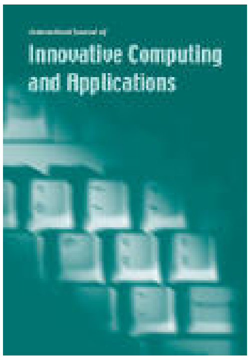

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New delay-independent exponential stability rule of delayed Cohen-Grossberg neural networks

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Abstract: This manuscript studies the stability for a class of Cohen-Grossberg neural networks (CGNNs) with variable delays. By practicing the scheme of Lyapunov function (LF), M-matrix (MM) theory, homeomorphism theory and nonlinear measure (NM) method, a new sufficient condition is obtained to ensure the existence, uniqueness and global exponential stability (GES) of equilibrium point (EP) for the studied network. As the condition is independent to delay, it can be applied to networks with large delays. The result generalises and improves the earlier publications. Finally, an example is supplied to exhibit the power of the results and less conservativeness over some earlier publications.

Keywords: stability; inequality; delay; homeomorphism.

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1 Introduction

The Cohen-Grossberg neural networks (CGNNs), originally submitted and scrutinised by Cohen and Grossberg (2014), has drawn increasing attention due to its promising applications in image and signal processing. Such applications heavily depend on the dynamic actions of the network. Therefore, the study of these actions is a necessary step for practicable design of the network. In hardware implementation, however, time delays occur due to finite switching speed of the amplifiers and communication time. Time lags may activate divergence, instability, or oscillation which may be hurtful to the network. Moreover, it was displayed that the converting of moving images requests the inclusion of delays in signal conducted over the networks. It is appreciated that consistent time lags in delayed response systems perform good approximation in uncomplicated circuits with a small number of cells. In most cases, delays are variable. Therefore, the research of networks with variable delays is more valuable than consistent delays. In past decades, many rules were developed to insure the global exponential stability (GES) of equilibrium point (EP) for CGNNs with variable delays (Huang et al., 2007a, 2007b; Li and Cao, 2006; Oliveira, 2011; Song and Cao, 2006; Xiong and Cao, 2005; Zheng et al., 2013, 2019).

Under different assumptions of amplification function, Zhang et al. (2009) inferred some rules for the global asymptotic stability (GAS) of a sort of CGNNs with varied time lags. On the basis of M-matrix (MM) theory, Oliveira (2011) setup sufficient rules to ensure the existence and global attractivity of an EP for CGNNs with varied delays, and the GES of the EP for CGNNs with variable delays. For a sort of CGNNs with variable delays, Lien et al. (2011) proposed delay-dependent and delay-independent stability conditions to ensure the robust stability and uniqueness of EP via LMI approach. By employing a Lyapunov-Krasovskii functional and equivalent descriptor system, Li et al. (2009) presented conditions to ensure the GES for a sort of CGNNs with varied delays. By formulating suitable Lyapunov function (LF) and integrating with matrix inequality procedure, Guo (2012) produced a rule for the GAS of a sort of CGNNs with variable delays. Without assuming the differentiability and boundedness of the activation functions, Gao and Cui (2009) derived some rules for the existence, uniqueness, and robust GES of the EP for a sort of interval CGNNs with variable delays. By exploiting H-matrix and MM theory, homeomorphism principle, LF method, and linear matrix inequality (LMI) technique, Du and Xu (2013) obtained some rules for the existence, uniqueness, and robust GES of the EP for a sort of interval CGNNs with varied delays. By virtue of LF and Halanay delay differential inequality, Wang and Qi (2013) proposed several rules in LMIs form for the GES in Lagrange sense of CGNNs with varied delays. By adopting the LF method and differential inequality techniques, Chen and Gong (2014) proposed some rules ensuring the exponential convergence of CGNNs with continuously distributed leakage delays. By proposing a generalised convex combination inequality to deal with multiple variable delays, Shan et al. (2013) obtained a new stability rule for a sort of CGNNs with variable delays.

Sparked by above argument, this paper deals with the issue of existence, uniqueness and the GES of the EP of CGNNs with variable delays. Our condition generalises and improves some earlier ones (Huang et al., 2007a, 2007b; Song and Cao, 2006; Xiong and Cao, 2005; Zhang et al., 2004). An example is demonstrated to reveal the efficiency of the obtained results.

Throughout this paper, set $\hat{\Upsilon} = \{1, 2, \ldots, \upsilon\}$. For vector $\xi = (\xi_1, \xi_2, \ldots, \xi_\upsilon)^T \in \mathbb{R}^\upsilon$, $\xi > 0$ symbolises $\xi_\iota > 0$, $\iota \in \hat{\Upsilon}$.. For matrix $\Sigma = (\sigma_{\iota\phi})_{\upsilon\times\upsilon}$, denote $|\Sigma| = (|\sigma_{\iota\phi}|)_{\upsilon\times\upsilon}$, $\tilde{\Sigma} = (\tilde{\sigma}_{\iota\phi})_{\upsilon\times\upsilon}$ with $\tilde{\sigma}_{\iota\iota} = \sigma_{\iota\iota}$ and $\tilde{\sigma}_{\iota\phi} = |\sigma_{\iota\phi}|$ for $\iota \neq \phi$, ι, $\phi \in \hat{\Upsilon}$.

2 Preparation

Examine the below CGNNs with variable delays:

$$\begin{cases} \dfrac{\mathrm{d}x_{\iota(t)}}{\mathrm{d}t} = -\tau_\iota\left(x_\iota(t)\right)\begin{bmatrix} \psi_\iota\left(x_\iota(t)\right) - \sum\limits_{\phi=1}^{\upsilon} \omega_{\iota\phi}\zeta_\phi\left(x_\phi(t)\right) \\ -\sum\limits_{\phi=1}^{\upsilon} \lambda_{\iota\phi}\zeta_\phi\left(x_\phi\left(t-\vartheta_{\iota\phi}(t)\right)\right) - \eta_\iota \end{bmatrix}, \\ x_\iota(t) = \mu_\iota(t), \qquad t \in [-\vartheta, 0], \qquad \iota \in \hat{\Upsilon}, \end{cases} \tag{1}$$

where $x_\iota(t)$ symbolises the case of the ι^{th} member, $\zeta_\phi(x_\phi(t))$ symbolises the activation function, $\tau_\iota(x_\iota(t))$ symbolises the amplification function, $\psi_\iota(x_\iota(t))$ symbolises the performed function, $\Omega = (\omega_{\iota\phi})_{\upsilon\times\upsilon}$, $\Lambda = (\lambda_{\iota\phi})_{\upsilon\times\upsilon}$ symbolise the strength of the neuron interconnections within the network, $\vartheta_{\iota\phi}(t)$ symbolises the variable delay with $0 \leq \vartheta_{\iota\phi}(t) \leq \vartheta$, $\vartheta'_{\iota\phi}(t) \leq 0$; η_ι symbolises the external bias, $\mu_\iota(t)$ is continuous on $[-\vartheta, 0]$.

Now, we make the following assumptions:

H1 (see Ozcan, 2018): $\tau_\iota(v)$ is continuous and there exist positive constants φ_ι and Φ_ι such that $\varphi_\iota \leq \tau_\iota(v) \leq \Phi_\iota$, $v \in \mathbb{R}$, $\iota \in \hat{\Upsilon}$.

H2 (see Lien et al., 2011; Xiong et al., 2017): $\psi'_\iota(v) \geq \psi_\iota > 0$, $v \in \mathbb{R}$, $\iota \in \hat{\Upsilon}$. Set $\Psi = diag\{\psi_1, \cdots, \psi_\upsilon\}$.

H3 (see Wang et al., 2018):

$$0 \leq \frac{\zeta_\iota(\theta) - \zeta_\iota(\rho)}{\theta - \rho} \leq \kappa_\iota, \quad \forall \theta, \rho \in \mathbb{R}, \theta \neq \rho, \iota \in \hat{\Upsilon}.$$

Set $K_\zeta = diag\{\kappa_1, \cdots, \kappa_\upsilon\}$.

Next, we require the below definitions.

Definition 1 (see Berezansky et al., 2014): $\Gamma = (\gamma_{\iota\phi})_{\upsilon\times\upsilon}$ is said to be MM if $\gamma_{\iota\phi} \leq 0 (\iota \neq \phi)$ and $\Gamma_\iota > 0$, where Γ_ι is the ι-order successive principal minor of Γ, ι, $\phi \in \hat{\Upsilon}$.

Definition 2 (see Zhang et al., 2014): A mapping $\Upsilon : \mathbb{R}^\upsilon \to \mathbb{R}^\upsilon$ is homeomorphic if Υ is continuous, bijection and the inverse mapping is also continuous.

Definition 3 (see Li and Cao, 2006): Suppose that $D \subset \mathbb{R}^\upsilon$ is open and $Z : D \to \mathbb{R}^\upsilon$. The constant

$$\varphi_D(Z) = \sup_{a,b \in D, a \neq b} \frac{\langle \Theta Z(a) - \Theta Z(b), a - b \rangle}{\| a - b \|_2^2}$$

is called the nonlinear measure of Z on D.

Definition 4 (see Zhang et al., 2004): A matrix A is said to belong to a class $\mathcal{P}$ if all principal minors of A are non-negative.

Lemma 1 (see Li and Cao, 2006): If $\varphi_D(Z) < 0$, then Z is injective. Especially, if $D = \mathbb{R}^\upsilon$, then Z is homeomorphic.

Lemma 2 (see Berman and Plemmons, 1979): Given $\Gamma = (\gamma_{\iota\phi})_{\upsilon\times\upsilon}$ with $\gamma_{\iota\iota} \leq 0$, $\iota \in \hat{\Upsilon}$. The following statements are equivalent to the statement that Γ is an MM:

a $\sigma^T\Gamma > 0$ for a vector $\sigma > 0$.

b $\Gamma\Theta + \Theta\Gamma^T > 0$ for some $\Theta = diag\{\theta_1, \theta_2, \ldots, \theta_\upsilon\} > 0$.

c $\gamma_{\iota\iota}\theta_\iota > \sum_{\phi=1,\iota\neq\phi}^{\upsilon} |\gamma_{\iota\phi}| \theta_\phi (\iota \in \hat{\Upsilon})$ for some $\Theta = diag\{\theta_1, \theta_2, \ldots, \theta_\upsilon\} > 0$.

3 Main results

Now we provide the result.

Theorem 1: Assume that (H1)–(H3) are satisfied. If $\Psi-\tilde{\Omega}K_{\zeta}-|\Lambda|K_{\zeta}$ is an MM, then system (1) has one unique EP x^* which is GES for any $\eta_\iota(\iota\in\hat{\Upsilon})$.

Proof: Define $\Xi:\mathbb{R}^{\upsilon}\to\mathbb{R}^{\upsilon}$ by $\Xi(x)=(\Xi_1(x), \Xi_2(x), \cdots, \Xi_{\upsilon}(x))^T$ with

$$\Xi_\iota(x)=-\psi_\iota(x_\iota)-\sum_{\phi=i}^{\upsilon}\omega_{\phi\iota}\zeta_\phi(x_\phi)-\sum_{\phi=1}^{\upsilon}\lambda_{\phi\iota}\zeta_\phi(x_\phi)+\eta_\iota,\quad \iota\in\hat{\Upsilon}.$$

Note that $x^*=(x_1^*, x_2^*, \cdots, x_{\upsilon}^*)^T$ is an EP of (1) if and only if $\Xi(x^*)=0$. Since $\Pi=\Psi-\tilde{\Omega}K_\zeta-|\Lambda|K_\zeta$ is an MM, one has $\Theta\Pi+\Pi^T\Theta>0$ for some $\Theta=diag\{\theta_1, \theta_2, \cdots, \theta_\upsilon\}>0$. Set $\pi=(\pi_1, \pi_2, \cdots, \pi_\upsilon)^T$, $\epsilon=(\epsilon_1, \epsilon_2, \cdots, \epsilon_\upsilon)^T$, then

$$\begin{aligned}
&\langle\Theta\Xi(\pi)-\Theta\Xi(\epsilon), \pi-\epsilon\rangle=\sum_{\iota=1}^{\upsilon}-\theta_\iota\Big\{\psi_\iota(\pi_\iota)-\psi_\iota(\epsilon_\iota)\\
&-\sum_{\phi=1}^{\upsilon}\omega_{\iota}\phi\big(\zeta_\phi|(\pi_\phi)-\zeta_\phi(\epsilon_\phi)\big)-\sum_{\phi=1}^{\upsilon}\lambda_{\iota\phi}\big(\zeta_\phi(\pi_\phi)-\zeta_\phi(\epsilon_\phi)\big)\Big\}(\pi_\iota-\epsilon_\iota)\\
&=\sum_{\iota=1}^{\upsilon}-\theta_\iota\Big\{|\psi_\iota(\pi_\iota)-\psi_\iota(\epsilon_\iota)||\pi_\iota-\epsilon_\iota|-\omega_{\iota\iota}|\zeta_\iota(\pi_\iota)-\zeta_\phi(\epsilon_\iota)||\pi_\iota-\epsilon_\iota|\\
&-\sum_{\phi\neq\iota}^{\upsilon}\omega_{\iota\phi}\big(\zeta_\phi(\pi_\phi)-\zeta_\phi(\epsilon_\phi)\big)(\pi_\iota-\epsilon_\iota)-\sum_{\phi=1}^{\upsilon}\lambda_{\iota\phi}\big(\zeta_\phi(\pi_\phi)-\zeta_\phi(\epsilon_\phi)\big)(\pi_\iota-\epsilon_\iota)\Big\}\\
&\le\sum_{\iota=1}^{\upsilon}-\theta_\iota\Big\{\psi_\iota|\pi_\iota-\epsilon_\iota|-\omega_{\iota\iota}|\zeta_\iota(\pi_\iota)-\zeta_\iota(\epsilon_\iota)|\\
&-\sum_{\phi\neq\iota}^{\upsilon}|\omega_{\iota\phi}||\zeta_\phi(\pi_\phi)-\zeta_\phi(\epsilon_\phi)|-\sum_{\phi=1}^{\upsilon}|\lambda_{\iota\phi}||\zeta_\phi(\pi_\phi)-\zeta_\phi(\epsilon_\phi)|\Big\}|\pi_\iota-\epsilon_\iota|\\
&=\sum_{\iota=1}^{\upsilon}-\theta_\iota\Big\{\psi_\iota|\pi_\iota-\epsilon_\iota|-\omega_{\iota\iota}|\zeta_\iota(\pi_\iota)-\zeta_\iota(\epsilon_\iota)|\\
&-\sum_{\phi\neq\iota}^{\upsilon}|\omega_{\phi\iota}||\zeta_\iota(\pi_\iota)-\zeta_\iota(\epsilon_\iota)|-\sum_{\phi=1}^{\upsilon}|\lambda_{\iota\phi}||\zeta_\phi(\pi_\phi)-\zeta_\phi(\epsilon_\phi)|\Big\}|\pi_\iota-\epsilon_\iota|\\
&\le\sum_{\iota=1}^{\upsilon}-\theta_\iota\Big\{\psi_\iota|\pi_\iota-\epsilon_\iota|-\omega_{\iota\iota}\kappa_\iota|\pi_l-\epsilon_\iota|-\sum_{\phi\neq\iota}^{\upsilon}|\theta_{\phi\iota}|\kappa_\iota|\pi_\iota-\epsilon_\iota|\\
&-\sum_{\phi=1}^{\upsilon}|\lambda_{\phi\iota}|\kappa_\iota|\pi_\iota-\epsilon_\iota|\Big\}|\pi_\iota-\epsilon_l|\\
&\le\sum_{\iota=1}^{\upsilon}-\theta_\iota\Big\{\psi_\iota|\pi_\iota-\epsilon_\iota|-\sum_{\phi=i}^{\upsilon}\tilde{\omega}_{\phi\iota}\kappa_\iota|\pi_\iota-\epsilon_\iota|-\sum_{\phi=1}^{\upsilon}|\lambda_{\phi\iota}|\kappa_\iota|\pi_\iota-\epsilon_\iota|\Big\}|\pi_\iota-\epsilon_\iota|\\
&=-\sum_{\iota=1}^{\upsilon}\sum_{\phi=1}^{\upsilon}\big(\theta_\iota\psi_\iota-\theta_\iota\tilde{\omega}_{\phi\iota}\kappa_\iota-\theta_\iota|\lambda_{\phi\iota}|\kappa_\iota\big)|\pi_\iota-\epsilon_\iota||\pi_\iota-\epsilon_\iota|\\
&=-(\pi-\epsilon)^T\Theta\Pi(\pi-\epsilon)\\
&=-\frac{1}{2}(\pi-\epsilon)^T(\Theta\Pi+\Pi^T\Theta)(\pi-\epsilon)\\
&\le-\frac{1}{2}\lambda_{\min}(\Theta\Pi+\Pi^T\Theta)\|\pi-\epsilon\|_2^2.
\end{aligned}$$

Thus, $\varphi_{\mathbb{R}^\upsilon}(\Theta\Xi)<0$. Thus, $\Theta\Xi$ is homeomorphic from Lemma 1. This implies that $\Theta\Xi(x^*)=0$ for a unique $x^*\in\mathbb{R}^\upsilon$. As Θ is invertible, $\Xi(x)=0$ has a unique solution x^*. Hence, system (1) has a unique EP x^*.

Next, we prove the GES of the EP x^* of (1). Set $\xi_\iota(t)=x_\iota(t)-x_\iota^*$, then system (1) becomes

$$\begin{aligned}\frac{\mathrm{d}\xi_\iota(t)}{\mathrm{d}t}=-\tilde{\tau}_\iota(\xi_\iota(t))\Big[&\tilde{\psi}_\iota(\xi_\iota(t))-\sum_{\phi=1}^{\upsilon}\omega_{\iota\phi}\tilde{\zeta}_\phi(\xi_\phi(t))\\
&-\sum_{\phi=1}^{\upsilon}\lambda_{\iota\phi}\tilde{\zeta}_\phi\big(\zeta_\phi\big(t-(\vartheta_{\iota\phi}(t))\big)\big)\Big],\end{aligned}\tag{2}$$

where $\tilde{\tau}_\iota(\xi_\iota(t))=\tau_\iota(x_\iota(t)+x_\iota^*)$, $\tilde{\psi}_\iota(\xi_\iota(t))=\psi_\iota(\xi_\iota(t)+x_\iota^*)-\psi_\iota(x_\iota^*)$, $\tilde{\zeta}_\phi(\xi_\phi(t))=\zeta_\phi(\xi_\phi(t)+x_\phi^*)-\zeta_\phi(x_\phi^*)$.

Note that Π is an MM, from Lemma 2, there exists $\sigma=(\sigma_1, \sigma_2, \cdots, \sigma_\upsilon)^T>0$ with

$$-\sigma_\iota\psi_\iota+\omega_{\iota\iota}\sigma_\iota\kappa_\iota+\sum_{\phi\neq\iota}|\omega_{\phi\iota}|\sigma_\phi\kappa_\iota+\sum_{\phi=1}^{\upsilon}|\lambda_{\phi\iota}|\sigma_\phi\kappa_\iota<0,\quad \iota\in\hat{\Upsilon}.\tag{3}$$

Denote

$$\begin{aligned}\varrho_\iota(\xi_\iota(t))=-\tilde{\psi}_\iota(\xi_\iota(t))+\sum_{\phi=1}^{\upsilon}\omega_{\iota\phi}\tilde{\zeta}_\phi(\xi_\phi(t))\\
+\sum_{\phi=1}^{\upsilon}\lambda_{\iota\phi}\tilde{\zeta}_\phi\big(\xi_\phi(t-\vartheta_{\iota\phi}(t))\big),\end{aligned}$$

$$\Phi_\iota(\xi_\iota, t)=\begin{cases}\varphi_\iota, & \varrho_\iota(\xi_\iota(t))<0,\\ \Phi_\iota, & \varrho_\iota(\xi_\iota(t))>0,\end{cases}$$

$$\begin{aligned}\beta_\iota=-\psi_\iota\Phi_\iota(\xi_\iota, t)\sigma_\iota+\Big(&\omega_{\iota\iota}\Phi_\iota(\xi_\iota, t)\sigma_\iota\kappa_\iota+\sum_{\phi\neq\iota}|\omega_{\phi\iota}|\Phi_\phi(\xi_\phi, t)\sigma_\phi\kappa_\iota\\
&+\sum_{\phi=1}^{\upsilon}|\lambda_{\phi\iota}|\Phi_\phi(\xi_\phi, t)\sigma_\phi\kappa_\iota\Big)^+,\end{aligned}$$

where $\delta^+=\max(\delta, 0)$, then inequality (3) is equivalent to $\beta_\iota<0$, $\iota\in\hat{\Upsilon}$. Set $\beta=-\max_{1\le\iota\le\upsilon}\beta_\iota$, then $\beta>0$.

Define an LF as

$$V(\xi(t))=\sum_{\iota=1}^{\upsilon}\sigma_\iota|\xi_\iota(t)|+\sum_{\iota=1}^{\upsilon}\sum_{\phi=1}^{\upsilon}\sigma_\iota\Phi_\iota(\xi_\iota, t)|\lambda_{\iota\phi}|\int_{t-\vartheta_{\iota\phi}(t)}^{t}|\tilde{\zeta}_\phi(x_\phi(s))|\mathrm{d}s.$$

Set $\Delta_1=\{t$: $t>0$ such that $x_\iota(t)-x_\iota^*$ for some $\iota\in\hat{\Upsilon}\}$, $\Delta_2=(0, +\infty)\setminus\Delta_1$. Calculating the upper right derivative of $V(\xi(t))$ along the solution of (2), from assumptions (H1)–(H3), for $t\in\Delta_2$, we get that

$$D^{+}V(\xi(t)) = \sum_{\imath=1}^{\upsilon}\sigma_{\imath}\Big\{\mathrm{sgn}(\xi_{\imath}(t))\Big(-\tilde{\tau}_{\imath}(\xi_{\imath}(t))\Big(\tilde{\psi}_{\imath}(\xi_{\imath}(t))$$
$$-\sum_{\phi=1}^{\upsilon}\omega_{\imath\phi}\tilde{\zeta}_{\phi}(\zeta_{\phi}(t)) - \sum_{\phi=1}^{\upsilon}\lambda_{\imath\phi}\tilde{\zeta}_{\phi}(\xi_{\phi}(t-\vartheta_{\imath\phi}(t)))\Big)\Big)$$
$$+\sum_{\phi=1}^{\upsilon}\Phi_{\imath}(\xi_{\imath},t)|\lambda_{\imath\phi}|\Big(\big|\tilde{\zeta}_{\phi}(\xi_{\phi}(t))\big|$$
$$-(1-\vartheta'_{\imath\phi}(t))\times\big|\tilde{\zeta}_{\phi}(\xi_{\phi}(t-\vartheta_{\imath\phi}(t)))\big|\Big)\Big\}$$
$$\le \sum_{\imath=1}^{\upsilon}\sigma_{\imath}\Big\{\Big(\Phi_{\imath}(\xi_{\imath},t)\Big(-\big|\tilde{\psi}_{\imath}(\xi_{\imath}(t))\big| + \omega_{\imath\imath}\big|\tilde{\zeta}_{\imath}(\xi_{\imath}(t))\big|$$
$$+\sum_{\phi\neq\imath}|\omega_{\imath\phi}|\big|\tilde{\zeta}_{\phi}(\xi_{\phi}(t))\big| + \sum_{\phi=1}^{\upsilon}|\lambda_{\imath\phi}|\big|\tilde{\zeta}_{\phi}(\xi_{\phi}(t-\vartheta_{\imath\phi}(t)))\big|\Big)\Big)$$
$$+\sum_{\phi=1}^{\upsilon}\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)|\lambda_{\imath\phi}|\Big(\big|\tilde{\zeta}_{\phi}(\xi_{\phi}(t))\big| - \big|\tilde{\zeta}_{\phi}(\xi_{\phi}(t-\vartheta_{\imath\phi}(t)))\big|\Big)\Big\}$$
$$= \sum_{\imath=1}^{\upsilon}\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)\Big(-\big|\tilde{\psi}_{\imath}(\xi_{\imath}(t))\big| + \omega_{\imath\imath}\big|\tilde{\zeta}_{\imath}(\xi_{\imath}(t))\big|$$
$$+\sum_{\phi\neq\imath}|\omega_{\imath\phi}|\big|\tilde{\zeta}_{\phi}(\xi_{\phi}(t))\big| + \sum_{\phi=1}^{\upsilon}|\lambda_{\imath\phi}|\big|\tilde{\zeta}_{\phi}(\xi_{\phi}(t))\big|\Big)$$
$$= \sum_{\imath=1}^{\upsilon}\Big(-\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)\big|\tilde{\psi}_{\imath}(\xi_{\imath}(t))\big| + \Big(\omega_{\imath\imath}\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)$$
$$+\sum_{\phi\neq\imath}|\omega_{\imath\phi}|\sigma_{\phi}\Phi_{\phi}(\xi_{\phi},t) + \sum_{\phi=1}^{\upsilon}|\lambda_{\imath\phi}|\sigma_{\phi}\Phi_{\phi}(\xi_{\phi},t)\Big)\big|\tilde{\zeta}_{\imath}(\xi_{\imath}(t))\big|\Big)$$
$$\le \sum_{\imath=1}^{\upsilon}\Big(-\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)\big|\tilde{\psi}_{\imath}(\xi_{\imath}(t))\big|$$
$$+\Big(\omega_{\imath\imath}\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t) + \sum_{\phi\neq\imath}|\omega_{\imath\phi}|\sigma_{\phi}\Phi_{\phi}(\xi_{\phi},t)$$
$$+\sum_{\phi=1}^{\upsilon}|\lambda_{\imath\phi}|\sigma_{\phi}\Phi_{\phi}(\xi_{\phi},t)\Big)^{+}\big|\tilde{\zeta}_{\imath}(\xi_{\imath}(t))\big|\Big)$$
$$\le \sum_{\imath=1}^{\upsilon}\Big(-\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)\psi_{\imath}|\xi_{\imath}(t)| + \Big(\omega_{\imath\imath}\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)$$
$$+\sum_{\phi\neq\imath}|\theta_{\imath\phi}|\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t) + \sum_{\phi=1}^{\upsilon}|\lambda_{\imath\phi}|\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)\Big)^{+}\big|\tilde{\zeta}_{\imath}(\xi_{\imath}(t))\big|\Big)$$
$$\le \sum_{\imath=1}^{\upsilon}\Big(-\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)\psi_{\imath} + \Big(\omega_{\imath\imath}\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)$$
$$+\sum_{\phi\neq\imath}|\omega_{\imath\phi}|\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t) + \sum_{\phi=1}^{\upsilon}|\lambda_{\imath\phi}|\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)\Big)^{+}\Big)\kappa_{\imath}|\xi_{\imath}(t)|$$
$$= \sum_{\imath=1}^{\upsilon}\beta_{\imath}|\xi_{\imath}(t)| \le \beta\sum_{\imath=1}^{\upsilon}|\xi_{\imath}(t)|.$$

Define

$$h_{\imath}(\rho) = \beta + \rho\sigma_{\imath} + \sum_{\phi=1}^{\upsilon}|\psi_{\phi\imath}|\sigma_{\phi}\Phi_{\phi}\kappa_{\imath}\rho\vartheta e^{\rho\vartheta}, \quad \imath\in\hat{\Upsilon}. \tag{4}$$

Obviously $h_{\imath}(0) = \beta < 0$, $\dot{h}_{\imath}(\rho) > 0$ and $h_{\imath}(\rho) \to +\infty$ as $\rho \to +\infty$. Therefore, $h_{\imath}(\rho_{\imath}) = 0$ for unique $\rho_{\imath} > 0$. Set $\rho = \min_{1\le\imath\le\upsilon}\rho_{\imath}$, then $\rho > 0$, and $h_{\imath}(\rho_{\imath}) \le 0$, $\imath\in\hat{\Upsilon}$.

From equations (2) and (3), for $t > \vartheta$

$$\begin{aligned} D^{+}\big(e^{\rho t}V(\xi(t))\big) &= \rho e^{\rho t}V(\xi(t)) + e^{\rho t}D^{+}V(\xi(t)) \\ &\le \rho e^{\rho t}\Big(\sum_{\imath=1}^{\upsilon}\sigma_{\imath}|\xi_{\imath}(t)| + \sum_{\imath=1}^{\upsilon}\sum_{\phi=1}^{\upsilon}\sigma_{\imath}\Phi_{\imath}(\xi_{\imath},t)|\lambda_{\imath\phi}|\int_{t-\vartheta_{\imath\phi}(t)}^{t}\big|\tilde{\zeta}_{\phi}(x_{\phi}(s))\big|\mathrm{d}s\Big) \\ &\quad + e^{\rho t}\beta\sum_{\imath=1}^{\upsilon}|\xi_{\imath}(t)| \\ &\le \sum_{\imath=1}^{\upsilon}(\beta+\rho\sigma_{\imath})e^{\rho t}|\xi_{\imath}(t)| + \sum_{\imath=1}^{\upsilon}\sum_{\phi=1}^{\upsilon}\sigma_{\imath}\Phi_{\imath}\kappa_{\imath}|\lambda_{\imath\phi}|\int_{t-\vartheta}^{t}|\xi_{\phi}(s)|\mathrm{d}s. \end{aligned} \tag{5}$$

For $t > \vartheta$, integrating the inequality (5) from 0 to t gives

$$\begin{aligned} e^{\rho t}C(\xi(t)) - V(\xi(0)) &\le \sum_{\imath=1}^{\upsilon}(\beta+\rho\sigma_{\imath})\int_{0}^{t}e^{\rho s}|\xi_{\imath}(s)|\mathrm{d}s \\ &\quad + \sum_{\imath=1}^{\upsilon}\sum_{\phi=1}^{\upsilon}\rho\sigma_{\imath}\Phi_{\imath}\kappa_{\phi}|\lambda_{\imath\phi}|\int_{0}^{t}e^{\rho s}\mathrm{d}s\int_{s-\vartheta}^{s}|\xi\phi(v)|\mathrm{d}v. \end{aligned} \tag{6}$$

Changing the order of the double integral in equation (6) gives that

$$\begin{aligned} \int_{0}^{t}e^{\rho s}\mathrm{d}s\int_{s-\vartheta}^{s}|\xi_{\imath}(v)|\mathrm{d}v &= \int_{-\vartheta}^{t}|\xi_{\phi}(v)|\mathrm{d}v\int_{\max(0,v)}^{\min(t,v+\vartheta)}e^{\rho s}\mathrm{d}s \\ &\le \int_{-\vartheta}^{t}\vartheta e^{\rho(v+\vartheta)}|\xi_{\phi}(v)|\mathrm{d}v \\ &\le \vartheta e^{\rho\vartheta}\int_{-\vartheta}^{0}|\xi_{\phi}(s)|\mathrm{d}s + \vartheta e^{\rho\vartheta}\int_{0}^{t}e^{\rho s}|\xi_{\phi}(s)|\mathrm{d}s. \end{aligned} \tag{7}$$

From equations (4), (6) and (7), one obtains

$$e^{\rho t}V(\xi(t)) - V(\xi(0)) \le \sum_{\imath=1}^{\upsilon}h_{\imath}(\rho)\int_{0}^{t}e^{\rho s}|\xi_{\imath}(s)|\mathrm{d}s + \varepsilon \le \varepsilon,$$

where

$$\varepsilon = \sum_{\imath=1}^{\upsilon}\sum_{\phi=1}^{\upsilon}\rho\sigma_{\imath}\varphi_{\imath}\kappa_{\phi}\vartheta e^{\rho\vartheta}|\lambda_{\imath\phi}|\int_{-\vartheta}^{0}|\xi_{\phi}(s)|\mathrm{d}s.$$

Thus, one has

$$V(\xi(t)) \le (\varepsilon + V(\xi(0)))e^{-\rho t}, \; t > \vartheta.$$

Finally, combining equations (3) and (6) yields

$$\sum_{\imath=1}^{\upsilon}\sigma_{\imath}|\xi_{\imath}(s)| \le (\varepsilon + V(\xi(0)))e^{-\rho t}, \; t > \vartheta. \tag{8}$$

It is easy to verify that inequality (8) also holds for $t \in \Delta_1$. This completes the proof.

4 Comparisons and example

To compare with the earlier publications, the results in Huang et al. (2007a, 2007b), Song and Cao (2006), Xiong and Cao (2005) and Zhang et al. (2004) are restated as follows: Theorem 2 (Xiong and Cao, 2005; Zhang et al., 2004). Assume that every activation function is partially Lipschitz continuous and monotone increasing function, then model (1) has a unique EP which is absolute exponential stable if

$$-\bar{\Omega} - |\Lambda| \in \mathcal{P},$$

where $\bar{\Omega} = (\bar{\omega}_{\iota\phi})_{\upsilon\times\upsilon}$ with $\bar{\omega}_{\iota\iota} = \max\{\omega_{\iota\iota}, 0\}$ and $\bar{\omega}_{\iota\phi} = |\omega_{\iota\phi}|$ for $\iota \neq \phi$.

Remark 1: Theorem 1 improves the result of Theorem 2. It is easy to see that

$$-\tilde{\Omega} - |\Lambda| = -\bar{\Omega} - |\Lambda| + \hat{\Omega},$$

where $\hat{\Omega} = diag\{\bar{\omega}_{11} - \omega_{11}, ..., \bar{\omega}_{\upsilon\upsilon} - \omega_{\upsilon\upsilon}\} \geq 0$. As Ψ and K_ζ are positive diagonal matrices, if $-\bar{\Omega} - |\Lambda| \in \mathcal{P}$, then $\Psi - \tilde{\Omega}K_\zeta - |\Lambda| K_\zeta$ is an MM. That is, the condition of Theorem 2 is stronger than Theorem 1.

Theorem 3 (Huang et al. (2007a, 2007b; Song and Cao, 2006): Assume that (H1)–(H2) are satisfied and every activation function is Lipschitz continuous. If $\Psi - |\Omega| K_\zeta - |\Lambda| K_\zeta$ is an MM, then system (1) has one unique EP which is GES for any $\eta_\iota (\iota \in \hat{\Upsilon})$.

Remark 2: Theorem 1 improves the result of Theorem 3. It is easy to see that

$$\Psi - \tilde{\Omega}K_\zeta - |\Lambda| K_\zeta = \Psi - |\Omega| K_\zeta - |\Lambda| K_\zeta + \breve{\Omega}K_\zeta,$$

where $\breve{\Omega} = diag\{|\omega_{11}| - \omega_{11}, ..., |\omega_{\upsilon\upsilon}| - \omega_{\upsilon\upsilon}\} \geq 0$. As $K_\zeta > 0$ is a diagonal matrix, if $\Psi - |\Omega| K_\zeta - |\Lambda| K_\zeta$ is an MM, then $\Psi - \tilde{\Omega}K_\zeta - |\Lambda| K_\zeta$ must be an MM. That is, the condition of Theorem 3 is stronger than Theorem 1.

Example 1: Set $\upsilon = 2$ and

$$\tau_1(x_1(t)) = \frac{1}{64}(2 + \cos(x_1(t))),$$
$$\tau_2(x_2(t)) = \frac{1}{64}(2 + \sin(x_2(t))),$$

$$\psi_1(x_1(t)) = 5x_1(t), \psi_2(x_2(t)) = 4x_2,$$
$$\zeta_1(v) = \zeta_2(v) = \tanh(v),$$

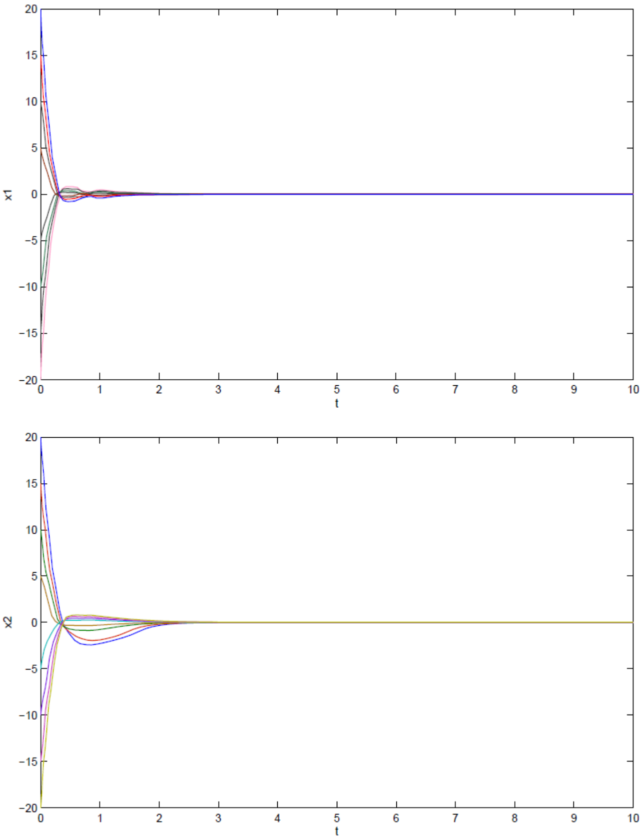
$$\Omega = \begin{pmatrix} -9 & -2 \\ 1 & -7 \end{pmatrix}, \Lambda = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix},$$

$$\vartheta_{11}(t) = \frac{1}{1+t}, \vartheta_{22}(t) = \frac{1}{2+t},$$
$$\vartheta_{12}(t) = \vartheta_{21}(t) = 0, t > 0,$$

in system (1). Then, (H1)–(H3) hold with

$$\vartheta = 1, \varphi_1 = \varphi_2 = \frac{1}{64}, \Phi_1 = \Phi_2 = \frac{3}{64},$$
$$\Psi = diag\{5, 4\}, K_\zeta = \mathbf{I}.$$

Figure 1 The state orbit of the network with eight random initial values in Example 1 (see online version for colours)



It is easy to verify that

$$\Psi - \tilde{\Omega}K_\zeta - |\Lambda| K_\zeta = \begin{pmatrix} 5 & 0 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} -9 & 2 \\ 1 & -7 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 11 & -2 \\ -1 & 9 \end{pmatrix}$$

is an MM, based on Theorem 1 one gets that system (1) has a unique EP that is GES for any η_1, η_2. Set $\eta_1 = \eta_2 = 0$, the time responses of the states are shown in Figure 1 with eight random initial values $\mu_\iota(t) \in [-20, 20] (\iota = 1, 2)$. We see that all the states are convergent to the unique and GES EP of the system. The EP is at the origin.

However, as

$$-\bar{\Omega} - |\Lambda| = -\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -3 & -2 \\ -1 & -2 \end{pmatrix} \notin \mathcal{P},$$

and

$$\Psi - |\Omega| K_\zeta - |\Lambda| K_\zeta = \begin{pmatrix} 5 & 0 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} 9 & 2 \\ 1 & 7 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -7 & -2 \\ -1 & -5 \end{pmatrix}$$

is not an MM, that is, nether Theorem 2 nor Theorem 3 holds. As a result, none of the conditions in Xiong and Cao (2005), Zhang et al. (2004), Huang et al. (2007a, 2007b) and Song and Cao (2006) can be applied to verify the stability. Thus one can conclude the condition of the paper is more effective and less conservative than those of Huang et al. (2007a, 2007b), Song and Cao (2006), Xiong and Cao (2005) and Zhang et al. (2004) for this system.

5 Conclusions

By constructing an LF, utilising MM theory, homeomorphism theory and nonlinear measure (NM) method, this paper established a condition to ensure the existence, uniqueness and GES of EP for a type of CGNNs with variable delays. As the condition independent of the delay, it can be applied to networks with large delays. An example is given to show the effectiveness of the results and less conservativeness over some earlier publications. One of our future research topics is to apply the method to analyse more complicated networks, namely genetic regulatory networks, reaction-diffusion networks, and semi-Markovian networks.

References

Berezansky, L., Braverman, E. and Idels, L. (2014) 'New global exponential stability criteria for nonlinear delay differential systems with applications to BAM neural networks', *Applied Mathematics and Computation*, 1 September, Vol. 243, pp.899–910, DOI: 10.1016/j.amc.2014.06.060.

Berman, A. and Plemmons, R.J. (1979) *Nonnegative Matrices in the Mathematical Sciences*, Academic Press, New York.

Chen, Z. and Gong, S. (2014) 'Exponential convergence of Cohen-Grossberg neural networks with continuously distributed leakage delays', *Journal of Inequalities and Applications*, DOI: 10.1186/1029-242X-2014-48.

Cohen, M. and Grossberg, S. (1983) 'Absolute stability and global pattern formation and parallel memory storage by competitive neural networks', *IEEE Transactions on Systems, Man and Cybernetics*, September/October, Vol. SMC-13, No. 5, pp.815–826, DOI: 10.1109/tsmc.1983.6313075.

Du, Y. and Xu, R. (2013) 'Global robust exponential stability and periodic solutions for interval Cohen-Grossberg neural networks with mixed delays', *Discrete Dynamics in Nature and Society*, DOI: 10.1155/2013/ 963897.

Gao, M. and Cui, B. (2009) 'Robust exponential stability of interval Cohen-Grossberg neural networks with time-varying delays', *Chaos, Solitons and Fractals*, 30 May, Vol. 40, No. 4, pp.1914–1928, DOI: 10.1016/j.chaos.2007.09.072.

Guo, Y. (2012) 'Global stability analysis for a class of Cohen-Grossberg neural network models', *Bulletin of the Korean Mathematical Society*, Vol. 49, No. 6, pp.1193–1198, DOI: 10.4134/BKMS.2012.49.6.1193.

Huang, T., Chan, A., Huang, Y. and Cao, J. (2007a) 'Stability of Cohen-Grossberg neural networks with time-varying delays', *Neural Networks*, October, Vol. 20, No. 8, pp.868–873, DOI: 10.1016/j.neunet.2007.07.005.

Huang, T., Li, C. and Chen, G. (2007b) 'Stability of Cohen-Grossberg neural networks with unbounded distributed delays', *Chaos Solitons and Fractals*, November, Vol. 34, No. 3, pp.992–996, DOI: 10.1016/j.chaos.2006.04.008.

Li, P. and Cao, J.D. (2006) 'Stability in static delayed neural networks: a nonlinear measure approach', *Neurocomputing*, August, Vol. 69, Nos. 13–15, pp.1776–1781, DOI: 10.1016/j.neucom.2005.12.031.

Li, T., Fei, S-M., Guo, Y-Q. and Zhu, Q. (2009) 'Stability analysis on Cohen-Grossberg neural networks with both time-varying and continuously distributed delays', *Nonlinear Analysis Real World Applications*, August, Vol. 10, No. 4, pp.2600–2612, DOI: 10.1016/j.nonrwa.2008.04.003.

Lien, C-H., Yu, K-W., Lin, Y-F., Chang, H-C. and Chung, Y-J. (2011) 'Stability analysis for Cohen-Grossberg neural networks with time-varying delays via LMI approach', *Expert Systems with Applications*, May, Vol. 38, No. 5, pp.6360–6367, DOI: 10.1016/j.eswa.2010.11.103.

Oliveira, J.J. (2011) 'Global stability of a Cohen-Grossberg neural network with both time-varying and continuous distributed delays', *Nonlinear Analysis Real World Applications*, October, Vol. 12, No. 5, pp.2861–2870, DOI: 10.1016/j.nonrwa.2011.04.012.

Ozcan, N. (2018) 'New conditions for global stability of neutral-type delayed Cohen-Grossberg neural networks', *Neural Networks*, October, Vol. 106, pp.1–7, DOI: 10.1016/ j.neunet.2018.06.009.

Shan, Q., Zhang, H., Yang, F. and Wang, Z. (2013) 'New delay-dependent stability criteria for Cohen-Grossberg neural networks with multiple time-varying mixed delays', *Soft Computing*, September, Vol. 17, No. 11, pp.2043–2052, DOI: 10.1007/s00500-013-1114-7.

Song, Q. and Cao, J. (2006) 'Stability analysis of Cohen-Grossberg neural network with both time-varying and continuously distributed delays', *Journal of Computational And Applied Mathematics*, 1 December, Vol. 197, No. 1, pp.188–203, DOI: 10.1016/j.cam.2005.10.029.

Wang, F-X., Liu, X-G., Tang, M-L. and Hou, M-Z. (2018) 'Improved integral inequalities for stability analysis of delayed neural networks', *Neurocomputing*, 17 January, Vol. 273, pp.178–189, DOI: 10.1016/ j.neucom.2017.07.054.

Wang, X. and Qi, H. (2013) 'New LMI-based criteria for Lagrange stability of Cohen-Grossberg neural networks with general activation functions and mixed delays', *International Journal of Computational Intelligence Systems*, September, Vol. 6, No. 5, pp.836–848, DOI: 10.1080/18756891.2013.805587.

Xiong, W. and Cao, J. (2005) 'Absolutely exponential stability of Cohen-Grossberg neural networks with unbounded delays', *Neurocomputing*, October, Vol. 68, pp.1–12, DOI: 10.1016/ j.neucom.2005.02.005.

Xiong, W., Shi, Y. and Cao, J. (2017) 'Stability analysis of two-dimensional neutral-type Cohen-Grossberg BAM neural networks', *Neural Computing and Applications*, April, Vol. 28, No. 4, pp.703–716, DOI: 10.1007/s00521-015-2099-1.

Zhang, H., Wang, Z. and Liu, D. (2009) 'Global asymptotic stability and robust stability of a class of Cohen- Grossberg neural networks with mixed delays', *IEEE Transactions on Circuits and Systems – I: Regular Papers*, March, Vol. 56, No. 3, pp.616–629, DOI: 10.1109/TCSI.2008.2002556.

Zhang, J., Suda, Y. and Iwasa, T. (2004) 'Absolutely exponential stability of a class of neural networks with unbounded delay', *Neural Networks*, April, Vol. 17, No. 3, pp.391–397, DOI: 10.1016/ j.neunet.2003.09.005.

Zhang, Z., Cao, J. and Zhou, D. (2014) 'Novel LMI-based condition on global asymptotic stability for a class of Cohen-Grossberg BAM networks with extended activation functions', *IEEE Transactions on Neural Networks and Learning Systems*, June, Vol. 25, No. 6, pp.1161–1172, DOI: 10.1109/ TNNLS. 2013.2289855.

Zheng, C-D., Gong, C-K. and Wang, Z. (2013) 'New delay-dependent exponential stability for discrete-time recurrent neural networks with mixed time-delays', *International Journal of Innovative Computing and Applications*, Vol. 5, No. 2, pp.65–75, DOI: 10.1504/ IJICA.2013.053173.

Zheng, C-D., Jia, H-H. and Shan, Q. (2019) 'New stochastic synchronisation condition of neutral-type Markovian chaotic neural networks under impulsive perturbations', *International Journal of Innovative Computing and Applications*, Vol. 10, No. 2, pp.100–106, DOI: 10.1504/IJICA.2019.102113.