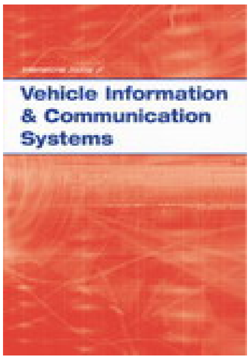

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Application of model predictive control on metro train scheduling problems

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Abstract: The arrangement of the metro train scheduling problem aims to maintain headway regularity and the number of passengers; therefore, it does not exceed capacity. In this study, the metro train traffic model was developed without referring to the nominal schedule, then the running time model and the dwell time model were added to the metro train traffic model, and considering changes in the number of passengers. A model predictive control is applied to control the metro train scheduling problems. The control adjusts the dwell time and the running time of the train. The optimisation problem in this study is a quadratic programming problem consisting of a quadratic cost function and linear constraints related to the train scheduling problems. Based on the simulation results, the headway deviation and the deviation of the number of train passengers in the metro train scheduling problem can be minimised using the model predictive control.

Keywords: train scheduling; metro train; model predictive control; quadratic programming.

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1 Introduction

Metro train plays an essential role in cross-urban transport, specifically in large cities. The existence of enhancement amount passenger impact on increasing transportation needs. Metro train has become the main transportation for people in large cities. Unfortunately, train delays still occur in the train performance. Several irregularities in the metro train schedules were affected by the train delays. The deviation of the train schedule could cause an accumulation of passengers, especially during peak hours. Considering the metro train scheduling problems, it makes sense to design a control methodology to compensate for the delays and adjust the onboard passenger load to enhance the required service efficiency and quality level.

Modelling and developing a system can be used to solve various real problems. Gandhi and Bhole (2013) proposed a unified empirical model to demonstrate the successful fabrication of several microstructures. In addition, Gandhi et al. (2013) developed a Microstereolithography System (MSL) to improve fabrication speed and high resolution. On the other hand, Assis and Milani (2004) introduced the metro train traffic model as the train headway model, which is the time interval between departure times of the consecutive trains without referring to the nominal schedule. However, their research did not consider the dwell time and running time models. Li et al. (2018) modelled the metro train traffic using the departure time model, which refers to the nominal schedule. Therefore, in their study, the train headway is modelled as the difference between actual departure time with nominal departure time, dwell time model and running time model. Another study by Wang et al. (2022) considered changes in the number of passengers in the metro train traffic model.

The previous metro train scheduling problems have been solved using the genetic algorithm method by Kang et al. (2015), Hassannayebi et al. (2014), Wu et al. (2015) and Wang et al. (2015). In addition, Shou et al. (2012) and Shi et al. (2017) solved the problem of scheduling metro trains using the heuristic programming method. Mor and Kumar (2022) used non-parametric techniques for traffic prediction on Toll Plaza traffic. Meanwhile, Assis and Milani (2004) proposed a Linear Programming-based Model Predictive Control (LP – MPC) for metro train scheduling problems. Shou et al. (2021) proposed a hierarchical control strategy based on MPC for optimal driving safety and comfort. Based on the results of their research, it was found that this strategy can effectively control vehicle speed to ensure driving safety and comfort. Zhang et al. (2019) proposed a Model Predictive Control (MPC) method based on quadratic programming for metro train scheduling problems. Based on the results of their research, it was found that MPC was effective in optimising metro train scheduling problems. But in his research, the metro train traffic model still refers to a nominal schedule; therefore, when a technical problem occurs in a metro line, that causes no could determined nominal schedule. Then it won't be easy to control the metro train schedule.

In this study, the metro train traffic is modelled without referring to the nominal schedule but as the departure time interval between successive trains (headway). Then, the dwell time model and the running time model were added to the train headway model. Besides that, the change of amount passengers on metro train scheduling problems is considered in this study. Moreover, the model predictive control is used to control the metro train scheduling problems. The method uses state space model characteristics of equality dynamic metro train to predict system output. Therefore, in this study, a model predictive control will be applied to the metro train scheduling problems.

The following presents the main ideas of the contribution of this study:

- In previous studies, the metro train traffic model still refers to the nominal schedule. When scheduling problems occur, and the nominal schedule is challenging to find, it is difficult to control train scheduling problems. In this study, metro train traffic arrangements related to the metro train traffic model without referring to the nominal schedule were developed. Still, metro train traffic is modelled based on the time intervals of successive train departures (headway).
- In previous studies, the train's running and dwell time were not modelled. It is essential to consider the model of the running time and the dwell time; therefore, in this study, the model of the running time and the dwell time is added based on Zhang et al. (2019) and Li et al. (2018). In addition, the changes in the number of passengers were considered in the scheduling problem of metro trains; namely, the amount of boarding passengers of the train is regulated; therefore, it does not exceed the capacity.
- This paper extends previous results in Assis and Milani (2004), which use model predictive control based on linear programming for train scheduling problems. A methodology for train scheduling problems using model predictive control based quadratic program (Zhang et al., 2019) is proposed. It also considered the constraint related to operational constraints, safety constraints and passengers constraints.

2 The metro train traffic model

The metro train traffic model is a dynamic traffic model describing train and passenger movements. The metro line consists of the M trains and N stations. Every train was supposed to stop at all stations. The dwell time of the train is assumed to increase proportionally to the time interval between the departure times of successive trains. The running time is assumed not influenced by the number of passengers. The total amount of boarding passengers is closely related to the dwell time between successive trains, and the number of alighting passengers is supposed to be proportional to the number of onboard passengers. Therefore, the system model of the metro train traffic comprises the headway and passenger load equations.

In this section, the headway equations are presented in Sub-section 2.1. The passenger load equations are presented in Sub-section 2.2. Moreover, the state space form related to this problem is presented in Sub-section 2.3.

2.1 Headway equations

In controlling metro train traffic, the only effort that can be made to maintain a regular schedule is to maintain regular intervals for the departure of successive trains (headway). In connecting with that, the headway relates to the departure time, the running time, and the dwell time of the train. The train headway model is formed based on what affects it in metro train traffic.

The departure time of train i at two succesive stations, which is station s and station $s+1$ affected by the departure time of train i at station s, which is symbolised with $t_{i,s}$, the running time of train i between station s and station $s+1$ which symbolised with $r_{i,s}$ and the dwell time of train i at station $s+1$ which symbolised with $d_{i,s+1}$. In other words, the departure time of train i at station $s+1$, which is symbolised with $t_{i,s+1}$ has been modelled by Assis and Milani (2004) as

$$t_{i,s+1} = t_{i,s} + r_{i,s} + d_{i,s+1}. \tag{1}$$

Besides that, introduced R_s is the nominal running time of station s to $s+1$ and $u_{1i,s}$ is controlled for adjustment of the running time. Based on that, in Zhang et al. (2019) the running time of train i between station s and $s+1$, which is symbolised with $r_{i,s}$ is modeled as

$$r_{i,s} = R_s + u_{1i,s} \tag{2}$$

$u_{1i,s} > 0$ means the running time is increased, and for $u_{1i,s} < 0$, it means the running time is decreased.

Temporarily, according to Li et al. (2018), the dwell time is assumed to increase proportionally to the time interval between departure times of successive trains. The dwell time of train i at station $s+1$, which is symbolised with $d_{i,s+1}$ is modeled as

$$d_{i,s+1} = \theta(t_{i,s+1} - t_{i-1,s+1}) + D_0 + u_{2i,s+1} \tag{3}$$

where θ is the delay rate, D_0 is the minimum dwell time at the station and $u_{2i,s}$ control for adjusting the dwell time. By substituting (2) and (3) to (1), the departure time of train i is described by

$$t_{i,s+1} = t_{i,s} + \theta(t_{i,s+1} - t_{i-1,s+1}) + R_s + D_0 + u_{1i,s} + u_{2i,s+1}. \tag{4}$$

Furthermore, the train headway is modelled by Assis and Milani (2004) as the time interval of successive train departures. The headway between train i and train $i-1$ is represented by $x_{i,s} = t_{i,s} - t_{i-1,s}$. Therefore, based on (4) the headway of train i at station $s+1$, which is symbolised with $x_{i,s+1}$ is

$$\begin{aligned} x_{i,s+1} &= t_{i,s+1} - t_{i-1,s+1} \\ &= t_{i,s} - t_{i-1,s} + \theta x_{i,s+1} - \theta x_{i-1,s+1} + R_s - R_s \\ &\quad + D_0 - D_0 + u_{1i,s} - u_{1i-1,s} + u_{2i,s+1} - u_{2i-1,s+1}. \end{aligned} \tag{5}$$

Suppose $u_{i,s} = \delta u_{1i,s} + \delta u_{2i,s+1}$ where $\delta u_{1i,s} = u_{1i,s} - u_{1i-1,s}$ and $\delta u_{2i,s+1} = u_{2i,s+1} - u_{2i-1,s+1}$, the headway (5) can be written as

$$x_{i,s+1} = \frac{1}{(1-\theta)} x_{i,s} - \frac{\theta}{(1-\theta)} x_{i-1,s+1} + \frac{1}{(1-\theta)} u_{i,s}. \tag{6}$$

Based on the headway equation, the headway deviation of train i at station s which symbolised with $h_{i,s}$ is defined as $h_{i,s} = x_{i,s} - x^r_{i,s}$ where $x^r_{i,s}$ is desirable headway. With this, based on the headway equation on (6), the headway deviation of consecutive trains can be written as

$$h_{i,s+1} = \frac{1}{(1-\theta)} h_{i,s} - \frac{\theta}{(1-\theta)} h_{i-1,s+1} + \frac{1}{(1-\theta)} u_{i,s} \tag{7}$$

where $h_{i,s}$ is the headway deviation of train i at station s, θ is the delay rate and $u_{i,s}$ is control of train *i* at station *s*.

Furthermore, after the headway deviation equations are getting, passenger load equations will be presented, describing the change in the number of passengers between successive trains. The passenger load equations are presented in the following section.

2.2 Passenger load equations

The number of train passengers during peak hours can become too crowded if not regulated. To anticipate this, it is necessary to regulate the number of boarding passengers not exceeding capacity. In connection with that, a model for the number of passengers is formed by considering the number of passengers inside the train, the number of boarding passengers and the number of alighting passengers from the train.

The number of passengers in train i who departed from station $s + 1$ is symbolised with $p_{i,s+1}$. Noticed that the number of passengers on train i, which leave from station $s+1$ is a summation of amount passengers inside train i at station s, which is symbolised with $p_{i,s}$ and the number of boarding passengers in train i at station $s+1$ symbolised by $pe_{i,s+1}$ reduced with the number of alighting passengers from train i at station $s+1$ symbolised by $ps_{i,s+1}$. Based on that, in Wang et al. (2022), $p_{i,s+1}$ is modeled as

$$p_{i,s+1} = p_{i,s} + pe_{i,s+1} - ps_{i,s+1}. \tag{8}$$

Noticed that according to Wang et al. (2022), the amount of boarding passengers in the train i at the station $s+1$ symbolised by $pe_{i,s+1}$ affected by the time interval of the departure consecutive trains at the station $s+1$ represented by $(x_{i,s+1})$ is

$$pe_{i,s+1} = \eta_{s+1} x_{i,s+1} \tag{9}$$

where η_{s+1} is the number of passengers per second, the amount of passengers who goes on the train in station $s+1$ and $x_{i,s+1}$ is headway of train i at station $s+1$. Wang et al. (2022) assumed that the number of alighting passengers is proportional to the number of onboard passengers. According to that assumption, the number of alighting passengers from train i in station $s+1$, which is symbolised with $ps_{i,s+1}$ could be expressed as

$$ps_{i,s+1} = \mu_{s+1} p_{i,s} \tag{10}$$

where μ_{s+1} is the coefficient proportionality of onboard passengers for train i at station $s+1$. Then, (9) and (10) are substituted for (8) to get

$$\begin{aligned} p_{i,s+1} &= p_{i,s} + \eta_{s+1} x_{i,s+1} - \mu_{s+1} p_{i,s} \\ &= (1-\mu_{s+1}) p_{i,s} + \eta_{s+1} x_{i,s+1}. \end{aligned} \tag{11}$$

Then, by substituting the headway equation $(x_{i,s+1})$ in (6) to (11), the following equation can be written as

$$p_{i,s+1} = (1-\mu_{s+1})\,p_{i,s} + \frac{\eta_{s+1}}{(1-\theta)}x_{i,s} - \frac{\theta\eta_{s+1}}{(1-\theta)}x_{i-1,s+1} + \frac{\eta_{s+1}}{(1-\theta)}u_{i,s}. \tag{12}$$

Suppose the passenger load deviation of train i at station s symbolised with $q_{i,s}$ is defined as $q_{i,s} = p_{i,s} - p^r_{i,s}$ where $p^r_{i,s}$ is the desirable passenger load. Based on the passenger load equations (12), the following passenger load deviation is

$$q_{i,s+1} = (1-\mu_{s+1})\,q_{i,s} + \frac{\eta_{s+1}}{(1-\theta)}h_{i,s} - \frac{\theta\eta_{s+1}}{(1-\theta)}h_{i-1,s+1} + \frac{\eta_{s+1}}{(1-\theta)}u_{i,s}. \tag{13}$$

Based on the previous description, the headway equations of the consecutive train and the passenger load equations have been formed. The two equations are combined and will be formed into the state-space equations. These equations will be used in applying predictive model control to the metro train scheduling problem. The formation of state-space equations is presented in the following section.

2.3 State-space equations

In this metro train traffic modeling, based on the headway equations and the number of passengers equations, state space equations are formed. The headway deviation and the passenger load deviation as a state. The dwell time and the running time of the train are the input. Meanwhile, the output is the headway deviation and the passenger load deviation.

Let $v_{i,s} = \left[h_{i,s}, q_{i,s}\right]^T$. According to (7) and (13), the metro train traffic model can be rewritten as

$$v_{i,s+1} = A_{s+1}v_{i,s} + C_{s+1}v_{i-1,s+1} + B_{s+1}u_{i,s} \tag{14}$$

where $A_{s+1} = \begin{bmatrix} \frac{1}{1-\theta} & 0 \\ \frac{\eta_{s+1}}{1-\theta} & 1-\mu_{s+1} \end{bmatrix}$, $C_{s+1} = \begin{bmatrix} \frac{-\theta}{1-\theta} & 0 \\ \frac{-\theta\eta_{s+1}}{1-\theta} & 0 \end{bmatrix}$, and $B_{s+1} = \begin{bmatrix} \frac{1}{1-\theta} \\ \frac{\eta_{s+1}}{1-\theta} \end{bmatrix}$.

According to (14), define $V(c) = \left[v_{c-1,1}, v_{c-2,2}, \ldots, v_{c-N,N}\right]$ as a state vector with the dimension $2N \times 1$ representing headway deviation and passenger load deviation. In addition, $U(c) = \left[u_{c,0}, u_{c-1,1}, \ldots, u_{c-N+1,N-1}\right]^T$ as control vector with the dimension $N \times 1$. Based on that, (14) can be expressed as the form of state space

$$V(c+1) = \bar{A}V(c) + \bar{B}U(c) \tag{15}$$

where $\bar{A} = \begin{bmatrix} C_1 & 0 & 0 & 0 & \dots \\ A_2 & C_2 & 0 & 0 & \dots \\ 0 & A_3 & C_3 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & A_N & C_N \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B_1 & 0 & 0 & 0 & \dots \\ 0 & B_2 & 0 & 0 & \dots \\ 0 & 0 & B_3 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & B_N \end{bmatrix}$

matrix A_s, B_s, C_s for $s = 1, 2, \dots, N$ is matrix with dimension 2×2, $\bar{A}$ is matrix with dimension $2N \times 2N$, $\bar{B}$ is matrix with dimension $2N \times N$ and c is time step.

3 Optimal scheduling problem

The regulation of metro train scheduling problems aims to minimise the train's headway deviation and passenger load deviation. The headway deviation is the difference between the actual and desired headway. Suppose the headway deviation of the train can be minimised. In that case, the actual headway will approach the desired headway to ensure the regularity of the headway and overcome deviations from the train departure schedule. Furthermore, the passenger load deviation is the difference between the number of train passengers and the desired number. If the passenger load deviation can be minimised, then the number of train passengers will approach the desired number of passengers to overcome the accumulation of passengers.

Since this study investigates an optimal scheduling problem of the metro train with considering the constraints related to that problem, the corresponding objective function is

$$J = \sum_i \sum_s \left\{ w v_{i,s}^2 + z u_{i,s}^2 \right\} \tag{16}$$

where w and z are given constant weights. The first term is related to headway and passenger load deviation summation. In addition, the second term is related to significant control used to reduce control costs.

Moreover, the constraints on the metro train scheduling problem consist of safety and operational constraints as follows:

- *Safety and operational limits*: Safety limits for trains that operate at every station related to the limits of headway and capacity of passengers. Associated with that, introduced $x_{i,s}^{\min}$ is minimum limit headway and $x_{i,s}^{\max}$ is maximum limit headway. Based on that, to ensure distance safe between two consecutive trains, the following constraints are given as

  $$x_{i,s}^{\min} \le x_{i,s} \le x_{i,s}^{\max}. \tag{17}$$

  Besides that, for the number of passengers in a train, no can exceed the maximum amount of passenger carriages. Let $p_{i,s}^{\max}$ be the maximum limit of the amount passengers on the train, therefore the following constraints are given as

  $$0 \le p_{i,s} \le p_{i,s}^{\max}. \tag{18}$$

- *Control constraints*: Restrictions for control $u_{i,s}$ related to the minimum allowable control limit symbolised with $u_{\min}$ and maximum control limit which allowed which symbolised with $u_{\max}$. According to that, the following constraints are given as

$$u_{\min} \leq u_{i,s} \leq u_{\max}. \tag{19}$$

Furthermore, based on state space form (15) the objective function (16) subject to (17)–(19) can be expressed as

$$\min_{U(c)} \sum_{c=c_0}^{c_f} \left\{ V(c)^T WV(c) + U(c)^T ZU(c) \right\} \tag{20}$$

subject to

$$V(c+1) = \bar{A}V(c) + \bar{B}U(c) \tag{21}$$

$$-R_1 V(c) \leq \left(x^r - x^{\min} \right) I_{N\times 1} \tag{22}$$

$$R_1 V(c) \leq \left(x^{\max} - x^r \right) I_{N\times 1} \tag{23}$$

$$R_2 V(c) \leq P(c) \tag{24}$$

$$-R_2 V(c) \leq M(c) \tag{25}$$

$$U_{\min} \leq U(c) \leq U_{\max} \tag{26}$$

where $I_{N\times 1}$ is matrix with dimension $N \times 1$ in which all the elements are 1

$$R_1 = [r_{is}], r_{is} = \begin{cases} 1, s = 2i-1 \\ 0, lainnya \end{cases}, R_2 = [r_{is}], r_{is} = \begin{cases} 1, s = 2i \\ 0, lainnya \end{cases},$$

$$P(c) = \left[p^{\max} - p^r_{c-1,1}, p^{\max} - p^r_{c-2,2}, \ldots, p^{\max} - p^r_{c-N,N} \right]^T,$$

$$U_{\min} = \left[u_{\min}, u_{\min}, \ldots, u_{\min} \right]^T, U_{\max} = \left[u_{\max}, u_{\max}, \ldots, u_{\max} \right]^T, dan$$

$$M(c) = \left[p^r_{c-1,1}, p^r_{c-2,2}, \ldots, p^r_{c-N,N} \right]^T.$$

4 Application of model predictive control

In this study, the Model Predictive Control (MPC) method was used. MPC is a control method that can bring the system output path to an ideal path desired by predicting the input along the prediction horizon. Then, the input prediction sequence that minimises the objective function is calculated. The strategy used in MPC is the receding horizon strategy. The calculated input line depends on the past's input sequence, and only the first input sequence is applied to the system (Maciejowski, 2000). Besides, MPC could handle systems with multiple inputs and outputs with existing constraints. For example, the input for metro train scheduling problems is the dwell time and the running time.

The output is headway deviation and passenger load deviation. Specifically, the application of the MPC method on the metro train scheduling problems is as follows.

Based on optimisation problem in (20) subject to (21)–(26), for every time step, the objective function and constraints in Prediction Horizon (H_p) can be written as

$$\min_{U(c+i)} \sum_{i=0}^{H_p-1} \left\{ V^T(c+i+1)WV(c+i+1) + U^T(c+i+1)ZU(c+i+1) \right\} \tag{27}$$

subject to

$$V(c+i+1) = \bar{A}V(c+i) + \bar{B}U(c+i) \tag{28}$$

$$-R_1V(c+i+1) \le \left(x^r - x^{\min}\right) I_{N\times 1} \tag{29}$$

$$R_1V(c+i+1) \le \left(x^{\max} - x^r\right) I_{N\times 1} \tag{30}$$

$$R_2V(c+i+1) \le P(c+i+1) \tag{31}$$

$$-R_2V(c+i+1) \le M(c+i+1) \tag{32}$$

$$U_{\min} \le U(c+i) \le U_{\max}. \tag{33}$$

The optimisation problem on (27) could be formulated as the form of quadratic programming. Define $\bar{V} = \left[V^T(c+1), V^T(c+2), \ldots, V^T\left(c+H_p\right)\right]^T$ and

$\bar{U} = \left[U^T(c), U^T(c+1), \ldots, U^T\left(c+H_p-1\right)\right]^T$ on every time step c for state prediction $V(c)$ until horizon prediction H_p therefore the following equation is given as

$$\bar{V} = \bar{F}V(c) + \Omega\bar{U} \tag{34}$$

where $\bar{F} = \begin{bmatrix} \bar{A} \\ \bar{A}^2 \\ \vdots \\ \bar{A}^{H_P} \end{bmatrix}$ and $\Omega = \begin{bmatrix} \bar{B} & 0 & 0 & \ldots \\ \bar{A}\bar{B} & \bar{B} & 0 & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \bar{A}^{H_p-1}\bar{B} & \bar{A}^{H_p-2}\bar{B} & \ldots & \bar{B} \end{bmatrix}$.

Noticed that the objective function on (27) could be written becomes

$$\bar{V}^T\bar{W}\bar{V} + \bar{U}^T\bar{Z}\bar{U} \tag{35}$$

where $\bar{W}$ and $\bar{Z}$ can be taken from equation (16). Then substitute (34) to (35) therefore (27) could be written as $\bar{U}^T\left[\Omega^T\bar{W}\Omega + \bar{Z}\right]\bar{U} + 2\bar{U}^T\left[\Omega^T\bar{W}\bar{F}V\left(c\right)\right] + \Phi$, where $\Phi = V^T(c)\bar{F}^T\bar{W}\bar{F}V(c)$ constant. Besides, constraints (28)–(33) could be rewritten in the form $\bar{U}$. Therefore, the optimisation problem (27) subject to (28)–(33) could be rewritten in form $\bar{U}$ as

$$\min_{\bar{U}} \bar{U}^T\left(\Omega^T\bar{W}\Omega + \bar{Z}\right)\bar{U} + 2\bar{U}^T\left(\Omega^T\bar{W}\bar{F}V(c)\right) + \Phi \tag{36}$$

where $\Phi = \bar{V}^T(c)\bar{F}^T\bar{W}\bar{F}V(c)$

subject to

$$-R\Omega\bar{U} \leq \left(x^r - x^{\min}\right) I_{H_p N\times 1} + R\bar{F}V(c) \tag{37}$$

$$R\Omega\bar{U} \leq \left(x^{\max} - x^r\right) I_{H_p N\times 1} - R\bar{F}V(c) \tag{38}$$

$$S\Omega\bar{U} \leq \bar{P} - S\bar{F}V(c) \tag{39}$$

$$-S\Omega\bar{U} \leq \bar{M} + S\bar{F}V(c) \tag{40}$$

$$I_{H_p N\times 1}\bar{U} \leq \bar{U}_{\max} \tag{41}$$

$$-I_{H_p N\times 1}\bar{U} \leq -\bar{U}_{\min} \tag{42}$$

where

$$R = diag\left(R_1, R_1, \ldots, R_1\right), S = diag\left(R_2, R_2, \ldots, R_2\right),$$

$$\bar{U}_{\max} = \left[U_{\max}^T(c+1), U_{\max}^T(c+2), \ldots, U_{\max}^T\left(c+H_p\right)\right]^T,$$

$$\bar{U}_{\min} = [U_{\min}^T(c+1), U_{\min}^T(c+2), \ldots, U_{\min}^T\left(c+H_p\right)]^T,$$

$$\bar{P} = \left[P^T(c+1), P^T(c+2), \ldots, P^T\left(c+H_p\right)\right]^T,$$

$$\bar{M} = \left[M^T(c+1), M^T(c+2), \ldots, M^T\left(c+H_p\right)\right]^T.$$

Furthermore, optimisation problem (36) subject to (37)–(42) is equivalent to

$$\min_{\bar{U}} \frac{1}{2}\bar{U}^T H\bar{U} + \bar{U}^T f + \Phi \tag{43}$$

subject to

$$\begin{bmatrix} -R\Omega \\ R\Omega \\ S\Omega \\ -S\Omega \\ I_{H_p N\times 1} \\ -I_{H_p N\times 1} \end{bmatrix} \bar{U} \leq \begin{bmatrix} \left(x^r - x^{\min}\right) I_{H_p N\times 1} + R\bar{F}V(c) \\ \left(x^{\max} - x^r\right) I_{H_p N\times 1} - R\bar{F}V(c) \\ \bar{P} - S\bar{F}V(c) \\ \bar{M} - S\bar{F}V(c) \\ \bar{U}_{\max} \\ -\bar{U}_{\min} \end{bmatrix} \tag{44}$$

where $H = 2\left\{\Omega^T\bar{W}\Omega + \bar{Z}\right\}$, $f = 2\left\{\Omega^T\bar{W}\bar{F}V(c)\right\}$, and $\Phi = \bar{V}^T(c)\bar{F}^T\bar{W}\bar{F}V(c)$.

5 Numerical simulation

In this study, a simulation was implemented for a single metro line which includes 14 stations and 40 trains. The delay rate (θ) is 0.02 based on Li et al. (2018). The initial condition of headway deviation and passenger load deviation of 14 stations are given as the vector $v_{0,s} = \left[3,3,3,4,4,4,5,5,6,8,8,9,10,10,2,4,4,6,7,8,8,8,9,9,10,10,10,10\right]^T$. According to Wang et al. (2022), the value of η_s and value μ_s are listed in the Tables 1 and 2.

Table 1 The value of η_s

s	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
η_s	0.8	0.3	0.5	0.4	0.6	0.5	0.3	0.4	0.3	0.3	0.2	0.2	0.2	0

Table 2 The value of μ_s

s	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
μ_s	0	0.02	0.04	0.02	0.2	0.08	0.05	0.1	0.08	0.15	0.02	0.02	0.2	1

Given limitation constraint for headway is $x_{i,s}^{\min} = 90$ seconds, which is the minimum headway, $x_{i,s}^{r} = 180$ seconds which is the desirable headway and $x_{i,s}^{\max} = 300$ seconds which is the maximum headway. The values of these parameters were taken from Assis and Milani (2004). Then, the limitations for amount passengers are given based on Wang et al. (2022). Let $p_{i,s}^{\max} = 2800$ passengers, which is the maximum of passengers load, and $p_{i,s}^{r} = 2400$ which is the desirable passenger load. Besides that, based on Zhang et al. (2019) given the limitation of input, $u_{\min} = -30$ seconds and $u_{\max} = 35$ seconds. It means adjustment time with enhancement did not exceed 35 seconds, and subtraction time did not exceed 30 seconds. Then, the weight $\overline{W}$ and $\overline{Z}$ on objective function is choosen, where $w = z = 1$.

The effectiveness of the proposed model and method is illustrated in the given numerical examples. The quadprog function in Matlab 2013a was used for this optimisation problem and simulated it on a Windows 10 operating system, Intel(R) Core(TM) i3-10110U CPU @2.10 GHz processor speed and 8 GB memory size. Simulation result are getting based on given weight $\overline{W}$ and $\overline{Z}$, prediction horizon $\left(H_p\right)$ and control horizon $\left(H_u\right)$. Let $H_u = H_p = 5$. Simulation conducted during peak hours is 7.00 a.m until 8.30 a.m with the time interval simulation of 135 seconds or equivalent to 40-time steps.

The headway deviation without control (see Figure 1) and passenger load deviation without control (see Figure 2) was simulated according to the given parameters' values. For Figures A1(a), A2(b), A3(c), A4(d), A5(e) and A6(f) are presented on Appendix. Based on Figures (1) and (2), simulation without control $(u = 0)$ means that only the safety and operational limits constraint in (17) and (18) were considered in scheduling problems at metro lines. Then, the corresponding operating conditions, such as headway deviation without control and passenger load deviation without control of all the stations at each stage can be shown, which is displayed in the below figure. Based on that result, simulation without control for headway deviation and passenger load deviation fluctuated over time. There are larger accumulated train delays for all the stations of the metro under the initial value, which negatively affect the operational efficiency of the metro traffic. This condition could have a bad impact on the metro train scheduling problem.

Figure 1 Headway deviation without control

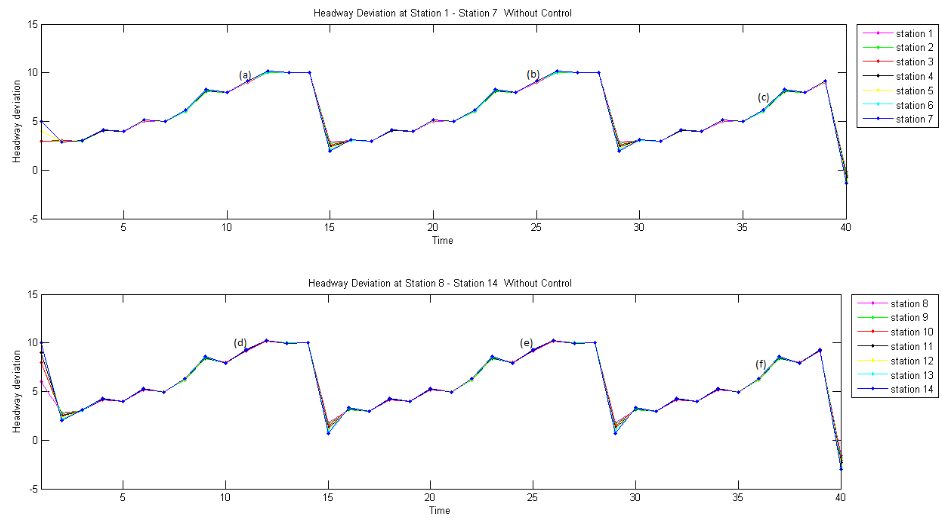
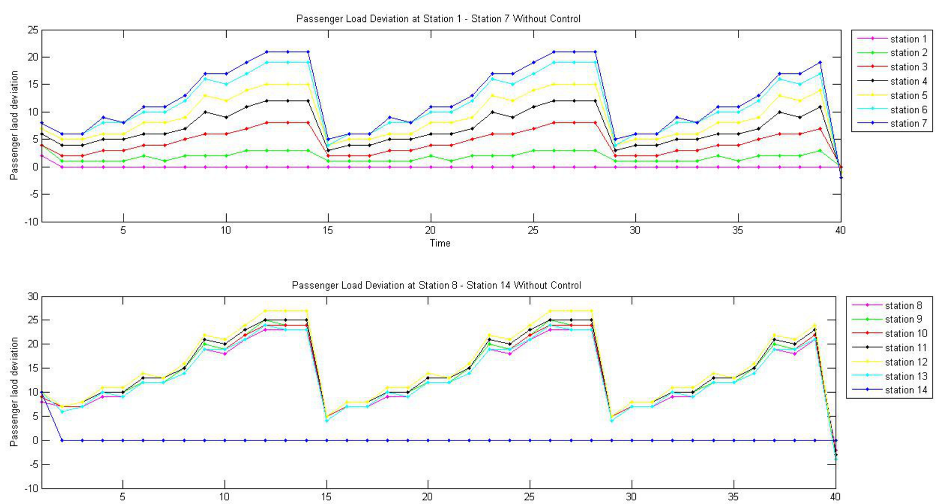


Figure 2 Passenger load deviation without control



Furthermore, the Model Predictive Control (MPC) method is applied to the metro train scheduling problems. The simulation results of headway deviation and passenger load deviation using MPC are given. Based on Figures (3) and (4), it can be seen that the train headway deviation and passenger load deviation at stations 1–14 can be minimised towards zero in the 2nd time step or approx. at 7.05 a.m. This means that train delays can be controlled. Hence, the headway deviation and the passenger load deviation become very small. The train can run precisely on time according to the desired schedule and does not exceed the maximum capacity of passengers.

Figure 3 Headway deviation using model predictive control

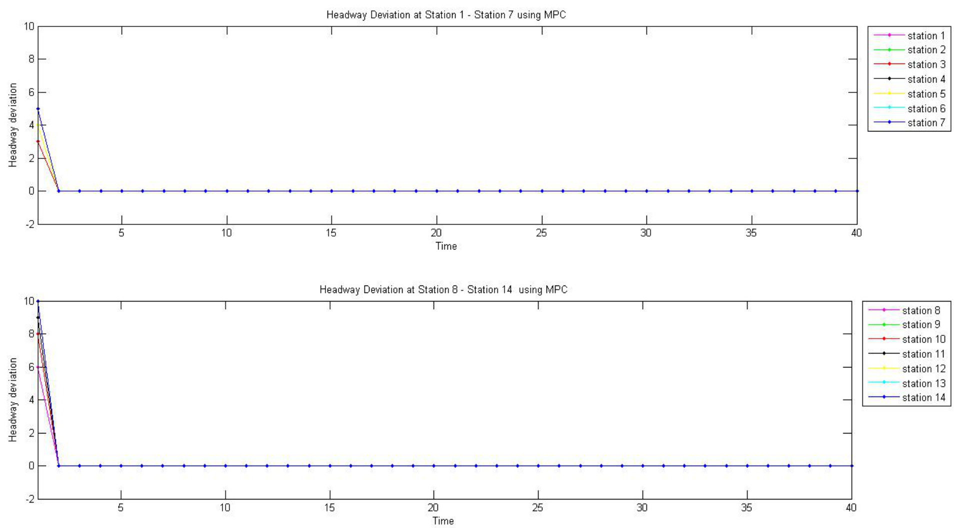
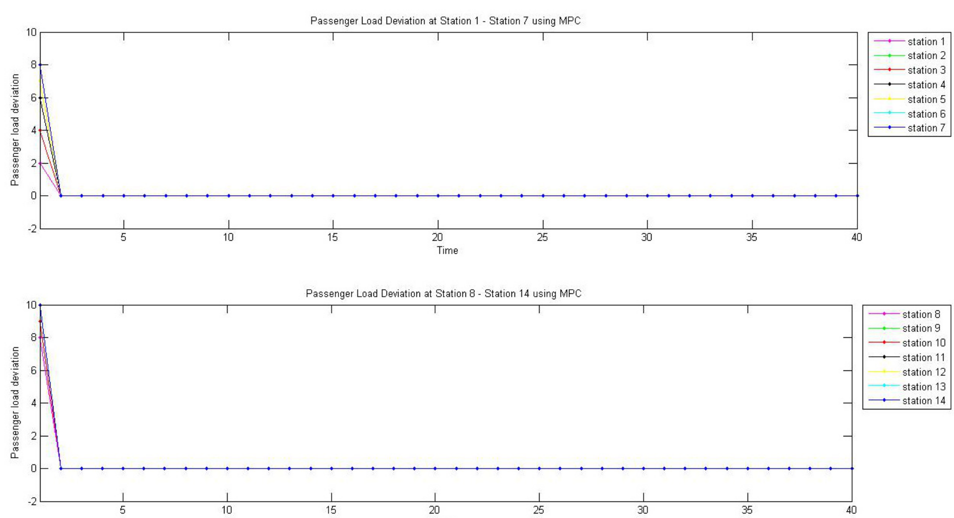


Figure 4 Passenger load deviation using model predictive control



Moreover, the proposed model (15) is formed. Then, the MPC is designed to solve the optimisation problem (20)–(26). Using the MPC, optimal input is obtained in the form of adjustments to the running time and the dwell time, which can minimise headway and passenger load deviation. Then, the corresponding headway and passenger load deviation are minimised after being controlled using MPC.

6 Conclusion and future work

Based on the research presented above, it could be seen that the application of model predictive control on metro train scheduling problems can bring headway deviation and the passenger load deviation to zero. This means that deviation can be controlled therefore it becomes minimal. On the other hand, model predictive control was successfully applied to these problems.

Furthermore, the metro train traffic model could be developed for a real-time model of a metro train system with multiple lines, and the objective function could be to consider the energy consumption of the metro train.

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Appendix

Figure A1 (a) of Figure 1

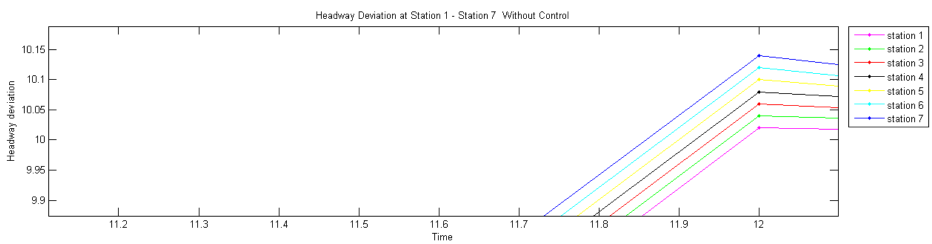

Figure A2 (b) of Figure 1

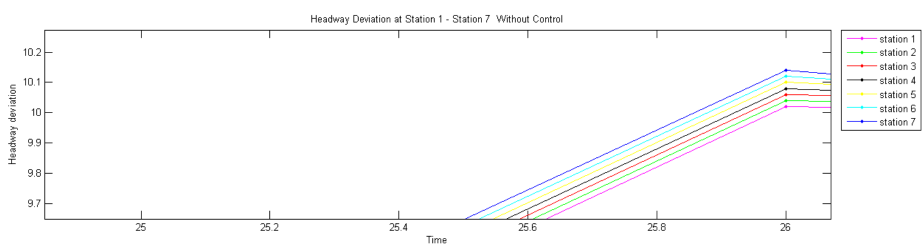

Figure A3 (c) of Figure 1

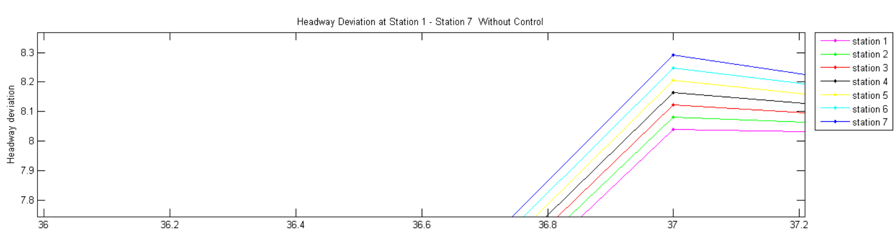

Figure A4 (d) of Figure 1

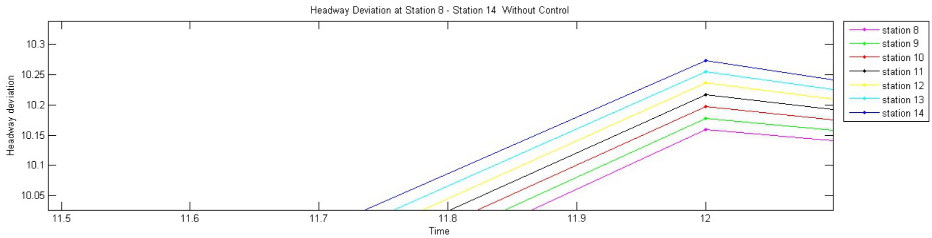

Figure A5 (e) of Figure 1

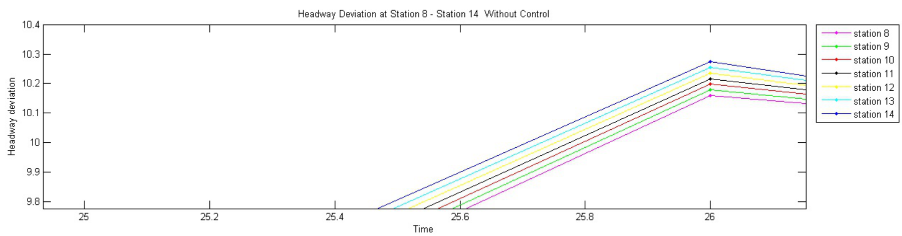

Figure A6 (f) of Figure 1