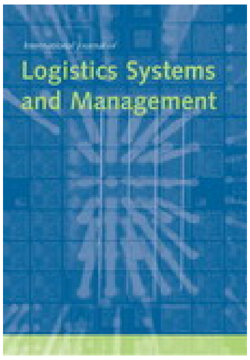


International Journal of Logistics Systems and Management

ISSN online: 1742-7975 - ISSN print: 1742-7967
https://www.inderscience.com/ijlsm

Planning the fuel distribution in urban centres using multi-compartment trucks under restrictions on the delivery period and the model of the truck

João Henrique Brunow Barbosa, Jéssica Salomão Lourenção, Rodrigo de Alvarenga Rosa

DOI: 10.1504/IJLSM.2022.10048414

Article History:

Received:	22 December 2021
Accepted:	02 May 2022
Published online:	09 January 2024

Planning the fuel distribution in urban centres using multi-compartment trucks under restrictions on the delivery period and the model of the truck

João Henrique Brunow Barbosa*,
Jéssica Salomão Lourenção and
Rodrigo de Alvarenga Rosa

Federal University of Espírito Santo,
Av. Fernando Ferrari, 514, Goiabeiras,
Vitória – ES, CEP 29075-910, Brazil
Email: joaohbrunow@gmail.com
Email: jessica_lourencao@hotmail.com
Email: rodrigo.a.rosa@ufes.br
*Corresponding author

Abstract: Distribution companies need to improve their distribution strategies to meet the increasing fuel stations demand, respecting restrictions on the delivery period and on the truck's model, both defined by fuel stations and/or local authorities. Currently, these companies plan one trip per truck and divide the whole distribution region in several small regions, on the other hand, our paper proposes to plan several trips per truck and consider one global distribution region. We propose a mathematical model and an algorithm integrating in the fuel replenishment problem characteristics as: multi-compartment, multi-trip, site dependent, time windows and heterogeneous fleet. Tests were done using data from a Brazilian fuel distribution company. Results showed that planning considering a global distribution region brought financial gains for the company. The access constraint highly impacted the transport costs.

Keywords: fuel replenishment problem; FRP; city logistics; urban logistics; vehicle routing problem; fuel distribution company; truck transport; management.

Reference to this paper should be made as follows: Barbosa, J.H.B., Lourenção, J.S. and Rosa, R.d.A. (2024) 'Planning the fuel distribution in urban centres using multi-compartment trucks under restrictions on the delivery period and the model of the truck', *Int. J. Logistics Systems and Management*, Vol. 47, No. 1, pp.51–81.

Biographical notes: João Henrique Brunow Barbosa has a Master's in Civil Engineer at the Federal University of Espírito Santo. He is also a researcher at the Laboratory of Mathematical Modelling Applied to Industrial Engineering (LAMMEP) at Federal University of Espírito Santo in Brazil. His topics of interest are operational research and metaheuristics applied to logistics problems. He is working as a consultant for a Brazilian logistics company.

Jéssica Salomão Lourenção has a Master's in Civil Engineer at the Federal University of Espírito Santo. She is also a researcher at the Laboratory of Mathematical Modelling Applied to Industrial Engineering (LAMMEP) at Federal University of Espírito Santo in Brazil. Her topics of interest are operational research and metaheuristics applied to logistics problems.

Rodrigo de Alvarenga Rosa is a Post-Doctor in Operational Research applied to Logistics. He works at Federal University of Espírito Santo, Brazil. He is a Professor at the Graduate Program in Civil Engineer. He also works as a Professor for the undergraduate courses of Industrial Engineer and Civil Engineer. He is also a coordinator and researcher of the Laboratory of Mathematical Modelling Applied to Industrial Engineering (LAMMEP) at Federal University of Espírito Santo. His topics of interest are operational research and metaheuristics applied to logistics problems.

1 Introduction

In 2019, world fuel consumption was 98.3×10^6 barrels/day, after an increase of 0.9% (924×10^3 barrels/day) compared to 2018. In 2019, the three main fuel consumers were: USA consuming 19.4×10^6 barrels/day (19.7% of the world total); China, consuming about 14.1×10^6 barrels/day (14.3% of the world total); India, consuming 5.3×10^6 barrels/day (5.4% of the world total). Brazil is the ninth consumer, consuming about 2.4 $\times 10^6$ barrels/day (2.4% of the world total), an increase of 0.9% in relation to 2018 (ANP, 2020).

Fuel distribution companies use trucks with multi-compartment, known as tank trucks, to transport different fuels from their distribution bases to the fuel stations. Using these trucks is possible to transport at the same time different fuels (gasoline, diesel and ethanol) in the same trip for one or several fuel stations. This transport has a great impact on the urban mobility, because the trucks are big and slow, contributing to increase the traffic jam in the urban centres.

To avoid traffic jam, first, policymakers had forbidden trucks to access certain regions of the city depending on their size (Brazilian government, established by Resolution No. 210 in 13th of November 2006 (CONTRAN, 2006) that a truck to access an urban centre must have its length smaller than 14.0 m). Second, they established periods in which trucks can visit the fuel stations and unload. In this way, distribution companies must plan the trucks trips to meet the fuel stations demand per day respecting all the aforementioned restrictions, aiming to reduce the transportation cost.

We assessed the problem faced by one distribution company named Raízen, which distributes different fuels from Shell in Brazil. It uses different models of multi-compartment trucks. For each model of the truck, the capacity of each compartment is known. In addition, it considers that certain models of trucks cannot travel to some regions of the city, either by legal restrictions or by operational restrictions of the fuel stations. It also considers that each fuel station can establish a period that the trucks can arrive and unload. This period may also be set considering the restrictions imposed by the authorities. It considers that each truck does one travel per day (in this paper, we will consider that each truck can perform more than one trip per day to assess possible financial gains for the company). All trips start and end in the company's distribution base. At the end of each trip, the truck's compartments are cleaned in the distribution base before being refilled. The total demand of each fuel station, considering all different fuels, must be met by only one truck. The different compartments of the trucks can be loaded with different fuels. But one compartment must be filled with just one fuel at the same time to avoid contaminations. It is necessary to control the total amount of fuel in each compartment all along the truck's trip.

Currently, the assessed fuel distribution company uses the strategy in which it divides the distribution region in smaller regions. Then, for each smaller region, they plan the distribution. This paper raises a hypothesis question: In terms of the transportation cost, planning the distribution considering one global region (the whole distribution region) is better than to divide it in smaller regions? To answer this main hypothesis, we assessed three key performance indicators (KPIs):

1 total cost

2 total number of trucks used

3 total distance travelled by the trucks.

To assess the hypothesis question, this paper proposes a new mathematical model and an algorithm to solve it to plan the trucks trips to delivery fuel to the fuel stations considering all the characteristics mentioned before, which represent the reality of the distribution company plan process. The proposed mathematical model was named as fuel replenishment problem (FRP) with multi-compartment, multi-trip, time windows (TWs), site dependent (SD) and heterogeneous fleet. It aims to minimise the transportation cost, i.e., the cost to buy and maintain the trucks needed to meet the fuel stations demand and the cost to travel the distance of all trips. As a second goal, it also aims to reduce the number of compartments used to improve the unloading process, which will be better explained next. This problem is NP-hard and to find an optimal solution by a commercial solver is very hard. Therefore, we also proposed an algorithm inspired by the metaheuristic simulated annealing (SA), which differs from the original SA by the number of movements needed to find new neighbourhoods. In the original SA there is one movement and, in our algorithm, we proposed five movements.

To assess the raised hypothesis question of this paper, data from the Raízen company, which is the Shell official fuel distribution company for the fuel stations in Espírito Santo (ES) State, Brazil, was gathered. The company has a distribution base located in Vitória, ES, which supplies 51 fuel stations, located in the cities of Vitória, Vila Velha, Cariacica, Serra and Viana, which are considered a single urban centre called Metropolitan Region of Vitória (RMG). Using the gathered data, it was assessed the impacts of each of above-mentioned restrictions and the strategy to plan the trip considering just one distribution region, without dividing it in smaller regions.

After a literature review, we found only one paper, Benantar et al. (2016), which deals with a similar problem of our problem. Our paper differs from theirs, for the following reasons:

1 We consider the possibility of a truck to travel several trips in one day.

2 We control the amount of each fuel for each fuel station in each compartment at the moment that a truck arrives at all fuel stations. This is important when the transport starts, helping the distribution company to control on-line the fuel in the truck, avoiding any fraud or wrong delivery.

3 Our objective function (OF) seeks to minimise not only the costs, but also the number of compartments used in each truck. This modification is important because it may reduce the operating time at the fuel station. The truck can be unloaded faster

if the fuel is transported using less compartments. This happens because the truck driver do not have to make coupling and uncoupling manoeuvres of the ducts that unload the fuel from the truck compartment to the fuel station tanks.

4 We consider that a fuel station cannot be visited by all trucks, only some models of trucks are allowed to visit it (these restrictions are called SD). These restrictions that we considered are related with urban centrrs restrictions, imposed by law and/or imposed by the fuel stations' location.

5 We deal with the period that the trucks can visit the fuel stations and unload, known as TWs restrictions.

This paper is structured as follows: Section 2 presents the relevant concepts and a literature review on the FRP. Section 3 presents the studied problem and how data was collected and the proposed test scenarios. In Section 4, the mathematical model is presented. In Section 5, the proposed algorithm is presented. Section 6 presents the results and analysis, and Section 7 presents the conclusions and future research.

2 Literature review

This paper deals with the FRP, also known as the petrol station replenishment problem (PSRP), which is basically the multi-compartment vehicle routing problem (MCVRP), which is a variant of the capacitated vehicle routing problem (CVRP), which aims to plan trips for each truck of a fleet with the lowest possible cost. CVRP considers that each client is visited exactly once by a truck, each truck starts and ends its trip in the depot, and the capacity of trucks cannot be exceeded (Braekers et al., 2016). Some of the variations of the CVRP used in this paper are: MCVRP, multi-trip vehicle routing problem (MTVRP), vehicle routing problem with time window (VRPTW), vehicle routing problem with heterogeneous fleet (HFVRP), and vehicle routing problem with site dependent (SDVRP) (Braekers et al., 2016).

In the MCVRP is used a fleet of trucks which has independent compartments, where each compartment can carry one or more type of cargo. Multi-compartment trucks are widely used in waste collection, distribution of oil products, refrigerated and non-refrigerated products. This model of truck is interesting, since the same truck can carry several products to one or more clients without any risk of cargo contamination. Although the MCVRP is based on the CVRP, it does require additional restrictions, such as: not exceeding the capacity of each compartment and different products should not be placed in the same compartment avoiding the possibility of contamination of the cargoes. MTVRP considers that a vehicle can perform in the planning period more than one trip, returning at the end of each trip to the depot.

In VRPTW, trucks must arrive at the client in a pre-defined time interval, known as TW, which may vary from client to client. Each TWs has a start and end moment set by the client (Braekers et al., 2016). There are two types of TWs:

1 hard

2 soft.

Hard TWs are the ones in which the truck must arrive at the client in a defined period. In this case, no delays are allowed and if the truck arrives before the TWs starting time, it must wait until the starting time to start to operate at the client. If it arrives later, the client will not accept the truck. In soft TWs, it is possible to violate the starting and end time of the TWs, but a penalty must be paid for advance or delay (Lin et al., 2014). In this paper it is considered the hard TW. In SDVRP, some clients may not be visited by all models of trucks due to its size or due to some legal restrictions imposed by any authority. The client must inform the model of the truck that it can receive (Lin et al., 2014).

Few scientific papers were published about FRP and MCVRP, despite the use of multi-compartment trucks in various practical problems of fuel distribution companies. Next, we present a literature review about the MCVRP. Abdelaziz et al. (2002) studied a special case of delivery of oil from a company in Tunisia to trucks with multiple compartments. They proposed the use of the variable neighbourhood search (VNS) metaheuristic to solve the problem. However, the authors clarified that the proposed model was far from capturing all the complexities of the practice because they were not considered the driver's workload and the client's TW. Chajakis and Guignard (2003) addressed the problem of trucks carrying various products, such as refrigerated and non-refrigerated. They used trucks with different sized partitions, working out two mathematical models and Lagrangian relaxation to solve the problem.

El Fallahi et al. (2008) proposed two metaheuristic memetic algorithm (MA) and Tabu search (TS). No access restrictions or TW were considered in the elaboration of the models. The theoretical methods were available in the literature. TS provided better results than MA. Muyldermans and Pang (2010) proposed a new set of local search procedures (LSP) for the MCVRP where the client can be serviced by more than one truck. They concluded that in many cases, the use of multi-compartment trucks is more advantageous than the use of single compartment trucks. Reed et al. (2014) solved the problem of collecting residential waste for recycling using the ant colony system (ACS) metaheuristic. They used the MCVRP where each compartment is destined to a type of waste like glass or paper.

Lahyani et al. (2015) suggested a mathematical model and the branch-and-cut (B&C) heuristic for MCVRP, multiple trips and heterogeneous loading in the collection of different types of olive oil in Tunisia. They tested actual scenarios, but the model proposed by them does not control the amount of load in each compartment when it reaches a client. Abdulkader et al. (2015) considered a homogeneous fleet and used the metaheuristic hybridised ant colony (HAC) algorithm. The authors did not address the issue with a focus on restricting truck access and clients' TWs. Henke et al. (2015) presented the problem in the context of collecting waste glass, which must be transported, segregated according to their hue. They proposed that the size of the compartments should be variable. In addition, a client can be attended by more than one truck since it is to collect glasses of different colours. They developed a metaheuristic VNS for the problem.

Chowmali and Sukto (2020) proposed a two-phase approach to solve the MCVRP applied do the FRP, but they did not apply to a real problem, and they did not consider TWs and SD. Wang et al. (2020) studied the MCVRP with split delivery and multi-trip applied to the FRP, but they did not consider the SD. They proposed an adaptive large neighbourhood search (ALNS) metaheuristic to solve the problem. Benantar et al. (2020) proposed the PSRP with adjustable demands (PSRP-AD) in which the distribution company may decide reductions on the demand of the fuel station in order to reduce the distribution costs.

Carotenuto et al. (2017) assessed the periodic petrol station replenishment problem (PPSRP), which belongs to the periodic vehicle routing problem (PVRP), together with inventory routing problems (IRP) and they solved the routing problem, CVRP, and the inventory at the fuel stations. Carotenuto et al. (2016) dealt with the PPSRP, which belongs to the PVRP with multi-compartment, applied to the FRP, considering a homogeneous fleet and multi-depot problem with a planning horizon of one week. Cornillier et al. (2008) dealt with the PPSRP. Triki and Al-Hinai (2016) studied the PPSRP considering in the OF overtime work and computational to solve it. Al-Hinai and Triki (2020) proposed a PPSRP, differently from the others, in which the distribution company can choose the frequency to visit the fuel stations. They named the problem as PPSRP with service choice (PPSRP-SC). Triki and Al-Hinai (2016) presented a survey about the PPSRP. Our present paper does not deal with PPSRP and its variants, our present paper deals with the FRP or PSRP.

Villegas and Albornoz (2016) dealt with PSRP, but different from our paper, they considered that each compartment is dedicated to one petrol station. They did not consider site dependency constraints like our paper. They proposed an insertion heuristic proposed by Solomon in 1987.

As we presented in the introduction of this paper, after a literature review, Benantar et al. (2016) were the ones that dealt with a problem close to our problem. We are different from them mainly because we consider the possibility of a truck to travel several trips in one day (multi-trip VRP). Other differences are:

1 We control the amount of each fuel for each fuel station in each compartment at the moment that a truck arrives at all fuel stations.

3 Our OF seeks to minimise not only the costs, but also the number of compartments used in each truck.

4 We consider that a fuel station cannot be visited by all trucks in a different way from them.

5 We also deal with the period allowed to the trucks to visit the fuel stations and unload, known as TWs restrictions.

6 We propose an algorithm with a complete different approach of them.

3 Assessed problem and data acquisition

We assessed the problem faced by Raízen S/A, which is a company that works in the fuel distribution sector, such as gasoline, diesel oil and ethanol. It distributes these fuels produced by Shell (2019) to fuel stations located in all Brazilian states, more specifically in the State of ES, in which we gathered the data for our tests. One of the situations that Raízen must consider in its daily plan is the restriction to access some fuel stations by some models of trucks, because there is a law prohibiting them to travel in some city regions to avoid traffic jam. In addition, another determining factor in fuel distribution plan is the period established to visit the fuel station and unload the truck. This factor, in some cases, causes that two nearby clients can not be in the same trip of a truck because the period established by both fuel stations do not allow the truck to leave one of them and travel to the other respecting both periods. Raízen provided data about the distribution of fuel to meet the demand of 51 fuel stations located in five nearby cities in the State of ES [Vitória, Vila Velha, Serra, Cariacica and Viana, called Metropolitan Region of Vitória (RMGV)]. Its distribution base is located at Serra city.

The fleet consists of three models of trucks: TR [Figure 1(a)] with a total capacity of 15.0 m^3 and three compartments of 5.0 m^3; BI [Figure 1(b)] with a total capacity of 25.0 m^3 and five compartments of 5.0 m^3; VA [Figure 1(c)] with a total capacity of 35.0 m^3 and seven compartments of 5.0 m^3.

Figure 1 Models of trucks: (a) model TR (b) model BI (c) model VA (see online version for colours)

(a)

(b)

(c)

The company considers two costs:

1 fixed cost, which is the price to buy the truck divided by its lifetime

2 variable cost, which is the cost to travel one kilometre calculated as the sum of the cost of the fuel, truck driver and maintenance.

The fixed cost for TR, BI and VA model of the truck is, respectively, US$2,000.00, US$3,500.00 and US$4,500.00. The variable cost for TR, BI and VA model of the truck is, respectively, US$2.60, US$2.30 and US$3.30. The company reported that the trucks travel with an average speed of 30.0 km/h for all models of trucks of the fleet. The time spent to unload the truck at the fuel station depends on the quantity of cubic meters unloaded in each fuel station. The company reported that the truck unloading rate at the fuel stations is 1.0 m^3/minute and the loading rate at the distribution base is 3.0 m^3/minute.

We developed the scenarios to test the proposed mathematical model and the proposed algorithm based on real data gathered with the Raízen company. In all scenarios we considered a heterogeneous fleet like the company's fleet. We tested two different situations:

1 for each fuel station is defined which model of the truck can visit it, known as SD restriction

2 for each fuel station is established a different period to be visited, known as TWs restriction.

These two situations were tested to evaluate their influence on the planned trips.

Therefore, five groups were defined:

- Group 1 represents the fuel stations of Vitória city
- Group 2 represents the fuel stations located at Vila Velha city
- Group 3 considers the fuel stations located at Serra city
- Group 4 deals with the fuel stations located at Viana city and Cariacica city.
- Group 5 considers all 51 fuel stations located at RMG (all five cities).

Table 1 presents the main characteristics of each proposed scenario. For comparison, each scenario, from 1 to 12, was created in three different situations:

1 without TW restriction and without SD restriction (scenarios 1, 4, 7 and 10)

2 without TW restriction and with SD restriction (scenarios 2, 5, 8 and 11)

3 without TW restriction and without SD (scenarios 3, 6, 9 and 12).

Scenario 13 represents all five cities, regions, of the RMG considering TW and SD restrictions. Scenarios 1, 4, 7 and 10 are the ones currently faced daily by the distribution company. The comparison of the sum of the results from scenarios 1, 4, 7 and 10 with the result of scenario 13 is used to answer the hypothesis question raised by this paper: planning considering a global distribution region (with all regions) is better than planning considering several smaller regions?

Table 1 Proposed scenarios

Group	*Scenarios*	*Number of fuel stations*	*Number of trucks available*			*With TW?*	*With SD?*
			TR	*BI*	*VA*		
1	1	13	3	2	3	Yes	Yes
	2					No	Yes
	3					No	No
2	4	14	3	4	3	Yes	Yes
	5					No	Yes
	6					No	No
3	7	14	3	5	4	Yes	Yes
	8					No	Yes
	9					No	No
4	10	10	1	4	4	Yes	Yes
	11					No	Yes
	12					No	No
5	13	51	10	15	14	Yes	Yes

4 Proposed mathematical model

This section presents the proposed mathematical model to plan the fuel distribution (gasoline, ethanol and diesel). The problem faced by the distribution company consists in planning the trucks trips in one day considering that:

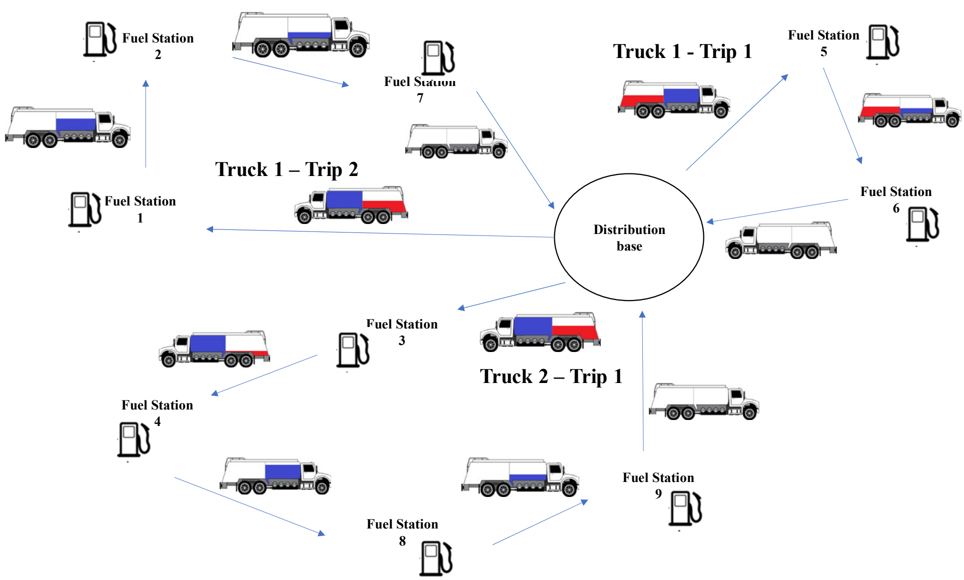
1. the fleet of the trucks is heterogeneous (there are different models of trucks)
2. the trucks have several compartments of fixed size
3. each fuel station is visited once by a truck
4. each fuel station can receive only specific models of trucks (SD restrictions)
5. each fuel station is only visited within a period established by it (TWs restrictions)
6. the fuels transported on the truck must not exceed the capacity of the truck and the compartment
7. a truck can carry more than one fuel if each one is in a different compartment
8. each compartment can carry only one fuel to prevent contamination of one fuel by other.

After the truck has visited all the fuel stations and does not have any fuel in its compartments, it returns to the distribution base to be loaded, and then start a new trip. It can perform as many trips as possible in a working day.

We consider that there are *nc* fuel stations and *nk* trucks with at most *nw* compartments and each truck can carry *np* fuels. Each truck departs from the distribution base carrying *np* different fuels to meet the demand of *nc* fuel stations. Each truck can daily travel *nr* trips. The distribution base is the node 0, where all trucks start their trips, and it is considered a virtual distribution base, node $nc + 1$, where all trucks must return after their trips. Figure 2 illustrates the problem, where truck 1 leaves the distribution base to visit several fuel stations, returns to the distribution base to load, and then starts another trip to visit others fuel stations. truck 2 performs just one trip.

Thus, considering that N represents the set of nodes to be served by a set K of trucks, the mathematical model can be represented by a complete graph $G(N; A)$; where $N = \{0\} \cup C \cup \{nc + 1\}$. Node $\{0\}$ represents the real distribution base and $\{nc + 1\}$ represents the virtual distribution base to complete the graph. After the previous explanations, it is presented next the sets, parameters, decision variables, OF and constraints of the proposed mathematical model.

Figure 2 Schematic drawing of the mathematical model (see online version for colours)



4.1 Sets

C Set of fuel stations, $C = \{1, \ldots, nc\}$.

CR Set of fuel stations which considers the distribution base, $C0 = \{\{0\} \cup C\}$.

CV Set of fuel stations which considers the virtual distribution base, $CV = \{C \cup \{nc + 1\}\}$.

CT Set of fuel stations which considers the distribution base and the virtual distribution base, $CT = \{\{0\} \cup C \cup \{nc + 1\}\}$.

K Set of trucks, $K = \{1, \ldots, nk\}$.

K^i Set of trucks that can visit fuel station $i \in C$, $K^i \in K$.

P Set of fuels, $P = \{1, \ldots, np\}$.

W Set of compartments, $W = \{1, \ldots, nw\}$.

R Set of trips, $R = \{1, \ldots, nr\}$.

4.2 Parameters

q_{ip} Demand of fuel station $i \in C$ for the fuel $p \in P$.

cc_{kw} Capacity of compartment $w \in W$ of the truck $k \in K$.

d_{ij} Distance between nodes $i \in N$ and $j \in N$.

tv_{ij} Travel time between nodes $i \in N$ and $j \in N$.

t_c Time to load the truck $k \in K$ to start a new trip.

t_l Maximum travel time limit, it is equal for all trucks.

td_p Time to unload the fuel $p \in P$ for all trucks.

twi_i Starting moment of the period that the truck can arrive at fuel station $i \in C$.

twf_i Ending moment of the period that the truck can arrive at fuel station $i \in C$.

cq_k Cost of the kilometre travelled by truck $k \in K$.

cf_k Fixed cost of truck $k \in K$.

M An arbitrarily large number for the logic of the model, it was set to 9,999.99.

m An arbitrarily small number for the logic of the model, it was set to 0.01.

φ Weight applied to the third part of the OF that aims to reduce the number of compartments used.

ϕ Weight applied to the fourth part of the OF that aims to reduce the initial time of customer service.

4.3 Decision variables

x_{ijkr} Binary variable that assumes value equal to 1 if truck $k \in K$ travels from node $i \in CT$ to node $j \in CT$ in the trip $r \in R$, and 0 otherwise.

z_{pwkr} Binary variable that assumes value equal to 1 if fuel $p \in P$ is in the compartment $w \in W$ of the truck $k \in K$ in the trip $r \in R$, and 0 otherwise.

qc_{ipwkr} Quantity of fuel $p \in P$ in the compartment $w \in W$ of the truck $k \in K$ when it arrives at node $i \in CT$ in the trip $r \in R$.

ta_{ikr} Moment that truck $k \in K$ arrives at node $i \in CT$ in the trip $r \in R$.

ut_k Binary variable that assumes value equal to 1 if the truck $k \in K$ is used, and 0 otherwise.

4.4 Objective function

$$\begin{aligned}
&\text{Minimise} \quad \sum_{k\in K} cf_k ut_k + \sum_{i\in CT}\sum_{j\in CT}\sum_{k\in K}\sum_{r\in R} d_{ij} cq_k x_{ijkr} \\
&+\varphi \sum_{p\in P}\sum_{w\in W}\sum_{k\in K}\sum_{r\in R} z_{pwkr} + \phi \sum_{i\in CT}\sum_{k\in K}\sum_{r\in R} ta_{ikr}
\end{aligned} \tag{1}$$

Subject to:

$$\sum_{j\in CV}\sum_{k\in K}\sum_{r\in R} x_{ijkr} = 1 \qquad \forall i\in C \tag{2}$$

$$\sum_{j\in CT} x_{0jkr} = 1 \qquad \forall k\in K, r\in R \tag{3}$$

$$\sum_{i\in CR} x_{i(nc+1)kr} = 1 \qquad \forall k\in K, r\in R \tag{4}$$

$$\sum_{i\in CR} x_{ihkr} - \sum_{j\in CV} x_{hjkr} = 0 \qquad \forall k\in K, h\in C, r\in R \tag{5}$$

$$x_{i0kr} = 0 \qquad \forall i\in CT, k\in K, r\in R \tag{6}$$

$$x_{(nc+1)jkr} = 0 \qquad \forall j\in Ct, k\in K, r\in R \tag{7}$$

$$\sum_{j\in C} x_{0jkr} = \sum_{i\in C} x_{i(nc+1)kr} \qquad \forall k\in K, r\in R \tag{8}$$

$$z_{pwkr} \le M \sum_{i\in C} qc_{ipwkr} \qquad \forall p\in P, w\in W, k\in K, r\in R \tag{9}$$

$$z_{pwkr} \ge m \sum_{i\in C} qc_{ipwkr} \qquad \forall p\in P, w\in W, k\in K, r\in R \tag{10}$$

$$\sum_{p\in P} z_{pwkr} \le 1 \qquad \forall w\in W, k\in K, r\in R \tag{11}$$

$$\sum_{w\in W} qc_{ipwkr} = q_{ip} \sum_{j\in CV} x_{ijkr} \qquad \forall i\in C, p\in P, k\in K, r\in R \tag{12}$$

$$\sum_{i\in C}\sum_{p\in P} qc_{ipwkr} \le cc_{kw} \qquad \forall w\in W, k\in K, r\in R \tag{13}$$

$$\sum_{i\in CR}\sum_{j\in CV} \left(tv_{ij} + q_{ip} td_p\right) x_{ijkr} \le tl \qquad \forall k\in K, r\in R \tag{14}$$

$$ta_{jk} \ge ta_{ik} + x_{ijkr} tv_{ij} + x_{ijkr} \sum_{p\in P} q_{ip} td_p + \left(1 - x_{ijkr}\right) M \quad \forall i\in CR, j\in CV, k\in K, r\in R \tag{15}$$

$$ta_{0kr} \ge ta_{(nc+1)k(r-1)} + tc \sum_{i\in C}\sum_{p\in P}\sum_{w\in W} qc_{ipwkr} \qquad k\in K, r\in R/\{1\} \tag{16}$$

$$ta_{ikr} \geq twi_i \sum_{j \in CV} x_{ijkr} \qquad \forall i \in C, k \in K, r \in R \qquad (17)$$

$$ta_{ikr} \leq twf_i \sum_{j \in CV} x_{ijkr} \qquad \forall i \in C, k \in K, r \in R \qquad (18)$$

$$ta_{0kr} \geq twi_0 \sum_{j \in C} x_{0jkr} \qquad \forall k \in K, r \in R \qquad (19)$$

$$ta_{0kr} \leq twf_0 \sum_{j \in C} x_{0jkr} \qquad \forall k \in K, r \in R \qquad (20)$$

$$ta_{(nc+1)kr} \geq twi_{(nc+1)} \sum_{i \in C} x_{i(nc+1)kr} \qquad \forall k \in K, r \in R \qquad (21)$$

$$ta_{(nc+1)kr} \leq twf_{(nc+1)} \sum_{i \in C} x_{i(nc+1)kr} \qquad \forall k \in K, r \in R \qquad (22)$$

$$ta_{(nc+1)kr} \leq tl \qquad \forall k \in K, r \in R \qquad (23)$$

$$ut_k \leq M \sum_{j \in C} \sum_{r \in R} x_{0jkr} \qquad \forall k \in K \qquad (24)$$

$$ut_k \geq m \sum_{j \in C} \sum_{r \in R} x_{0jkr} \qquad \forall k \in K \qquad (25)$$

$$\sum_{k \in K} ut_k \geq 1 \qquad (26)$$

$$\sum_{k \in K} ut_k \leq nk \qquad (27)$$

$$x_{ijkr} = 0 \qquad \forall i \in C, j \in CT, k \notin K^i, r \in R \qquad (28)$$

$$x_{ijkr} \in \{0, 1\} \qquad \forall i, j \in CT, k \in K, r \in R \qquad (29)$$

$$z_{pwkr} \in \{0, 1\} \qquad \forall p \in P, w \in W, k \in K, r \in R \qquad (30)$$

$$ta_{ikr} \in \mathbb{R}^+ \qquad \forall i \in CT, k \in K, r \in R \qquad (31)$$

$$qc_{ipwkr} \in \mathbb{R}^+ \qquad \forall i \in CT, p \in P, w \in W, k \in K, r \in R \qquad (32)$$

$$ut_k \in \{0, 1\} \qquad \forall k \in K \qquad (33)$$

The OF [equation (1)] is divided in four parts. The first two parts represent the sum of the fixed cost, represented by the sum of the cost of all trucks used, plus the variable cost, calculated as the sum of the total distance travelled by the trucks multiplied by the cost per kilometre. These two costs, indeed, is what matters to the transportation cost.

The third part is important to the operational aspect, it represents the sum of the number of compartments used in each truck multiplied by a weight φ to reduce its importance in the OF, it is set to 0.01. This part is intended to put all the demand for a fuel that the truck carries in the minimum number of compartments to reduce the unload time at the fuel station. The fourth part intends to reduce the total time travelled by the trucks, if this part is not considered, the mathematical model may set the travelling time to its limit, even when the truck travels less than the established limit. It is multiplied by a weight ϕ to reduce its importance in OF, it is set to 0.001. The OF must be minimised.

Constraint (2) ensures that each fuel station is served by a single truck in a single trip. Constraints (3) and (4) define that the truck in a single trip must start their trip in the distribution base and finish it in the virtual distribution base. Constraints (5) are the flow conservation constraint. Constraints (6) ensure that no trucks in all trips finish its trip at the distribution base. Constraints (7) ensure that no trucks in all trips start its trip at the virtual distribution base. Constraints (8) impose that if a truck left the distribution base in a trip, it must also return in this trip to the virtual distribution base.

Constraints (9) and (10) ensure that variable z_{pwkr} assumes value equal to 1 if the fuel p is in the compartment $w \in W$ of the truck $k \in K$ in trip $r \in R$; and 0 otherwise. Constraints (11) ensure that no more than one fuel $p \in P$ is in the truck $k \in K$ in trip $r \in R$ in its compartment w, $z_{pwkr} < 1$, or no fuel $p \in P$ is in the truck $k \in K$ in trip $r \in R$ in its compartment w ($z_{pwkr} = 0$). Constraints (12) ensure that in case the fuel station is visited by a truck $k \in K$ in trip $r \in R$, then its total demand for fuels must be in the compartments of the truck. Constraints (13) ensure that the total fuel in each truck $k \in K$ compartment in trip $r \in R$ must be less than the compartment's capacity.

Constraints (14) ensure that each trip of each truck does not exceed the maximum travel time limit of each trip. Constraints (15) guarantee that the starting time of trip r ($r \geq 2$) of truck $k \in K$ at the distribution base must be equal or greater than the ending time of the previous trip plus the time to load the truck with the fuels which will meet the demand of all clients of the new trip. Constraints (16) ensure that if truck $k \in K$ travels from node $i \in CT$ to node $j \in CT$, then the arrival time at node $j \in CT$ must be greater than the arrival time at node $j \in CT$ plus the travel time between them plus the time to unload at node $i \in CT$. Constraints (17)–(18) ensure that a truck in a trip r must arrive in the period between the starting moment and the ending moment of the period that fuel station $i \in C$ specifies to be visited. Constraints (19)–(20) ensure that the period that a truck in trip $r \in R$ can leave the distribution base is respected. Constraints (21) and (22) ensure that the period that a truck in trip $r \in R$ must return to the distribution base is respected. Constraints (23) ensure that the truck $k \in K$ in trip $r \in R$ cannot visit fuel station $i \in C$ if its model is not accepted by the fuel station.

Constraints (24) and (25) ensure that if the truck $k \in K$ visits at least one customer in any trip, then it is considered used, ($ut_k = 1$). Constraints (26) and (27) ensure that at least one truck must be used, and the total trucks used must be equal or smaller than nk, i.e., the number of available trucks in the fleet. Constraints (29)–(33) define the domain of the decision variables. The proposed model has $|K|(|CT|^2|R| + |CT||R| + 1)$ binary decision variables and $|CT||K||R|(|P||W| + 1)$ real decision variables.

5 Proposed algorithm

The proposed algorithm is inspired by the metaheuristic SA. Our algorithm differs from the original SA by the number of movements needed to find new neighbourhoods, in the original SA there is one movement, and, in our algorithm, we propose five movements, which will be presented next. The pseudocode of the proposed algorithm is presented in Figure 3.

Figure 3 Pseudocode of the proposed algorithm

```
InputParameters(T0, ft, imx, nr, α)
sC ← InitialSolution()
sC.OF ← CalculateOF(sC)
CopySolution(sC, sB)
T ← T0
r ← 0
WHILE (r < nr)
    WHILE (T > ft)
        FOR i ← 1 TO imx
            sN ← Create_Neighbour(sC)
            IF (Create_Neighbour(sC) == feasible)
                sN.OF ← CalculateOF(sN)
                IF (sN.OF < sC.OF)
                    CopySolution(sN, sC)
                    IF (sN.OF < sB.OF)
                        CopySolution(sN, sB)
                    END IF
                END IF
                ELSE
                    Δ ← sN.OF – sC.OF
                    Ψ = e^(−Δ/T)
                    IF (Ψ > Randomise())
                        CopySolution(sN, sC)
                    END IF
                END ELSE
            END IF
        END FOR
    T ← α T
    END WHILE
    r ← r + 1
    T = T0/e^(log(T0/100)/(NR–r+1))
    T0 ← T
    CopySolution(sB, sC)
END WHILE
Print(sB)
```

Initially, the algorithm receives some parameters: the initial temperature (*T*0), the freezing temperature (*ft*), the maximum number of iterations (*imx*), the number of reannealing cycles (*nr*) and the cooling rate (α). It is considered in the proposed algorithm three solutions:

1 best solution (*sB*)

2 current solution (*sC*)

3 neighbour solution (*sN*).

Then, an initial solution is generated by function *InitialSolution*() using data of the instance. To create the initial solution, first the clients are sorted according to their TWs. Them the function tries to allocate the client in a truck. If the allocation is successful, meaning the truck has the capacity and all of the clients constrains are satisfied, the client is put in the truck's route. If the allocation is not successful, the function chooses another truck and tries to allocate the client again. This process is repeated until the all the clients are allocated in the trucks. This solution will be the start point of the algorithm, allocating all the fuel stations in the trucks respecting all constraints, without concern for the lowest cost. This solution will be used as the first *sC*. Function *CopySolution*(*sC*, *sB*) is used to copy solution *sC* to solution *sB*. Function *Create_Neighbour*(*sC*) randomly choose one of the five possible movements to reach new neighbourhoods in each iteration (Figures 4, 5, 6, 7, and 8):

1 movement 1

2 movement 2

3 movement 3

4 movement 4

5 movement 5.

After the movement is done, it verifies if all constraints were respected. The first constrain to be verified is the capacity of the truck. It is tested if the compartments of the truck are able to accommodate the client's order while respecting the condition of only having one type of fuel per compartment. To verify the TW constraint, the time of arrival at the client by the vehicle, that had been modified by the movement are calculated. Then, the time of arrival in the client is compared to the TW of each client. To verify the site dependency constraint, every time a movement makes a client change trucks it is verified if the client is able to receive the type of truck it was put in. If they were respected, then a feasible new neighbourhood solution *sN* is created. If at lest one constraint was not respected, then the solution *sN* is not accepted and the movement is not considered to the algorithm. Next, we present all five movements.

Movement 1 (Figure 4) consists in the exchange in the visiting sequence of two different fuel stations in the same truck's trip. A truck and one of its trips are chosen at random. After that, two fuel stations of the truck's trip are randomly chosen and have their visiting sequence changed between them. In movement 2 (Figure 5), the algorithm chooses at random two trucks, and for each of them, it chooses at random one trip and then, for the first truck's trip chosen, it chooses at random a fuel station, and then it moves it to the second truck's trip chosen. In movement 3 (Figure 6), the algorithm chooses at random two trucks, and for each of them, it chooses at random one trip. For each truck's trip chosen, it chooses at random one fuel station, and then it switches the fuel station of the first truck's trip with the second truck's trip chosen, respecting the same original visiting sequence in each truck's route.

Figure 4 Movement 1 (see online version for colours)

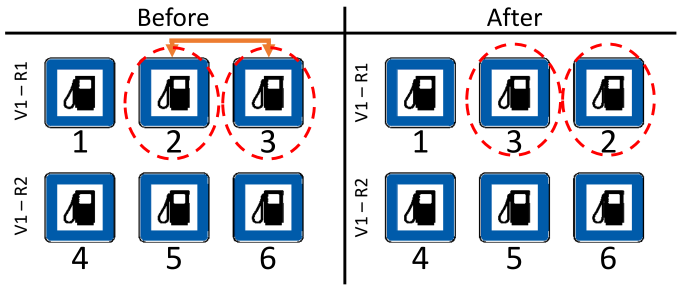


Figure 5 Movement 2 (see online version for colours)

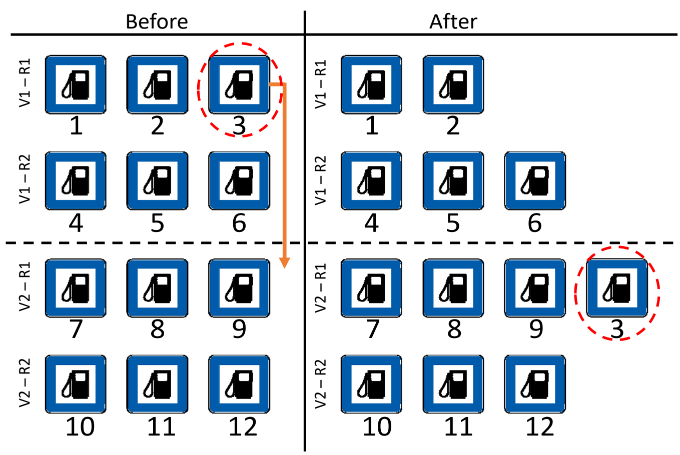


Figure 6 Movement 3 (see online version for colours)

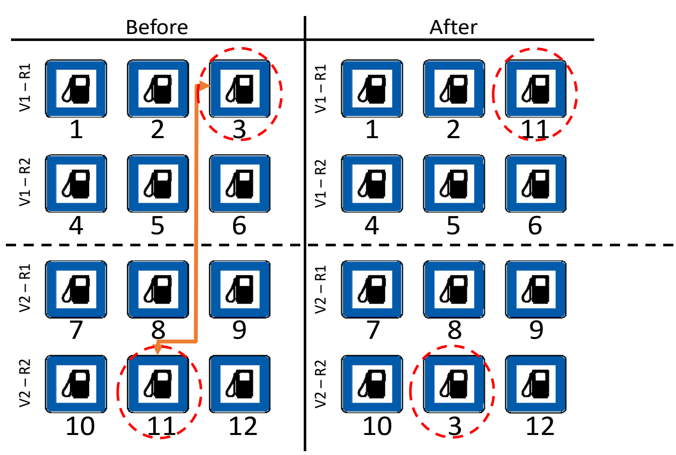


Figure 7 Movement 4 (see online version for colours)

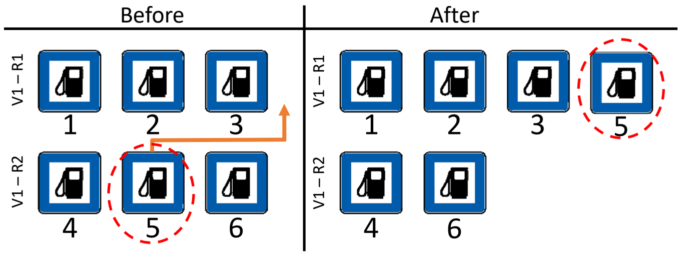
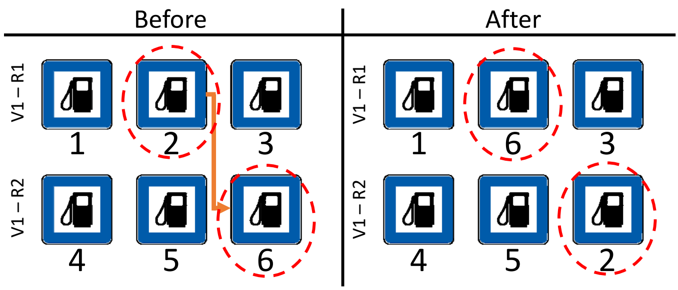


Figure 8 Movement 5 (see online version for colours)



In movement 4 (Figure 7), the algorithm chooses at random one truck and chooses at random two trips. Then, it chooses at random one fuel station from the first trip chosen, and then it moves this fuel station to the second trip chosen. In movement 5 (Figure 8), the algorithm chooses at random one truck and chooses at random two trips. Then, it chooses at random one fuel station from each truck's trip chosen, and then it switches the fuel station between trips, respecting the visiting sequence at each chosen trip.

In movements 2, 3, 4 and 5, when a fuel station is switched from one truck to other truck, or from one trip to another, first it is necessary to sum the total cargo of each type that this fuel station has in the truck considering all compartments. After that, the algorithm must check if the new truck has capacity to receive all different fuels of the fuel station. To do so, first for each type of fuel is verified if the truck is already transporting it. If it is transporting, it checks if there is any free volume in the compartments to store this fuel from the fuel station. This process must be done for both switched trucks.

To store the fuel in the truck, the algorithm checks if there is a compartment containing the same type of fuel being used. If there is volume available in that compartment, the algorithm will store the fuel there until all the fuel is stored or the compartment is full. If the capacity of the compartment was not enough for all the fuel or there was not a compartment containing the same type of fuel available, an unused compartment will receive the fuel or rest of the fuel that could not be stored in the previous compartment.

Likewise, when unloading the fuel, the algorithm unload first from the compartment with the least amount of the intended fuel. Only when the compartment is empty that a new compartment with the same type of fuel is chosen to have its fuel unloaded.

With a new feasible solution *sN* at hand, comes the function *CalculateOF*(*sN*) that have the sole purpose of calculating the OF of the new neighbour solution found. The OF is calculated by the sum of the fixed cost of all used trucks plus the sum, for all used trucks, of the multiplication of the truck's variable cost by its respective total travelled distance. If the neighbour solution *sN* is smaller than the current solution *sC*, the neighbour solution *sN* is copied to the current solution *sB* and if it smaller than the best solution *sB*, it is also copied to the best solution *sB*. If the neighbour solution *sN* is equal or greater than the current solution *sC*, a condition is applied to give the chance of this new worse solution to be accepted. The probability of a worse solution to be accepted is given by the equation $\Psi = e^{-\frac{\Delta}{T}}$, in which Δ is the difference between *sC.OF* and *sN.OF* and T is the current temperature of the cycle. The value of Ψ is compared to a value generated by function *Randomise*(), which generates a random value between 0 and 1. If Ψ is higher than the number generated by function *Randomise*(), the solution is accepted and the neighbour solution *sN* is copied to the current solution *sC*. This routine of accepting a worse solution is the mechanism used to scape from local minimums.

After reaching maximum number of iterations (*imx*), the temperature is decreased according to the cooling rate, $T = \alpha T$. This is done until the freezing temperature (*ft*) is reached. Reannealing is applied to search a better solution after the algorithm finished the cooling phase. The new initial temperature is given by the formula $T = T0/e^{(\log(T0/100)/(NR-r+1))}$, and then starting another cooling cycle copying the best solution to current solution.

The algorithm input parameters were chosen aiming first to improve the solution, minimising it, and second aiming to reduce the algorithm running time. To set these input parameters, four instances, which considers both restrictions (SD restrictions and TWs restrictions) were chosen (instances 1, 4, 7 e 10), and for each of them we run the algorithm five times. First, we set the cooling rate at 0.975 and the number of iterations to 1,500. Then the algorithm was tested with four different temperatures, 40,000, 50,000, 60,000 and 70,000. We note that any increase further than 50,000 was not leading to improvements in the solution, so the initial temperature was set to 50,000.

Having set the initial temperature, we tested the cooling rate with three different values, 0.950, 0.975 and 0.980. We noticed that all three values used for the cooling rate tested brought the same solution. The difference in process time between the lowest and the highest cooling rate was less than two seconds. Being so, we decided to use the intermediary value of 0.975.

With both the initial temperature and the cooling rate set, we tested the number of iterations. Three different values were tested, 1,000, 1,500 and 2,000. The tests showed that setting the iterations to 1,000 reached solutions worse than the ones obtained with 1,500 iterations. With 2,000 iterations there were no improvements to the solution. Therefore, we decided to set the number of iterations to 1,500. The freezing temperature was set to 0.01. Regarding the number of reannealing, we noticed that no better solution was found after the first cooling cycle. Although no better solution was found during the reannealing process, the number of reannealing was set to 2 to try to find any improvement of the best solution found in the first cooling process.

6 Results and analysis

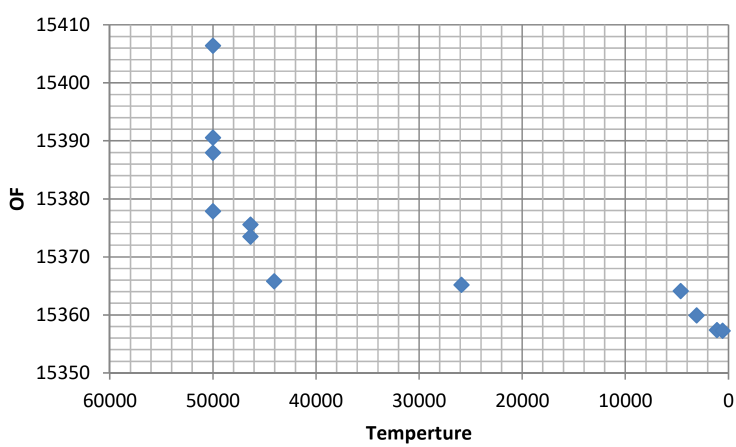
Results obtained by the proposed algorithm were compared to the results obtained by the solver CPLEX 12.8 (IBM, 2019) running the mathematical model. We used a computer with two processors Intel® Xeon® CPU E5-2640 v3 @2.60 GHz and 64 GB RAM. It was established for CPLEX a limit runtime of 86,400 seconds, or 24 hours.

We present next the results divided in two parts. First, we present the performance results for CPLEX running the mathematical model and for the proposed algorithm. Second, we present the operational results and insights for logistics managers.

6.1 Performance results

Table 2 presents the performance results achieved by CPLEX and by proposed algorithm and the comparison between both. CPLEX found the optimal solution, gap = 0.0%, for seven scenarios, scenarios 1, 2, 7, 8, 10, 11 and 12. For all these seven scenarios, the proposed algorithm found solutions with the same values found by CPLEX, with deviation equal to zero. For the five scenarios, scenarios 3, 4, 5, 6 and 9, in which CPLEX did not find the optimal solution, but found a feasible solution, we used the UB as the solution to compare with the proposed algorithm. The proposed algorithm found solutions with the same values or better found by CPLEX. It is worthy to say that CPLEX stopped after 86,400 seconds, 24 hours, with high gaps in scenarios 3, 6 and 9, respectively, 98.9%, 98.8% and 99.1%. For scenarios 3 and 6, the proposed algorithm found better solutions than CPLEX.

Figure 9 Temperature × OF: scenario 2 (see online version for colours)



When we analysed the deviations obtained by the proposed algorithm, we find that it is stable, because we observe that in scenarios 1 to, 2, 3, 4, 5, 7, 8, 10, 11 and 12 the algorithm presents deviations of 0.000%, in scenarios 6 and 9 it presents deviations smaller than 0.003% and in scenario 13 it presents deviation equal to 0.367%. For scenario 13, CPLEX did not found a LB for scenario 13 after the maximum execution time of 24 hours and in the other hand the proposed algorithm obtained the solution at a runtime of 77.0 seconds.

Table 2 Proposed algorithm and CPLEX performance results

Group	Scenario	CPLEX					Proposed algorithm					Gains of the proposed algorithm × CPLEX	
		OF (US$)	*UB (US$)*	*LB (US$)*	*Time (s)*	*GAP (%)*	*Average (US$)*	*Best (US$)*	*Worst (US$)*	*Deviation (%)*	*Exec. time (s)*	*OF (US$)*	*Exec. time (s)*
1	1	15,357.3			340.6	0.0	15,357.3	15,357.3	15,357.3	0.000	6.0	0.0	334.6
	2	15,357.3			1,070.6	0.0	15,357.3	15,357.3	15,357.3	0.000	6.0	0.0	1,064.6
	3	11,724.6	11,724.6	175.0	80,000.0	98.9	11,703.8	11,703.8	11,703.8	0.000	6.0	21.2	80,000.0
2	4	24,753.1	24,753.1	19,531.6	80,000.0	42.9	24,753.1	24,753.1	24,753.1	0.000	7.0	0.0	80,000.0
	5	24,753.1	24,753.1	21,177.8	80,000.0	14.4	24,753.1	24,753.1	24,753.1	0.000	6.0	0.0	80,000.0
	6	21,183.1	21,183.1	11,505.2	80,000.0	98.8	21,166.3	21,165.6	21,172.4	0.003	7.0	16.8	80,000.0
3	7	32,364.5			2,237.1	0.0	32,364.5	32,364.5	32,364.5	0.000	11.0	0.0	1,274.4
	8	32,364.5			8,721.9	0.0	32,364.5	32,364.5	32,364.5	0.000	10.0	0.0	8,711.9
	9	23,674.1	23,674.1	4,370.5	80,000.0	99.1	23,674.5	23,674.1	23,677.3	0.001	11.0	–0.4	80,000.0
4	10	27,402.4			7,337.3	0.0	27,402.4	27,402.4	27,402.4	0.000	6.0	0.0	7,331.0
	11	27,402.4			3,312.3	0.0	27,402.4	27,402.4	27,402.4	0.000	6.0	0.0	3,306.3
	12	20,698.0			51,960.0	0.0	20,698.0	20,698.0	20,698.0	0.000	6.0	0.0	51,954.0
5	13	nf	nf	nf	80,000.0	nf	76,513.4	76,233.6	76,558.5	0.367	77.0	-	-

Note: nf – not found.

Figure 10 Temperature × OF: scenario 3 (see online version for colours)

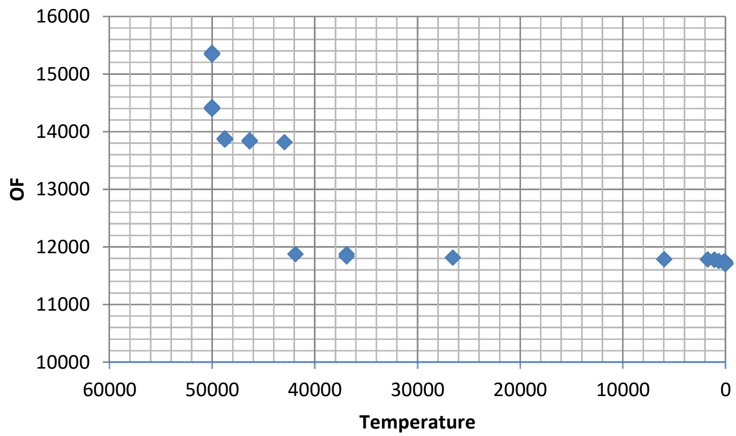


Figure 11 Temperature × OF: scenario 4 (see online version for colours)

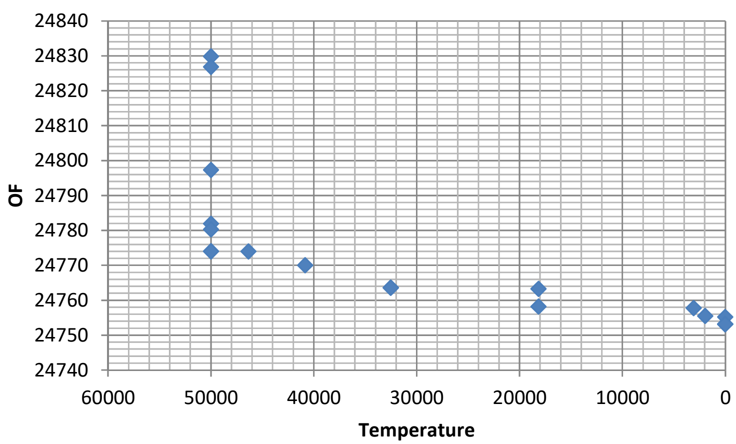
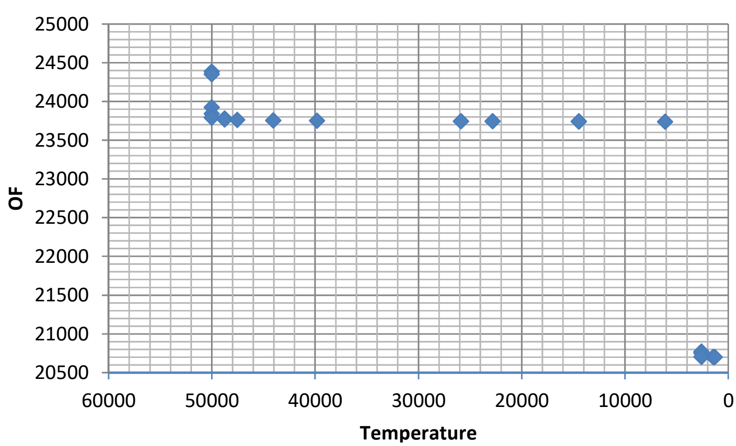


Figure 12 Temperature × OF: scenario 12 (see online version for colours)



In order to observe the behaviour of the proposed algorithm during its execution, we recorded the temperature in which the algorithm was when it found each new best solution. In Figures 9, 10, 11 and 12, we plotted the behaviour of the proposed algorithm regarding temperature × OF value for scenarios 2 (Figure 9), 3 (Figure 10), 4 (Figure 11) and 12 (Figure 12). As each scenario was executed 10 times, the data chosen to draw the graph was the scenario which was presented the best OF.

We noticed that OF value decreases hard when the proposed algorithm finds a solution with at least one truck less. The gains obtained through the reduction of the distance is still important, but they are small when compared with the reduction of the number of trucks. It was also noticed that there is no relation between the temperature and the reduction of the number of trucks used. A possible reason for that is because the algorithm chooses the movements in random way.

6.2 Operational results

To assess the operational results, we used the average solution found by the proposed algorithm. As an example, to understand the output of the proposed algorithm, Table 3 shows trip 1 of scenario 1. In Scenario 1, trip 1, five trucks were used of the seven available trucks (Table 1), one truck of model VA, two trucks of model BI and two trucks of model TR. In Table 3, trucks' trip starts and finishes at the distribution base (db), the number in each trip represents the number of the client.

Table 3 Trip 1 of truck 1 in scenario 1

Truck no.	*Model of the truck*	*Route*	*Fuels in each compartment (m^3)*						
			1	*2*	*3*	*4*	*5*	*6*	*7*
1	VA	db -> 49 -> 40 -> 45 -> 42 -> db	4.8	5.0	5.0	3.4	4.9	4.9	5.0
2	VA	Not used							
3	BI	db -> 51 -> 41 -> 47 -> db	2.7	4.6	4.5	5.0	1.3		
4	BI	db -> 39 -> 48 -> 43 -> db	2.8	4.6	5.0	5.0	1.1		
5	TR	Not used							
6	TR	db -> 46 -> 50 -> db	1.0	4.2	1.8				
7	TR	db -> 44 -> db	3.9	1.0	1.6				

Figure 13 presents the routing scheme of the trip 1 of each truck in scenario 1, no trucks travels 2 trips in this scenario. Figure 14 presents trip 1 of truck 1 with a schematic representation of the fuels in its compartment. It is important to highlight that the proposed mathematical model and the proposed algorithm controls the total amount of the fuels in each compartment when the truck arrives at each fuel station on the trip. This is important to allow the distribution company to control its operations, verifying online at each fuel station on the trip the amount of the fuel in each compartment is equal to the quantity planned, avoiding any fraud or wrong deliveries.

Table 4 and Figure 14 presents the operational results of the proposed algorithm for scenarios 1 to 12 to assess the impact of TWs restrictions and access restrictions (SD) as described for the scenarios in Table 1.

Figure 13 Trips of the five trucks used in scenario 1 (see online version for colours)

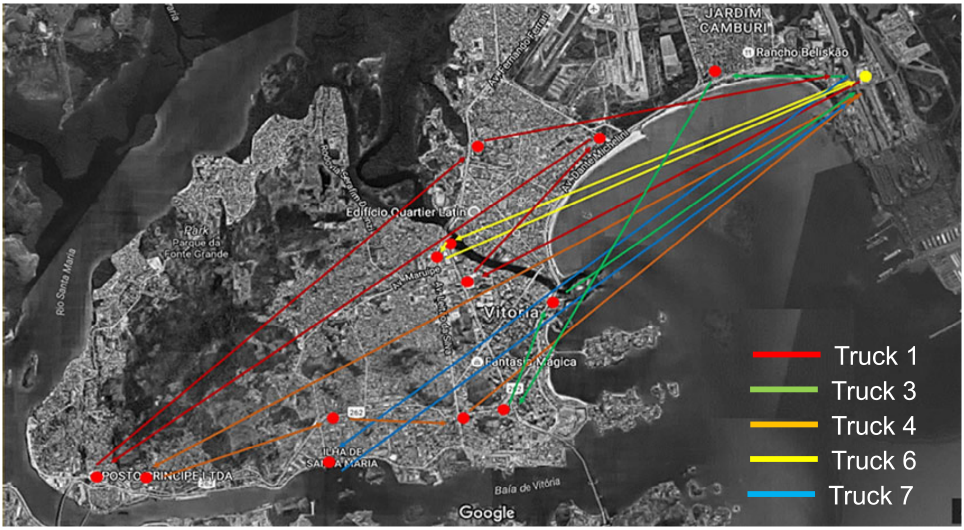
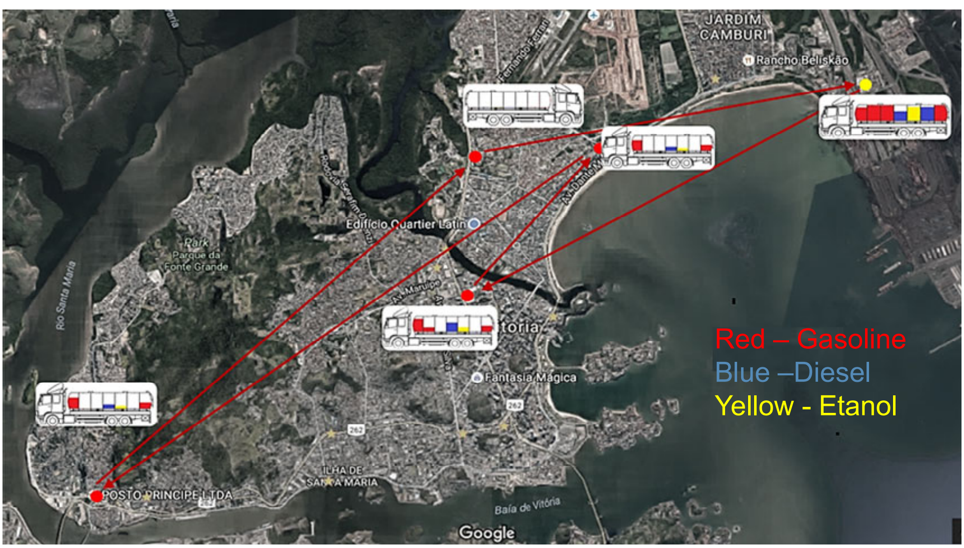


Figure 14 Fuel in each compartment of truck 1, trip 1, in scenario 1 (see online version for colours)

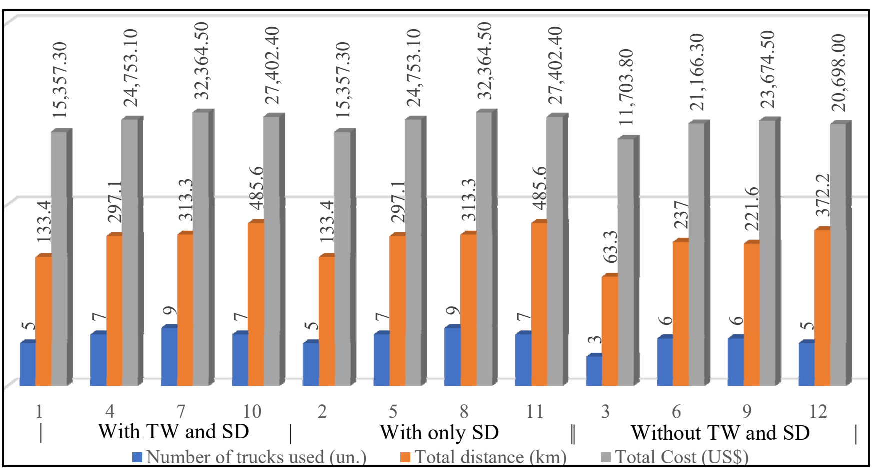


Based on Table 4 and Figure 15, it can be noticed that scenarios 1, 4, 7 and 10 (which consider TW and SD restrictions) when compared, respectively, with scenarios 2, 5, 8 and 11 (which do not consider TW restrictions) had no difference between their operational results, showing that the there is no impact of the TW restrictions for the scenarios assessed. One possible explanation for these results is that we are assessing an urban centre, and, under these circumstances, the distance and the travel time between fuel stations and the distribution base are small and, therefore, not impacting the trucks to arrive in the period established by the fuel stations to be visited.

Table 4 Operational results found by the proposed algorithm

Restriction	*Scenario*	*Number of trucks used (un.)*				*Total distance (km)*	*Fixed cost (US$)*	*Variable cost (US$)*	*Total cost (US$)*
		TR	*BI*	*VA*	*Total*				
With TW and SD restrictions	1	2	2	1	5	133.4	15,000.0	357.3	15,357.3
	4	1	4	2	7	297.1	24,000.0	753.1	24,753.1
	7	1	5	3	9	313.3	31,500.0	864.5	32,364.5
	10	0	4	3	7	485.6	26,000.0	1,402.4	27,402.4
With only SD restrictions	2	2	2	1	5	133.4	15,000.0	357.3	15,357.3
	5	1	4	2	7	297.1	24,000.0	753.1	24,753.1
	8	1	5	3	9	313.3	31,500.0	864.5	32,364.5
	11	0	4	3	7	485.6	26,000.0	1,402.4	27,402.4
Without TR and SD restrictions	3	0	1	2	3	63.3	11,500.0	203.8	11,703.8
	6	1	3	2	6	237.0	20,500.0	666.3	21,166.3
	9	0	2	4	6	221.6	23,000.0	674.5	23,674.5
	12	0	1	4	5	372.2	19,500.0	1,198.0	20,698.0

Figure 15 Operational results of the scenarios by the type of restriction imposed (see online version for colours)



Comparing scenarios 3, 6, 9 and 12 (without TW and SD restrictions), respectively, with scenarios 1, 4, 7 and 10 (with TW and SD restrictions) and, respectively, with scenarios 2, 5, 8 and 11 (with only SD restrictions), it is possible to note that the number of trucks used, the total distance travelled, the fixed cost, the variable cost and the total cost are reduced. The reason of this reduction is due to the use of bigger trucks when no SD restrictions were considered, and, thus, more fuel stations that are located nearby can be visited by one big truck, leading to a reduction of all operational results. Results indicate that larger trucks, even when SD restrictions are respected, bring better results. Therefore, for the scenarios tested, SD restrictions were the one that impacts more the fuel distribution plan in the context of urban logistics.

All trucks in all scenarios did just one trip, a possible reason for this is the time spent to travel in urban centres, plus the time to return the distribution base, plus the time to load the truck to a second trip overcome the maximum travel time limit. However, for the future expansion plans of the distribution company, which considers working in two shifts, day and night, the maximum time to travel may not be a limitation to do more than one trip and, therefore, multi-trip plan might worthy for them.

The main hypothesis raised in this paper was: planning considering a global distribution region (plan the trucks trips considering just one distribution region) reaches better results than planning considering several small regions? To answer this main hypothesis, we assessed three KPIs:

1 total cost

2 total number of trucks used

3 total distance travelled by the trucks to meet the demand of all fuel stations located at RMG considering TW and SD restrictions.

First, we put together the values of the three KPIs for scenarios 1, 4, 7 and 10 in Table 5. The sum of them represents the KPIs for the global distribution region (sum of the five cities of RMG) planned separately (last line of Table 5). After Table 5 is presented, it is possible to show Table 6, which was built to assess the KPIs for both strategies:

1 planning considering several small regions [same values of the line total (all four scenarios) of Table 5]

2 planning considering a global distribution region (only scenario 13, which represents the entire RMG region, considering a single global distribution region with the five cities).

Figure 16 summarises the results for the three KPIs.

Table 5 Scenarios representing the five cities of RMG region considering TW and SD restrictions

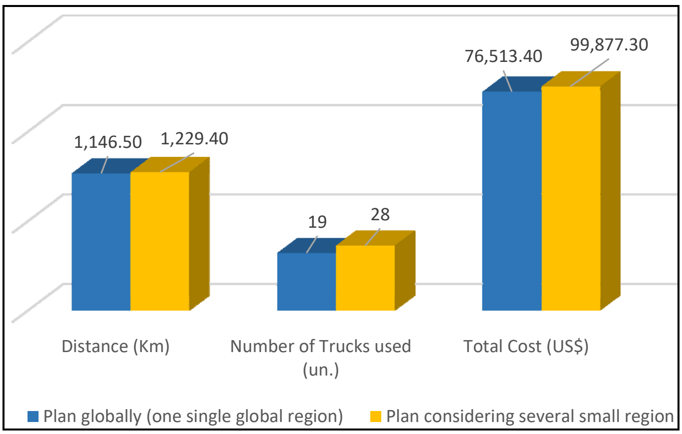
Scenario	*Total cost (US$)*	*Distance (Km)*	*Number of trucks used (un.)*			
			TR	*BI*	*VA*	*Total*
1	15,357.30	133.4	2	2	1	5
4	24,753.10	297.1	1	4	2	7
7	32,364.50	313.3	1	5	3	9
10	27,402.40	485.6	0	4	3	7
Total (all 4 scenarios)	99,877.30	1,229.4	4	15	9	28

Table 6 and Figure 16 show that the hypothesis raised in this paper, planning considering a global distribution region (one single global region), is better than planning considering several small regions. It can be seen that the strategy to plan considering a global distribution region reduces per day the number of trucks used per day (nine trucks less) and the total distance travelled by the fleet per day (82.9 km less), leading to a total transportation cost reduction of US$23,363.90. Therefore, in a year basis, considering 20 working days a month, it is possible to estimate a reduction of US$5,607,336.00 in the total transportation cost of the company.

Table 6 Comparison of the strategies

Strategy	*Total cost (US$)*	*Distance (Km)*	*Number of trucks used (un.)*			
			TR	*BI*	*VA*	*Total*
Planning considering several small regions	99,877.3	1,229.4	4	15	9	28
Planning considering a global distribution region (one single global region) – scenario 13	76,513.4	1,146.5	0	6	13	19
Reductions achieved comparing planning considering a global distribution region over planning considering several small regions	–23,363.90	–82.90	–4.00	–9.00	4.00	–9.00

Figure 16 KPIs assessed to compare the two strategies (see online version for colours)



This reduction in the number of trucks is possible because the merging of all the regions allows fuel stations that are in different regions, but close to each other, to be visited by the same truck. This allows one truck to visit fuel stations that are close to each other instead of having two trucks visiting them. This can be seen when we assessed the number of trucks by model. While in the strategy to plan considering a global distribution region it was used zero TR (capacity of 15.0 m^3), six BI (capacity of 25.0 m^3) and 13 VA (capacity of 35.0 m^3), in the strategy to plan considering several small regions it was used four TR, 15 BI and nine VA. Then it can be said that the strategy to plan considering a global distribution region tends to choose bigger trucks, which visit more fuel stations, respecting SD restrictions, leading to a reduction of the total number of trucks and raising the occupancy of the truck.

It is worthy to say that the proposed algorithm can be used by any fuel distribution company in the whole world. The proposed algorithm can solve real problems in a fast way, less than 77 seconds, allowing the distribution company to plan several times, as much as they want, adjusting the plan to its necessity. It is interesting to note that the proposed algorithm can be used by other distribution companies which use multi-compartment vehicles, like the ones operating in the solid recyclable waste reverse logistics.

7 Conclusions

This paper assessed the problem faced by a fuel distribution company in Brazilian urban centres to delivery different fuels to meet the demand of the fuel stations, considering restrictions on the delivery period and the model of the truck. The distribution company uses different models of multi-compartment trucks which can travel one or more trips daily and certain models of trucks cannot travel to some regions of the city, either by legal restrictions or by operational restrictions of the fuel stations. Each fuel station can establish a period that the trucks can arrive and unload. The different compartments of the trucks can be loaded with different fuels, but one compartment must be filled with just one fuel at the same time to avoid contaminations.

To solve this problem, this paper proposes a mathematical model, not yet proposed in the published literature, which was named as FRP with multi-compartment, multi-trip, TWs, SD and heterogeneous fleet. This problem is NP-hard and to find an optimal solution by a solver is very hard. Therefore, we also proposed an algorithm inspired by the metaheuristic SA, which differs from the original SA by the number of movements needed to find new neighbourhoods, in the original SA there is one movement and in our algorithm, we proposed five movements.

The fuel distribution company currently uses the strategy in which it divides the distribution region in smaller regions. Then, for each smaller region, they plan the distribution. The main hypothesis raised in this paper was: planning considering a global distribution region (plan the trucks trips considering just one distribution region) reaches better results than planning considering several small regions? To answer this question, we sum scenarios 1, 4, 7 and 10, which together represent the global distribution region (sum of the five cities of RMG) and compared with scenario 13, which represents the entire RMG region (considering a single global distribution region with the five cities). We assessed three KPI:

1 total cost

2 total number of trucks used

3 total distance travelled by the trucks to meet the demand of all fuel stations located at RMG considering TW and SD restrictions.

Results showed that the main hypothesis raised in this paper, planning considering a global distribution region, is better than planning considering several small regions. Results showed that the strategy to plan considering a global distribution region reduces the number of trucks used per day (nine trucks less) and the total distance travelled by the fleet per day (82.9 km less), leading to a total daily transportation cost reduction of US$23,363.90. In a year basis, it is possible to estimate a reduction of US$5,607,336.00 in the total transportation cost of the company.

This reduction in the number of trucks was possible because the merging of all the regions allows fuel stations that are in different regions, but close to each other, to be visited by the same truck. This allows one truck to visit fuel stations that are close to each other instead of having two trucks visiting them. It can be said that the strategy to plan considering a global distribution region tends to choose bigger trucks, which visit more fuel stations, respecting SD restrictions, leading to a reduction of the total number of trucks and raising the occupancy of the truck.

All trucks in all scenarios did just one trip, a possible reason for this is the time spent to travel in urban centres, plus the time to return the distribution base, plus the time to load the truck to a second trip overcomes the maximum travel time limit. However, for the future expansion plans of the distribution company, which considers working in two shifts, day and night, the maximum time to travel may not be a limitation to do more than one trip and, therefore, multi-trip planning might be worthy for them.

Results indicate that TW restrictions did not impact the number of trucks used and the distance travelled, maybe this happened because the distribution occurs in an urban centre, where the distance between the fuel stations is small and the travelling time is also short. In the other hand, SD restrictions strongly impact the results in the context of urban logistics, forcing the company to use more trucks, raising its transportation cost.

We evaluate the performance of the CPLEX to solve the proposed mathematical and the performance of the proposed algorithm. Second, we assessed operational results regarding to the impact of the restrictions on the delivery period (called TW restrictions) and the model of the truck (called SD restrictions). The proposed algorithm found solutions equal to the ones found by CPLEX and in three cases, found better solutions. CPLEX run for 24 hours to find a solution, even if it was not the optimal solution, while the proposed algorithm run, in the worst case, for 77 seconds. The proposed algorithm proved to be stable, after 10 runs, it showed deviations lower than 0.003% and in just one case, it had deviations lower than 0.367%.

Lastly, it is worthy of mention that the proposed algorithm can be used by any fuel distribution company in the whole world. The proposed algorithm can solve real problems in a fast way, less than 77 seconds, allowing the distribution company to plan several times, as much as they want, adjusting the plan to its necessity. It is interesting to note that the proposed algorithm can be used by other distribution companies which use multi-compartment vehicles, like the ones operating in the solid recyclable waste reverse logistics.

As managerial implications, this paper shows that, in urban areas, to do several trips in a working shift is not an interesting approach, because urban areas are usually far from the distribution base and the trucks face a traffic jam in the cities. If the fuel stations ask to establish a period to be visited, this causes little impact to the distribution plan. In the other hand, the trucks' limitation to visit some fuel stations causes high impact to the distribution plan, raising the distribution cost. The last managerial implication deals with the distribution area. By the results, managers shall change the actual strategy to distribute using several small areas to a single global area.

The main limitation of our paper is that we did not consider a specific time for the drivers' lunch, because the drivers use any time stopped to unload at the fuel station to eat, and this would be very hard to be put in the mathematical model or in the metaheuristic.

It is suggested for future research to consider two situations which are not faced by the studied company in this paper:

1 the distribution company has more than one distribution base

2 using the available fleet and considering all the constraints, it might happen that some fuel stations cannot be visited, and thus, must be chosen which fuel station will be visited and which ones will not be visited.

References

Abdelaziz, F.B., Roucairol, C. and Bacha, C. (2002) 'Deliveries of liquid fuels to SNDP fuel stations using trucks with multiple compartments', in *2002 IEEE International Conference on Systems, Man and Cybernetics*, IEEE, October, Vol. 1, pp.478–483.

Abdulkader, M.M., Gajpal, Y. and ElMekkawy, T.Y. (2015) 'Hybridized ant colony algorithm for the multi compartment truck routing problem', *Applied Soft Computing*, Vol. 37, pp.196–203.

Al-Hinai, N. and Triki, C. (2020) 'A two-level evolutionary algorithm for solving the petrol station replenishment problem with periodicity constraints and service choice', *Annals of Operations Research*, Vol. 286, No. 1, pp.325–350.

ANP (2020) *Anuário Estatístico Brasileiro do Petróleo, Gás Natural e Biocombustíveis*.

Benantar, A., Ouafi, R. and Boukachour, J. (2016) 'A petrol station replenishment problem: new variant and formulation', *Logistics Research*, Vol. 9, No. 1, p.6.

Benantar, A., Ouafi, R. and Boukachour, J. (2020) 'An improved Tabu search algorithm for the petrol-station replenishment problem with adjustable demands', *INFOR: Information Systems and Operational Research*, Vol. 58, No. 1, pp.17–37.

Braekers, K., Ramaekers, K. and Van Nieuwenhuyse, I. (2016) 'The truck routing problem: state of the art classification and review', *Computers & Industrial Engineering*, Vol. 99, pp.300–313.

Carotenuto, P., Giordani, S. and Celani, D. (2017) 'Planning retail distribution of fuel oils', *Transportation Research Procedia*, Vol. 27, pp.484–491.

Carotenuto, P., Giordani, S., Massari, S. and Vagaggini, F. (2016) 'A multi-depot periodic vehicle routing model for petrol station replenishment', in *Advanced Concepts, Methodologies and Technologies for Transportation and Logistics*, pp.421–437, Springer, Cham.

Chajakis, E.D. and Guignard, M. (2003) 'Scheduling deliveries in trucks with multiple compartments', *Journal of Global Optimization*, Vol. 26, No. 1, pp.43–78.

Chowmali, W. and Sukto, S. (2020) 'A novel two-phase approach for solving the multi-compartment vehicle routing problem with a heterogeneous fleet of vehicles: a case study on fuel delivery', *Decision Science Letters*, Vol. 9, No. 1, pp.77–90.

CONTRAN (2006) *Resolução CONTRAN No. 210 de 13/11/2006* [online] https://www.legisweb.com.br/legislacao/?id=104243 (accessed 17 October 2019).

Cornillier, F., Boctor, F.F., Laporte, G. and Renaud, J. (2008) 'A heuristic for the multi-period petrol station replenishment problem', *European Journal of Operational Research*, Vol. 191, No. 2, pp.295–305.

El Fallahi, A., Prins, C. and Calvo, R.W. (2008) 'A memetic algorithm and a Tabu search for the multi-compartment truck routing problem', *Computers & Operations Research*, Vol. 35, No. 5, pp.1725–1741.

Henke, T., Speranza, M.G. and Wäscher, G. (2015) 'The multi-compartment truck routing problem with flexible compartment sizes', *European Journal of Operational Research*, Vol. 246, No. 3, pp.730–743.

IBM (2019) *CPLEX Optimization Studio V12.8* [online] https://www.ibm.com/support/pages/cplex-optimization-studio-v128 (accessed 20 December 2019).

Lahyani, R., Coelho, L.C., Khemakhem, M., Laporte, G. and Semet, F. (2015) 'A multi-compartment truck routing problem arising in the collection of olive oil in Tunisia', *Omega*, Vol. 51, pp.1–10.

Lin, C., Choy, K.L., Ho, G.T., Chung, S.H. and Lam, H.Y. (2014) 'Survey of green truck routing problem: past and future trends', *Expert Systems with Applications*, Vol. 41, No. 4, pp.1118–1138.

Muyldermans, L. and Pang, G. (2010) 'On the benefits of co-collection: experiments with a multi-compartment truck routing algorithm', *European Journal of Operational Research*, Vol. 206, No. 1, pp.93–103.

Reed, M., Yiannakou, A. and Evering, R. (2014) 'An ant colony algorithm for the multi-compartment truck routing problem', *Applied Soft Computing*, Vol. 15, pp.169–176.

Shell (2019) *Sobre a Shell* [online] http://www.shell.com.br/ (accessed 18 October 2019).

Triki, C. and Al-Hinai, N. (2016) 'Optimisation techniques for planning the petrol replenishment to retail stations over a multi-period horizon', *International Journal of Operational Research*, Vol. 27, Nos. 1–2, pp.341–355.

Villegas, P.A. and Albornoz, V.M. (2016) 'Large scale petrol station replenishment problem with time windows: a real case study', in *ICORES*, pp.408–415.

Wang, L., Kinable, J. and van Woensel, T. (2020) 'The fuel replenishment problem: a split-delivery multi-compartment vehicle routing problem with multiple trips', *Computers & Operations Research*, Vol. 118, p.104904.