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## **A review on reactive power measurement in harmonic environment**

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**Abstract:** This paper reviews evolving methods of reactive power measurement in a harmonic environment. First, the definitions of reactive power under harmonic condition are presented. The regulatory aspects of reactive power billing of various countries are analysed indicating the necessity of an accurate method for reactive power monitoring. Several measurement techniques adopted by reactive power meters are discussed and their accuracy in harmonic environment is evaluated based on experimental investigations reported in research articles. The paper highlights the necessity of a standardised method for measurement of reactive power accounting for disturbance from harmonic sources.

**Keywords:** reactive power; non-active power; harmonic distortion; measurement techniques; power quality; measurement errors.

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## 1 Introduction

Reactive power has its existence with the advent of ac systems. It indicates the amplitude of the oscillating power component flowing between the sources and loads, contributing to energy loss in the line. It holds a key role in increasing the transmission and distribution efficiency of power systems. Transformers and motors require reactive power to create electromagnetic field essential for their existence. Power loss in the line will be affected if this requirement has to be met by utility. So, a better practice is to generate reactive power near customers demanding it as mentioned in Zhang et al. (2014). This is most suited for the current scenario where a greater number of distributed generators are penetrating into power system.

Energy billing of consumers involves both active energy and reactive energy. Active energy indicates cost of actual energy delivered to consumer and reactive energy represents the oscillating component which affects the system capacity to handle the load. If the reactive power drawn by the consumers is more, then the capacity of the system has to be increased to meet the same load. So reactive power monitoring is essential in billing perspective. Earlier, induction type energy meters were used which have been replaced by static meters on account of better stability and accuracy. Smart meters are preferred in a smart grid environment because of their capability of transferring data between consumers and utility with the help of various communication technologies. Smart meters enable the utility to monitor consumer data remotely. Currently, researches are focussing on to implementing artificial intelligent techniques like ANN, fuzzy logic control (FLC), and adaptive neural fuzzy inference system (ANFIS) in smart meters for consumer load management (Saqib et al., 2020).

Static meters employed for reactive power metering, is based on different methods as discussed in Section 4. Under sinusoidal conditions these meters give accurate measurement (Vieira et al., 2017). With the penetration of more nonlinear loads into the network, harmonic distortion has increased to a great extent. These meters give meaningless measurements when subjected to such a harmonic environment (Shklyarskiy et al., 2020). There is no compatibility with the readings provided by various meters. The reactive power measurement obtained from static meters is compared with several power theories resulting in varying performance of the instrument (Demerdziev and Dimchev, 2023). Reactive power can also be used as an indicator for detecting the location of harmonic sources (Sinha et al., 2017). Many countries adopt penalty if the reactive power usage is exceeded beyond a limit. This may lead to unfair

reactive power billing. Several investigations have been reported in literatures regarding the performance of static meters in a harmonic environment. All these studies point to the fact that harmonics do have an effect on reactive power measurement. So, a unique method of measurement of reactive power is essential under harmonic condition in billing perspective.

Several theories have been proposed by engineers in the past both in time domain and frequency domain to define the reactive power in a distorted environment. Nicolae et al. (2022) presented several theories related to power definitions for measuring power and power factor in non-sinusoidal three phase system.

This paper highlights the reactive power definitions for non-sinusoidal system in the next section. The regulatory framework of reactive power billing adopted by different countries is presented in Section 3. The reactive power measurement techniques adopted by static meters are explained in Section 4. Finally, an analysis on the experimental investigations reported in research articles is included in Section 5.

## 2 Reactive power definitions

Under sinusoidal condition, the reactive power is expressed as

$$Q = VI \sin \phi \tag{1}$$

where  $V$  is the r.m.s value of voltage,  $I$  is the r.m.s value of current and  $\phi$  is the phase angle between voltage and current signals. The above equation no longer holds good for a distorted environment. The reactive power definitions proposed by engineers in the past is detailed as follows.

### 2.1 Definition proposed by C. Budeanu

Constantin Budeanu was a Romanian electrical engineer whose contribution to electric network analysis had a great impact on the power system. He was the first scientist to give a three-dimensional approach to apparent power in non-sinusoidal systems. According to Budeanu, apparent power is expressed as (Emanuel, 2010):

$$\begin{aligned}
 S^2 &= P^2 + Q_B^2 + D_B^2 \\
 P &= \sum_{h=1}^n V_h I_h \cos \phi_h \\
 Q_B &= \sum_{h=1}^n V_h I_h \sin \phi_h
 \end{aligned}
 \tag{2}$$

where  $P$  is the total active power,  $Q_B$  is the reactive power and  $D_B$  is the distortion power. This necessitates in determining the harmonic components of voltage and current signals using Fourier transform. Budeanu’s reactive power suffers the disadvantage that power factor cannot be made unity if  $Q_B$  is made zero by passive components.

## 2.2 Definition proposed by S. Fryze

A time domain approach was proposed by Fryze in which the current is decomposed into active current  $i_a$  and reactive current  $i_b$  (Emanuel et al., 2012). The current having exact replica of voltage is the active current and the remaining current constitute reactive current. Reactive power is defined as

$$Q_F = Vi_b \quad (3)$$

Fryze's approach led to the power triangle concept stating that

$$\begin{aligned} S^2 &= P^2 + Q_F^2 \\ pf &= \frac{Q_F}{S} \end{aligned} \quad (4)$$

Even though power factor can be made unity by making  $Q_F = 0$ , it does not give any information on how compensation can be achieved with passive components. Fryze's model falls short of providing information on the fundamental frequency active and reactive power components,  $P_1$  and  $Q_1$ , which are the most important power quantities to be measured. It is only a simplified model, not revealing all the details of the instantaneous power and not representing the true mechanism of energy transfer to the load.

## 2.3 Definition proposed by N.L. Kusters and W.J.M. Moore

N.L. Kusters and W.J.M. Moore decomposed current into three components namely active component, inductive or capacitive reactive component and residual reactive component with the apparent power equation as given by (Kusters and Moore, 1980):

$$S^2 = P^2 + Q_l^2 + Q_{lr}^2 \quad (5)$$

where  $Q_l$  is the capacitive or inductive reactive power that can be compensated by selecting suitable passive components.

## 2.4 Definition proposed by Manfred Depenbrock

Depenbrock and Staud (1998) improved on Fryze's method by separating the fundamental and harmonic components of voltage and current signals. The apparent power is expressed as

$$\begin{aligned} S^2 &= P^2 + Q_F^2 \\ &= P^2 + (D^2 + Q^2) \end{aligned} \quad (6)$$

where  $P$  is the active power,  $D$  is the distortion power and  $Q$  is the reactive power given by the expression:

$$Q^2 = (VI_{q1})^2 = (V_1^2 + V_H^2)I_{q1}^2 = Q_1^2 + Q_c^2 \quad (7)$$

$V_1$  is fundamental voltage,  $V_H$  is harmonic voltage and  $I_{q1}$  is the fundamental reactive current. Depenbrock defines two reactive powers, fundamental reactive power  $Q_1$  and complementary reactive power  $Q_c$ . The drawback is that just like Fryze’s method, no information is given about fundamental active power  $P_1$  and the complementary reactive power  $Q_c$  seems to be only a mathematical artifice without a solid physical base.

*2.5 Definition proposed by L. Czarnecki*

As a modification of Fryze’s model, Czarnecki (1984) divided current into active and non-active currents

$$i = i_a + i_b \tag{8}$$

Further separating non-active current into reactive component  $i_r$  and scattered component  $i_s$  such that the three currents  $i_a$ ,  $i_s$  and  $i_r$  are mutually orthogonal, hence the r.m.s values can be written as

$$I^2 = I_a^2 + I_s^2 + I_r^2 \tag{9}$$

Reactive power proposed by Czarnecki is defined as

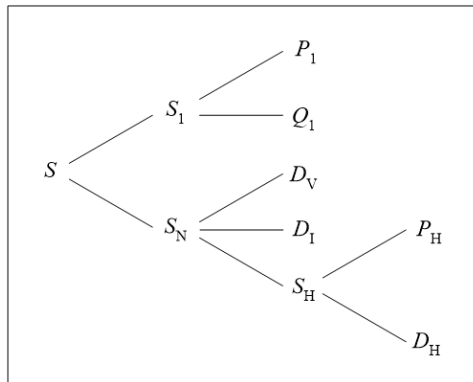
$$Q_r = VI_r \tag{10}$$

*2.6 Definition proposed in IEEE Standard 1459-2010*

The approach in IEEE Standard 1459-2010 separates fundamental active power  $P_1$  and reactive power  $Q_1$  from rest of powers. In this method, voltage and current signals are separated into fundamental and harmonic components

$$\begin{aligned} V^2 &= V_1^2 + V_H^2 \\ I^2 &= I_1^2 + I_H^2 \end{aligned} \tag{11}$$

**Figure 1** Apparent power decomposition according to IEEE 1459 Std



From Figure 1, apparent power is given by the expression

$$\begin{aligned} S^2 &= V^2 I^2 = (V_1^2 + V_H^2)(I_1^2 + I_H^2) \\ &= (V_1 I_1)^2 + (V_1 I_H)^2 + (V_H I_1)^2 + (V_H I_H)^2 \\ &= S_1^2 + D_I^2 + D_V^2 + S_H^2 \end{aligned} \quad (12)$$

Fundamental apparent power

$$S_1 = V_1 I_1 = \sqrt{P_1^2 + Q_1^2} \quad (13)$$

Non-fundamental apparent power

$$S_N^2 = D_I^2 + D_V^2 + S_H^2 \quad (14)$$

where  $D_I$  is current distortion power indicating the amount of VAR due to current distortion and is given as

$$D_I = V_1 I_H = S_1 \times THD_I \quad (15)$$

$D_V$  is voltage distortion power revealing the amount of VAR due to voltage distortion and is given as

$$D_V = V_H I_1 = S_1 \times THD_V \quad (16)$$

$S_H$  is harmonic apparent power given as

$$S_H = V_H I_H = S_1 \times THD_I \times THD_V = \sqrt{P_H^2 + D_H^2} \quad (17)$$

$P_H$  is harmonic active power and  $D_H$  is harmonic distortion power. The non-active power includes fundamental reactive power  $Q_1$ ,  $D_I$ ,  $D_V$  and  $D_H$ . None of the present-day reactive power meters (varmeters) gives same readings for all the four non-active powers creating a big challenge to the billing authority.

Emanuel et al. (2012) explain the power components mentioned in the IEEE Std.1459 using Poynting vectors. It is emphasised that the power components are caused by the interaction of fundamental and harmonic components of electric and magnetic fields expressed as Poynting vectors. The actual energy is quantified by the fundamental positive-sequence powers and the remaining components quantify the harmonic pollution created by either consumer or utility. The actual phenomenon of energy flow by electromagnetic waves in transmission line conductors is well explained in Emanuel (2004b) and Emanuel et al. (2012) indicating the importance of Poynting vector in power theory. Emanuel (2005) presents three problems related to finding effective voltage and current for calculating apparent power as mentioned in 1459 IEEE std. The concept of three phase apparent power using effective voltage and current is illustrated in Emanuel (2004a) and Pajic and Emanuel (2006) along with a case study and found all the non-active powers. Montoya et al. (2020a, 2020b) define non-active power using geometric algebra. The application of geometric algebra in networks with different types of loads both in sinusoidal and non-sinusoidal conditions are explained

with suitable examples. This paper gives a new method for solution of electric circuits in non-sinusoidal systems.

Even though several definitions exist to account reactive power in non-sinusoidal conditions, still the utilities systems adopt earlier sinusoidal concept of measurement. There may be several reasons to address this issue, especially the interdependency of voltage and current waveforms in a distorted environment.

### **3 Regulatory aspects of reactive power billing**

Each country has some regulatory framework. Some countries such as Denmark, Czech Republic and Greece do not impose any penalty for reactive power usage by the consumers (da Silva et al., 2020; Vieira et al., 2017). Countries like Brazil exempt household consumers from reactive power penalty. In addition, some countries have rate structures imposing penalty if power factor or equivalent parameters has exceeded limit. This section will discuss the rate structures given by different countries for reactive power to see its effectiveness under non-sinusoidal conditions.

#### *3.1 Brazil*

In Brazil, the household consumers are charged for the consumption of active power and no regulations apply for the reactive power drawn by them. While, medium and large consumers are charged both for active power and reactive power drawn from the utility. The penalty imposed on reactive power usage depends on power factor. The limiting value of power factor is 0.92 (da Silva et al., 2020). The power factor is observed in the time intervals, i.e., between 11:30 PM and 06:30 AM. Consumers have to meet the minimum power factor limit. As the reactive power usage increases, the power factor reduces from 0.92 and the consumers are penalised for the excess reactive power drawn by them.

#### *3.2 Egypt*

In Egypt, consumers with connected loads more than 500 kW are imposed penalty depending on their average annual power factor. The purpose of imposing penalty is to motivate the consumers for installing power factor correction equipment at their premises (Abdel Aziz et al., 2003). Annual average power factor,

$$pf = \frac{kWh}{\sqrt{kWh^2 + kVArh^2}} \tag{18}$$

The penalty  $P$ , when the pf is between 0.9 and 0.7 is expressed as follows:

$$P = (0.9 - pf) \times kWh \tag{19}$$

The penalty when the pf is less than 0.7 is given by.

$$P = [0.2 + 1.5(0.7 - pf)] \times kWh \tag{20}$$

The penalty increases if the consumer is not taking measures to improve pf leading to discontinue of service. Also, the consumer will receive bonus if pf is between 0.92 and 0.95.



### 3.3 Spain

In Spain, consumers are charged a penalty if the power factor goes below 0.90 (Vieira et al., 2017). This is applicable to consumers with connected load of 15 kW and above. The power factor considered here is true power factor. The consumers are also eligible for a rebate if their power factor goes above 0.95.

### 3.4 Canada (British Columbia)

In Canada, consumers are charged depending on active, reactive and apparent powers. A PF limit of 0.90 is considered in the Province of British Columbia. When the power factor goes below 0.9, the consumer is liable to pay surcharge. The surcharge amount depends on consumer average power factor in a particular period (<https://app.bchydro.com/accounts-billing/rates-energy-use/electricity-rates/power-factor.html>). The percentage of surcharge applied for various power factors is shown in Table 1.

**Table 1** Surcharge for various power factor

<i>Range of power factor</i>	<i>Percentage of surcharge</i>
0.90–1.0	None
0.88–0.90	2%
0.85–0.88	4%
0.80–0.85	9%
0.75–0.80	16%
0.70–0.75	24%
0.65–0.70	34%
0.60–0.65	44%
0.55–0.60	57%
0.50–0.55	72%
0.50	80%

### 3.5 India

In India, tariff structure varies from region to region. In general, consumers are charged for active energy consumption. Reactive energy is accounted by providing penalty when the power factor goes below 0.90 and incentive if the power factor exceeds 0.95. This is done to enable the consumers to improve their power factor to unity. For commercial and industrial consumers, tariff has changed to kVAh billing instead of kWh billing (Mahadiscom, 2020). The new tariff system will be considering the total energy supplied to the consumer by the utility which is represented by the apparent power. This system offers advantage to utility, at the same time will be treating all consumers equally irrespective of the harmonic pollution created by them.

### 3.6 Australia

In Australia, reactive power billing depends on the installed capacity and voltage level (Vieira et al., 2017). For low voltage installations, the power factor limits are listed in

Table 2. The power factor limits are between 0.75 and 0.85. The surcharge is applied when the active power is 50% above of the maximum power allowed for that particular voltage level.

**Table 2** Power factor limits for L.V installations in Australia

<i>Installed capacity</i>	<i>Inductive</i>	<i>Capacitive</i>
<100 kVA	0.75	0.8
>100 kVA < 2 MVA	0.8	0.8
>2 MVA	0.85	0.85

### 3.7 France

In France, consumers with installed capacity above 36 kVA are penalised for reactive power usage. Reactive power billing is based on a parameter called phi tangent ratio. Phi tangent ratio is the ratio of reactive power to active power (da Silva et al., 2020). This ratio plays a key role for the safe operation of the electricity grid. The limiting value of phi tangent ratio is 0.4. It means that the reactive power usage of consumer is up to 40% of their active power consumed. Any excess reactive power drawn by consumer need to be paid in the form of reactive power billing. The power factor is limited to 0.928 when phi tangent ratio is 0.4.

### 3.8 USA – California

Low voltage consumers in California, are penalised for reactive power when the measured reactive power is greater than 40% of the active power (Vieira et al., 2017). The surcharge is applicable only for consumers whose installed capacity is more than 20 kW.

From above, it is understood that the countries in spite of different regulations adopted, gives importance to reactive power billing aiming to improve power quality of the system.

## 4 Reactive power measurement methods

Today, static meters have been employed for the measurement of reactive power. Different methods are being used by these meters for the measurement (Emanuel, 2012). In this section, various methods on which varmeter works under harmonic conditions are discussed.

### 4.1 The 90° shift method

Varmeters work on the basis of shifting of either voltage or current signals by 90°. This shift can be achieved by integrating or differentiating the respective phasors. Suppose the voltage and current is

$$\begin{aligned}
 v(t) &= V \cos(\omega t + \alpha) \\
 i(t) &= I \cos(\omega t + \beta)
 \end{aligned}
 \tag{21}$$

Taking the integral of voltage,

$$v't = \int V \cos(\omega t + \alpha) dt = \frac{V}{\omega} \sin(\omega t + \alpha) \quad (22)$$

Reactive power measured by varmeter is

$$Q_{int} = \frac{\omega}{kT} \int_{\tau}^{\tau+kT} v' i dt = \frac{VI}{kT} \int_{\tau}^{\tau+kT} \sin(\omega t + \alpha) \cos(\omega t + \beta) d\omega t \quad (23)$$

$$Q_{int} = \frac{VI}{2} \sin(\alpha - \beta) = VI \sin \theta$$

Suppose the phase shift is achieved by taking the derivative of voltage, we get

$$v'(t) = \frac{dv}{dt} = \omega V \sin(\omega t + \alpha) \quad (24)$$

Reactive power measured will be

$$Q_{dif} = \frac{1}{\omega kT} \int_{\tau}^{\tau+kT} -v' i dt = \frac{VI}{kT} \int_{\tau}^{\tau+kT} \sin(\omega t + \alpha) \cos(\omega t + \beta) d\omega t \quad (25)$$

$$Q_{dif} = \frac{VI}{2} \sin(\alpha - \beta) = VI \sin \theta$$

From above we see that varmeters give satisfactory results under sinusoidal conditions. Such meters will give erroneous readings when voltage and current become distorted. Under distorted conditions, voltage and current is represented as

$$v(t) = V_1 \cos(\omega t + \alpha_1) + \sum_{h \neq 1} V_h \cos(h\omega t + \alpha_h) \quad (26)$$

$$i(t) = I_1 \cos(\omega t + \beta_1) + \sum_{h \neq 1} I_h \cos(h\omega t + \beta_h)$$

Taking integral of voltage and substituting in equation (4) gives the reactive power as

$$Q_{int} = V_1 I_1 \sin(\alpha_1 - \beta_1) + \sum_{h \neq 1} \frac{1}{h} V_h I_h (\sin(\alpha_h - \beta_h))$$

$$= Q_1 + \Delta Q_{int}$$

$$\Delta Q_{int} = \sum_{h \neq 1} \frac{Q_h}{h} \quad (27)$$

Using derivative of voltage, reactive power is

$$Q_{dif} = V_1 I_1 \sin(\alpha_1 - \beta_1) + \sum_{h \neq 1} h V_h I_h \sin(\alpha_h - \beta_h)$$

$$= Q_1 + \Delta Q_{dif}$$

$$\Delta Q_{dif} = \sum_{h \neq 1} h Q_h \quad (28)$$

Thus, in integration method, the overall reactive power decreases by a factor inversely proportional to the order of harmonics while in differentiation method, the overall reactive power increases by a factor proportional to the order of harmonics. Hence errors exist when compared to fundamental reactive power in both cases.

#### 4.2 The quarter cycle delay method

In this method, voltage or current signals are shifted digitally by providing a time delay. The time delayed voltage signal is

$$v\left(t - \frac{T}{4}\right) = V_1 \cos\left(\omega t + \alpha_1 - \frac{\omega T}{4}\right) + \sum_{h \neq 1} V_h \cos\left(h\omega t + \alpha_h - \frac{h\omega T}{4}\right) \quad (29)$$

Reactive power is

$$\begin{aligned} Q_{qcv} &= \frac{1}{kT} \int i(t)v\left(t - \frac{T}{4}\right) = Q_1 + \Delta Q_{qcv} \\ \Delta Q_{qcv} &= -P_2 - Q_3 + P_4 + Q_5 - P_6 - \dots \end{aligned} \quad (30)$$

The time delayed current signal will give the reactive power as

$$\begin{aligned} Q_{qci} &= -\frac{1}{kT} \int v(t)i\left(t - \frac{T}{4}\right) = Q_1 + \Delta Q_{qci} \\ \Delta Q_{qci} &= P_2 - Q_3 - P_4 + Q_5 + P_6 - \dots \end{aligned} \quad (31)$$

#### 4.3 Budeanu's method

The reactive power proposed by Budeanu is expressed as

$$\begin{aligned} Q_B &= \sum_h V_h I_h \sin \theta_h = Q_1 + \Delta Q_B \\ \Delta Q_B &= \sum_{h \neq 1} Q_h \end{aligned} \quad (32)$$

#### 4.4 Power triangle method

In this method, apparent power is obtained by multiplying root mean square values of current and voltage. Total active power is expressed as

$$P^2 = (P_1 + P_H)^2 = P_1^2 + P_H^2 + 2P_1P_H \quad (33)$$

Reactive power is expressed as

$$Q = \sqrt{S^2 - P^2} \quad (34)$$

From the above, it is understood that the errors produced by varmeters depends on the method employed and increases with harmonic distortion. Researches are going on today for finding a unique measurement method applicable in all situations.

## 5 Varmeter performance analysis

In this section, the impact of nonlinear loads on the reactive power measurement proposed in research articles is analysed.

In Brito et al. (2016), the impact of electronic loads on the reactive power measurement of Brazilian Electric Power companies is analysed. The authors emphasises that the measured reactive power indicates both reactive power and distortion power as given by CPT (Conservative Power Theory). So, reactive power billing creates some mismatch which needs to account for distortion in the system. Another technique of reactive power measurement in distorted networks explained in Kulinich et al. (2018) uses instantaneous power derivative to determine reactive power. This method eliminates the problem of extracting the harmonics of current and voltage signals. The analysis of various powers under non-sinusoidal conditions according to IEEE standard 1459-2010 is presented in Sychev et al. (2020b). The article calculated various components of apparent power in different harmonic conditions which can be used to identify the origin of nonlinear distortion and also for selecting devices for harmonic elimination.

In Yin et al. (2018), instantaneous reactive power theory is used for measurement of reactive power. In this, the three phase quantities are converted into p-q coordinate system. By rotating p-q coordinates with speed of various harmonic components and filtering, the positive and negative sequence harmonic components of respective voltage and current signals are generated. These sequence components can be used to calculate positive and negative sequence reactive and active power of each harmonic component which can be converted to three phase power, thereby obtaining power in harmonic environment.

A new expression for determining the power factor of a system supplied by both non-sinusoidal source as well as nonlinear load is presented in Sychev et al. (2020a). It shows the amount by which the power factor is reduced due to the nonlinearity of the system. In Langella and Testa (2007), a circular convolution-based algorithm used for power measurements under harmonic conditions is explained with a computational burden same as that of DFT.

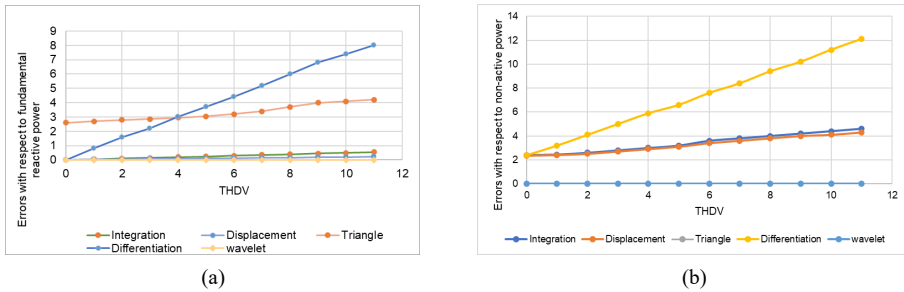
Cataliotti et al. (2008) develop a PC-based instrument which gives powers as specified in IEEE Std.1459. A time-domain approach is used to detect the fundamental positive sequence components of current and voltage signals which is subtracted from actual signals to get harmonic voltages and currents. All the powers are computed from these harmonic quantities without conducting any spectral analysis. Several algorithms like Newton type algorithm (Terzija et al., 2005), algorithm based on the Walsh function (Aliyu et al, 2014) are proposed by many authors for measuring reactive power in a network containing both linear and nonlinear type of loads. Osipov et al. (2016) used wavelet transform for finding powers of non-sinusoidal systems.

Zhijian and Zhongdong (2009) discuss the active power flow in a system where different types of consumers are connected. The active power drawn by consumers with linear load includes both the fundamental and harmonic active power produced by nonlinear loads of another consumer. This enables linear consumers to pay more for the active power drawn by them. Authors also points to the fact that energy measurement using both induction type and electronic meters creates errors while measuring harmonic active power. These meters consider the fundamental and harmonic power as same and increase the metering errors. In Pinzon et al. (2016), all the non-active powers were measured as per the IEEE Std 1459 on a microgrid with solar PV, batteries and

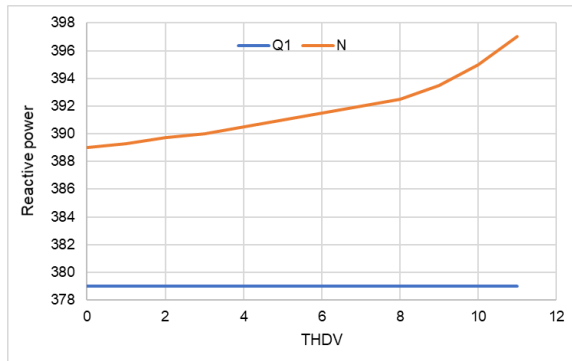
DG systems. The measurements indicated the harmonic pollution of microgrid and the necessity of filters for enhancing the power quality of the system.

The meters provided by various manufacturers use different techniques for calculating reactive power giving compatible results under sinusoidal conditions but meaningless results with distorted current and voltage waveforms. Gallo et al. (2015) proposed a test signal for identifying the approach adopted by manufacturers in various meters. Coelho and Brito (2023) proposed a method of power measurement using discrete Stockwell transform suitable for non-sinusoidal conditions.

**Figure 2** Variation of errors of various reactive power measurement methods with respect to the, (a)  $Q_1$  (b) non-active power (see online version for colours)



**Figure 3** Variation of non-active power and  $Q_1$  as a function of  $THD_V$  at source side (see online version for colours)



The experimental investigation conducted by Vieira et al. (2017) uses current signal at a residence measured at two different time intervals for reactive power calculation. The errors of each measurement technique are detailed along with a sensitivity analysis to verify the extent of these errors. Figure 2(a) shows the errors of each measurement methods with respect to  $Q_1$  for different  $THD_v$  of source voltage. It is found that the power triangle technique gives an error of 2.5% for zero value of  $THD_v$  while the rest of the methods produce no error with respect to fundamental reactive power. As the source side distortion increases, it is seen that the errors also increase with the highest error for differentiation method and least error for integration and displacement methods. Figure 2(b) shows the errors of each measurement methods with respect to non-active

power for different  $THD_v$  of source voltage. It is seen that all the methods except power triangle and wavelet gives an error greater than 2% since the power triangle method measures the non-active power. As the source side distortion increases, it is seen that the errors also increase with the highest error for differentiation method and least error for power triangle method. Hence according to Vieira et al. (2017), if non-active power definition is used for reactive power billing, then power triangle technique is most suitable. However, if fundamental reactive power is used for billing, then displacement and integration methods are preferred.

The variation of fundamental reactive power and non-active power with respect to  $THD_v$  is also analysed in Vieira et al. (2017). From Figure 3, it is understood that fundamental reactive power remains constant throughout whereas the non-active power increases with increase in voltage distortion. Hence, if billing is done via non-active power, the reactive power measured at the consumer connected to a high  $THD_v$  system will be more compared to a consumer connected to a low  $THD_v$  system. With this in view, the authors suggest fundamental reactive power as a reliable quantity to be used for billing purposes.

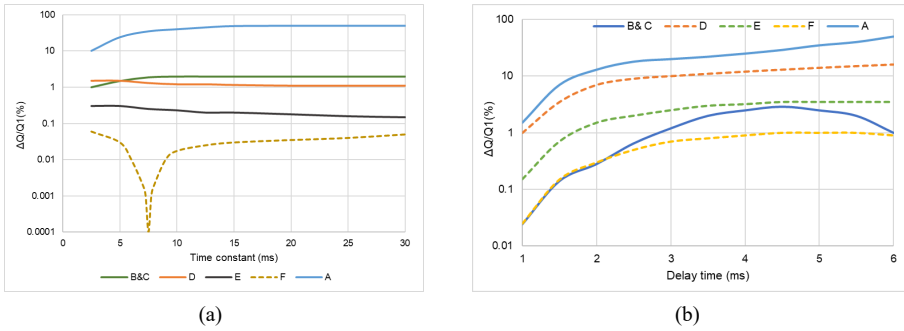
Zhang et al. (2015) measure reactive power of nonlinear loads by various techniques and computes error with respect to fundamental reactive power. Table 3 gives reactive power measurement methods considered for analysis. For a single-phase rectifier supplying RL load, it is observed that the errors produced by the methods A, B and C is more compared to other methods. The error increases with increase in time constant for method A and remains almost same for other methods as seen in Figure 4(a). Another measurement was carried out for a thyristorised phase controller feeding RL load for varying the firing angle. It is seen from Figure 4(b) that the error with respect to fundamental reactive power increases with increase in firing angle.

**Table 3** Methods of reactive power measurement

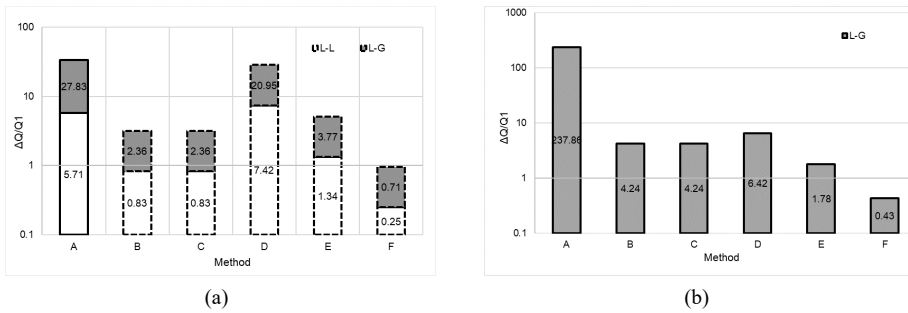
<i>Symbol</i>	<i>Method name</i>
A	Vector
B	Quarter-cycle voltage delay
C	Quarter-cycle current delay
D	Differential phase shift
E	Budeanu
F	Integral phase shift

In addition to single phase loads, analysis was also done for three phase nonlinear loads. Figure 5(a) shows measurement error with respect to fundamental reactive power for a three phase six pulse rectifier with an RL load. Measurements are done for both line-line voltage and line-ground voltage. It is found that methods A and D produce more error compared to other methods. The measurement is also carried for an adjustable speed drive operating in a distorted environment. It is seen from Figure 5(b), that method A is having largest error totally disqualifying vector method. Zhang et al. (2015) point to the fact that meters based on the above measurement methods, while operating in non-sinusoidal systems, gives inaccurate measurements, and should not be used for the purpose.

**Figure 4** (a) Variation of  $\frac{\Delta Q}{Q_1}$  with time-constant for a rectifier fed by RL load (b) Variation of  $\frac{\Delta Q}{Q_1}$  with delay time for a controlled rectifier fed by RL load (see online version for colours)



**Figure 5** (a) Variation of  $\frac{\Delta Q}{Q_1}$  for a six-pulse rectifier fed by RL load (b) Variation of  $\frac{\Delta Q}{Q_1}$  for an induction motor drive



The operating principles of static meters are not specified by many manufactures. In this context, Cataliotti et al. (2007) analysed the behaviour of different static meters under various operating conditions. An inductive type meter of class 3 accuracy and five static meters of accuracy classes 2 and 0.5 were used for testing purpose. The main issues analysed are the percentage error in harmonic conditions. All static meters gave measurements which are compatible with measurement errors within the accuracy limits in sinusoidal conditions as shown in Figure 6(a). In harmonic condition, the meters gave values which are not compatible for the same test conditions with measurement errors outside the accuracy limits as mentioned in Figure 6(b). The reactive power obtained by these static meters cause different penalisation to customers for the same usage of energy. So, the authors emphasise that all the meters should operate on a common standard for measuring reactive energy under all operating conditions.

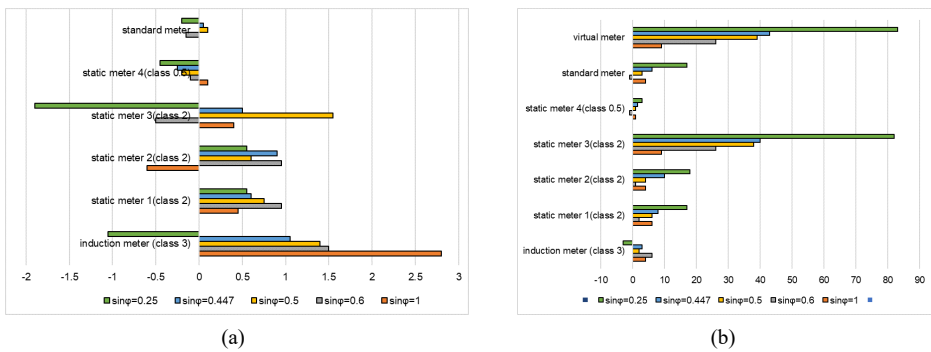
Vasconcellos et al. (2015) evaluate the behaviour of electronic meters, of the Brazilian Institute of Metrology, in a harmonic environment. The errors of measurement of reactive power under various harmonic conditions are analysed. From Figure 7, it is seen that the measurement errors increase with respect to THD for all the meters tested. This shows the dependency of meters on the harmonics present in



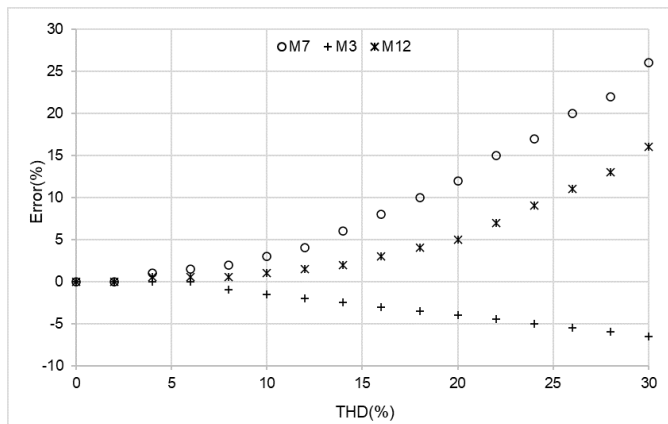
the grid. Table 4 gives a summary of reactive power measurement by varmeters under non-sinusoidal conditions.

All the above analyses point to the fact that meters behave differently in a harmonic environment (Kutija and Pravica, 2021). An investigation has been performed by Xavier et al. (2020) to validate the divergence in the readings provided by various reactive power meters under harmonic conditions. Tests were performed on 19 electronic meters and the results pointed out the fact that different techniques are adopted by meters for measuring reactive power. So there is a need of standardisation for the method of measuring reactive power. Also due to system impedance, it can happen that a linear consumer may be penalised for the harmonic pollution created by other consumers or utility. As mentioned in IEEE Standard 519-2014, both utility and customers are responsible to keep the harmonic distortion within limits. So, it is essential to identify the amount of harmonic contribution and distortion power created by each consumer to ensure fair billing. In Anu and Fernandez (2020), waveform comparison is adopted to identify harmonic contribution of each consumer eliminating the possibility of penalising linear consumers. Also, metering should be able to measure actual reactive power drawn by nonlinear loads.

**Figure 6** Percentage errors of the static meters, (a) sinusoidal case (b) non-sinusoidal case (see online version for colours)



**Figure 7** Measurement errors with respect to THD for static meters



**Table 4** Summary of articles of reactive power measurement by varmeters

<i>Authors</i>	<i>Paper insights</i>
Brito et al. (2016)	Impact of electronic loads on the reactive power measurement of Brazilian electric power companies is analysed.
Kulinich et al. (2018)	Instantaneous power derivative is used to determine reactive power.
Sychev et al. (2020b)	The analysis of various powers under non-sinusoidal conditions according to IEEE Standard 1459-2010 is presented.
Yin et al. (2018)	Instantaneous reactive power theory is used for measurement of reactive power.
Sychev et al. (2020a)	A new expression for determining the power factor of a system supplied by both non-sinusoidal source as well as nonlinear load is presented.
Langella and Testa (2007)	A circular convolution-based algorithm used for power measurements under harmonic conditions.
Cataliotti et al. (2008)	A PC-based instrument which gives powers as specified in IEEE Std. 1459 is developed.
Terzija et al. (2005)	Reactive power in a network containing both linear and nonlinear type of loads is measured using Newton type algorithm.
Aliyu et al (2014)	Reactive power in a network containing both linear and nonlinear type of loads is measured using Walsh functions transform.
Osipov et al. (2016)	Wavelet transform is used for finding powers of non-sinusoidal systems.
Zhijian and Zhongdong (2009)	The active power drawn by consumers with linear load includes both the fundamental and harmonic active power produced by nonlinear loads of another consumer.
Pinzon et al. (2016)	All the non-active powers were measured as per the IEEE Std 1459 on a microgrid with solar PV, batteries and DG systems.
Gallo et al. (2015)	A test signal for identifying the approach adopted by manufacturers in various meters is proposed.
Coelho and Brito (2023)	A method of power measurement using discrete Stockwell transform suitable for non-sinusoidal conditions is proposed.
Vieira et al. (2017)	The paper compares reactive power measurement by different methods under non-sinusoidal environment
Zhang et al. (2015)	The paper measure reactive power of nonlinear loads by various techniques and computes error with respect to fundamental reactive power.
Cataliotti et al. (2007)	The paper analyses the behaviour of different static meters under various operating conditions.
Vasconcellos et al. (2015)	The authors evaluate the behaviour of electronic meters, of the Brazilian Institute of Metrology, in a harmonic environment.
Xavier et al. (2020)	The authors validate the divergence in the readings provided by various reactive power meters under harmonic conditions.
Panoiu et al. (2023)	The authors uses adaptive neuro fuzzy inference systems (ANFIS) for the prediction of reactive power produced by the ac powered locomotive.

Artificial intelligence techniques can also be used for the prediction of reactive power as explained in Panoiu et al. (2023). In this article, the reactive power produced by the ac powered locomotive is predicted using adaptive neuro fuzzy inference systems (ANFIS).

The training of ANFIS was done with the actual data obtained from the train supply substation under different situations. The article highlights the usefulness of reactive power prediction using ANFIS enabling to take actions for limiting the factors leading to poor power quality.

## 6 Conclusions

For increasing the system efficiency, accurate monitoring and control of reactive power is essential. The penalties introduced by countries for controlling the reactive power drawn by consumers depend on the accuracy of reactive power meters. Static meters employed to measure reactive power behave differently in a harmonic environment. Manufacturers adopt different techniques to measure reactive power in static meters which are suitable for sinusoidal conditions. The analysis done by various authors indicate the need for standardisation of static meters for reactive power measurement under harmonic conditions. Hence a unique reactive power measurement strategy is essential under non-sinusoidal conditions so that reactive power billing can be done in a fair manner avoiding controversies between customers and utilities. Also, the presence of non-active powers increases losses in the system, hence monitoring and control of the same is also beneficial. The regulatory framework for reactive power billing needs to be modified for non-sinusoidal conditions. A standardised measurement method enables utilities to impose reactive power billing making consumers aware of their harmonic contribution and adopt measures to mitigate it. Hence the power quality of the system can be improved.

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