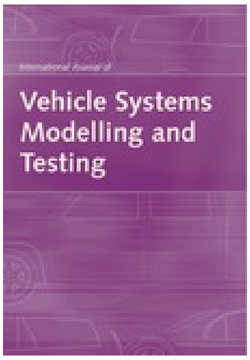

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Analytical model to predict electro-mechanical steering gear performance based on gears mesh quality

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Abstract: In the present paper, an analytical model to predict the performance of a mechanical rack and pinion steering gear is presented. One of the peculiarities of the gears used in those systems is that the rack axis cannot be fixed during the meshing to avoid jamming in the steering effort. The consequence is a need of an operational clearance that at the same time must be kept as small as possible to achieve satisfactory noise performance, avoiding gear rattle. The results will be used by gear designers to select the correct quality class of the gears, and of each individual gear parameter, that will guarantee to meet the functional performance requirements, imposed by the vehicles constructor, leading to cost reduction of gear manufacturing. Multibody simulations on reverse engineered components and experimental data collected from functional bench tests have been used to validate this study.

Keywords: rack-and-pinion; steering system; gears; analytical model; multibody simulation; functional test; predictive model.

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Biographical notes: Tobia Freddi received his Master's Degree in Mechanical Engineering from University of Brescia in 2019. He joined ZF Steering Division to develop his master's thesis work and continued to work with University of Brescia and ZF to complete the study on gears performance.

Angelo Piantoni received his Master's Degree in Mechanical Engineering from University of Brescia in 2011. In the same year, he joined ZF Steering Division initially as a University Researcher on friction, tribology and surface, finish of steering gears and from 2012 as R&D Engineer. He is currently working as core engineering team lead for mechanical steering gears and part of ZF Global Rack and Pinion team.

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1 Introduction

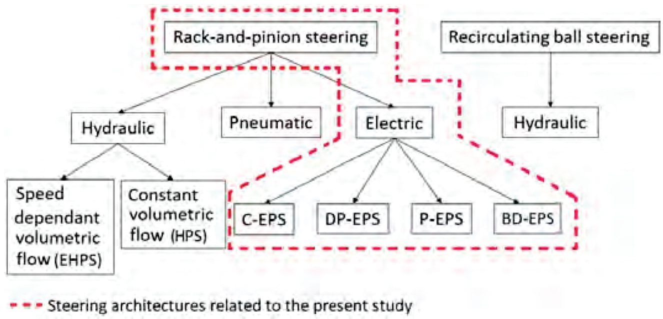
The main task of the steering system is to afford the control of lateral vehicle dynamics, by means of the rotation of the steering wheel performed by the driver, and to guarantee the safety of passengers on the vehicle. It is very important to have a high-standard functional performance to provide these tasks, and for this reason the requests of vehicle manufacturers are significantly severe.

This work deals with the performance of mechanical rack-and-pinion steering systems, the most common steering configuration in the automotive industry. In particular, the purpose of this study is to create an analytical model to predict the steering gear performance based on manufacturing errors of rack and pinion components to fulfil those requests. The performance of a steering gear with regard to friction and noise is strongly linked to the distance variation between rack and pinion axis, called center distance variation (CDV) or Box Center variation (ΔBC). The higher is the manufacturing precision of the components and the lower is the variation of this dimension; in the theoretical case of perfect components the distance variation will be equal to zero, and this corresponds as well with the ideal case for steering performance in which the gear can operate without jamming and backlash, minimising noise.

1.1 Steering systems

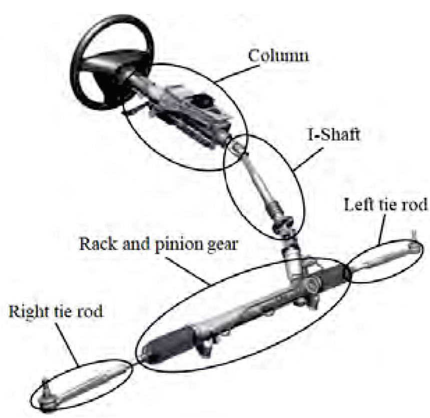
The two standard designs of mechanical steering system are recirculating ball steering and rack-and-pinion steering. Because of its reduced steering force, recirculating ball steering is used basically in high weight segments. That is the case of commercial vehicles and in some SUVs, while for passenger car segment the most common drive solution is rack-and-pinion system. The rack and pinion steering superseded over the recirculating ball gears in most of passenger cars market with independent suspension of the front axle, because of the less need for space, less weight of the full steering system, lower steering elasticity and lower production costs. The cons of the rack-and-pinion steering, like less damping of externally excited power pulses (bumpiness), the curve of the steering ratio and the lateral forces from the tie rods, are compensated by constructive measures. The necessity to develop power steering systems was caused by the increase of vehicles' weight and the need of improved vehicle steerability. The current technologies of powered steering include hydraulic power steering (HPS), electro-hydraulic power steering (EHPS), and electromechanical electric power steering (EPS). Figure 1 shows a schematic classification of the main steering systems, with evidence of those affected by this study.

Figure 1 Classification of main steering systems (see online version for colours)



In rack-and-pinion steering the torque applied by the driver is transferred to the pinion which, gearing with the rack, converts the rotative motion to linear motion. The rack is connected to the uprights with tie rods, allowing the steering of the wheels. The main components of the rack-and-pinion steering system are the steering wheel, column, intermediate shaft (I-shaft), rack-and-pinion gear and tie rods, shown in Figure 2

Figure 2 Main components of a steering assembly



Rack-and-pinion system constitutes a type of helical gears coupling that has the following features:

- crossed (non-orthogonal) axis
- variable centre distance (floating axis)
- overlap factor higher than 1 (always more than one tooth in contact)
- zero backlash between teeth (double flank contact).

Because of these peculiarities, these systems are poorly studied in literature, the main features of steering systems design can be found in the Steering Handbook by Harrer and Pfeffer (2016).

1.2 State of art

Scientific literature on analytical models of steering systems does not accurately address this topic, but it focuses mainly on manufacturing and design, kinematic analysis, synthesis of linkage systems and, more recently, steering-by-wire systems. Recent developments in the vehicle steer-by-wire system have been collected in the work of Mortazavizadeh (2020) while Yuhara's paper deals about the concept of steer-by-wire-oriented steering system design (2021).

The subject theme for the most recent literature in the field of design and manufacturing is the variable transmission ratio gears, e.g., Zheng et al. (2021) studies on geometric characteristics and tooth modification for variable speed pinion-rack drive, and Grabovic et al. (2021) published a paper regarding a hybrid analytical/Boolean approach to the generation of rack and pinion drives with variable transmission ratio. Modelling and methods for gear shaping process and cutting force prediction of variable transmission ratio rack has been analysed by Xu et al. (2020), while Song et al. (2022) have developed a study on the electric power assisted steering systems in vehicle from a CAE simulation point of view. A study on the optimisation of steering linkage has been proposed by Sleesongsom and Bureerat (2016); Huan et al. have developed a reliability sensitivity analysis methodology for the kinematic accuracy of rack-and-pinion steering linkages. Computation and optimisation of rack and pinion steering mechanism considering kingpin parameters and tyre side slip angle has been studied by Zhang et al. (2023).

The influence of gear manufacturing errors on rack and helical pinion meshing has been extensively studied by Marano et al. (2017 and 2018) with analytical and numerical solution, that includes reverse engineering (performed by CMM measurement) to obtain the rack geometry. Scientific literature on multibody simulation, on the effects of machining, assembly and transmission errors is fully reported in his study.

The following analytical model is the extension of the work done by Marano et al. (2018) to predict the centre distance variation. In fact, this model includes the contribution of pinion's manufacturing errors, to complete the study on the entire steering system.

This study has led to the definition of an overall equation that includes the superposition of the effects of rack and pinion manufacturing errors, which can be used to fully predict the centre distance variation. Multibody and analytical models are validated through experimental measurements, resulting in better agreement than Marano's study.

1.3 Main components of rack-and-pinion systems and their functions

1.3.1 Pinion

The pinion (Figure 3) is an helical gear which is connected to column and steering wheel by the intermediate steering shaft, and it is meshing with the rack. It has a fixed axis of rotation and it is constrained by means of two bearings. The main task of the steering pinion is to transform the rotation of the steering wheel into a translation of the rack. As the steering wheel can be turned in both directions (CW and CCW) it is necessary that the gear mesh is designed with teeth in double flank contact without backlash. This allow the driver to quickly change the direction of rotation without clearance in the components meshing that could lead to loss of steering feel (non-constant steering transmission or delays) and customer dissatisfaction due to metallic noise.

Figure 3 Cut of pinion and his constraints (see online version for colours)

To have a better transmission of the motion, the pinion also must be helicoidal, in order to have a contact that is distributed over a larger surface, and the minimum diameter.

1.3.2 Rack and rack guide

The rack converts the rotation, given by the steering wheel on the pinion, into the translation of itself and the tie rod. The rack must transfer the highest applying steering forces and tie rod forces in axial and radial direction.

The rack is guided on one side from teeth meshing with the pinion and on the other side by a spring-loaded rack yoke. The rack is constrained in this way because it must be always in contact with pinion but also free to move to compensate meshing errors. The consequence of this is that the rack has a floating axis. In fact, if both rack and pinion axis would be rigidly constrained a meshing error would create an hyperstatic reaction which would excessively increase friction and consequently effort to rotate the pinion.

The rack yoke almost completely surrounds the diameter of the rack on the side that is turned away from the pinion (looking at the cross section) and presses the rack against the pinion by means of a compression spring. The structure of the rack yoke and its components is shown in Figures 4 and 5.

The other constraint of the rack is the rack bushing. It is a sliding fit which has the task to support the lateral forces from the tie rods in the rack and to guarantee the low-friction and noiseless translation of the rack.

Figure 4 Rack yoke's components (see online version for colours)

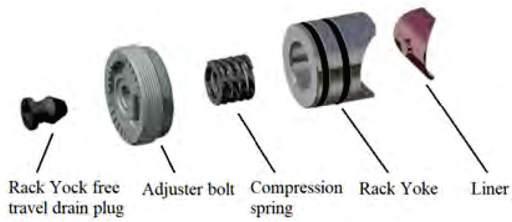
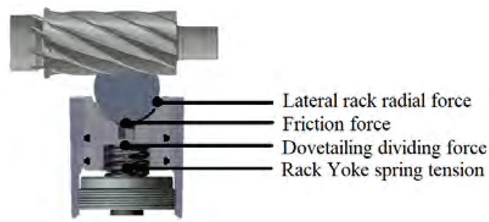


Figure 5 Rack yoke and forces acting on it (see online version for colours)



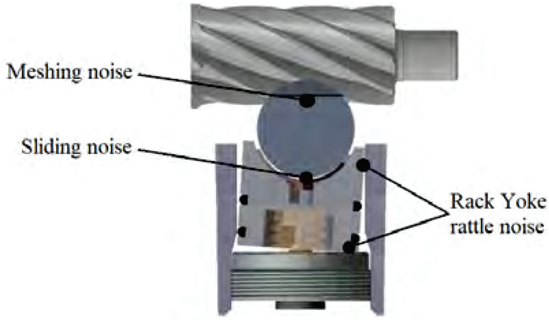
1.4 Influence of centre distance variation on steering performance

To guarantee the correct functioning of the rack support it is required an axial free travel of the rack yoke that is directly related to manufacturing errors of the gears. This free travel can be ensured by a clearance between the rack yoke and the adjuster bolt called yoke clearance. To avoid noises of the rack yoke, it is necessary to have this free travel as little as possible. See Figure 6 for the different possible types of noise.

The axial bearing clearance is given by the adjuster bolt for the rack yoke clearance that basically represents a stop unit for the rack yoke, preventing additional movement. Here again, the free travel should be as low as possible. Every rack and pinion couple has its own potential range of yoke clearance set, where the minimum is always given by the center deviation that is the sum of the manufacturing errors of that specific pair of gears, but that is not known and not predictable upfront. If the setting of the rack yoke clearance is offering an insufficient free travel, it will lead to a significant increase of the friction in the rack, which the driver will perceive as very displeasing and may cause an increased steering effort or jamming; on the other hand, instead, if the highest limitation of the clearance is exceeded, there is risk of steering rattle noise.

In the theoretical case of perfect gears, without machining errors, this clearance could be theoretically set equal to zero as the backlash will be negligible. In the real case this situation is not possible due to the manufacturing errors of the components. Therefore, the rack yoke clearance has to be adjusted in such a way that it can also compensate the permissible deviations of the run-out deviation of the pinion, the permissible static sag of the rack and the tolerance differences. Altogether, the setting of rack yoke and its parts allow tuning the steering qualities, such as noise, damping and response of the mechanical gear.

Figure 6 Possible causes of noise due to rack yoke movements (see online version for colours)



2 Analytical model

The centre distance variation is calculated by a validated analytical model that process the real measurements of components with their errors. This analytical model is the superposition of the theory to estimate the operating center distance developed in Marano et al. (2018) for of rack's deviations and a novel model that provide a prediction of pinion's deviations contributes. Rack deviations contribute to ΔBC is reported in the following equation (Marano et al., 2018, p.192):

$$\Delta BC_{rack} = \left|\frac{\Delta p_{rack}}{2tan(\alpha_n)}\right| + \left|\frac{tan(K \cdot \Delta\beta_{rack})}{2tan(\alpha_n)}\right| + \Delta h_{overoller} \tag{1}$$

Where K is:

$$\frac{Rack\ facewidth\ (normal\ to\ axis)}{cos(\beta_{rack})} \tag{2}$$

2.1 Pinion parameters affecting the centre distance variation

2.1.1 Radial single-ball measure

The radial single-ball dimension, M_{rK}, is the distance between the gear axis and, in the case of an external gear, the outermost point of a measuring sphere of diameter DM, which lies in a tooth space in contact with both tooth flanks; see Figure 7.

The theoretical M_{rK} dimension with a standard diameter sphere is calculated as shown in the following steps.

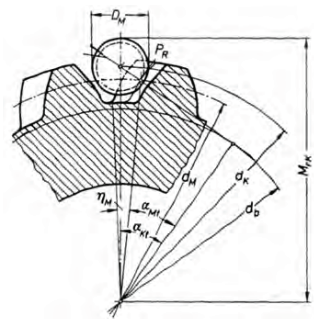
$$inv(\alpha_{Kt}) = \frac{D_M}{z \cdot m_n \cdot cos\alpha_n} - \eta + inv(\alpha_t) \tag{3}$$

Once the $inv(\alpha_{Kt})$ is calculated, α_{Kt} can be obtained by reversing the involute function (which is not analytically reversible, therefore it must by reversed using a numerical method).

The next step is to calculate the diameter of the circle on which the centre of the measuring ball lies, d_K, which can be found as:

$$d_K = d \cdot \frac{cos(\alpha_t)}{cos(\alpha_{Kt})} \tag{4}$$

Figure 7 M_{rK} measurement (ISO 21771)



Finally, the theoretical radial single-ball dimension, M_{rK}, is given by:

$$M_{rK} = \frac{1}{2} \cdot (d_K + D_M) \tag{5}$$

The ΔBC due to a radial single-ball measure deviation needs a correction coefficient because the transversal pressure angle due to contact between tooth and ball is different from the one which derives from tooth and rack contact.

$$M_{rK} \text{ correction coefficient} = \frac{sen(\alpha_{Mt})}{sen(\alpha_{Kt})} \tag{6}$$

where:

- α_{Mt} is the transversal pressure angle at d_k diameter due to rack-tooth contact;
- α_{Kt} is the transversal pressure angle at d_k diameter due to ball-tooth contact.

The ΔBC affected by this parameter is explicated in the next equation.

$$\Delta BC_{M_{rK}} = \Delta M_{rK} \cdot \frac{sen(\alpha_{Mt})}{sen(\alpha_{Kt})} \tag{7}$$

2.1.2 Pinion helix angle

The effect of the helix error on the centre distance variation can be evaluated from the projection of the helix on a plane, which as a nominal inclination β and its related error $\Delta\beta$ (Figure 8).

The contribution to the centre distance variation can be then calculated as expressed in the following equations.

$$l = \frac{Rack\ facewidth\ (normal\ to\ axis)}{cos(\beta_{mid})} \tag{8}$$

$$x = \frac{l}{2} \cdot tan(\Delta\beta_{pinion}) \tag{9}$$

$$Y_{pinion} = \left| \frac{x}{tan(\alpha_{n-mid})} \right| = \left| \frac{l \cdot tan(\Delta\beta_{pinion})}{2tan(\alpha_{n-mid})} \right| \tag{10}$$

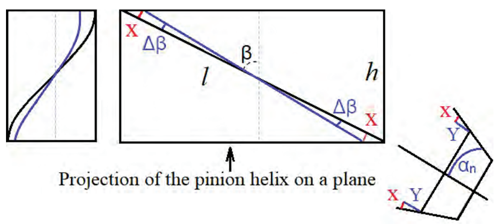
$$\Delta BC = \left| \frac{Rack\ facewidth\ (normal\ to\ axis)}{cos(\beta_{mid})} \cdot \frac{tan(\Delta\beta_{pinion})}{2tan(\alpha_{n-mid})} \right| \tag{11}$$

where $-mid$ values are intended as values at *Pinion Mid Contact Gauge Point Diameter*, d_{mid}, which is the diameter where the contact occurs on average and it has been calculated by means of the following equation:

$$d_{mid} = 2 \cdot \sqrt{\left(\frac{d_b}{2}\right)^2 + \left[\sqrt{\left(\frac{d_{out}}{2}\right)^2 - \left(\frac{d_b}{2}\right)^2} - \left(\frac{L_p}{2}\right)\right]^2} \tag{12}$$

where L_p is the *Length of Path Contact*.

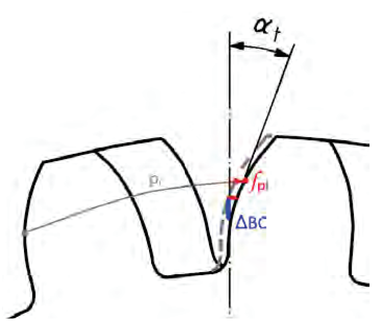
Figure 8 Pinion helix angle deviation (see online version for colours)



2.2 Pinion pitch deviation

The pitch error f_{pi} is defined as the difference between an actual size and the nominal size of an individual transverse pitch of the right or left flank (Figure 9).

Figure 9 Pinion pitch deviation (see online version for colours)



The centre distance variation caused by this deviation is determined by the equation:

$$\Delta BC = \frac{|f_{pi}|}{2tan(\alpha_{t-mid})} = \frac{|f_{pi} \cdot cos(\beta)|}{2tan(\alpha_{n-mid})} \tag{13}$$

2.3 Pinion global equation

The centre distance variation caused by the parameters described above can be written in one equation that includes all of them. This equation is the linear superposition of every independent effect and it is valid only in the theoretical situation in which the pinion (the real pinion, with its errors) meshes with a perfect rack, called *master rack*.

$$\begin{aligned}\Delta BC_{pinion} = \Delta M_{rK} \cdot \frac{sen(\alpha_{Mt})}{sen(\alpha_{Kt})} + \frac{|f_{pi} \cdot cos(\beta)|}{2tan(\alpha_{n-mid})} + \\ + \left| \frac{Rack\ facewidth}{cos(\beta_{mid})} \cdot \frac{tan(\Delta\beta)}{2tan(\alpha_{n-mid})} \right| \end{aligned} \tag{14}$$

2.4 Rack and pinion global equation

To estimate the centre distance variation of rack and pinion systems due to components manufacturing errors a global equation is proposed. It is based on the superposition of the effects of every single error/deviation taken with their signs in order to consider properly compensations or amplifications caused by the pairing of the two components.

To obtain a more accurate simulation of the teeth meshing behaviour, the gear mesh total contact ratio ϵ_γ has been introduced to the equation. This parameter affects the meshing errors that contributes to the equation, i.e., pitch deviations and helix angle deviations.

$$\begin{aligned}\Delta BC = \Delta M_{rK} \cdot \frac{sen(\alpha_{Mt})}{sen(\alpha_{Kt})} + \Delta h_{overoller} + \frac{|f_{pi} \cdot cos(\beta_{pinion}) - \Delta p_{rack}|}{2tan(\alpha_{n-mid}) \cdot \epsilon_\gamma} + \\ + \frac{Rack\ facewidth}{cos(\beta)} \cdot \frac{|tan(\Delta\beta_{pinion} - \Delta\beta_{rack})|}{2tan(\alpha_{n-mid}) \cdot \epsilon_\gamma} \end{aligned} \tag{15}$$

This equation can be seen as the union of the contributes of three macro-deviations:

- Radial deviations = $\Delta M_{rK} \cdot \frac{sen(\alpha_{Mt})}{sen(\alpha_{Kt})} + \Delta h_{overoller}$
- Pitch deviations = $\frac{|f_{pi} \cdot cos(\beta) - \Delta p_{rack}|}{2tan(\alpha_{n-mid}) \cdot \epsilon_\gamma}$
- Helix angle deviations = $\frac{Rack\ facewidth}{cos(\beta)} \cdot \frac{|tan(\Delta\beta_{pinion} - \Delta\beta_{rack})|}{2tan(\alpha_{n-mid}) \cdot \epsilon_\gamma}$.

3 Multibody simulations to validate single errors effect on ΔBC

The validation of the analytical model was performed in two separate steps comparing the results with:

1 multibody simulation with imposed deviation on single parameters

2 experimental test performed with gears fully measured.

If there will be an evidence of a correlation between the analytical model and the multibody simulation and then between model and bench test, consequentially there will be a correlation between multibody simulation and real case.

The kinematic model is developed by means of multibody software ®FunctionBay RecurDyn (multi-body dynamics software based on recursive formulation).

To obtain the results necessary for this case study it is not essential to use a model in which all the components of the steering guide are present, but it is better to simplify the system to obtain more flowing and faster simulations. Therefore, the ultimate multibody setting of the system is reduced to an only rack and pinion model (Figure 10), analysed from a kinematic point of view. Contact parameters (Tables 1 and 2), surface patch and friction settings have been set as reported in Marano et al. (2017, 2018).

Figure 10 Simplified model of rack and pinion system (see online version for colours)

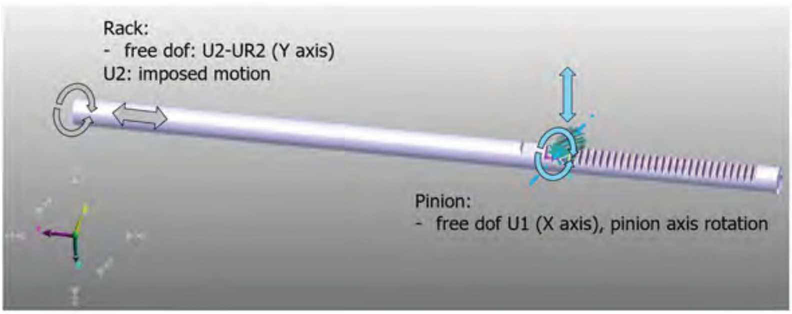


Table 1 Multibody contact parameters

Interface	*K [N/mm]*	*C [Ns/mm]*	*a*	*b*	*c*
Rack-pinion	3.45 E6	0.1	1	0	0
Rack-yoke liner	1.44 E6	0.1	1.5	0	0
Yoke-plug	1 E8	0.1	1.5	0	0

Table 2 Multibody contact parameters

Patch parameter	*Rack*	*Pinion*	*Liner*	*Yoke*	*Plug*
Surface type	Triangle	Triangle	Triangle	Triangle	Triangle
Plane tolerance factor	3	0.5	3	3	3
Max facet size factor	2	0.1	2	2	2

The sliding force can be considered constant and independent from the other friction sources, as explained in Gritti et al. (2017) and Wou et al. (2001).

The two components are represented as rigid bodies, where the movement imposed by the software is the rack sliding. It slides so that the pinion passes from one end to the other of the rack, then the imposed movement invert his direction and comes back to the initial position.

3.1 Reverse engineering of R&P

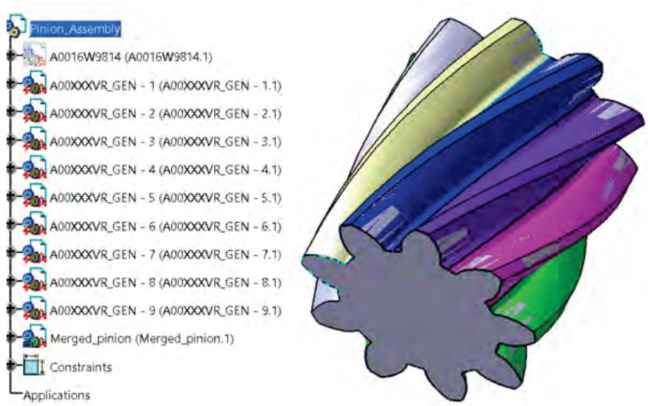
The racks and pinions used in these simulations are parametric 3D models with real geometry obtained from reverse engineering of real components.

The CMM measurement is provided by Zeiss ®Contura CMM for racks and by the evolventimeter Mahr PRIMAR MX4 for pinions, which are able to do a fully automatic measuring of gear parameters.

These particular machines have a tool probe that touches teeth flanks and create a virtual geometry that will be compared with the theoretical one. When all the teeth are inspected, the data are elaborated by a computer that outputs a report with the values of the measurement. The detected parameters are helix and pressure angles, transverse pitch, overoller measurement and M_{rK} measurement. Once these measurements are obtained, they are used as input for a parametric model of the component, shown in Figure 11.

In these models every tooth is independent from each other with its own helix angle, pitch measure and over pin/ball quote, hence the simulations replicate the real components' behaviour.

Figure 11 Parametric 3D model of pinion (see online version for colours)



3.2 *Tests on single pinion errors*

The single parameters analysed are helix angle and pitch errors. Radial errors do not require a special own simulation to be validate; in fact, because of their nature, they are already in the centre distance direction, therefore their behaviour is easily understandable. To have a correct comparison material, six different $\Delta\beta$ and four f_{pi} deviations have been set (Table 3):

Table 3 $\Delta\beta$ f_{pi} deviations setting

$\Delta\beta$ [minutes]	f_{pi} [mm]
$\pm$ 1	$\pm$ 0.01
$\pm$ 2	$\pm$ 0.02
$\pm$ 5	

To see the variation between the ideal center distance and the one with deviations, helix modification was imposed on four of the nine teeth.

The pitch errors cannot be uniform for every tooth, otherwise there will be a compenetration between teeth. To avoid this situation, a set of errors with sequence $-3x$; $+x$; $+x$; $+x$; 0;0; 0; 0; was imposed on the eight pitches of a nine-toothed gears.

Simulations results for two examples of the cases examined are exposed in the following graphs, one showing the $\Delta\beta_{pinion} = 1'$ (Figure 12) and the other showing $f_{pi} = 0.01$ mm (Figure 13).

Figure 12 Helix error's multibody simulation report (see online version for colours)

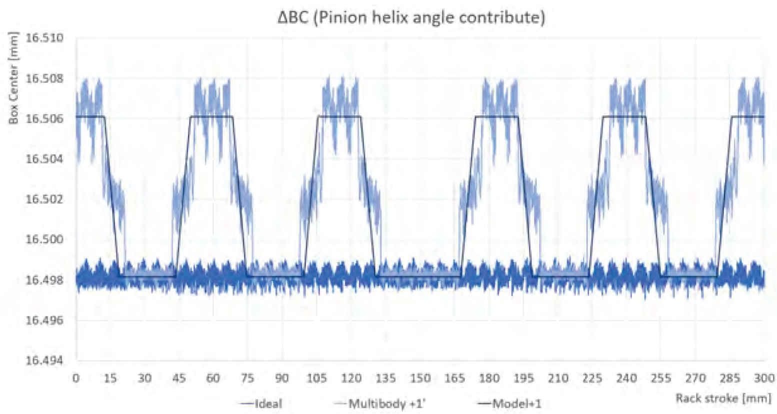
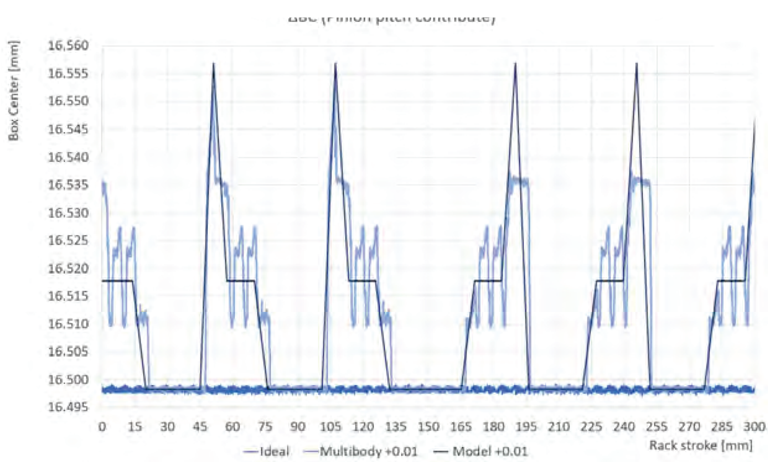


Figure 13 Pitch error's multibody simulation report (see online version for colours)



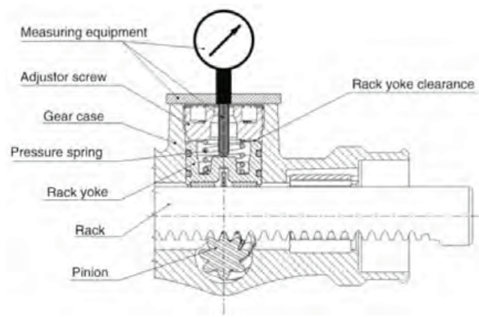
4 Set-up of bench test

The bench test used to validate the model is the yoke clearance variation functional test as it was considered the best way to evaluate the box centre variation: in fact, an increase of the center distance means a rack movement in the direction opposite to the pinion, compressing the pressure spring, and hence the yoke clearance will decrease of the same amount and vice versa. Therefore, the two functions are the opposite of one other.

$$\Delta\ Box\ Center = -\Delta\ Yoke\ Clearance \tag{16}$$

This test is performed as shown in Figure 14, actuating the rack and recording the yoke position as a function of rack axial position by means of a linear transducer. The rack is not subject to external loads, but it is only forced to move in one direction and then in the other way to return at its initial position.

Figure 14 Section along the rack yoke axis to show the set-up of bench tests



To study properly the influence of the manufacturing errors and to validate the model in the right way, is been set a DOE (Design Of Experiments). the selected input variables are manufacturing errors, i.e.:

- rack teeth convergence[1]
- rack lead angle
- pinion runout
- pinion helix angle.

These variables are the same included in the analytical model, except for the rack and pinion pitch errors which are not included in this set of variables because it is the most stable parameter in rack-and-pinion manufacturing process.

Every manufacturing errors have been set on two levels:

- *Level 1*: Parameters are set at their nominal tolerance range (green in Figure 15).
- *Level 2*: Parameters are set to a value that differs from the nominal, chosen arbitrarily to ensure that should cause a significant variation in the Δ *box center* (red in Figure 15).

The output variable in this DOE is the yoke clearance variation. Two repetition of the tests have been done for every error combination. The totality of the components, racks and pinions, have been measured to have all the data needed by the analytical model. Figure 15 summarises the set of tests done.

Figure 15 Set of conditions combination of DOE (see online version for colours)

Steering Gear		Convergence [mm]	Rack Lead Angle [minutes]	Pinion Runout [mm]	Pinion Helix Angle [minutes]
1	2				
6 H	58 H	0	0	0	0
63 H	3 H	0.03	0	0	0
31 H	44 H	0	5	0	0
41 H	23 H	0.03	5	0	0
64 H	21 H	0	0	0.015	0
10 H	61 H	0.03	0	0.015	0
34 H	24 H	0	5	0.015	0
12 H	53 H	0.03	5	0.015	0
46 H	17 H	0	0	0	-3
42 H	8 H	0.03	0	0	-3
27 H	49 H	0	5	0	-3
26 H	39 H	0.03	5	0	-3
5 H	59 H	0	0	0.015	-3
30 H	52 H	0.03	0	0.015	-3
15 H	62 H	0	5	0.015	-3
1 H	36 H	0.03	5	0.015	-3

5 Results obtained

The following graphs show the most significant comparisons between the centre distance variation obtained by bench tests and the analytical results given by the model explained in Section 2, which includes the rack's and pinion's deviations obtained by CMM measurements.

The two cases shown in Figures 16 and 17, one with convergence error only (No. 3H) and the other one with pinion run-out (No. 64H), are well predicted by the analytical model. Furthermore, as on rack strokes includes circa three pinion's revolutions it can be noticed a periodic trend along the rack stroke.

Figure 16 Comparison between analytical model and bench test results for steering gear No. 3H

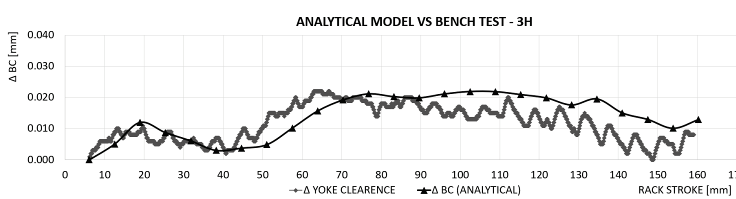
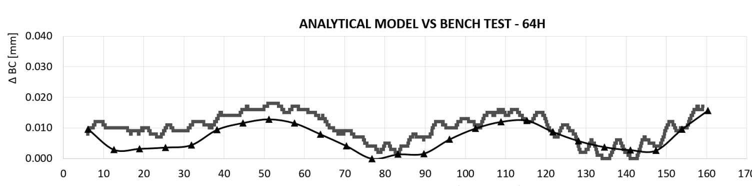
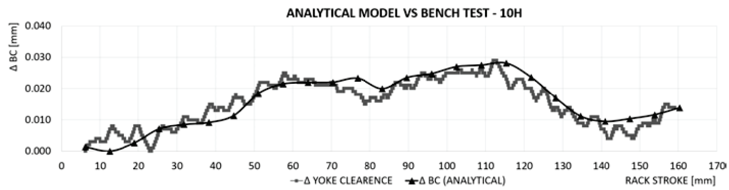


Figure 17 Comparison between analytical model and bench test results for steering gear No. 64H



The steering gear 10H (Figure 18) has both run-out and convergence errors. The maximum value of the centre distance variation is higher than both previous cases. This is due to the fact that radial errors are adding one to each other resulting in a higher centre distance variation measure.

Figure 18 Comparison between analytical model and bench test results for steering gear No. 10H



Gears No. 44H (Figure 19) and No. 46H (Figure 20) have rack angle error and pinion helix error, respectively.

Figure 19 Comparison between analytical model and bench test results for steering gear No. 44H

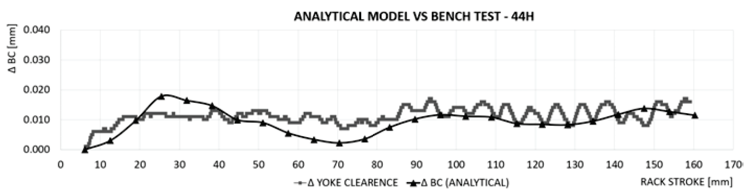


Figure 20 Comparison between analytical model and bench test results for steering gear No. 46H

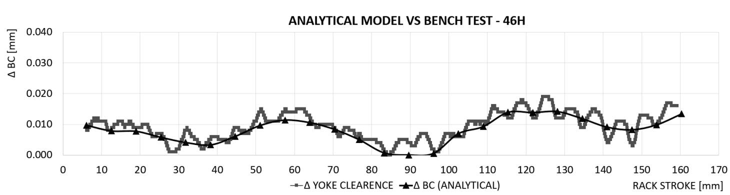


Figure 21 represent the results of gear No. 49H, which has both helix and rack angles deviations. Also in this case the model reflects the previous results, giving an optimal output of the trend and it is confirming that angular errors in opposite direction are not compensating, but increasing the centre distance variation.

Both radial and meshing errors has been applied to gear No. 8h (Figure 22), which shows a good correlation between bench test and analytical equation's result.

As it emerges from the comparison graphs, the model predicts with good approximation the experimental data. A numerical comparison between the analytical prediction and the bench tests can be performed analysing the *range of centre distance variation*, intended as the difference between maximum and minimum value along the full rack stroke, of both cases. Therefore, every steering gear have two range values, one for the model and one for the test. Quality of the model's prediction has been estimated with the dispersion chart shown in Figure 23, where the model ranges are on the x axis and the test ranges are on the y axis and every point on this chart indicates a steering gear tested.

Figure 21 Comparison between analytical model and bench test results for steering gear No. 49H

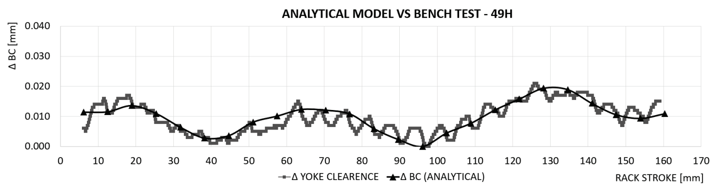


Figure 22 Comparison between analytical model and bench test results for steering gear No. 8h

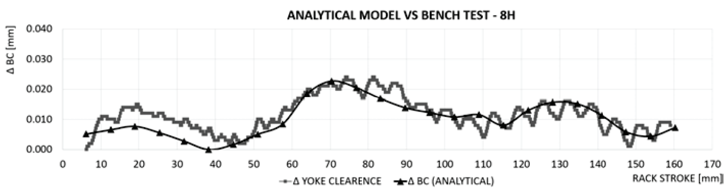
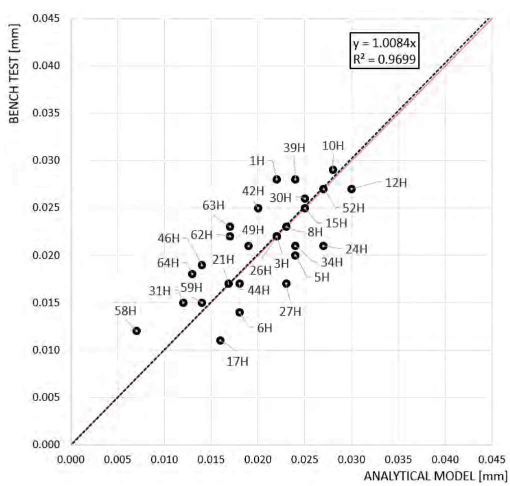


Figure 23 Dispersion chart that shows the quality of model prediction (see online version for colours)



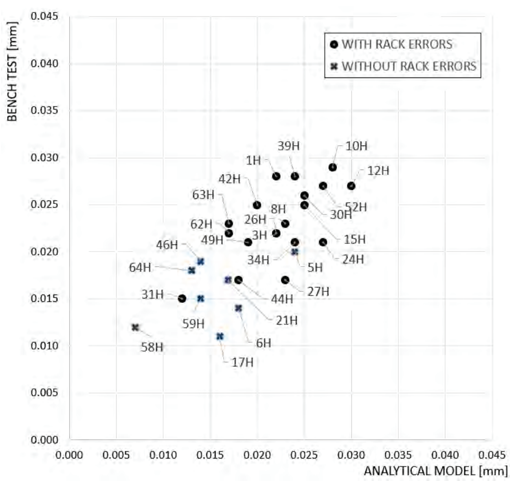
It is evident that in case of perfect prediction the two ranges will have the same values, therefore it will be seen as a line with equation $y = x$, shown in red on the chart. The trend line of the experimental points from the DOE compared to analytical solution (with imposed intersection in the origin) has equation $y = 1.0084x$ with a coefficient of determination

$R^2 = 0.9699$. These results prove that the analytical model is in good agreement with experimental results obtained from bench test.

The differences between measured and analytical results could be related to the fact that the model cannot perfectly predict the compensation or amplification of the combined meshing errors as it is considering some simplified assumptions than reality. Moreover, there are parameters that were not taken into consideration (e.g., pressure angle deviations) that may have a significant influence and may cause the oscillatory trend of bench tests graphs.

Analysing the spread of the points of the DOE in the chart emerges that the most influential factor that affects the centre distance variation is the radial error, while meshing error have less impact on the results. Another point to highlight is that rack errors have a greater impact, in fact gears without them (i.e., 6H - 17H - 46H - 58H - 5H - 64H - 21H - 59H) are never above 20 μ m (Figure 24). This is due to the fact that rack's manufacturing process provides lower accuracy to the component itself.

Figure 24 Centre distance variation: comparison between gears without rack errors and gears with rack errors (see online version for colours)



6 Conclusions

In the present paper, a novel methodology to characterise the functional performances of a rack and pinion steering gear is proposed.

A multi-body model that includes both rack and pinion manufacturing errors obtained from reverse engineering of real measured components has been developed.

The numerical model has been used to correlate the single effect of the manufacturing errors, which lead to a meshing error that causes the centre distance variation, with the test rig experimental data and the analytical equation.

A design of experiment has been conducted to validate the model; the global equation for the center distance variation related to rack and pinion manufacturing tolerances has shown a good correlation with the real measurements obtained from the test bench.

The expected advantages of this study will be:

- Gear designers will be able to select the correct tolerance ranges for each gear parameter to achieve the desired performance at system level, increasing customer satisfaction.
- These results are independent from the rack and pinion gear configuration (number of teeth, helix angle, etc.) because it is based on the geometric error of the single parameter, and it is parametrised on the deviation of each value for each parameter, not on the absolute values.
- Prioritise the parameters with stronger influence on the center distance variation, especially where the manufacturing process can accept tighter tolerances without increasing costs significantly.
- Increasing tolerances of the parameters with lower leverage on center distance variation, resulting in cost reduction of gear manufacturing.
- Compensate manufacturing errors between rack and pinion by means of selective assembly or matching, resulting in scrap reduction or opportunity of achieving best performance at system level without changing gear manufacturing process.

One of the possible future developments of this study could be the implementation of a parameter into the equation that counts with better approximation the superposition of the meshing errors of the two components.

Another possible improvement on the analytical model could be the consideration of other parameters that now have been considered negligible for the current level of approximation, like the pressure angle.

A potential further development of this study could be to check its applicability also for gears with different tolerances (quality class). This could also offer a potential extension to other applications rather than automotive steering gears.

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Note

[1]The *convergence* is a teeth height modification to have an overoller quote variation along the rack stroke with a convex trend. This parameter is strictly related with *Rack Overoller Dimension*.