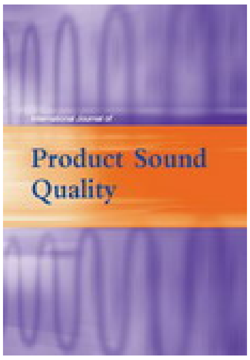


International Journal of Product Sound Quality

ISSN online: 1742-6766 - ISSN print: 1742-6758
https://www.inderscience.com/ijpsq

Vibration-based crack detection in cantilever beams using Rao-1 algorithm

S. Manikanta Reddy, M. Senthilkumar, T.G. Sreekanth

DOI: 10.1504/IJPSQ.2023.10058167

Article History:

Received:	02 December 2022
Last revised:	09 February 2023
Accepted:	09 February 2023
Published online:	28 September 2023

Vibration-based crack detection in cantilever beams using Rao-1 algorithm

S. Manikanta Reddy*, M. Senthilkumar and T.G. Sreekanth

Department of Production Engineering,
PSG College of Technology,
Coimbatore, Tamil Nadu, India
Email: manikantaslv@gmail.com
Email: msk.prod@psgtech.ac.in
Email: sreekanthtg007@gmail.com
*Corresponding author

Abstract: The prediction of crack location and crack depth in a cantilever beam is considered as an optimisation problem. In this paper, the location and depth of the crack are predicted by minimising the cost function. The cost or objective function is formulated based on the difference between the first three experimental and calculated natural frequencies. The natural frequencies are calculated from the equations which are developed based on the numerical analysis. Rao-1 algorithm is used for predicting the crack location and depth. The Rao-1 algorithm does not have any algorithm-specific parameters to tune them to get optimised performance. Results obtained from the Rao-1 algorithm are compared with those obtained by particle swarm optimisation (PSO). PSO algorithm detected the crack with an average error of 4.94% and the crack depth with 5.96% whereas the Rao-1 algorithm has predicted the location and depth of the crack with an average error of 4.85% and 6.08%. Rao-1 algorithm is also robust and can be used in damage detection to avoid tuning algorithm-specific parameters for better prediction.

Keywords: crack; beam; objective function; optimisation; Rao-1 algorithm; natural frequency.

Reference to this paper should be made as follows: Reddy, S.M., Senthilkumar, M. and Sreekanth, T.G. (2023) 'Vibration-based crack detection in cantilever beams using Rao-1 algorithm', *Int. J. Product Sound Quality*, Vol. 1, No. 1, pp.46–58.

Biographical notes: S. Manikanta Reddy has completed his research on crack detection in composites using artificial intelligence techniques. His current research includes composite materials and damage detection.

M. Senthilkumar is a Professor of Production Engineering at the PSG College of Technology. He received his Doctorate in Composite Materials from the Anna University. His current research interests include composite materials and application of smart materials in vibration control.

T.G. Sreekanth is an Assistant Professor of Production Engineering in the PSG College of Technology. His area of interest includes health monitoring of composites and advanced composite materials.

1 Introduction

Beams have been mostly used in many areas of engineering applications and have been used in modelling civil and mechanical engineering problems. The early detection of structural damage is one of the biggest concerns in the automobile, aerospace, civil and mechanical industries. Early damage detection helps in decreasing the downtime, to assess the safety, and to avoid catastrophic failures. The concept of structures' health monitoring helps in detecting the damage in structures and repairs the damage to increase the service life. Damage can be defined as any alteration in structural integrity that causes unwanted stresses and vibrations in the structure.

Damage may occur in the structure due to manufacturing defects, accidental impacts, fatigue loads, hail storms, run-away debris, and other external factors (Senthilkumar et al., 2021). The damage to the structure causes a change in the dynamic response of the system. The change in the dynamic response can be used in damage detection. The damage identified in beam-like structures using flexibility matrix changes (Pandey and Biswas, 1994). The effect of the location and size of crack on natural frequencies is studied to detect the crack in beam-type structures (Kim and Stubbs, 2003). Free and forced vibration analysis was carried out on single- and two-edge cracked beams using a finite element method. The changes in natural frequencies and harmonic responses are used to evaluate the crack parameters (Orhan, 2007). An analytical model was developed for cracked beams to predict changes in natural frequencies for several crack parameters including various loadings and boundary conditions. The crack is modelled using a rotational spring approach based on fracture mechanics theory (Gomes and Almeida, 2014).

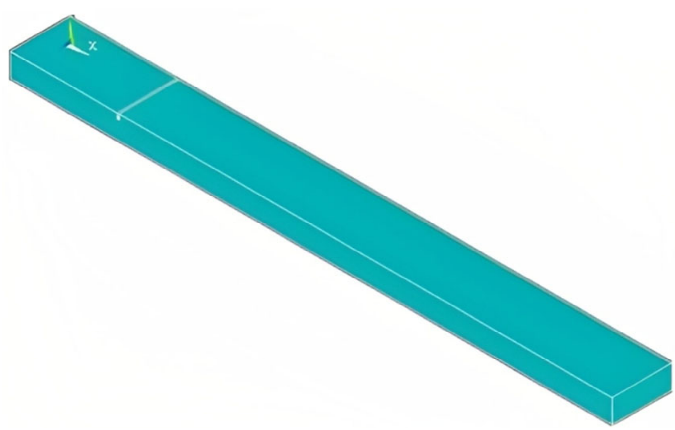
In recent years, many methods have been developed to identify the location and estimate the severity of the damage to structures. Artificial neural networks, optimisation techniques, and other artificial intelligence tools have been used in damage detection (Tran-Ngoc et al., 2019; Sreekanth et al., 2022a, 2022b). Feedforward neural networks were used to detect damages such as cracks and delamination in composite laminates (Senthilkumar et al., 2022; Sreekanth et al., 2021). Another kind of approach is employing optimisation algorithms to detect the damage. In this approach, an objective function is to be minimised, which is formulated based on the variations between the modal parameters. Conventional optimisation techniques are gradient-based and they fall into a local minimum. Frequency-based crack detection method coupled with a genetic algorithm (GA) is used to detect the crack location and its severity in a cracked multi-span beam (Mungla et al., 2017). GA and frequency-based methods were used to detect a crack in the clamped-clamped beam. It was observed that the frequency-based method predicted the crack parameters with less error compared to the GA (Mungla et al., 2016). Particle swarm optimisation (PSO) is a population-based metaheuristic algorithm and it was used to detect an open crack in beam-like structures using by minimising the difference between measured and calculated natural frequencies (Khatir et al., 2018). To locate the damage in structures PSO algorithm is used along with FRF as input response. The algorithm is tested on a cantilever beam and plane frame for various damage conditions. The prediction capability of the algorithm is compared with PSO and GA considering frequency-based objective function. PSO has quick convergence and better local search capability over GA (Mohan et al., 2013). To improve the convergence speed and accuracy of PSO, it is combined with the artificial immunity system. The immunity enhanced PSO was used in the damage detection of structures. The objective is based on natural frequencies and mode shapes. It has been observed that IEPSO is quite efficient and robust in damage detection (Kang et al., 2012). To accelerate the searching process by minimising the search space, a modified PSO (MPSO) is proposed. This MPSO is employed in cantilever beams to detect cracks through the inverse method. The crack is identified using MPSO by minimising the objective function based on the difference between the actual and estimated frequencies (Jena and Parhi, 2015). There are many optimisation techniques emerged in recent years, in those techniques Rao-1 algorithm is a metaphor less – heuristic search method and it has been recently used in various fields (Bhukya et al., 2021).

In this paper, the Rao-1 algorithm is employed to solve the crack detection problem. In Section 2, a numerical study is conducted to extract the natural frequencies to identify a crack using an inverse approach. The methodology of the Rao-1 algorithm is described in Section 3. Section 4 describes the free vibration testing on cantilever beams. The results obtained by the Rao-1 algorithm and PSO algorithm are discussed in Section 5. The article is concluded in Section 6.

2 Numerical analysis

The finite element method is used to study and analyse the behaviour of damaged and undamaged beams. The numerical analysis has been carried out using the ANSYS software. The dimensions of the beam are 250 mm in length, 25 mm in width, and 10 mm thick. The element type used in the analysis is 20 nodes solid 186.

Figure 1 Finite element model of the cracked beam (see online version for colours)



The crack is created at every 5 mm starting from 2.5 mm to 225.5 mm from the fixed. The crack depths are varied from 0.5 mm to 3 mm with a step size of 0.5 mm and the cracked beam is shown in Figure 1. The numerical analysis was used to study the effect of crack on natural frequencies. The second mode shape of the cracked cantilever beam is shown in Figure 2.

The Young's modulus of the beam is 69 GPa, the density is 2,730 kg/m^3 and Poisson's ratio is 0.33. For various crack parameters, the natural frequencies are calculated and are listed in Table 1.

Figure 2 Second mode shape of the cracked cantilever beam (see online version for colours)

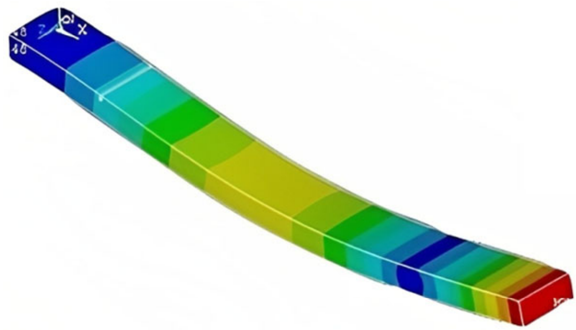
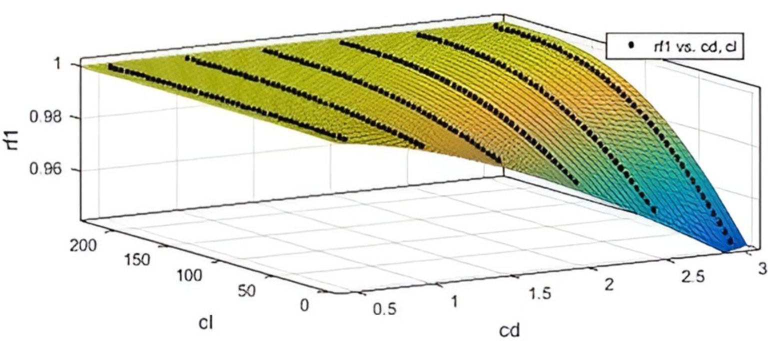


Table 1 Natural frequencies for various crack depths

Crack location at (mm)	*Crack depth (mm)*	*Frequency (Hz)*		
		1st mode	*2nd mode*	*3rd mode*
-	-	123.71	769.42	2,130.0
27.5	0.5	123.41	768.83	2,129.9
27.5	1.0	122.79	767.61	2,129.5
27.5	1.5	121.80	765.67	2,128.9
27.5	2.0	120.51	763.19	2,128.1
27.5	2.5	118.87	760.10	2,127.1

Figure 3 Plot of relative natural frequency variation with crack location and crack depth (see online version for colours)



The first three bending frequencies are obtained from the numerical analysis. Figure 3 shows the variation of relative natural frequency for various crack locations and crack depths. The relative frequency (rf) is the ratio of the cracked beam frequency to the intact beam frequency. When the crack is located near the fixed end, the decrease in first natural frequency is more with increasing crack depth. When the crack is located far from the fixed end the reduction in the first natural frequency is less. The natural frequencies, the corresponding crack locations, and crack sizes are used to construct an equation. The polynomial equations are constructed using cftool in MATLAB. The polynomial equation constructed from the first natural frequencies is shown in equation (1).

The polynomial equation is in the form of

$$\begin{aligned} rf1 = {} & p001 + p101 * x(i) + p011 * y(j) + p201 * x(i)^2 + p111 * x(i) * y(j) \\ & + p021 * y(j)^2 + p301 * x(i)^3 + p211 * x(i)^2 * y(j) + p121 * x(i) * y(j)^2 \\ & + p031 * y(j)^3 + p401 * x(i)^4 + p311 * x(i)^3 * y(j) + p221 * x(i)^2 * y(j)^2 \\ & + p131 * x(i) * y(j)^3 + p041 * y(j)^4 + p501 * x(i)^5 + p411 * x(i)^4 * y(j) \\ & + p321 * x(i)^3 * y(j)^2 + p231 * x(i)^2 * y(j)^3 + p141 * x(i) * y(j)^4 \\ & + p051 * y(j)^5 \end{aligned} \quad (1)$$

Three polynomial equations are constructed and the relative natural frequencies are calculated for various crack parameters. The objective of the inverse method is to predict the unknown crack location and depth by using an optimisation algorithm that minimises the difference between experimentally measured frequencies and calculated natural frequencies.

The inverse problem is solved by minimising the objective function

$$\min f(l, d) = \sum_{i=1}^{n} \left(rf_i^a - rf_i^c \right) \quad (2)$$

rf_i^a actual (experimental) relative natural frequency and rf_i^c is the calculated relative natural frequency. n is the number of natural frequencies used and 'i' indicates the i^{th} natural frequency.

3 Rao-1 algorithm

Rao-1 algorithm is a simple metaphor-less and algorithm-specific parameter-less optimisation algorithm proposed by Rao and it is not motivated by the social interactions of a flock of birds, the colony of bees, and ants (Rao, 2020). Let there are 'm' number of design variables and 'n' number of candidate solutions. Let the objective function to be minimised is $f(x)$. The best candidate obtains the best fitness value in the entire candidate solutions and the worst candidate obtains the worst fitness value in the entire candidate solutions. The design variable value of the candidate is modified using the equation

$$X_{new} = X_i + rand * \left(X_{best} - X_{worst} \right) \quad (3)$$

X_{best} is the value of the variable for the best candidate and X_{worst} is the value of the variable for the worst candidate. X_{new} is the updated value of X_i.

3.1 Implementation of Rao-1 algorithm

The steps followed in crack detection using the Rao-1 algorithm are as follows: the steps are elaborated below, using the flow chart as shown in Figure 4.

Step 1 The design variables in the crack detection problem are crack location and crack depth, represented by the position of the candidates. The initial population of the crack parameters is created randomly within the bounds of the candidates as

$$X = LB + rand * (UB - LB) \quad (4)$$

where *LB* and *UB* are the lower and upper bounds of the design variables.

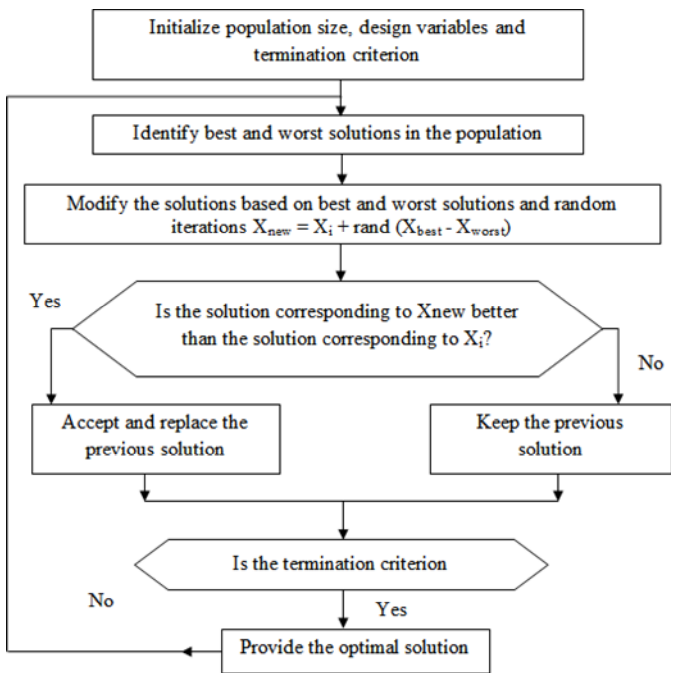
Step 2 The objective function is evaluated for various candidates from equation (2) to obtain the crack parameters.

Step 3 Update the population by equation (3) and also update the bounds of the crack parameters.

Step 4 Evaluate the fitness values of updated crack parameters and compare them with the previous best to update it. Store the crack parameters corresponding to the best fitness value. Find the crack parameters related to the least value of the fitness to give optimum crack parameters.

Step 5 The algorithm will be terminated if the termination criterion is reached, otherwise it is continued from step 3.

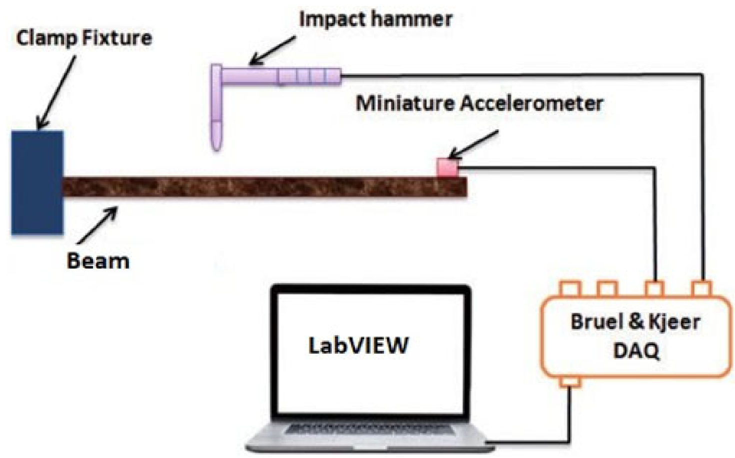
Figure 4 Flowchart of Rao-1 algorithm



4 Free vibration test

Free vibration test has been carried out on metallic beams to obtain the natural frequencies. The beam with dimensions of 250 mm length and 25 mm width and 10 mm thickness is used to measure the natural frequencies. The experimental setup is shown in Figure 5 and the beam is mounted in the fixed-free configuration. Experiments were conducted on a cantilever beam with surface crack and the natural frequencies were measured using an accelerometer sensor the values or displayed in the LabVIEW software. The surface crack shown in Figure 6 is generated in the metallic beam using laser cutting to get a uniform depth and width of the crack. The width of the crack is maintained at 1 mm.

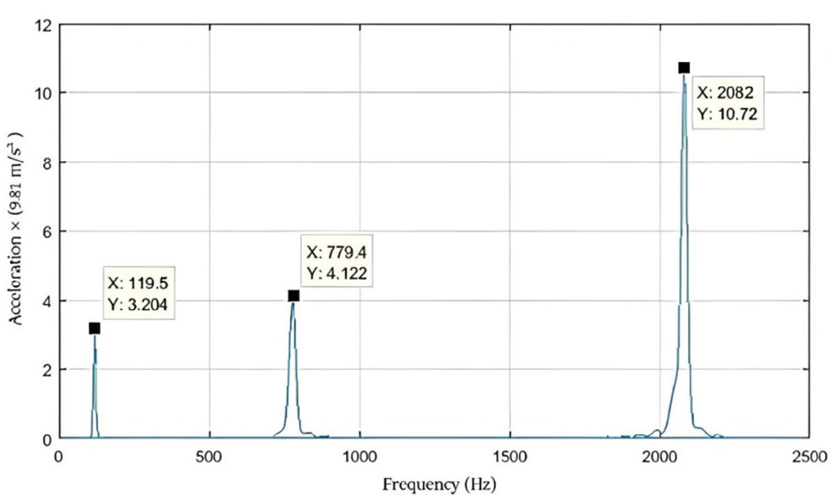
Figure 5 Experimental free vibration setup (see online version for colours)



Source: Munde et al. (2019)

Figure 6 Beam with surface crack [scale 1:2] (see online version for colours)

Figure 7 Frequency response of the uncracked beam (see online version for colours)

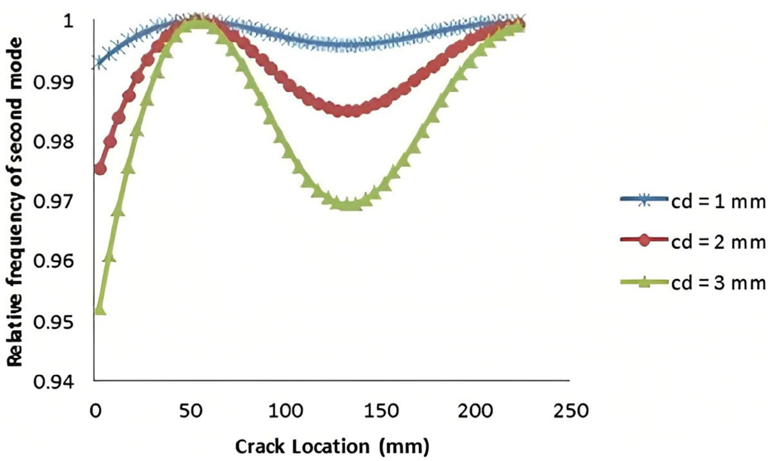


The cantilever beam was given impulse by an impact hammer and the response is measured using the triaxial accelerometer sensor. The beam's vibration response can be observed by roving hammer and roving accelerometer methods. Here, the beam vibration response was measured by mounting the accelerometer near the fixed end of the beam and exciting at the free end. The accelerometer was mounted on the beam using wax as adhesive. The block diagram is drawn in LabVIEW software to plot time and frequency domain responses. Each experiment was conducted multiple times and the average value of the natural frequency was considered. The frequency response of the intact beam is shown in Figure 7 and the first three modes are considered.

5 Results and discussion

The natural frequencies are obtained for various crack locations and crack depths using the finite element method. The variation in relative natural frequency values with crack location and crack depth are studied.

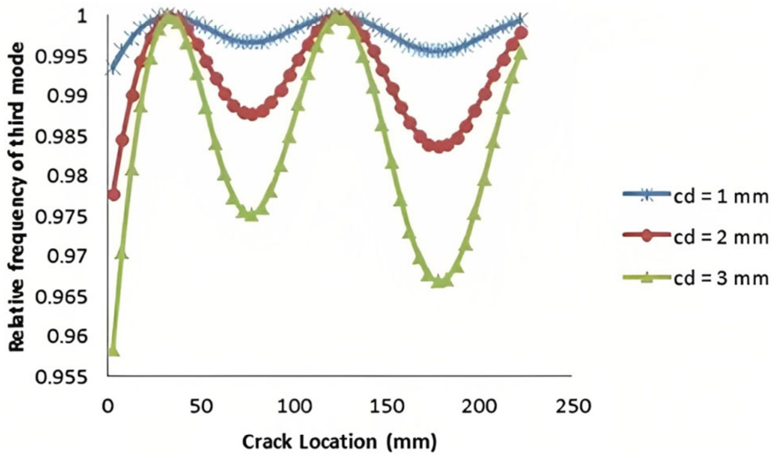
Figure 8 Variation of the relative frequency of the second mode for various crack locations and three crack depths (see online version for colours)



The first mode's relative frequency decreases when the crack is located near the fixed end. If the crack depth increases then there will be more reduction in relative frequency in any mode as shown in Figure 3. The second and third mode relative frequencies of a cracked beam at the various locations for three different crack depths are shown in Figures 8 and 9.

Minimum and maximum values of relative natural frequencies occur in second and third modes in different crack locations. The minimum value of relative natural frequency occurs when the crack is located at a more curvature region (maximum or minimum displacement of mode shape). If the crack is located in a more curvature region, there will be more loss in stored strain energy due to the highest bending of the beam leading to a decrease in relative natural frequency. The maximum value (damaged beam frequency reaches closer to intact beam frequency) occurs even if the crack is located at the vibration node of the particular mode because there will be no bending of the beam causing (stored strain energy loss will not be there) no change in frequency value.

Figure 9 Variation of the relative frequency of the third mode for various crack locations and three crack depths (see online version for colours)



The natural frequencies for various crack parameters are obtained from the finite element method. This data (270 sets of natural frequencies and crack parameters) is used to develop the equations which establish the relationship between frequencies and crack parameters. The frequency values are the dependent variables and the crack parameters are the independent variables. The natural frequencies are calculated from the developed equations. Actual values (relative frequency values and crack parameters) are not used in constructing the equations. The optimisation algorithms are used to minimise the difference between actual and calculated frequencies. The predicted crack location and depth using Rao-1 and PSO algorithms are provided in Tables 2 and 3, respectively.

$$Error = \frac{actual\ value - predicted\ value}{actual\ value} \tag{5}$$

The error in the prediction of crack location and crack depth is calculated using equation (5). Rao-1 algorithm has predicted the crack parameters with an average error of 4.85% and 6.08% as shown in Table 2. Both the algorithms can predict the crack parameters with sufficient accuracy.

Table 2 Predicted crack location and crack depth from Rao-1 algorithm

Actual values		*Predicted values using Rao-1 algorithm*			
Crack location (mm)	*Crack depth (mm)*	*Crack location (mm)*	*Error in %*	*Crack depth (mm)*	*Error in %*
35	1.8	33.459	4.40	1.928	5.93
100	2.3	104.776	4.77	2.661	16.2
115	2.3	105.435	8.31	2.197	4.06
154	2.1	150.402	2.33	2.133	0.79
179	1.8	171.031	4.44	1.883	3.46
Average error			4.85	-	6.08

The computational efficiency and prediction capability of the Rao-1 algorithm are verified by comparing its results with the results obtained by PSO.

Table 3 Predicted crack location and crack depth from PSO algorithm

Actual values		*Predicted values using PSO algorithm*			
Crack location (mm)	*Crack depth (mm)*	*Crack location (mm)*	*Error in %*	*Crack depth (mm)*	*Error in %*
35	1.8	33.459	4.40	1.928	5.93
100	2.3	104.776	4.77	2.661	16.2
115	2.3	104.925	8.76	2.211	3.44
154	2.1	150.402	2.33	2.133	0.79
179	1.8	171.031	4.45	1.883	3.46
Average error %			4.94	-	5.96

PSO algorithm detected the crack with an average error of 4.94% and the crack depth with 5.96%.

The algorithm specific parameters (inertia and acceleration constants) selected in performance evaluation of PSO algorithm are $w_{\max} = 0.9$, $w_{\min} = 0.4$, $c_1 = 1$ and $c_2 = 1$.

Table 4 Comparison of actual and estimated values of frequencies and crack parameters

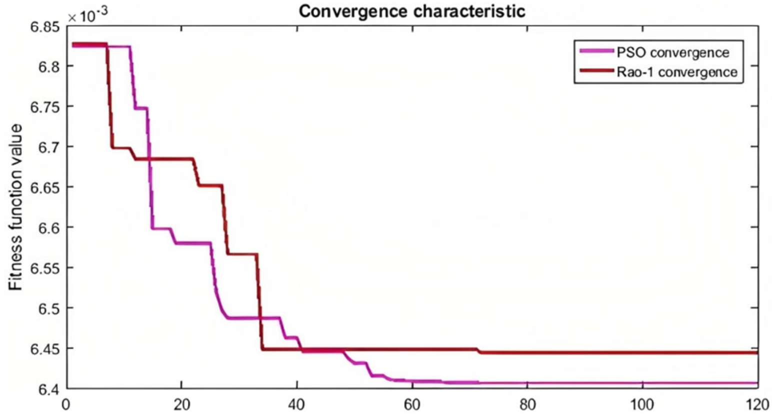
Actual relative frequencies			*Calculated relative frequencies*			*Actual crack parameters in (mm)*	
rf_1	rf_2	rf_3	rf_1	rf_2	rf_3	*Location*	*Depth*
0.9821	0.9970	0.9992	0.9841	0.9977	0.9976	35	1.8
0.9905	0.9820	0.9930	0.9920	0.9874	0.9918	100	2.3
0.9933	0.9867	0.9990	0.9944	0.9835	0.9931	115	2.3

rf_1, rf_2, and rf_3 are the relative natural frequencies of first, second, and third modes.

From Table 4 the inference is that, if the actual and calculated relative frequency values are almost equal in all the modes then the predicted crack parameters will be closer to the actual values. Relative frequencies calculated from the equations are not exactly equal to the actual frequencies. The deviation in the actual and calculated relative frequencies causes the prediction error. Inaccurate measurement of frequencies and difficulty in reproducing the ideal boundary conditions are also attributed to the prediction error. This error can be minimised by using more amount of data in constructing the equations to get calculated relative frequencies closer to the actual relative frequencies. It is very important to measure the frequency values accurately to detect the crack in the beam elements while conducting experiments.

Both the algorithms are taking the same amount of computational time, but the PSO is converging faster than the Rao-1 algorithm as shown in Figure 10. The PSO algorithm is taking 60 iterations to converge whereas the Rao-1 algorithm is converging at 72 iterations. Selection of algorithm specific parameters like appropriate weight, acceleration factors and velocity vectors made the PSO algorithm to converge faster where as in the TLBO algorithm computation of the mean might cause the slow and stepwise convergence. In the first 60 iterations both the algorithms are converging towards global optimum, but PSO convergence is steep because of the algorithm specific parameters.

Figure 10 Convergence plot of Rao-1 and PSO algorithm (see online version for colours)



6 Conclusions

The inverse-based crack detection in cantilever beams was carried out using the Rao-1 algorithm. The natural frequencies of the beams are determined by using the finite element model. The presence of crack causes changes in natural frequencies and these changes are used to detect a crack using the Rao-1 algorithm by minimising the objective function. The objective function is the difference between the experimentally measured and numerically calculated frequencies. The experimental study was conducted on the cantilever beam to observe the performance of the Rao-1 algorithm in crack detection. PSO algorithm detected the crack with an average error of 4.94% whereas the Rao-1 algorithm has predicted the location with an average error of 4.85%. The obtained results indicate the Rao-1 algorithm has predicted crack location better than PSO. TLBO algorithm can be used instead of PSO to avoid selecting fine-tuned algorithm specific parameters. Based on this study, it can be concluded that the Rao-1 algorithm is capable of detecting cracks in structures.

References

Bhukya, L., Annamraju, A. and Nandiraju, S. (2021) 'A novel maximum power point tracking technique based on Rao-1 algorithm for solar PV system under partial shading conditions', *Int. Trans. Electr. Energy Syst.*, Vol. 31, No. 9, p.e13028.

Gomes, H.M. and Almeida, F.J.F. (2014) 'An analytical dynamic model for single cracked beams including bending, axial stiffness, rotational inertia, shear deformation and coupling effects', *Appl. Math. Model.*, Vol. 38, No. 3, pp.938–948.

Jena, P.K. and Parhi, D.R. (2015) 'A modified particle swarm optimization technique for crack detection in Cantilever Beams', *Arab. J. Sci. Eng.*, Vol. 40, No. 11, pp.3263–3272.

Kang, F., Li, J.J. and Xu, Q. (2012) 'Damage detection based on improved particle swarm optimization using vibration data', *Appl. Soft Comput.*, Vol. 12, No. 8, pp.2329–2335.

Khatir, S., Dekemele, K., Loccufier, M., Khatir, T. and Wahab, M.A. (2018) 'Crack identification method in beam-like structures using changes in experimentally measured frequencies and particle swarm optimization', *Comptes Rendus Mécanique*, Vol. 346, No. 2, pp.110–120.

Kim, J.T. and Stubbs, N. (2003) 'Crack detection in beam-type structures using frequency data', *J. Sound Vib.*, Vol. 259, No. 1, pp.145–160.

Mohan, S.C., Maiti, D.K. and Maity, D. (2013) 'Structural damage assessment using FRF employing particle swarm optimization', *Appl. Math. Comput.*, Vol. 219, No. 20, pp.10387–10400.

Munde, Y.S., Ingle, R.B. and Siva, I. (2019) 'A comprehensive review on the vibration and damping characteristics of vegetable fiber-reinforced composites', *J. Reinf. Plast. Compos*, Vol. 38, No. 17, pp.822–832.

Mungla, M.J., Sharma, D.S. and Trivedi, R.R. (2016) 'Identification of a crack in clamped-clamped beam using frequency-based method and genetic algorithm', *Procedia Eng.*, Vol. 144, pp.1426–1434.

Mungla, M.J., Sharma, D.S. and Trivedi, R.R. (2017) 'Inverse method to identify crack parameters in multi-span beam using genetic algorithm', *Nondestruct. Test. Eval.*, Vol. 32, No. 3, pp.301–318.

Orhan, S. (2007) 'Analysis of free and forced vibration of a cracked cantilever beam', *NDT & E Int.*, Vol. 40, No. 6, pp.443–450.

Pandey, K. and Biswas, M. (1994) 'Damage detection in structures using change in flexibility', *J. Sound Vib.*, Vol. 169, No. 1, pp.3–17.

Rao, R. (2020) 'Rao algorithms: three metaphor-less simple algorithms for solving optimization problems', *Int. J. Ind. Eng. Comput.*, Vol. 11, No. 1, pp.107–130.

Senthilkumar, M., Reddy, S.M. and Sreekanth, T.G. (2022) 'Dynamic study and detection of edge crack in composite laminates using vibration parameters', *Trans. Indian Inst. Met.*, Vol. 75, No. 2, pp.361–370.

Senthilkumar, M., Sreekanth, T.G. and Reddy, S.M. (2021) 'Nondestructive health monitoring techniques for composite materials: a review', *Polym. Polym. Compos.*, Vol. 29, No. 5, pp.528–540.

Sreekanth, T.G., Senthilkumar, M. and Reddy, S.M. (2021) 'Vibration-based delamination evaluation in GFRP composite beams using ANN', *Polym. Polym. Compos.*, Vol. 29, No. 9_suppl, pp.S317–S324.

Sreekanth, T.G., Senthilkumar, M. and Reddy, S.M. (2022a) 'Natural frequency based delamination estimation in GFRP beams using RSM and ANN', *Frat. Ed Integrita Strutt.*, Vol. 16, No. 61, pp.487–495.

Sreekanth, T.G., Senthilkumar, M. and Reddy, S.M. (2022b) 'Artificial neural network based delamination prediction in composite plates using vibration signals', *Frat. Ed Integrita Strutt.*, Vol. 17, No. 63, pp.37–45.

Tran-Ngoc, H., Khatir, S., De Roeck, G., Bui-Tien, T. and Wahab, M.A. (2019) 'An efficient artificial neural network for damage detection in bridges and beam-like structures by improving training parameters using cuckoo search algorithm', *Eng. Struct.*, Vol. 199, p.109637.

Appendix

Figure 11 PSO convergence in MATLAB (see online version for colours)

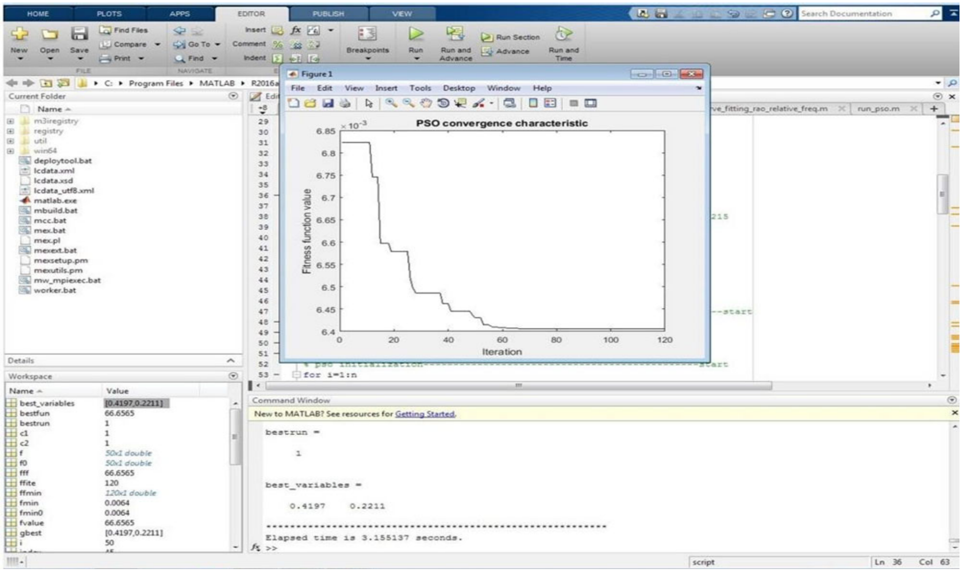


Figure 12 Rao-1 convergence in MATLAB (see online version for colours)