buyer has unit demand. Wang et al. (2013) extend the dynamic pricing model by relaxing the unit demand assumption of Varma and Vettas (2001). They aim to maximise the seller's revenue by determining the optimal posted price.

There are also studies where the performance is compared between posted price and bargaining. Wang (1995) compares bargaining using generalised Nash bargaining solution (GNBS) and posted price mechanism for a single object. He finds that the bargaining is better than the posted price. Roth et al. (2006) also compare bargaining and posted price considering customisation of services and its impact on pricing decision rule. They show that posted price and negotiating price are suitable for standardised and customised services, respectively. Kuo et al. (2011) model together negotiation using GNBS and posted price in a finite horizon period with limited inventory. They find that when the inventory is low, the retailer prefers posting the price to negotiate. When there is high inventory, the retailer prefers negotiation to posting the price. GNBS assumes buyers disclose their valuation honestly, whereas in this model, buyers' valuations are private information as in Myerson (1985) and Bhandari and Secomandi (2011).

Also, this paper considers dynamic pricing (DP) setting as in Varma and Vettas (2001) and compares it with the above bargaining problems. This paper analyses the performance of dynamic pricing with the above four bargaining mechanisms with and without refusal cost. Without refusal cost, dynamic pricing performs similarly to SPM. They together dominate other mechanisms when there is sufficiently low inventory and more time remaining. When the refusal cost increases, dynamic pricing starts dominating SPM. This paper conducts simulation experiments to validate the findings of the MDP model. Simulation results also produce the same order preference of the mechanisms as in the MDP model.

The organisation of this paper is as follows. The literature related to the bilateral bargaining mechanisms is highlighted in Section 2. Sections 3 and 4 describe the bargaining problem in static and dynamic contexts, respectively. Dynamic pricing model is discussed in Section 5. The solution procedure is discussed in Section 6. Section 7 describes the simulation-based approach. Numerical illustration is shown in Section 8 and discussed the solution in Section 9. Section 10 concludes with possible extensions to this study.

2 Literature review

The two types of approaches in bargaining games are the *sequential approach* and the *mechanism design approach*. This study limits only to the mechanism design approach. This section highlights literature on the bilateral bargaining mechanism, summarised in Table 1. The key and early studies in the mechanism design approach are Chatterjee and Samuelson (1983), Myerson and Satterthwaite (1983), and Myerson (1985).

Chatterjee and Samuelson (1983) model bilateral bargaining in a situation where one player knows his/her valuation of the object being sold and is uncertain about the other player's valuation. Both players submit offers simultaneously and the trade occurs if the buyer's offer is more than the seller's offer. They analyse offer strategies or behaviour of bargaining, which is increasing in individual valuation. They analyse equilibrium offers for special cases by solving bargaining problems (uniform distribution case) of 'identical' bargain and 'non-identical' bargain. They point out that Myerson and Satterthwaite (1983) prove that in the 'identical' case, comparing all the bargaining rules, STM rule is an optimal bargaining mechanism. In the 'non-identical' bargain, STM is no longer the optimal mechanism for maximising the total gains from trade.

Myerson and Satterthwaite (1983) study and prove impossibility results on bargaining problem relating to efficiency. In bilateral bargaining of single object, two parties (buyer and seller) have private valuations for the object being sold. They prove that in a Bayesian direct mechanism, there does not exist any bargaining game satisfying ex-post efficient and budget balance. Also, they provide the subsidy amount to make bargaining ex-post efficient. They design a direct mechanism that maximises the expected total profit from the trade. They also design a mechanism with the broker and analyse the optimal mechanism when sale happens through the broker only.

Myerson (1985) studies bilateral bargaining problem under the SUTP setting, where each player (buyer and seller) knows their private valuation and do not know about other player's valuation. He analyses different bargaining mechanisms namely neutral bargaining solution (NBM), buyer posted price (BPM), seller posted price (SPM), and split the difference (STM).

To test the properties of sealed-bid bargaining mechanisms, Radner and Schotter (1989) conduct a set of experiments. They find that the buyers and sellers behave consistent with the linear equilibrium bidding strategies and can get more potential gains from trade. Matthews and Postlewaite (1989) study unmediated communication-bidding game where it has two stages: a message where agents communicate followed by a bidding stage where agents bid in a double auction. They find that unmediated communication enlarges the equilibrium set.

Gresik (1991a) analyses the Myerson and Satterthwaite (1983) model with the ex-post individually rational mechanism. He finds that his mechanism maximises the ex-ante expected gains from trade and attains the Chatterjee-Samuelson linear equilibrium. Gresik (1991b) develops the ex-post individually rational mechanism by identifying a new payment rule in the Myerson and Satterthwaite (1983) model. In this new payment rule, the transfer of money happens only in the case of trade happens. They find that the resulting equilibrium is equivalent to linear equilibrium of a k-double auction. A k-double auction is a generalised STM mechanism with k ranges from 0 to 1. When k is equal to $\frac{1}{2}$, it is the STM mechanism.

Valley et al. (2002) present a double auction with two-sided private information where a seller and a buyer submit simultaneously an asking price and an offer price, respectively. They allow preplay communication before bargaining, and the players communicate either through writing or face-face. Myerson and Satterthwaite (1983) highlight that even if the players are allowed to communicate before bargaining, there is no equilibrium that gives higher than the expected gain of Chatterjee-Samuelson linear equilibrium. They analyse the role of communication in enhancing trade efficiency. Results show that when there is communication, the trade incidence is larger than the measured in the no-communication case and Chatterjee-Samuelson linear equilibrium. McGinn et al. (2003) consider double auction with preplay communication similar to Valley et al. (2002) model and find that they can achieve nearly full efficiency. They analyse the enhancement of efficiency by examining dyadic interaction – disclosure and reciprocity.

Saran (2011) modifies Myerson and Satterthwaite (1983) model by considering the agents as naive. To study the effect of naive traders in bilateral bargaining, he considers two approaches:

1 in a mechanism design, deciding the naive traders' proportion in order to maximise efficiency

2 given the mechanism, examine the impact of naive traders.

Saran (2012) allows communication before the double auction by assuming that the agents can be 'naive'. He finds that there is an increase in efficiency.

Flesch et al. (2016) study bilateral bargaining problem with discrete set of valuations. They analyse the existence of ex-post efficiency and interim implementation. Bayrak et al. (2019) study the Myerson and Satterthwaite (1983) problem through a risk-neutral intermediator with the assumption of discrete valuations. The objective of their study is to maximise the intermediator's expected gain using linear programming duality. Also, they study the bilateral problem with risk-averse intermediator and analyse the effect of the intermediator avoiding risk.

Table 1 Bilateral bargaining literature

Author(s) and Year	*Description*	*Environment*		*Allocation*		*Result(s)*
		Static	*Dynamic*	*Efficient*	*Optimal*	
Myerson and Satterthwaite (1983)	Study Bayesian individual rational and incentive compatible bargaining mechanism.	X		X		Myerson-Satterthwaite theorem.
Chatterjee and Samuelson (1983)	Consider bilateral bargaining under incomplete information.	X		X		Split the difference (STM) provides the highest expected total profit.
Myerson (1985)	Analyses lemon and SUTP problems.	X		X		Trade probability and expected payment functions for four bilateral bargaining mechanisms.
Radner and Schotter (1989)	Conduct a set of experiments to test the sealed-bid mechanism.	X		X		Huge profit from the trade.
Matthews and Postlewaite (1989)	Consider an unmediated communication-bidding game.	X		X		Equilibrium outcome extension in double auctions.
Gresik (1991a)	Analyses Myerson and Satterthwaite (1983) model with ex-post individually rational mechanism.	X		X		Achieves Chatterjee-Samuelson linear equilibrium.
Gresik (1991b)	Modifies payment rule in Myerson and Satterthwaite (1983) model in order to make mechanism ex-post individually rational.	X		X		Equilibrium existence is equivalent to *k*-double auction.
Valley et al. (2002)	Communication via written or face-face between agents is allowed before bargaining.	X		X		Due to the communication effect, efficiency is increased.
McGinn et al. (2003)	Study double auction with preplay communication and they analyse dyadic behaviours – disclosure and reciprocity.	X		X		When players are allowed to pre-play communication, full efficiency is achieved.
Saran (2011)	Consider agents as naive in the Myerson and Satterthwaite (1983) model.	X		X		Increase in efficiency.
Saran (2012)	Study double auction with pre-play communication with a positive proportion of naive traders.	X		X		Increase in efficiency.

Table 1 Bilateral bargaining literature (continued)

Author(s) and Year	*Description*	*Environment*		*Allocation*		*Result(s)*
		Static	*Dynamic*	*Efficient*	*Optimal*	
Bhandari and Secomandi (2011)	Study four bilateral bargaining mechanisms from the seller's point of view to maximise the expected revenue in an infinite time horizon.		X		X	Seller prefers SPM mechanism.
Dufalla (2014)	Analyses Bhandari and Secomandi (2011) model over finite time horizon.		X		X	Seller prefers STM, while other mechanisms behave similarly when the inventory levels are sufficiently high and sufficiently less time remaining. Seller prefers SPM, which dominates STM, followed by BPM when the inventory levels are sufficiently low and sufficiently more time remaining. NBM behaves similar to SPM when the seller has less inventory and more time remaining. NBM behaves similar to BPM when the seller has more inventory and less time remaining.
Flesch et al. (2016)	Analyse discrete bilateral bargaining problem.	X		X		Myerson and Satterthwaite (1983) impossibility result is not valid with discrete values.
Bayrak et al. (2019)	Consider bilateral bargaining problem through a risk-neutral intermediator with finite set of valuations using linear programming and compare with the assumption of risk-averse intermediator.	X		X		Risk-neutral intermediator problem gives higher objective function value than risk-averse problem.
Ramkishore and Amit (2019)	Extend Bhandari and Secomandi (2011) model with refusal cost.		X		X	Seller prefers SPM for lower refusal cost and NBM or BPM for higher refusal cost.
Yoon (2020)	Considers contingent contracts in Myerson and Satterthwaite (1983) model.	X		X		No more existence of Myerson-Satterthwaite impossibility result.
Pan and Wang (2021)	Introduce linear contract via a broker in Myerson and Satterthwaite (1983).	X		X		Permits broker to achieve more profits than Myerson and Satterthwaite (1983) model.

Yoon (2020) introduces contingent contracts in Myerson and Satterthwaite (1983) setting and represents the span of viable mechanisms that satisfy incentive compatibility, individual rational, ex-post efficiency, and budget balance. Pan and Wang (2021) analyse the Myerson and Satterthwaite (1983) problem with a broker introducing a linear contract. They design an optimal mechanism that gains more profits than achieved in Myerson and Satterthwaite (1983).

The above studies examine the bargaining problem in a static context related to efficiency and assume the seller's valuation as exogenous. Bhandari and Secomandi (2011) initiate revenue management with a bargaining problem in the dynamic context in an infinite horizon time period. They compare four bilateral bargaining mechanisms analytically as well as numerically from the seller's perspective and find that SPM dominates other mechanisms.

Dufalla (2014) modifies the model of Bhandari and Secomandi (2011) as the finite horizon problem. She finds that the seller selects STM, while the other three mechanisms behave similarly when the inventory levels are sufficiently high and sufficiently less time remaining. Seller prefers SPM, which dominates STM, followed by BPM when the inventory levels are sufficiently low and sufficiently more time remaining. NBM behaves similar to SPM when the seller has less inventory and more time remaining and behaves similar to BPM when the seller has more inventory and less time remaining. Ramkishore and Amit (2019) extend the Bhandari and Secomandi (2011) model with refusal cost. They find that the seller prefers SPM for lower refusal cost and NBM or BPM for higher refusal cost. This paper extends the model of Dufalla (2014) with the consideration of the refusal cost.

3 Bargaining mechanisms in static context

This paper considers a bargaining situation in a dynamic setting between a seller and an arriving buyer and analyses it as a direct revelation game. To build the basic block, this study depends on Myerson and Satterthwaite (1983), Chatterjee and Samuelson (1983), and Myerson (1985).

Myerson and Satterthwaite (1983) use the direct mechanism concept for bilateral trading. Consider a seller who endows an object and a buyer who values the object. The seller knows his valuation v_s and considers the buyer's valuation as a random variable $\tilde{v_b}$ which is distributed according to probability density function $f_b(\cdot)$ and cumulative distribution function $F_b(\cdot)$ with support $[a_b,b_b]$. Similarly, the buyer knows his valuation v_b and considers the seller's valuation as a random variable $\tilde{v_s}$ which is distributed according to probability density function $f_s(\cdot)$ and cumulative distribution function $F_s(\cdot)$ with support $[a_s,b_s]$. The distributions are common knowledge.

According to the revelation principle, attention is restricted to *direct* mechanisms, which is truthful. In a direct bargaining mechanism, the seller and the buyer submit their valuations v_s and v_b, respectively to the mediator. He decides whether the object can be sold and the price paid from the buyer to the seller. Therefore, a direct mechanism consists of two functions: trade probability $p(v_s, v_b)$ and price paid $y(v_s, v_b)$.

Given the player's types being v_s and v_b, the seller's and buyer's probability of trade and expected payment are:

$$\bar{p}_s(v_s) = E[p(v_s, \tilde{v_b})] = \int_{a_b}^{b_b} p(v_s, t_b) f_b(t_b)\, dt_b$$

$$\bar{y}_s(v_s) = E[y(v_s, \tilde{v_b})] = \int_{a_b}^{b_b} y(v_s, t_b) f_b(t_b)\, dt_b$$
$$\bar{p}_b(v_b) = E[p(\tilde{v_s}, v_b)] = \int_{a_s}^{b_s} p(t_s, v_b) f_s(t_s)\, dt_s$$
$$\bar{y}_b(v_b) = E[y(\tilde{v_s}, v_b)] = \int_{a_s}^{b_s} y(t_s, v_b) f_s(t_s)\, dt_s$$

Seller's and buyer's *interim* expected utilities are

$$\bar{u}_s(v_s) = \bar{y}_s(v_s) - v_s \bar{p}_s(v_s)$$
$$\bar{u}_b(v_b) = v_b \bar{p}_b(v_b) - \bar{y}_b(v_b)$$

Mechanism is individually rational if

$$\bar{u}_s(v_s) \geq 0,\ \forall v_s \in [a_s, b_s] \text{ and } \bar{u}_b(v_b) \geq 0,\ \forall v_b \in [a_b, b_b]$$

Mechanism is incentive compatible if

$$\bar{u}_s(v_s) \geq \bar{y}_s(\hat{v_s}) - v_s \bar{p}_s(\hat{v_s}),\ \forall v_s, \hat{v_s} \in [a_s, b_s] \text{ and}$$
$$\bar{u}_b(v_b) \geq v_b \bar{p}_b(\hat{v_b}) - \bar{y}_b(\hat{v_b}),\ \forall v_b, \hat{v_b} \in [a_b, b_b]$$

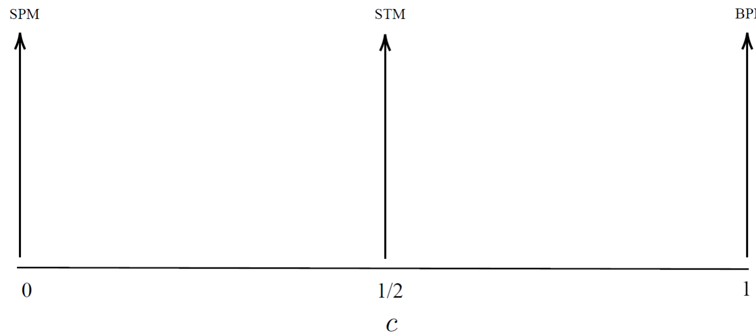
Chatterjee and Samuelson (1983) study a bargaining mechanism under incomplete information where the buyer and the seller submit their sealed offers b and s, respectively. In the case of an 'identical' bargain, the seller's valuation v_s and buyer's valuation v_b are uniformly distributed on $[0, 1]$. Bargaining happens only if $b \geq s$ and at price p,

$$p = cb + (1 - c)s, \quad 0 \leq c \leq 1 \tag{1}$$

If the buyer's and seller's valuations are independent random variables and uniformly distributed between [0, 1], this type of bargaining problem is called symmetric uniform trading problem (SUTP). In the SUTP setting, Myerson (1985) analyses different bargaining mechanisms namely neutral bargaining solution (NBM), buyer posted price (BPM), seller posted price (SPM), and split the difference (STM).

When $c = 0$ in equation (1), it is equivalent to SPM where the seller decides the price for his/her item, at which the buyer can either accept to buy or reject it. When $c = 1$ in equation (1), it is equivalent to BPM where the buyer decides the price to buy for an item at which the seller can either accept to sell or reject it. When $c = \frac{1}{2}$ in (1), it is known as STM mechanism where the buyer's price and the seller's price would be averaged. Under the sealed bid bargaining studied by Chatterjee and Samuelson (1983) in the case of an 'identical' bargain for $c = \frac{1}{2}$, the equilibrium strategies for the buyer and the seller are $\frac{2}{3}v_b + \frac{1}{12}$ and $\frac{2}{3}v_s + \frac{1}{4}$, respectively. The price to be paid is averaged to split the difference. Chatterjee and Samuelson (1983) highlight that the STM rule maximises the expected total gains from trade. Therefore, this STM rule is attractive for symmetric and identical bargain. Figure 1 shows the corresponding mechanism with respect to different c values.

Figure 1 Different mechanisms with respect to c values



Nash (1950) develops Nash bargaining solution mechanism for two-player, risk-neutral bargaining game under complete information. Myerson (1984) extends Nash bargaining solution to an incomplete information game and develops NBM mechanism. Myerson (1985) highlights that for the SUTP setting, NBM may be a good model for face-to-face negotiations. When the buyer's valuation v_b is close to 0 and if the trade does not happen, he/she will incur a minimum loss. Therefore, the buyer is said to be having strong bargaining power. If v_b is closer to 0 than v_s is closer to 1 i.e. $v_b < 1 - v_s$, then the buyer who has strong bargaining power than the seller will posts the price. Similarly, when the seller's valuation is close to 1, he is said to be having strong bargaining power. If v_s is closer to 1 than v_b closer to 0 i.e. $v_b > 1 - v_s$, then the seller who has strong bargaining power than the buyer will posts the price.

4 Model

This paper considers a monopoly seller who owns an initial capacity of I identical and discrete items of a product to sell in a predetermined selling period. Time n represents a discrete-time period where there are n time units to go in the sales horizon, and it is indexed by $n = N$, $N - 1$, …, 1 (reversed time index). In any time period n, at most a non-strategic customer who has private valuation v_b arrives with unit demand. Let α denotes the customer arrival probability, independent of the other periods and i denotes the number of units remaining in any period n. In each period, the seller's discount factor is $\delta \in [0, 1)$ irrespective of the time period, the number of remaining units, and the arrival of the customer. N, i, I, α, and δ are seller's private information.

Seller's valuation is denoted as v_s which depends endogenously on the remaining inventory i and remaining time period n. From the buyer's perspective, v_s is an independent random variable which is distributed according to cumulative distribution function $F_s(\cdot)$ with support $V_s := [0, 1]$. From the perspective of the seller, v_b is an independent random variable which is distributed according to cumulative distribution function $F_b(\cdot)$ with support $V_b := [0, 1]$.

When a buyer arrives, the mediator applies a direct and feasible mechanism k. In the direct mechanism shown in Figure 2, the seller and the buyer submit their valuations v_s and v_b, respectively to the mediator. He decides whether trade happens (indicated by the function $1\{f^k(v_s, \tilde{v_b}) = 1\}$) or not $(1\{f^k(v_s, \tilde{v_b}) = 0\})$ and the price paid from the buyer to the seller. If the item does not get transferred, the seller faces a refusal cost r. The customer type determines the refusal cost r and it is assumed as constant. A direct mechanism k consists of two functions: trade probability $p^k(v_s, v_b) \in [0, 1]$ and price paid $y^k(v_s, v_b) \in R$. Transaction (trade) happening condition and price paid for four mechanisms are shown in Table 2. Therefore, in the context of the seller,

$$\text{Trade probability is } \bar{p_s}^k(v_s) = \int_{v_b \in V_b} p^k(v_s, v_b) dF_b(v_b)$$

$$\text{Expected payment is } \bar{y_s}^k(v_s) = \int_{v_b \in V_b} y^k(v_s, v_b) dF_b(v_b)$$

Figure 2 Bilateral bargaining mechanism over finite horizon

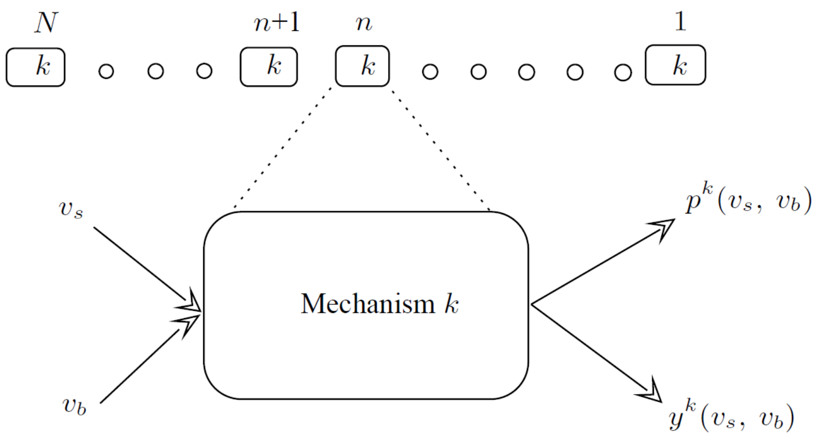


Table 2 BPM, SPM, NBM, and STM mechanisms in the SUTP setting

Mechanism k	*Transaction happening situation*	$y^k(v_s, v_b)$
SPM	$v_b \geq (1 + v_s)/2$	$(1 + v_s)/2$
STM	$v_b \geq v_s + 1/4$	$(v_s + v_b + 1/2)/3$
BPM	$v_b/2 \geq v_s$	$v_b/2$
NBM	$v_b \geq 3v_s$ or $3v_b - 2 \geq v_s$	$1\{v_b \leq 1 - v_s\}v_b/2$ $+1\{v_b > 1 - v_s\}(1 + v_s)/2$

Source: Myerson (1985) and Bhandari and Secomandi (2011)

The bargaining situation is modelled as a MDP with the inventory remaining and the time period remaining as state variables. This paper studies the seller's problem that maximises the optimal total expected discounted revenue $U_r^k(i, n)$ as a function of the remaining amount of i, remaining n period, and refusal cost r under mechanism k. Equation (2) determines the seller's valuation as a function of remaining inventory i and time n to go. The functional equation for the finite horizon is

$$\begin{aligned} U_r^k(i, n) = &(1 - \alpha)\delta U_r^k(i, n - 1) + \alpha\ max_{v_s} E[y^k(v_s, \tilde{v_b}) \\ &+ \delta U_r^k(i - 1, n - 1)1\{f^k(v_s, \tilde{v_b}) = 1\} \\ &+ (-r + \delta U_r^k(i, n - 1))1\{f^k(v_s, \tilde{v_b}) = 0\}], \quad \forall\ i \in I \end{aligned} \tag{2}$$

with boundary conditions $U_r^k(0, n) := 0$ for $n = 1, 2, \cdots, N$ and $U_r^k(i, 0) := 0$ for $i > 0$.

4.1 Effect of r on optimal value functions under bilateral bargaining mechanisms

Without considering refusal cost, Dufalla (2014) gives the seller's preference order of bargaining mechanisms. With refusal cost r, it influences the changes in value functions. As a consequence, when r increases, the order preference of mechanisms changes. Mechanisms k and l perform in the similar fashion at refusal cost r^*_{kl}. Lemma 1 determines the refusal cost r^*_{kl} for given i and n.

Lemma 1: If $U_r^k(i,n) = U_r^l(i,n)$ for all i and n time period to go at $r = r^*_{kl}$, therefore the r^*_{kl} value is derived as

$$r^*_{kl} = \frac{\delta U_r^k(i,n-1) - \delta U_r^l(i,n-1)}{\alpha[(1-\bar{p_s}^k(v^*_{s,r}(i,n))) - (1-\bar{p_s}^l(v^*_{s,r}(i,n)))]} + \frac{\alpha\begin{bmatrix}\bar{y_s}^k(v^*_{s,r}(i,n)) - \bar{y_s}^l(v^*_{s,r}(i,n)) \\ -\delta\Delta U_r^k(i,n-1)\bar{p_s}^k(v^*_{s,r}(i,n)) \\ +\delta\Delta U_r^l(i,n-1)\bar{p_s}^l(v^*_{s,r}(i,n))\end{bmatrix}}{\alpha[(1-\bar{p_s}^k(v^*_{s,r}(i,n))) - (1-\bar{p_s}^l(v^*_{s,r}(i,n)))]}$$

Proof: At time n, in terms of marginal inventory value, let $\Delta U_r^k(i,n) = U_r^k(i,n) - U_r^k(i-1,n)$, then rewriting equation (2) as,

$$U_r^k(i,n) = \delta U_r^k(i,n-1) + \alpha \max_{v_s} E[y^k(v_s,\tilde{v_b}) - \delta\Delta U_r^k(i,n-1)1\{f^k(v_s,\tilde{v_b})=1\} - r1\{f^k(v_s,\tilde{v_b})=0\}]$$

From $E[y^k(v_s,\tilde{v_b})] \equiv \bar{y_s}^k(v_s)$, $E[1\{f^k(v_s,\tilde{v_b})=1\}] \equiv \bar{p_s}^k(v_s)$, the above equation can be rewritten as,

$$U_r^k(i,n) = \delta U_r^k(i,n-1) + \alpha \max_{v_s}[\bar{y_s}^k(v_s) - \delta\Delta U_r^k(i,n-1)\bar{p_s}^k(v_s) - r(1-\bar{p_s}^k(v_s))]$$

Let

$$v^*_{s,r}(i,n) \in \text{argmax}_{v_s}[\bar{y_s}^k(v_s) - \delta\Delta U_r^k(i,n-1)\bar{p_s}^k(v_s) - r(1-\bar{p_s}^k(v_s))]$$

Therefore, the value function for a given r as a function of i for remaining time n under mechanism k is

$$U_r^k(i,n) = \delta U_r^k(i,n-1) + \alpha[\bar{y_s}^k(v^*_{s,r}(i,n)) - \delta\Delta U_r^k(i,n-1)\bar{p_s}^k(v^*_{s,r}(i,n)) - r(1-\bar{p_s}^k(v^*_{s,r}(i,n)))]$$

The value function for a given r as a function of i for remaining time n under mechanism l is

$$U_r^l(i,n) = \delta U_r^l(i,n-1) + \alpha[\bar{y_s}^l(v^*_{s,r}(i,n)) - \delta\Delta U_r^l(i,n-1)\bar{p_s}^l(v^*_{s,r}(i,n)) - r(1-\bar{p_s}^l(v^*_{s,r}(i,n)))]$$

Let $U_r^k(i,n) = U_r^l(i,n)$ for given i and n,

$$\begin{aligned}
&\delta U_r^k(i,n-1) + \alpha[\bar{y_s}^k(v_{s,r}^*(i,n)) \\
&- \delta\Delta U_r^k(i,n-1)\bar{p_s}^k(v_{s,r}^*(i,n)) - r(1-\bar{p_s}^k(v_{s,r}^*(i,n)))] \\
&\quad = \delta U_r^l(i,n-1) + \alpha[\bar{y_s}^l(v_{s,r}^*(i,n)) \\
&\quad - \delta\Delta U_r^l(i,n-1)\bar{p_s}^l(v_{s,r}^*(i,n)) - r(1-\bar{p_s}^l(v_{s,r}^*(i,n)))] \\
&\delta U_r^k(i,n-1) - \delta U_r^l(i,n-1) + \alpha[\bar{y_s}^k(v_{s,r}^*(i,n)) - \bar{y_s}^l(v_{s,r}^*(i,n)) \\
&- \delta\Delta U_r^k(i,n-1)\bar{p_s}^k(v_{s,r}^*(i,n)) + \delta\Delta U_r^l(i,n-1)\bar{p_s}^l(v_{s,r}^*(i,n))] \\
&\quad = \alpha r[(1-\bar{p_s}^k(v_{s,r}^*(i,n))) - (1-\bar{p_s}^l(v_{s,r}^*(i,n)))]
\end{aligned}$$

Therefore,

$$\begin{aligned}
r_{kl}^* = &\frac{\delta U_r^k(i,n-1) - \delta U_r^l(i,n-1)}{\alpha[(1-\bar{p_s}^k(v_{s,r}^*(i,n))) - (1-\bar{p_s}^l(v_{s,r}^*(i,n)))]} \\
&+ \frac{\alpha\begin{bmatrix} \bar{y_s}^k(v_{s,r}^*(i,n)) - \bar{y_s}^l(v_{s,r}^*(i,n)) \\ -\delta\Delta U_r^k(i,n-1)\bar{p_s}^k(v_{s,r}^*(i,n)) \\ +\delta\Delta U_r^l(i,n-1)\bar{p_s}^l(v_{s,r}^*(i,n)) \end{bmatrix}}{\alpha[(1-\bar{p_s}^k(v_{s,r}^*(i,n))) - (1-\bar{p_s}^l(v_{s,r}^*(i,n)))]}
\end{aligned}$$

5 Dynamic pricing

Consider a seller who has I units of identical and discrete items of a product to sell in a finite time period N. The time period is discrete and it is indexed by $n = N$, $N-1$, ..., 1 (reversed time index). Therefore, smaller n values represent the later time periods. At any time n, at most one buyer who is non-strategic arrives with probability α. Arriving buyers demand one unit and have a valuation of v_b for that unit demand. The seller believes that v_b is an independent random variable distributed according to $F_b(\cdot)$ with support [0, 1]. In each time n, the seller posts the price p for each unit of a product. Each arriving buyer decides whether to purchase the product at price p or not. Trade or transaction happens only if buyer's valuation $v_b \geq p$. When a transaction does not materialise, the seller is charged a refusal cost of r.

Let i denotes the inventory remaining in each time period. Let $U_r(i,n)$ be the seller's value function given the state i and refusal cost r when there is n time period to go. Let $\delta \in [0,1)$ be the discount factor of the seller's one-time period. Seller's private information are I, i, α, δ, and N. This paper studies the problem from the perspective of the seller that maximises the optimal total expected discounted revenue. Equation (3) gives the optimal price p as a function of remaining inventory i and time n to go.

$$\begin{aligned}
U_r(i,n) = (1-\alpha)\delta U_r(i,n-1) + \alpha \max\nolimits_p[(1-F(p))(p+\delta U_r(i-1,n-1)) \\
+ (-r+\delta U_r(i,n-1))F(p)], \quad \forall i \in 1,2,\ldots I
\end{aligned} \tag{3}$$

with boundary conditions $U_r(0,n) := 0$ for $n = 1,2,\ldots,N$ and $U_r(i,0) := 0$ for $i \geqslant 0$. Given the price p, $1-F(p)$ is the probability of trade happens.

6 Solution procedure

To solve bargaining and dynamic pricing models that compute value functions and optimal policies, backward induction is used which is described below:

1 Set $n = 0$ and $U_r^k(i, 0) = 0, \ \forall \ i$

2 Substitute $n + 1$ for n, therefore when $n = 1$, compute $U_r^k(i, 1), \ \forall \ i$.

Select $v_{s,r}^*(i, 1) \in U_r^k(i, 1)$, $\forall i$.

3 If $n = N$, stop. Otherwise, goto step 2.

7 Simulation approach

This section develops a simulation-based approach to validate the findings of the MDP model as shown below:

Step 1 Assume inventory level I, N time period to go, and refusal cost r.

Step 2 For each mechanism, run the backward induction algorithm to find the optimal policy v_s which is the input for the simulator for all inventory levels and time periods to go.

Step 3 Number of mechanisms are 4. Therefore $M = 4$.

Step 4 Assume number of runs or iterations IT.

Step 5 Let $it = 1$.

Step 6 Generate random numbers for customer arrival and buyer's valuation for all possible states (i, n).

Step 7 Let $m = 1$.

Step 8 Let $i = 0$.

Step 9 Let $i = i + 1$, $revenue(i) = 0$, and $n = 0$.

Step 10 Let $n = n + 1$.

Step 11 Check the customer arrival random number with the customer arrival probability.

Step 12 If the buyer arrives, check the buyer's valuation v_b with seller valuation v_s whether trade happens or not for the respective mechanism. If the item gets sold, inventory is reduced by one unit and the revenue is calculated as

$$revenue(i) = revenue(i) + payment.$$

Step 13 If the item does not get sold, inventory i remains, and the revenue is calculated as,

$$revenue(i) = revenue(i) - r$$

Step 14 If $n < N$, then goto step 10.

Step 15 If $i < I$, then goto step 9 else $m = m + 1$.

Step 16 If $m <= M$, then goto step 8.

Step 17 Let $it = it + 1$.

Step 18 if $it <= IT$, then goto step 6.

Step 19 Average revenue of each mechanism m for varies inventory levels i is calculated as,

$$average\ revenue(i) = \sum_{it=1}^{IT} revenue(i)/IT \quad \forall i = 1, 2, ...I$$

Step 20 Stop.

The simulation study is conducted for IT number of iterations. Each iteration represents the seller having a fixed inventory level to sell over finite time periods. Revenue is calculated in each iteration, and the average revenue is obtained by summing up the revenues from each iteration and dividing the total sum by the number of iterations. In order to compare four mechanisms, common random numbers are used for all mechanisms in each iteration.

8 Numerical investigation and sensitivity analysis

This section investigates the problem numerically to compare the performance of the seller under the four bargaining mechanisms with the inclusion of refusal cost r. Sensitivity study is performed to find the significance of model parameters on value function. In this section, dynamic pricing is compared with four bilateral bargaining mechanisms with and without the refusal cost r. Also, the relative performance of the four bilateral bargaining mechanisms with refusal cost is discussed.

8.1 Comparison among bargaining mechanisms

Considering a refusal cost r to be incurred by the seller where r varies from 0.05 to 0.45 per rejection, this paper assumes data as follows: N (time period) – 100 to go, i (inventory levels) – varies from 0 to 100, α (probability of arrival) – 0.30, and δ (discount factor) – 0.9998.

For the lower value of r [$r = 0.05$ in Figure 3(a)], when the inventory is low comparing the time period to go, SPM dominates STM, which dominates BPM. NBM performs similar to SPM when the seller has low inventory comparing the remaining time period to go and similar to BPM when the seller has high inventory comparing the remaining time period to go. When the inventory level increases compared to the remaining time period to go, STM dominates NBM, which dominates BPM and SPM. When the inventory level further increases, BPM and NBM behave in the same manner and dominate SPM.

Figure 3 Value function for different refusal costs when $N = 100$, (a) $r = 0.05$ (b) $r = 0.20$ (c) $r = 0.35$ (d) $r = 0.45$

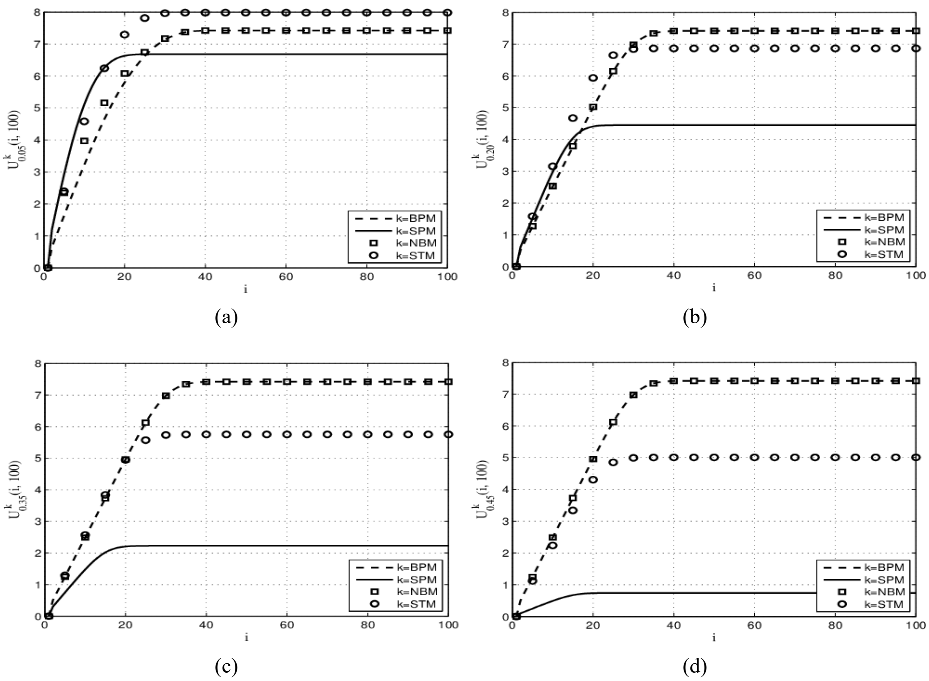


Figure 4 Sensitivity analysis modifying $N = 500$ when $\alpha = 0.30$ and $\delta = 0.9998$, (a) $r = 0.05$ (b) $r = 0.20$ (c) $r = 0.35$ (d) $r = 0.45$

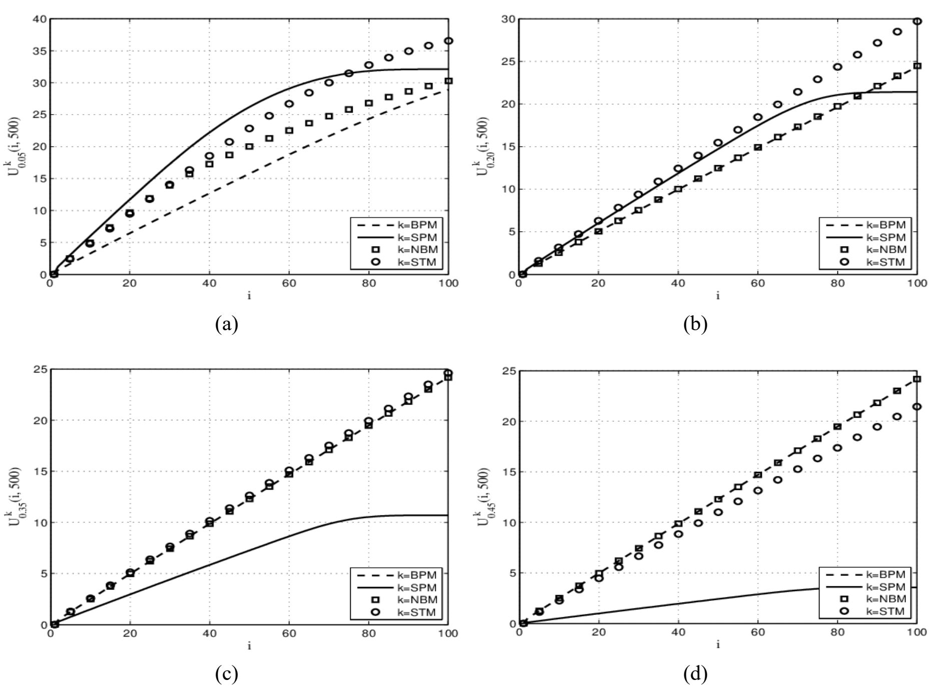


Figure 5 Sensitivity analysis modifying α when $N = 100$ and $\delta = 0.9998$, (a) $\alpha = 0.60$ and $r = 0.05$ (b) $\alpha = 0.60$ and $r = 0.20$ (c) $\alpha = 0.90$ and $r = 0.05$ (d) $\alpha = 0.90$ and $r = 0.20$

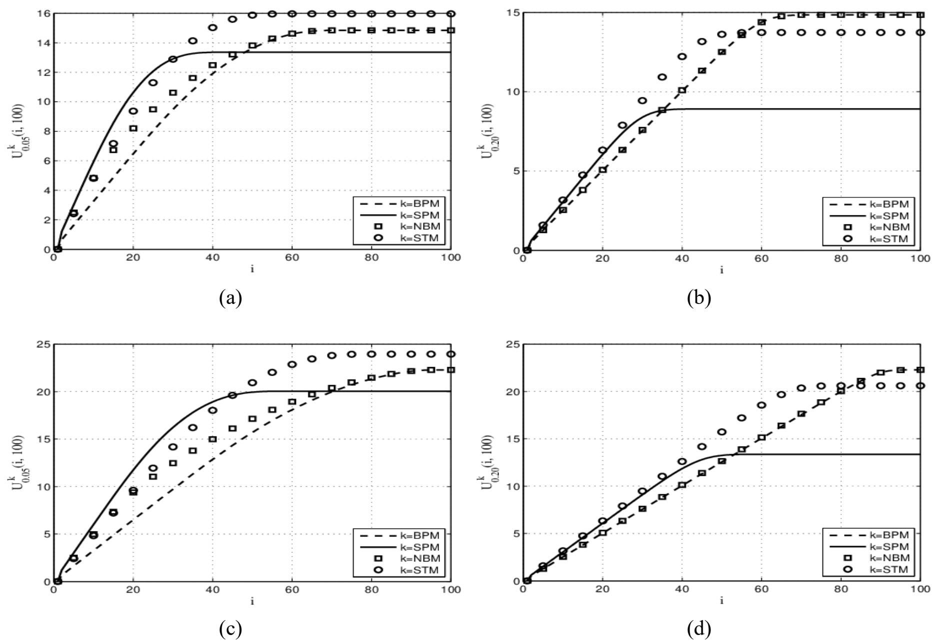
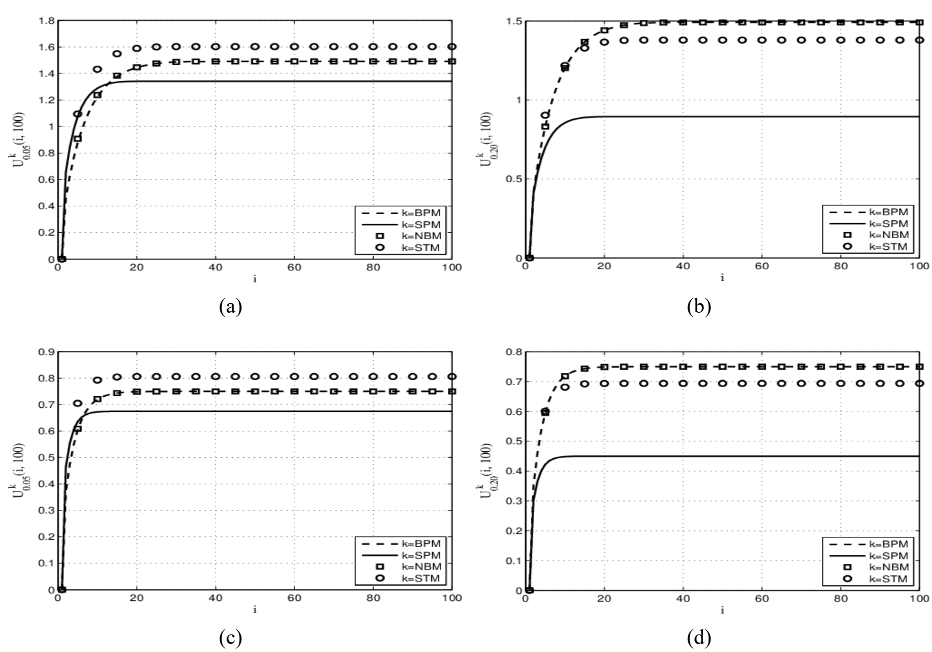


Figure 6 Sensitivity analysis modifying δ when $N = 100$ and $\alpha = 0.30$, (a) $\delta = 0.95$ and $r = 0.05$ (b) $\delta = 0.95$ and $r = 0.20$ (c) $\delta = 0.90$ and $r = 0.05$ (d) $\delta = 0.90$ and $r = 0.20$



When there is an increase in the r to 0.20 [Figure 3(b)], STM dominates SPM, which dominates NBM and BPM for low inventory level comparing time periods to go. When the inventory increases, NBM dominates BPM, which dominates STM and SPM. When the inventory level further increases, NBM and BPM behave in the same manner and together dominate STM and SPM. When the r further increases to 0.35 [Figure 3(c)], for low inventory, STM dominates NBM and BPM. SPM is least preferred irrespective of inventory level. When the inventory increases, NBM and BPM together dominate STM, followed by SPM. From Figure 3(d), for the higher value of r ($r = 0.45$), NBM and BPM together dominate STM, followed by SPM irrespective of inventory level.

Figure 7 Seller's preferences over cost when inventory level $i = 10$

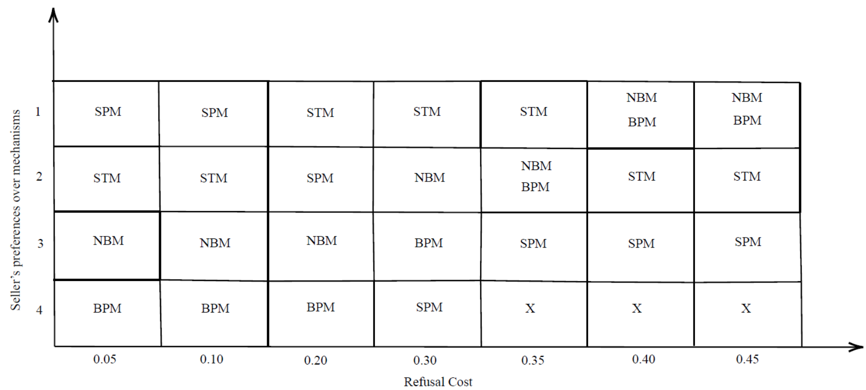
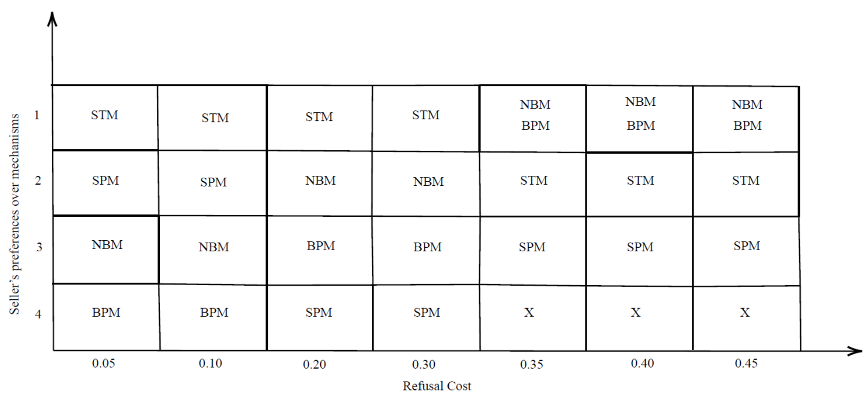


Figure 8 Seller's preferences over cost when inventory level $i = 20$



A sensitivity study is examined to check the influence of modifications in N, α, and δ parameters on the value function (Figures 4–6). It is carried out by modifying only one parameter at a time and maintaining all other parameter values unchanged. The preference of the seller might not be responsive to time N. However, it is found that α and δ impact the seller's preference. Figures 5(a) and 5(c) show that the increase in the probability of buyer arrival implies that the seller is in higher bargaining power for the same refusal cost. When the discount factor is more, the seller will ignore the future revenue. Therefore, the optimal policy of the seller is to sell the inventory soon.

He reduces his valuation for the trade to happen. Therefore, he is indifferent between negotiation and buyer posted price. Figures 7–9 show the summary of the preferences of the seller for various refusal costs and inventory levels when $N = 100$ periods to go.

Figure 9 Seller's preferences over cost when inventory level $i = 40$ to $i = 100$

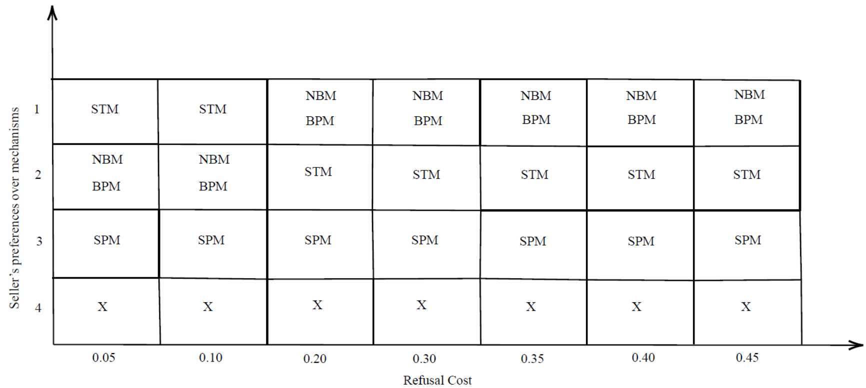
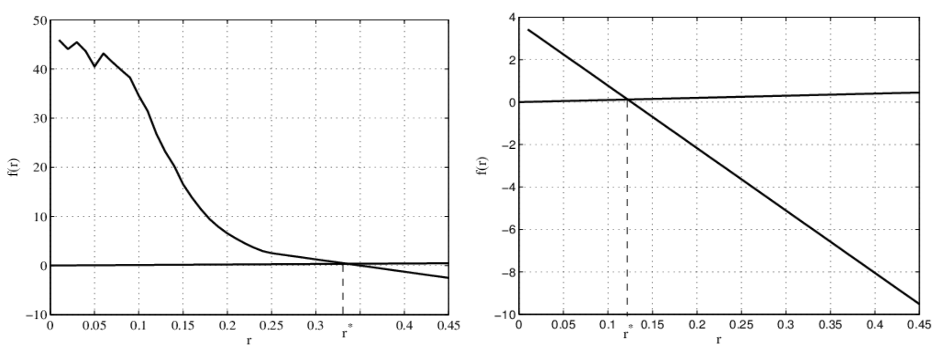
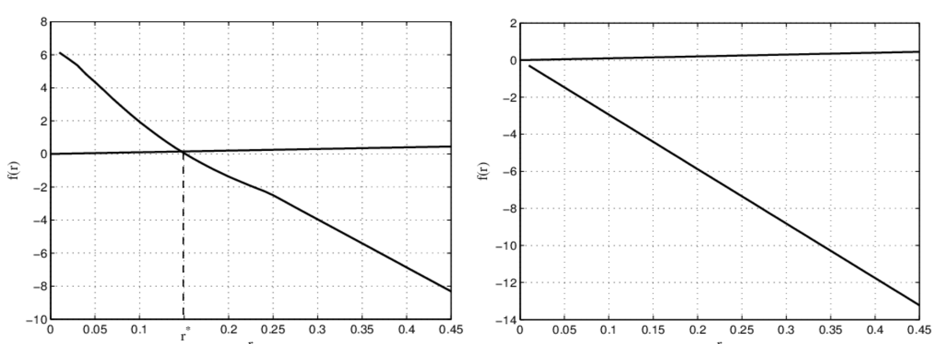


Figure 10 Comparison between STM and NBM mechanism, (a) $i = 20$ (b) $i = 70$



(a) (b)

Figure 11 Comparison between SPM and BPM mechanism, (a) $i = 20$ (b) $i = 70$



(a) (b)

To illustrate Lemma 1, Figures 10 and 11 show the comparison of mechanisms between k and l. Figure 10(a) shows STM dominates NBM when $r < 0.33$ for $i = 20$. Figure 10(b) shows STM dominates NBM when $r < 0.12$ for $i = 70$. At $r^* = 0.33$, NBM and STM perform analogously for $i = 20$ and at $r^* = 0.12$ for $i = 70$. From Figure 11(a), BPM and SPM perform analogously at $r^* = 0.15$ for $i = 20$. Figure 11(b) shows BPM dominates SPM for all r when $i = 70$.

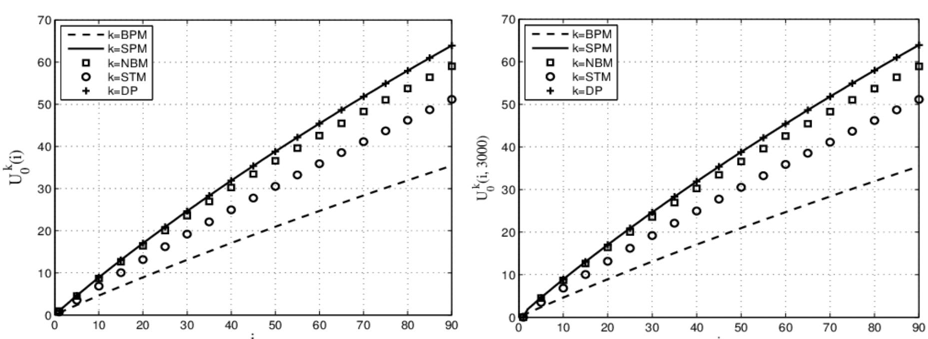
8.2 Comparison of dynamic pricing and bargaining mechanisms

Table 3 shows the seller's preference when comparing dynamic pricing (DP) with bargaining mechanisms without the refusal cost. When there is a very low inventory i remaining compared to the time period to go, SPM and dynamic pricing together dominate other mechanisms. When there are sufficient inventories, the seller prefers STM. Without the refusal cost, dynamic pricing and SPM perform in a similar fashion.

Table 3 Seller's preference when comparing dynamic pricing with bargaining mechanisms without refusal cost

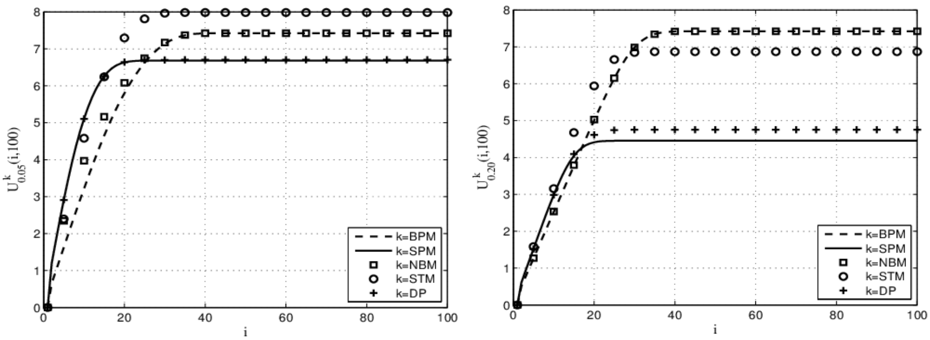
	Inventory i						
Time n	*50*	*100*	*200*	*400*	*600*	*800*	*1,000*
50	STM	STM	STM	STM	STM	STM	STM
100	STM	STM	STM	STM	STM	STM	STM
200	STM	STM	STM	STM	STM	STM	STM
400	SPM&DP	STM	STM	STM	STM	STM	STM
600	SPM&DP	STM	STM	STM	STM	STM	STM
800	SPM&DP	SPM&DP	STM	STM	STM	STM	STM
1,000	SPM&DP	SPM&DP	STM	STM	STM	STM	STM

Figure 12 Convergences to infinite horizon result when N = 3,000, (a) infinite horizon (b) finite horizon



(a) (b)

Figure 13 Comparison of dynamic pricing and bargaining mechanisms with refusal cost, (a) $r = 0.05$ (b) $r = 0.20$



(a) (b)

Figure 12(a) shows the performance of dynamic pricing with bargaining mechanisms in an infinite time period shown in Bhandari and Secomandi (2011) model. When there are large time periods to go (say $N = 3{,}000$), the comparison shown in Figure 12(b) converges to an infinite time period result. For the low refusal cost (say $r = 0.05$) shown in Figure 13(a), dynamic pricing and SPM perform in the same manner. From Figure 13(b), when the refusal cost increases, dynamic pricing dominates SPM.

8.3 Simulation experiments

Simulation study is performed under various experiments by changing parameters such as refusal cost r, inventory level i, buyer arrival probability α, and discount factor δ as shown in Table 4. To illustrate the simulation approach, each experiment is simulated for 1000 iterations. In each iteration, revenue is recorded for various inventory levels. The average revenue of inventory level i in each mechanism is computed by taking the revenue of inventory level i over all iterations. As an illustration, a comparison of mechanisms using simulation from experiment 1 to experiment 8 is shown in Figure 14. Table 5 shows the percentage gap between average revenue and p-value for each experiment. As given in Gocgun et al. (2011), the % gap in average revenue is calculated by,

$$= \frac{\text{avg. revenue (best mechanism)} - \text{avg. revenue (other mechanisms)}}{\text{avg. revenue (best mechanism)}} \times 100$$

In each experiment, the mechanism which achieves the highest average revenue is the best, and it varies as a function of i, r, α, and δ. Based on average revenue at a 5% significance level, the p-value is calculated using a one-tailed paired t-test. It is achieved by analysing the best mechanism performance with the mechanism that attains the least % gap.

$$H_0 : \mu_{Best} - \mu_{SecondBest} \leq 0$$
$$H_a : \mu_{Best} - \mu_{SecondBest} > 0$$

The simulation study results imply that the best mechanism in each experiment is the same as the preferred mechanism in the MDP model.

Table 4 Experiments by changing parameters when 100 periods to go

Exp no.	r	i	α	δ
1	0.05	10	0.3	0.9998
2	0.05	90	0.3	0.9998
3	0.20	10	0.3	0.9998
4	0.20	90	0.3	0.9998
5	0.35	10	0.3	0.9998
6	0.35	90	0.3	0.9998
7	0.45	10	0.3	0.9998
8	0.45	90	0.3	0.9998
9	0.05	10	0.6	0.9998
10	0.05	50	0.6	0.9998
11	0.20	10	0.6	0.9998
12	0.20	50	0.6	0.9998
13	0.05	10	0.9	0.9998
14	0.05	50	0.9	0.9998
15	0.20	10	0.9	0.9998
16	0.20	50	0.9	0.9998
17	0.05	50	0.3	0.95
18	0.05	90	0.3	0.95
19	0.20	50	0.3	0.95
20	0.20	90	0.3	0.95
21	0.05	50	0.3	0.90
22	0.05	90	0.3	0.90
23	0.20	50	0.3	0.90
24	0.20	90	0.3	0.90

Table 5 Simulation study results

Exp no.	*Best mechanism*	*SPM*	*STM*	*BPM*	*NBM*	*p-value*
1	SPM	–	10.4	37.4	22	$< 10^{-5}$
2	STM	16.8	–	7.98	7.98	$< 10^{-5}$
3	STM	4.3	–	19.76	19.31	$< 10^{-5}$
4	BPM&NBM	37.84	6.82	–	–	$< 10^{-5}$
5	STM	38.88	–	5.03	5.03	$< 10^{-5}$
6	BPM&NBM	69.74	22.04	–	–	$< 10^{-5}$
7	BPM&NBM	80.01	11.39	–	–	$< 10^{-5}$
8	BPM&NBM	91.65	32.85	–	–	$< 10^{-5}$
9	SPM	–	19.00	45.07	18.08	$< 10^{-5}$
10	STM	16.03	–	14.29	13.22	$< 10^{-5}$
11	STM	3.84	–	20.05	19.45	$< 10^{-5}$
12	STM	34.22	–	8.01	7.85	$< 10^{-5}$
13	SPM	–	19.29	45.81	16.93	$< 10^{-5}$
14	STM	4.78	–	24.98	18.07	$< 10^{-5}$
15	STM	4.45	–	19.85	19.37	$< 10^{-5}$
16	STM	16.58	–	20.49	19.85	$< 10^{-5}$
17	STM	17.64	–	8.69	8.69	$< 10^{-5}$
18	STM	14.05	–	6.64	6.64	$< 10^{-5}$
19	BPM&NBM	36.49	5.21	–	–	0.00
20	BPM&NBM	39.03	6.71	–	–	$< 10^{-5}$
21	STM	18.3	–	7.23	7.23	$< 10^{-5}$
22	STM	18.4	–	7.25	7.25	$< 10^{-5}$
23	BPM&NBM	41.94	12.38	–	–	$< 10^{-5}$
24	BPM&NBM	37.73	7.29	–	–	0.00

Figure 14 Comparison of mechanisms using simulation approach, (a) $r = 0.05$ (b) $r = 0.20$ (c) $r = 0.35$ (d) $r = 0.45$

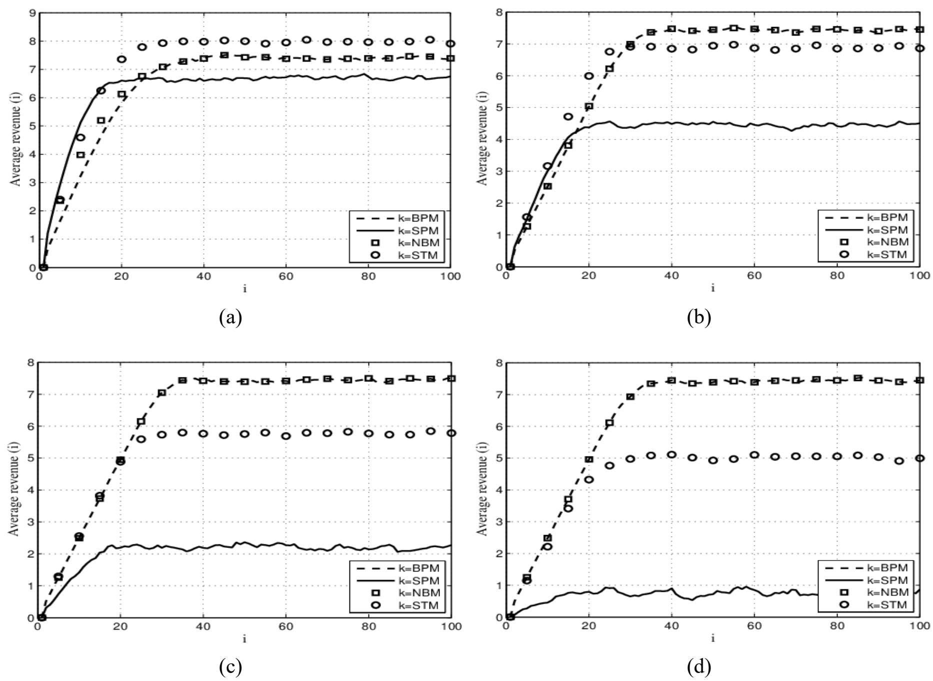
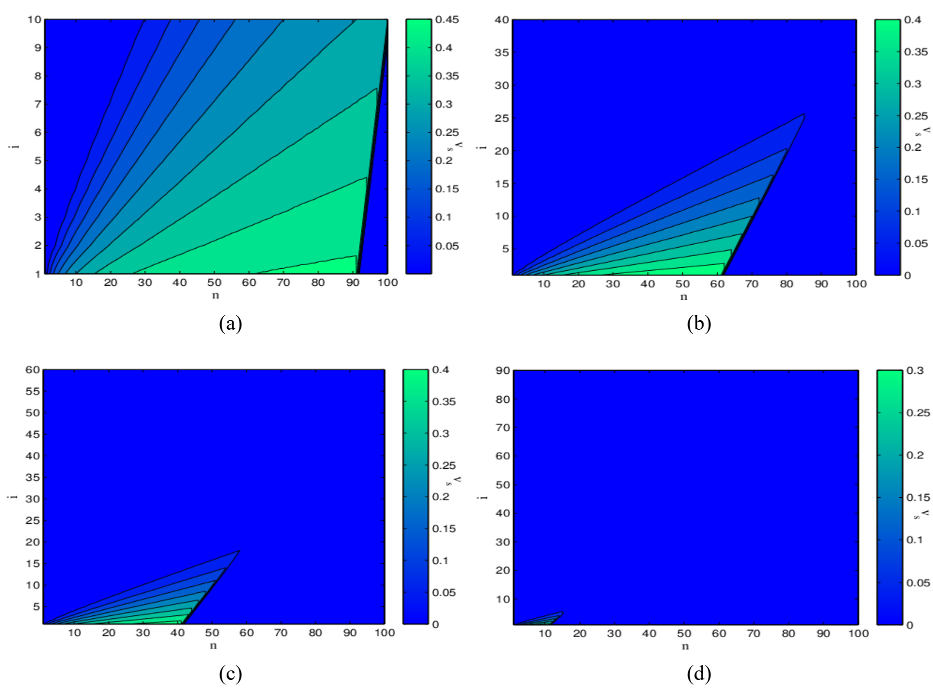


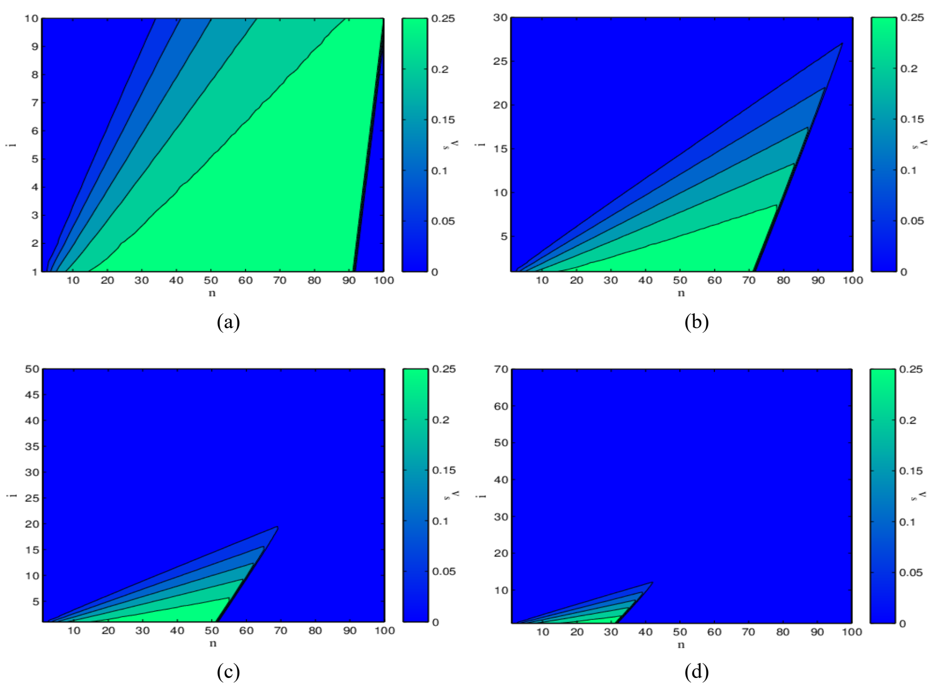
Figure 15 Seller's valuation v_s for various inventory levels i when $r = 0$, (a) $i = 10$ (b) $i = 40$ (c) $i = 60$ (d) $i = 90$ (see online version for colours)



9 Discussion

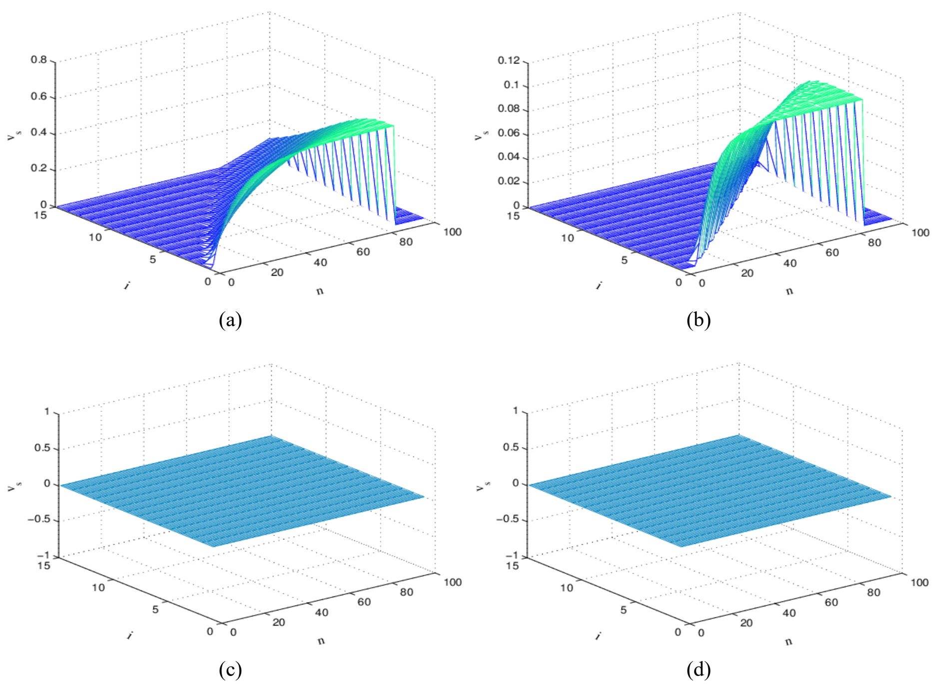
In the Dufalla (2014) model without refusal cost ($r = 0$), the seller prefers SPM when there is sufficiently low inventory and sufficiently more time period to go. Seller prefers STM, and the other three mechanisms behave similarly when there is sufficiently more inventory and sufficiently less time period to go. Seller's valuation approaches zero when there is sufficiently more inventory and sufficiently less time periods to go. Therefore, when the inventory level increases, the value function of these mechanisms does not change and remains constant. For illustration, Figure 15 shows the seller's valuation for different inventory levels and time period remaining without refusal cost. As shown in Figure 15, due to the valuations of the seller being zero, the expected payment function of BPM, SPM, and NBM mechanisms becomes $\frac{1}{4}$ and the expected payment function of STM mechanism becomes $\frac{9}{32}$. Since the magnitude of the STM payment function is more than other mechanisms, STM dominates other mechanisms when there is sufficiently more inventory and sufficiently less time remaining.

Figure 16 Seller's valuation v_s for various inventory levels i when $r = 0.05$, (a) $i = 10$ (b) $i = 30$ (c) $i = 50$ (d) $i = 70$ (see online version for colours)



The probability of no trade (the probability of an item not getting sold) and the expected price (payment) for four mechanisms are shown in Table 6. For the lower value of cost ($r = 0.05$), when there is a sufficiently low inventory with sufficiently high time-to-go, the order of preferences is the same as the case of no refusal cost. Without refusal cost, when there is sufficient inventory, STM dominates other mechanisms, whereas SPM, BPM, and NBM mechanisms behave similarly. The value function of the seller weakly decreases with r for all mechanisms. The seller's expected payment and the probability of trade depend on his valuation. As shown in Figure 16, when there is sufficiently more inventory and sufficiently less time-to-go, the seller's valuation v_s decreases and approaches zero, thereby the value function becomes constant. With refusal cost, the value function of the four bargaining mechanisms decreases in a different manner due to the product of the probability of no trade and refusal cost (probability of no trade function). From Table 6, it is evident that when $v_s = 0$, for both NBM and BPM mechanisms, the expected payment is $\frac{1}{4}$ with zero probability of no trade. Therefore, for both NBM and BPM mechanisms, the value functions converge to the same value. Since the probability of no trade function is zero for BPM and NBM mechanisms, value functions remain the same even when the refusal cost increases. In the case of SPM and STM mechanisms, if $v_s = 0$, the probability of no trade function becomes $r/2$ and $r/4$, respectively, thereby the value function decreases. Since the probability of no trade function for SPM is higher than the STM mechanism, its value function decreases by a significant amount.

Figure 17 Seller's valuation v_s for different r when i from 1 to 15, (a) $r = 0.05$ (b) $r = 0.20$ (c) $r = 0.35$ (d) $r = 0.45$ (see online version for colours)



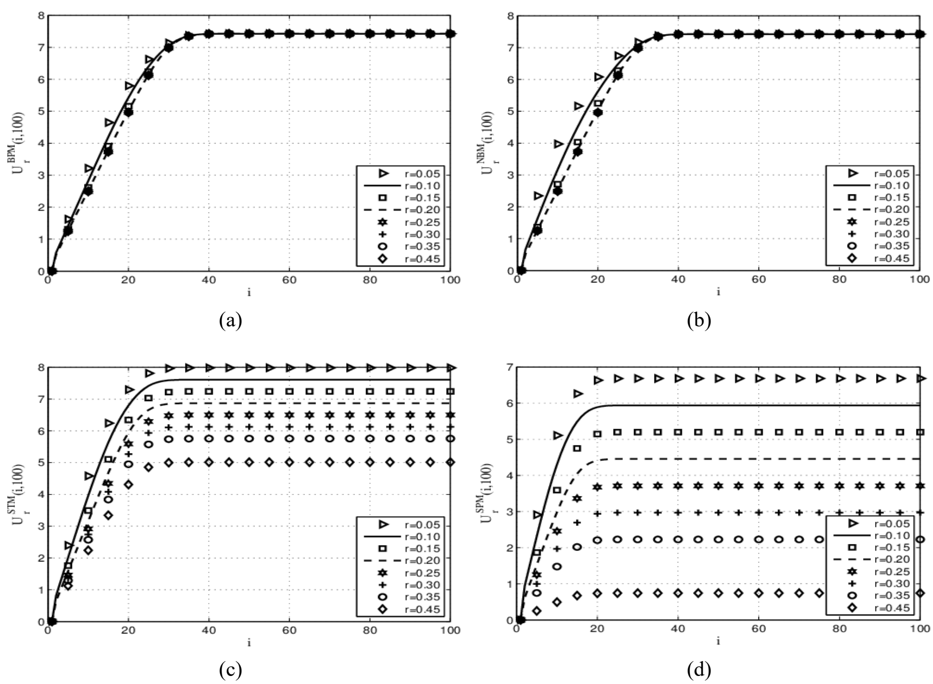
(a) (b) (c) (d)

Figure 17 shows that the seller's valuation v_s weakly decreases and approaches zero with the refusal cost for all inventory levels. As mentioned above, the probability of no trade function for SPM is higher than the STM mechanism. Therefore, when r increases (say $r = 0.20$), for sufficient low inventory and sufficient high time-to-go, STM dominates the SPM mechanism. Since there is no impact of refusal cost for BPM and NBM mechanisms, when there is sufficient inventory, BPM and NBM mechanisms remain constant and dominate STM, followed by the SPM mechanism. Similar to the case of $r = 0.05$, value functions for SPM decrease significantly, and it is the least selected mechanism.

Table 6 Probability of an item not getting sold and expected price for four bargaining mechanisms

Mechanism k	*No trade probability* $1 - \bar{p_s}^k(v_s)$	*Expected payment* $\bar{y_s}^k(v_s)$
SPM	$\frac{1+v_s}{2}$	$\frac{1-v_s^2}{4}$
STM	$\frac{1+4v_s}{4}$	$\frac{9}{32} - \frac{v_s^2}{2}$
BPM	$2v_s$	$\frac{1-4v_s^2}{4}$
NBM	$\begin{cases} 3v_s, & \text{if } v_s \leq 0.25 \\ \frac{2+v_s}{3}, & \text{otherwise} \end{cases}$	$\begin{cases} \frac{(1-8v_s^2-2v_s)}{4} + \frac{(v_s+v_s^2)}{2}, & \text{if } v_s \leq 0.25 \\ \frac{1-v_s^2}{6}, & \text{otherwise} \end{cases}$

Figure 18 Value function for different r under four mechanisms, (a) BPM (b) NBM (c) STM (d) SPM



When there is an increase in the r, the valuation of the seller becomes ‘zero’ for all the inventory levels [Figures 17(c) and 17(d)]. As r increases to 0.35, STM dominates BPM and NBM together, followed by the SPM mechanism when there is sufficiently less inventory remaining and sufficiently high time-to-go. Figure 18 shows the value function for different r under four mechanisms. As there is no effect of refusal cost for BPM and NBM, they remain the same even when the inventory increases, and they together start dominating the STM mechanism. Irrespective of the increase in refusal cost (say $r = 0.45$), BPM and NBM remain the same for all inventory. Therefore, they dominate STM followed by the SPM mechanism. When r increases to 0.50, the expected payment and probability of no trade function become equal for the SPM mechanism. The value function of SPM becomes zero. Therefore, the possible range of refusal cost to analyse the four bilateral bargaining mechanisms varies from 0.01 to 0.49.

10 Conclusions

This paper studies bilateral bargaining between each arriving buyer and a seller who has a finite number of items to sell in a finite time period. In every period, at most a buyer who is non-strategic turns up with unit demand. The buyer's and seller's valuations are private information to each other. A mediator determines the bargaining mechanism by considering the valuations of the buyer and the seller. This paper models the bargaining problem as a MDP and finds the marginal valuation of the seller endogenously based on inventory level. The seller faces a cost of refusal if the item does not get sold. When there is low inventory and more time period to go, then the seller is said to be in a strong position. When there is more inventory remaining and less time period to go, then the seller is said to be in a weak position. This paper numerically compares the performance of the seller under four bilateral bargaining mechanisms: split the difference, seller posted price, neutral bargaining solution, and buyer posted price. For low refusal cost, the seller chooses SPM when he is in a strong position. The seller prefers STM when he is in a weak position. When the refusal cost increases, STM is the preferred mechanism when there is sufficient low inventory and more time to go. The seller is indifferent between NBM and BPM when the inventory increases for a given time period. For high refusal cost, the seller is indifferent between NBM and BPM, irrespective of the time period and the inventory level. Also, this paper compares dynamic pricing with the above four bilateral bargaining mechanisms with and without the refusal cost. Without and with lower refusal cost, dynamic pricing and SPM perform in a similar fashion. When refusal cost increases, dynamic pricing starts dominating SPM. This paper conducts simulation experiments to evaluate the MDP model. Simulation results also give the same order preference of the mechanisms in each experiment.

In various industries, managers need to choose the pricing strategy. The pricing scheme is important as it increases revenue. Also, managers may incur the cost of penalty or refusal if the item does not get sold. In a strong position, this study suggests that the managers should announce the price if the loss of not satisfying the demand is low. Managers should choose to split the difference between their and the buyer's valuation if the loss of not satisfying the demand increases. In a weak position, this study suggests that the managers should choose to split the difference between their and the buyer's valuation if the loss of not satisfying the demand is low. In a strong/weak position, the managers should allow the buyers to choose the price or negotiate if the loss of not satisfying the demand is high.

The possible extensions to this work are arriving buyers have multi-unit demand and they are strategic. As a finite horizon model, deadlines are assumed as private, whereas in firms like airlines and hotels, deadlines are known. This paper assumes constant refusal cost, whereas, in real life, the seller incurs different refusal costs according to the type of buyer. Also, it can be assumed as unknown valuation distributions of each arriving buyer and the seller.

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