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A modified method for solving the unbalanced TP

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Abstract: Most of the methods suggested for the unbalanced transportation problems in the literature are based on first adding a dummy source/destination with zero cost to make it a balance transportation problem to then obtain the basic feasible solution (BFS). The present paper suggests a modified algorithm for finding a BFS to an unbalanced transportation problem through which we get the optimum solution without adding the dummy source/destination. The method is presented in an algorithmic form and implemented on several sets of input data to test the performance and effectiveness of the algorithm. A comparison is also made with the existing approach and it is found that the suggested algorithm shows better performance.

Keywords: unbalanced transportation problem; UTP; Vogel's approximation method; VAM; initial basic feasible solution; IBFS; basic feasible solution; BFS; optimal cost.

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1 Introduction

Transportation problem is a particular class of linear programming, which is associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another. The study of optimal transportation and allocation of resources is, transportation theory or transport theory (Gillett, 2000). The French Mathematician Gaspard Monge formalised this problem in 1781. In fact, Monge formulated it and solved it by geometrical means. Major advances were made in this field during World War II by the Soviet Mathematician and Economist. In 1941, Soviet Mathematician and Economist, Leonid Vitaliyevich Kantorovich was formulated this. He is known for his theory and development of techniques for the optimal allocation of resources. He is regarded as the founder of linear programming. He was the winner of the Stalin Prize in 1949 and the Nobel Memorial Prize in Economic Sciences in 1975. Consequently, the problem as it is stated is sometimes known as the Monge-Kantorovich transportation problem. The transportation problem itself was first formulated by Hitchcock (1941), and developed the basic transportation problem; however, it could be solved for optimally as answers to the complex business problem only in 1951, when Dantzig (1963) applied the concept of Linear programming in solving the transportation model. The linear programming formulation of the transportation problem is explained by Cooper and Henderson (1953) and is also known as the Hitchcock-Koopmans transportation problem.

The transportation problem represents a particular type of linear programming problem (LPP) used for allocating resources in an optimal way; it is a highly useful tool for managers and supply chain engineers for optimising costs. Transportation plays a leading role in the working of any economic system. It not only helps in increasing proficiency in communication for better managerial control but also acts as a restorative to the process of ongoing and emerging economic activities by moving raw materials, finished products, and people from one place to another within a reasonable period of time. This emphasises the need for a systems approach to solving the problems of transportation. Applications of transportation exists in real life everywhere and economic allocation of land is a significant action in agricultural development in this area. Suvarna and Hebbar (2020) deals the allocation of land for the various crops by the farmer to maximise their income. For this analysis the factors like market price, cost of crops and weather conditions on yield were considered for maximising the income. In one of the other applications of transportation Cantillo et al. (2020) discussed about a complex real-world problem of dispatch planning and cargo loading for highly irregular products with a heterogeneous fleet of trucks. For the solution the researcher used the sequential optimisation approach. The approach focuses on the case of goods with 'low-density values', highly diverse with large travel distances.

2 Review literature

The transportation problem is famous in operations research due to its wide applications in different walks of life. Many solution procedures have been developed in the literature for solving balanced transportation problem and unbalanced transportation problem (UTP). Gamal et al. (2020) proposed the constructive heuristic method to solve location-allocation problems by using Euclidean-distance metric under the consideration that number of users is much larger than the number of facilities. Geetha and Anandhi (2018) proposed the standard deviation method (SDM) for finding an initial basic feasible solution (IBFS) for an UTP. Industries require planning in transporting their products from production centres to the users end with minimal transporting cost to maximise profit. This process is known as Transportation Problem which is used to analyse and minimise transportation cost. Development is a continuous and endless process to find the best among the bests. Keeping in view of these factors Ahmed et al. (2016) proposed an algorithm called 'incessant allocation method' to obtain IBFS. In modern times, due to globalisation and industrial recession, more and more transportation problems turn out to be UTPs. It means that either supply is more, or demand is less or vice-a-versa. Many researchers (Girmay and Sharma, 2013; Kulkarni and Datat, 2010) have been solved the UTP by defining the different solution approach. Transportation problem is an extremely important problem faced by any government, and in most practical situations it is imbalanced in the sense that the total supply is less than the total demand. Moreover, usual transportation method maximises only single objective, the transportation cost which is not always the case in real world situations (Hitchcock, 1941). Mukhopadhyay and Singh (2000) proposed the compromise solution using goal programming technique for finding the IBFS of the UTPs. Khanizad and Montazer (2021) studies about the optimal allocation of human resources. This study presented a model of interaction between intelligent agents to solve the problem. Organisational units were considered agents in this analysis, who exploited human resources and negotiation for job delivery to discover their benefits. Transportation problems can also be solved by using different soft computing techniques. One of them is fuzzy soft computing technique. In a traditional transportation problem, supply, demand and cost are certainly crisp numbers. Therefore, in this situation, the decision maker can estimate transportation costs accurately. In contrast, real-world transport problems are accompanied by an uncertain amount of cost due to various factors such as fuel, traffic jams, weather variations in mountainous areas, etc. In such situations, the decision maker may not be able to accurately predict transportation costs. This situation will force the decision-maker to hesitate. To counter the uncertainties Kumar (2020) has designed a transport problem in which supply, demands are crisp numbers and cost is an intuitionistic fuzzy number. To deal with uncertainty and hesitation in transport problems, the author proposed the intuitionistic fuzzy zero-point method to find the optimal solution. Also, Iskander (2020) proposed a new approach for solving fuzzy capacitated location problems. For the solution of this approach both the capacity and the demand constraints are considered fuzzy while the objective function is not fuzzy. The models which are in the form of mixed zero-one nonlinear programs are transformed to their equivalent linear ones.

Hospitals, health centre and civic health interventions can be very different from other work environments. Healthcare systems are complex and there are many things we need to know about it such as hospital management systems, patient care, insurance, healthcare facility location, healthcare providers and legal issues, etc. Healthcare facility location decisions are of great importance due to their impact on the direct and social costs of wellbeing of people in an area. Ouyang et al. (2020) presents three mathematical models that follow a grid-based location approach and consider current and future demands in providing optimal location-allocation decisions. Luis and Imran (2019) Investigated, capacitive planar multi-facility location-allocation problem. For the solution of the model the number of facilities located in it is specified and each lacks capacity is analysed. It has a two-step method to deal with the problem, where in the first step a technique uses constant space in discrete cells, and in step 2, a variable neighbourhood search to improve the quality of the solution obtained by the previous step (VNS) has been implemented.

3 Definitions

3.1 Transportation problem

The transportation problem is a special type of LPP where objective is to determine the number of units that are to be transported from source *i* to destination *j*, so that the total transportation cost is to be minimised. The transportation problem can be described by using a linear programming mathematical model and usually its mathematical formulation is given as follows:

$$Minimise\ z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

subject to,

$$\sum_{j=1}^{n} X_{ij} = A_i \quad \text{for } i = 1, 2, 3, \ldots, m \tag{1}$$

$$\sum_{i=1}^{m} X_{ij} = B_j \quad \text{for } i = 1, 2, 3, \ldots, n \tag{2}$$

where $Xij \geq 0$.

Normally there are two types of transportation problem, one is balanced transportation problem, i.e., when $\sum_{i=1}^{m} A_i = \sum_{j=1}^{n} B_j$ and other is UTP, i.e., when $\sum_{i=1}^{m} A_i \neq \sum_{j=1}^{n} B_j$. The two sets of constraint will be consistent if these are equal. This is necessary as well as sufficient condition for a transportation problem to have a feasible solution. In transportation problem, the objective is to find X_{ij} in order to minimise Z, where some common notations are as follow:

m number of sources

n number of destinations

A_i supply units available at each source point ($i = 1, 2, 3, \ldots, m$)

B_j demand units required at each destination point ($j = 1, 2, 3, \ldots, n$)

C_{ij} the unit of transportation cost for transporting the units from source i to destination j.

3.2 *Feasible solution*

A feasible solution to a transportation problem is a set of non-negative allocations, X_{ij} that satisfies the row and column restrictions.

3.3 *Basic feasible solution*

A feasible solution to a transportation problem is said to be basic feasible solution (BFS) if it contains not more than $m + n - 1$ non-negative allocations, where m is the number of rows and n is the number of columns of the transportation problem.

3.4 *Initial basic presumable solution*

A feasible solution to a transportation problem is said to be an initial basic presumable solution (IBPS), while by adding the arbitrary supply/demand to make it a balanced problem, and must contains not more than $(m + n - 1)$ non-negative allocations.

3.5 *Optimal solution*

A feasible solution, not necessarily basic that minimises the transportation cost is called an optimal solution.

3.6 *Degenerate BFS*

A BFS in which the total number of non-negative allocations is less than $(m + n - 1)$ is called degenerate BFS.

4 Assumptions

- The unit transportation cost from the origin to a destination is known and certain.
- The unit cost is independent of the number of goods transported.
- The objective is to minimise the total transportation cost.
- The total quantity available at all the sources are not equal to the total quantity required by all the destinations.

5 Objective

The transport problem is a special type of LPP that connects origin/factories to destination/customers by shipping the commodities. The purpose of the transportation problem is to determine the shipping cost along with the schedule by which the total shipping or transportation cost is to be minimised while satisfying the supply and demand limits. Consider the UTP which consist of m sources and n destinations. The unit shipping cost from the i^{th} origin to j^{th} destination is known, shipped quantity from i^{th} origin to j^{th} destination and supply/demand availability at i^{th} origin/j^{th} destination is known and mentioned in UTP matrix *UTPM*(,). The objective is to determine the optimal transportation cost.

6 Proposed method

Consider the UTP, i.e., for which $\sum_{i=1}^{m} A_i \neq \sum_{j=1}^{n} B_j$. Now compute η or δ by using equation (3) given below.

$$\eta \text{ or } \delta = \left| \sum_{i=1}^{m} A_i - \sum_{j=1}^{n} B_j \right| \tag{3}$$

Now increase the supply/demand by δ/η as follows:

1 If $\sum_{i=1}^{m} A_i < \sum_{j=1}^{n} B_j$ then identify minimum A_i which occurs at the k^{th} origin and corresponding k^{th} origin supply will be increased by δ units by using equation (3.1)

$$A_i' = A_i + \delta \tag{3.1}$$

If tie occurred then increase supply by δ in equal integer proportions by using equation (3.2)

$$\delta = \Delta\delta_1 + \Delta\delta_2 + \ldots \tag{3.2}$$

2 If $\sum_{i=1}^{m} A_i > \sum_{j=1}^{n} B_j$ then identify minimum B_j which occurs at the r^{th} destination and corresponding r^{th} destination demand will be increased by η units by using equation (3.3)

$$B_j' = B_j + \eta \tag{3.3}$$

If tie occurred then increase demand by η in equal integer proportions by using equation (3.4)

$$\eta = \Delta\eta_1 + \Delta\eta_2 + \ldots \tag{3.4}$$

After getting the balanced transportation problem obtain its IBFS by using Vogel's approximation method (VAM) (Taha, 1968). For finding initial transportation cost for the given UTP, find the total transportation cost and subtract it from arbitrary amount δ

supply/η demand $[(A_i'/B_j')]$ of the allocated value which belongs to corresponding to this cell. This gives the IBFS. Using modified distribution (MODI) method by Taha (1968) for optimality test we find the optimum solution that has been observed that through the modified algorithm our IBFS is optimum as well.

7 Algorithm

The proposed algorithm gives the IBFS for both types of unbalance transportation problem, i.e., less supply/less demand.

Step 1 Set the cost matrix $C_{ij}(,)$ of given UTP.

Step 2 Find the sum of supply/demand and check for $\sum_{i=1}^{m} A_i \neq \sum_{j=1}^{n} B_j$.

Step 3 Calculate the required supply (δ)/demand (η) to convert it into balanced transportation problem as follows:

- Step 3.1 If $\sum_{i=1}^{m} A_i < \sum_{j=1}^{n} B_j$ then identify minimum A_i which occurs at the k^{th} origin and corresponding k^{th} origin supply will be increased by δ units using equation (3.1) or if there is tie then increase δ by using equation (3.2).
- Step 3.2 If $\sum_{i=1}^{m} A_i > \sum_{j=1}^{n} B_j$ then identify minimum B_j which occurs at the r^{th} destination and corresponding r^{th} destination demand will be increased by η units using equation (3.3) or if there is tie then increase η by using equation (3.4).

Step 4 Now find the IBFS by using VAM.

- Step 4.1 Find the cells having smallest and next to smallest cost in each row and write the difference (called penalty) along the side of the table in row penalty.
- Step 4.2 Find the cells having smallest and next to smallest cost in each column and write the difference (called penalty) along the side of the table in each column penalty.
- Step 4.3 Select the row or column with the maximum penalty and find cell that has least cost in selected row or column. Allocate as much as possible in this cell. If there is a tie in the values of penalties, then select the cell where maximum allocation can be possible.
- Step 4.4 Adjust the supply and demand and cross out (strike out) the satisfied row or column.
- Step 4.5 Repeat this step until all supply and demand values become zero.

Step 5 Check the number of allocations. Is it equal to $(m + n - 1)$? If yes, find the IBFS by using the next step as the case received in step 3. If it is not equal to $(m + n - 1)$ go to step 7 for degenerate solution.

Step 5.1 If we have step 3.1 then for finding arbitrary amount of cost δX_{ij}, select the allocated value in largest supply (A_i') as follows:

a If there are only one allocation in A_i' then take the cost of that cell δX_{ij} and subtract it from the initial basic presumable solution to get IBFS. This IBFS is obtained by equation (4.1).

$$IBFS = IBPS - \delta X_{ij} \tag{4.1}$$

b If there are more than one allocation in A_i' then calculate the cost δ by allocated cell by using equation (4.2).

$$\delta = \delta_1 \left(Max_1 X_{ij} \right) + \delta_2 \left(Max_2 X_{ij} \right) + \ldots \tag{4.2}$$

Now then subtract it from the initial basic presumable solution to get IBFS given by equation (4.3).

$$IBFS = IBPS - \left\{ \delta_1 \left(Max_1 X_{ij} \right) + \delta_2 \left(Max_2 X_{ij} \right) + \ldots \right\} \tag{4.3}$$

c If there are more than one allocation in A_i' and there be tie in the allocated cell with $MaxX_{ij}$ then choose allocated cell having maximum supply and then move to the next allocated cell and subtract it from the initial basic presumable solution to get IBFS by using equation (4.4).

$$IBFS = IBPS - \left\{ \delta_{Max_1 A_i'} \left(Max_1 X_{ij} \right) + \delta_{Max_2 A_i'} \left(Max_1 X_{ij} \right) + \ldots \right\} \tag{4.4}$$

Step 5.2 If in step 3, we have case 2, i.e., 3.2 then for finding arbitrary amount of cost ηX_{ij}, select the allocated value in largest demand (B_j') as follows:

a If there are only one allocation in B_j' then take the cost of that cell ηX_{ij} and subtract it from the initial basic presumable solution to get IBFS. This IBFS is obtained by equation (4.5).

$$IBFS = IBPS - \eta X_{ij} \tag{4.5}$$

b If there are more than one allocation in B_j' then calculate the cost η by allocated cell by using equation (4.6).

$$\eta_1 \left(Max_1 X_{ij} \right) + \eta_2 \left(Max_2 X_{ij} \right) + \ldots \tag{4.6}$$

Now then subtract it from the initial basic presumable solution to get IBFS given by equation (4.7).

$$IBFS = IBPS - \left\{ \eta_1 \left(Max_1 X_{ij} \right) + \eta_2 \left(Max_2 X_{ij} \right) + \ldots \right\} \tag{4.7}$$

c If there are more than one allocation in B'_j and there be tie in the allocated cell with $MaxX_{ij}$ then choose allocated cell having maximum demand and then move to the next allocated cell and subtract it from the initial basic presumable solution to get IBFS by using equation (4.8).

$$IBFS = IBPS - \left\{ \eta_{Max_1 B'_j} \left(Max_1 X_{ij} \right) + \eta_{Max_2 B'_j} \left(Max_1 X_{ij} \right) + \ldots \right\} \quad (4.8)$$

Step 6 Test the procedure for optimality to find whether making an allocation in it reduces the total transportation cost. For this modified distribution method (MODI) is used which proceed as follows:

Step 6.1 Setup a cost matrix containing the unit costs associated with the cells for which allocations have been made.

Step 6.2 Set dual variables corresponding to the supply A_i and demand B_j so that there will be $m + n$ dual variables, i.e., $\alpha_i + \beta_j$. These dual variables are obtained for all such occupied cells by using equation (5) such that

$$\alpha_i + \beta_j = C_{ij} \quad (5)$$

Step 6.3 Calculate implicit cost $\alpha_i + \beta_j$ for each unallocated cell and then find the difference from the actual cost C_{ij} for each unallocated cell by using equation (6)

$$C'_{ij} = C_{ij} - \left(\alpha_i + \beta_j \right) \quad (6)$$

Step 6.4 Test

If all $C'_{ij} \geq 0$, then solution is optimum and alternative allocation exist with same optimum solution.

If any $C'_{ij} < 0$, then the solution is not optimal.

Step 6.5 Repeat the steps 6.1 to 6.4 till to get optimum solution.

Step 7 Find IBFS for degenerate allocation according to steps 3.1 or 3.2 as the case obtained.

Step 8 Choose the cell having least unit cost for arbitrary allocation (if there is tie then select randomly).

Step 9 Go to step 6 for optimality test and repeat the procedure till get optimum solution.

Step 10 Stop.

8 Flow chart

Figure 1 Flowchart for finding IBFS

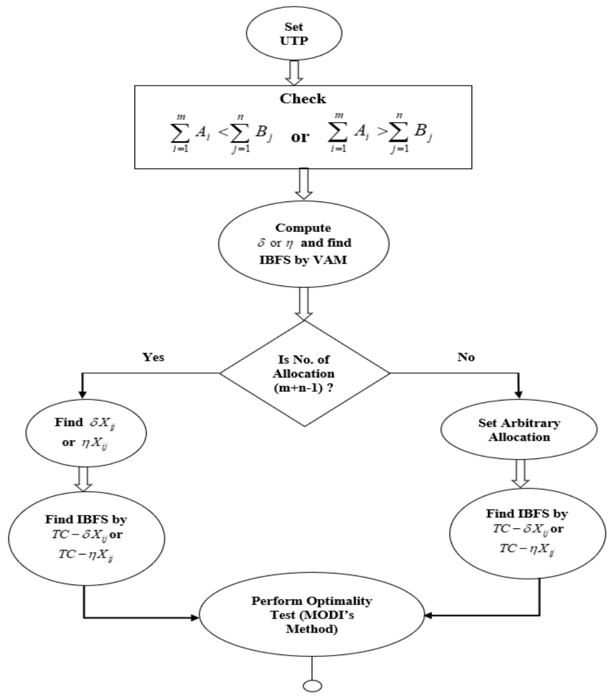
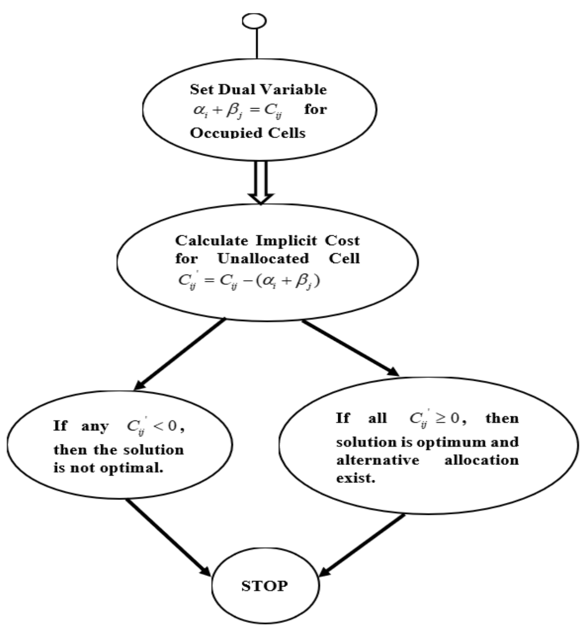


Figure 2 Flowchart for representing optimality test



9 Implementation

Example 1: Let us consider the following UTP:

	D_1	D_2	D_3	A_i
F_1	3	4	6	100
F_2	7	3	8	80
F_3	6	4	5	90
F_4	7	5	2	120
B_j	110	110	60	

1 The cost matrix $C_{ij}(,)$ of given UTP is as,

$UTPM(,) =$

	D_1	D_2	D_3	A_i
F_1	3	4	6	100
F_2	7	3	8	80
F_3	6	4	5	90
F_4	7	5	2	120
B_j	110	110	60	

2 Find the sum of supply and demand as follows:

$$Sum\ of\ supply \sum_{i=1}^{4} A_i = 100 + 80 + 90 + 120 = 390$$

$$Sum\ of\ demand \sum_{j=1}^{3} B_j = 110 + 110 + 60 = 280$$

3 Here $\sum_{i=1}^{4} A_i > \sum_{j=1}^{3} B_j$, i.e., demand will be increased by η units using equation (3.3) such that $B_3' = 110$.

4 Now find the IBFS by using VAM as follows:

	D_1	D_2	D_3	A_i
F_1	$\boxed{100}$ 3	4	6	100
F_2	7	$\boxed{80}$ 3	8	80
F_3	$\boxed{10}$ 6	$\boxed{30}$ 4	$\boxed{50}$ 5	90
F_4	7	5	$\boxed{120}$ 2	120
B_j	110	110	170	390

5 Here number of allocations is equal to $(m + n - 1)$ so that IBFS is obtained by using Step 5.2b. Here more than one allocation in corresponding B_3' therefore IBFS is obtained by using equations (4.6) and (4.7).

$$\eta_1\left(Max_1 X_{33}\right)+\eta_2\left(Max_2 X_{43}\right)=50\times5+60\times2=370\ units$$

Now,

$$IBPS=100\times3+80\times3+10\times6+30\times4+50\times5+120\times2=1,210\ units$$

$$IBFS=IBPS-\left\{\eta_1\left(Max_1 X_{33}\right)+\eta_2\left(Max_2 X_{43}\right)\right\}=1,210-370=840\ units$$

6 Perform optimality test by using modified distribution method as mentioned in steps 6.1 to 6.4.

- Setup a cost matrix containing the unit costs associated with the cells for which allocations have been made.

	v_1	v_2	v_3
u_1	$\boxed{100}$ 3		
u_2		$\boxed{80}$ 3	
u_3	$\boxed{10}$ 6	$\boxed{30}$ 4	$\boxed{50}$ 5
u_4			$\boxed{120}$ 2

- Set dual variables corresponding to the supply A_i and demand B_j.

	$v_1=6$	$v_2=4$	$v_3=5$
$u_1=-3$	$\boxed{100}$ 3		
$u_2=-1$		$\boxed{80}$ 3	
$u_3=0$	$\boxed{10}$ 6	$\boxed{30}$ 4	$\boxed{50}$ 5
$u_4=-3$			$\boxed{120}$ 2

- Calculate implicit cost $\alpha_i + \beta_j$ for each unallocated cell by using equation (5) and then find the difference from the actual cost C_{ij} for each unallocated cell using equation (6).

	$v_1 = 6$	$v_2 = 4$	$v_3 = 5$
$u_1 = -3$	$\boxed{100}$ 3	3	4
$u_2 = -1$	2	$\boxed{80}$ 3	4
$u_3 = 0$	$\boxed{10}$ 6	$\boxed{30}$ 4	$\boxed{50}$ 5
$u_4 = -3$	4	4	$\boxed{120}$ 2

- Here all $C'_{ij} \geq 0$ then solution is optimum and alternative allocation exist with same optimum solution.

7 Here IBFS is the optimum solution and the optimum transportation cost is 840 units.

8 Stop.

Example 2: Consider another UTP:

	D_1	D_2	D_3	D_4	D_5	A_i
F_1	5	8	6	6	3	800
F_2	4	7	7	6	5	500
F_3	8	4	6	6	4	900
B_j	400	400	500	400	800	

1 The cost matrix $C_{ij}(,)$ of given UTP is as,

$UTPM(,) =$

	D_1	D_2	D_3	D_4	D_5	A_i
F_1	5	8	6	6	3	800
F_2	4	7	7	6	5	500
F_3	8	4	6	6	4	900
B_j	400	400	500	400	800	

2 Find the sum of supply and demand as follows:

$$Sum\ of\ supply \sum_{i=1}^{3} A_i = 800 + 500 + 900 = 2,200$$

$$Sum\ of\ demand \sum_{j=1}^{5} B_j = 400 + 400 + 500 + 400 + 800 = 2,500$$

3 Here $\sum_{i=1}^{3} A_i < \sum_{j=1}^{5} B_j$, i.e., supply will be increased by δ units by using equation (3.1) such that $A'_i = 300$.

4 Now find the IBFS by using VAM, as follows:

	D_1	D_2	D_3	D_4	D_5	A_i
F_1	5	8	6	6	$\boxed{800}$ 3	800
F_2	$\boxed{400}$ 4	7	7	$\boxed{400}$ 6	5	800
F_3	8	$\boxed{400}$ 4	$\boxed{500}$ 6	6	4	900
B_j	400	400	500	400	800	

5 Here number of allocations is not equal to ($m + n - 1$) so that IBFS is to be obtained by using equation (4.1).

$$IBFS = IBPS - \delta X_{ij}$$

Now,

$$IBPS = 800 \times 3 + 400 \times 4 + 400 \times 6 + 400 \times 4 + 500 \times 6 = 11,000 \; units$$

$$\{\delta \times (MaxX_{24})\} = 300 \times 6 = 1,800 \; units$$

Therefore

$$IBFS = 11,000 - 1,800 = 9,200 \; units.$$

6 Now choose the cell having least unit cost for arbitrary allocation to make it equal to 7 allocation.

	D_1	D_2	D_3	D_4	D_5	A_i
F_1	5	8	6	6	$\boxed{800}$ 3	800
F_2	$\boxed{400}$ 4	7	7	$\boxed{400}$ 6	$\boxed{\Delta}$ 5	800
F_3	8	$\boxed{400}$ 4	$\boxed{500}$ 6	6	$\boxed{\Delta}$ 4	900
B_j	400	400	500	400	800	

7 Perform optimality test by modified distribution method (MODI) using steps 6.1 to 6.4 as follows:

- Setup a cost matrix containing the unit costs associated with the cells for which allocations have been made.

	v_1	v_2	v_3	v_4	v_5
u_1					$\boxed{800}$ 3
u_2	$\boxed{400}$ 4			$\boxed{400}$ 6	$\boxed{\Delta}$ 5
u_3		$\boxed{400}$ 4	$\boxed{500}$ 6		$\boxed{\Delta}$ 4

- Set dual variables corresponding to the supply A_i and demand B_i.

	$v_1 = -1$	$v_2 = 0$	$v_3 = 2$	$v_4 = 1$	$v_5 = 0$
$u_1 = 3$					$\boxed{800}$ 3
$u_2 = 5$	$\boxed{400}$ 4			$\boxed{400}$ 6	$\boxed{\Delta}$ 5
$u_3 = 4$		$\boxed{400}$ 4	$\boxed{500}$ 6		$\boxed{\Delta}$ 4

- Calculate implicit cost $\alpha_i + \beta_j$ for each unallocated cell by using equation (5) and then find the difference from the actual cost C_{ij} for each unallocated cell using equation (6).

	$v_1 = -1$	$v_2 = 0$	$v_3 = 2$	$v_4 = 1$	$v_5 = 0$
$u_1 = 3$	3	5	1	2	$\boxed{800}$ 3
$u_2 = 5$	$\boxed{400}$ 4	2	0	$\boxed{400}$ 6	$\boxed{\Delta}$ 5
$u_3 = 4$	5	$\boxed{400}$ 4	$\boxed{500}$ 6	1	$\boxed{\Delta}$ 4

- Here all $C'_{ij} \geq 0$ then solution is optimum and alternative allocation exist with same optimum solution.

8 Here IBFS is the optimum solution and the optimum transportation cost is 9,200 units.

9 Stop

10 Comparative analysis

In this paper, the optimum solution to an UTP has been workout and the proposed IBFS discussed here without adding dummy supply or demand. From the example, as illustrated in the implementation section one can conclude that the proposed algorithm takes less calculation and gives best optimum solution as compare to VAM and other methods discussed earlier in the literature. There are number of problems solved by the proposed algorithm and it is recorded that in most of the cases the optimum solution obtained shows better performance. The comparative analysis of the proposed algorithm is given in Table 1.

Table 1 Comparative analysis of IBFS

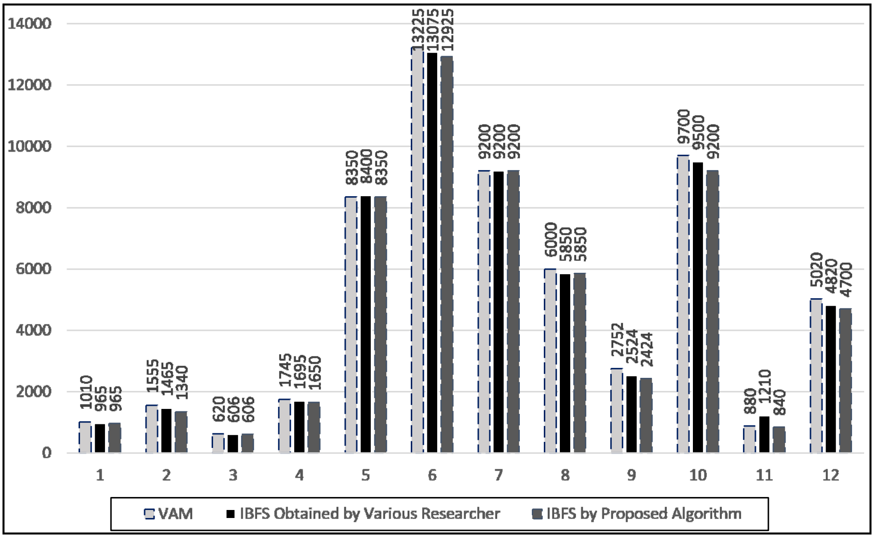
VAM	*IBFS by author's*	*IBFS by proposed algorithm*
1,010	965	965
1,555	1,465	1,340
620	606	606
1,745	1,695	1,650
8,350	8,400	8,350
13,225	13,075	12,925
9,200	9,200	9,200
6,000	5,850	5,850
2,752	2,524	2,424
9,700	9,500	9,200
880	1,210	840
5,020	4,820	4,700

11 Conclusions

The transport costs play the vital role for any industry, where commodities are to be shipped from one place to another place. It is the special case of the linear programming with the motto to provide the optimal transportation cost. Here the proposed algorithm provides an optimal solution directly, (or) a better optimal solution with minimum number of iterations for UTP. Also, through this algorithm we found that most of the time the IBFS is the optimum feasible solution as well. The method developed here ensure a solution which is exceptionally good to find the IBFS of UTP.

Finally, it can be claimed that the proposed algorithm may provide a noteworthy IBFS by ensuring minimum transportation cost. The graphical representation of the IBFS obtained by suggested algorithm and earlier discussed algorithms has been shown in Figure 3. These results are advanced and better in contrast to earlier research findings by other researchers. From the computational analysis here, it appears that 'arbitrary supply and demand' helps in working out the cost by which the initial cost of transportation can be reduced. This estimate suggests that studies using VAM as well as some types of non-dimensional transport costs may be a useful research direction. As a further futuristic research, it may be important to implement this solution method to solve the real-life transportation problem with more constraint and other parameter.

Figure 3 Graphical representation of comparative analysis



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