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A fuzzy random periodic review mixture inventory model with backorder price discount

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Abstract: In this paper, a periodic review inventory model with a mixture of backorders and lost sales is developed under mixed fuzzy random environment. It is assumed that the supplier provides some price discount to control the backorder rate and gives an incentive to the customers to wait for the arrival of their orders rather than take their orders elsewhere. The annual customer demand is considered to be continuous fuzzy random variable following normal distribution. The model is analysed under three scenarios – no price discount, fixed price discount, and controllable price discount. An algorithm is presented to simultaneously determine the optimal values of the review period, the target inventory level, and the backorder price discount, so that the total annual cost is minimised. Numerical examples show that the case of controllable backorder price discount leads to the system incurring lowest operational costs.

Keywords: inventory; periodic review; backorder price discount; continuous fuzzy random variable; normal distribution.

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1 Introduction

In the (R, T) periodic review system, the inventory level is reviewed periodically at regular intervals of time T and at that time an order is placed so that it brings the inventory up to a target level R. In such inventory systems, traditionally it is assumed that, when shortage occurs, the stock-out items are either completely backordered or completely lost. However in reality it is not so. In case of shortage, it is often observed that while some customers wait for their orders to arrive with the arrival of the next batch of orders (backorder), some may be unwilling to do so (lost sales). That is, it is a mixture of backorders and lost sales. However, lost sales not only leads to loss of profit for the supplier, it also causes loss of customer goodwill. Therefore, the supplier may offer the backordered items at a discounted price to increase the backorder rate. This price discount serves as an incentive for the customers to wait and not take their order elsewhere. Obviously a higher price discount from the supplier implies a greater incentive to the customers. But excessive price discount is infeasible and undesirable. Hence, it is subject to control.

Inventory model with a mixture of backorders and lost sales was proposed by Montgomery et al. (1973). Since then this model has been a focal area of research in inventory modelling. Various researchers, viz., Ouyang et al. (1996), Ouyang and Wu (1997), Moon and Chio (1998), Hariga and Ben-Daya (1999), Ouyang and Chung (2001), etc. have investigated inventory models assuming the partial lost sales rate as constant and then explored its optimal solutions in stochastic environment. Pan and Hsiao (2001) considered an inventory model with backorder discounts and variable lead-time. Ouyang et al. (2003) analysed a periodic review inventory model with the impact of backorder price discounts under the stochastic framework where the backorder discount and protection interval are control variables. An integrated inventory model with controllable lead time and backorder discount was described by Pan and Hsiao (2005). Lee (2006) described an optimal investment to reduce lost sales rate in periodic review inventory system. Lin (2008a) developed an periodic review inventory model with optimal review period and backorder rate with controllable lead time. A number of other researchers, viz., Cheng (2009), Huang et al. (2011), Sarkar et al. (2014), Jindal and Solanki (2016), etc. also developed inventory models with backorder rate under stochastic framework. An EOQ model for deteriorating items with time varying shortages under stochastic environment was developed by Tripathi and Uniyal (2015). Castellano et al. (2017) analysed a periodic review policy with quality improvement, setup cost reduction, backorder price discount, and controllable lead time. Ordering policies for non-instantaneous deteriorating items under hybrid partial prepayment, partial trade credit and partial backordering was developed by Lashgari et al. (2018). An inventory model with backorder price discount and stochastic lead time was developed by Kim et al. (2018). A distribution-free model with variable setup cost, backorder price discount and controllable lead time was analysed by Malik and Sarkar (2018). Kurdhi et al. (2020) developed a periodic review inventory system under partially backlogged shortages. Castellano et al. (2020) studied a single vendor-multiple buyer integrated inventory model with controllable lead time and backorders-lost sales mixture. Recently a two-echelon supply chain model with variable backorder was developed by Sarkar and Giri (2021). Also, a partial backlogging inventory model was investigated by Palanivel and Sugany (2021) under stochastic framework.

But most of the existing literature analyses the various backorder price discount inventory models under stochastic framework. However the stochastic models do not take into account any imprecision that may also be present in the real life inventory situation. In these inventory models, the ambiguity present in real life is considered but the imprecision appearing due to vague information is not taken into account. Thus to capture this type of information, fuzzy set theory (Zadeh, 1965) was introduced in the analysis of several inventory models. For instance, a periodic review inventory model involving fuzzy expected demand short and fuzzy backorder rate was developed by Lin (2008b). Vijayan and Kumaran (2008) analysed an inventory models with a mixture of backorders and lost sales under fuzzy cost. Wang (2012) described a continuous review inventory model with a mixture of backorders and lost sales under fuzzy demand. A periodic review inventory model with stock dependent demand, and backorder price discount was investigated by Pal and Chandra (2014). Saga et al. (2017) developed a periodic review integrated inventory model with controllable setup cost, imperfect items, and inspection errors under service level constraint. An inventory model for imperfect quality item, where shortages are allowed and are backlogged was developed in fuzzy environment by Kumar (2018). Fathalizadeh et al. (2019) developed a fuzzy inventory model with partial backordering for deteriorating items. Recently, De and Mahata (2020) analysed a fuzzy inventory supply chain model with partial backordering. However, these models did not take into consideration the inherent randomness of an inventory problem.

In most real life inventory situations, the decision makers have to deal with both imprecision and uncertainty. For instance, in case of new and/or highly seasonal products (winter garments, seasonal fruits, etc.) where historical data is scarce, experts may provide information about the customer demand in terms of linguistic expressions and this information may also vary randomly between the experts. Also in case of market fluctuation, climatic variability, etc. there may be such inventory situations which contain sufficient or even abundant information about customer demand (Khan and Dey, 2017, 2018). This statistical information itself may contain both fuzziness and randomness. In such situations, to quantify both types of uncertainty simultaneously fuzzy random variable acts as a suitable tool. With this view, several researchers, viz., Dutta et al. (2005), Chang et al. (2004), Chang et al. (2006), Lin (2008b), Dey and Chakraborty (2009, 2011, 2012), Zheng and Liu (2011), Panda et al. (2014), Bhuiya and Chakraborty (2015), Dey et al. (2016), Chakraborty and Bhuiya (2017), etc. to name a few, developed inventory models under fuzzy random framework. A periodic review inventory model with controllable lead time and backorder rate in fuzzy-stochastic environment was described by Soni and Joshi (2015). A fuzzy periodic review integrated inventory model involving stochastic demand with partial backordering, and adjustable production rate was studied by Jauhari et al. (2017). A fuzzy random continuous (Q, r, L) inventory model involving controllable back-order rate and variable lead-time with imprecise chance constraint was developed by Chakraborty et al. (2018). Very recently, Bhuiya and Chakraborty (2020) described a distribution-free continuous-review production-inventory model with mixture of backorders and lost sells under fuzzy random environment. But most of the existing fuzzy random inventory models consider the annual customer demand to be a discrete fuzzy random variable of the form $\tilde{D}_i = (\underline{D}i, Di, \overline{D}i)$ with associated probability $p_i,\ i = 1, 2, ..., n$. While this representation is quite suitable to encode fuzzy random demand information for new and/or highly seasonal products where there is scarcity of data, this is not very appropriate otherwise (Khan and Dey, 2017). In order to quantify abundant fuzzy-random information, continuous fuzzy random variable is more suitable. In this vein, Dey and Chakraborty (2011) analysed a continuous review inventory model with continuous fuzzy random variable customer demand following uniform distribution. But it is well established that, compared to the uniform distribution, the normal distribution is better suited to describe stochastic demand information. In the same vein, a normally distributed fuzzy random variable is better suited compared to a uniformly distributed fuzzy random variable to represent fuzzy random demand information. Keeping this in mind, Khan and Dey (2017) developed the fuzzy random periodic review system with the annual customer demand and the lead-time plus one period's demand taken to be normally distributed continuous fuzzy random variables with associted fuzzy probability density function (pdf). Khan and Dey (2018) also developed the continuous review system along the same lines. However, neither of these models allowed partial backordering. But, as stated earlier, it is more realistic to assume a mixture of backorders and lost sales with the supplier trying to control the backorder rate in terms of offering a backorder price discount. That is, the supplier offers the backordered items at a discounted price to encourage the customers to wait for their orders instead of taking the orders elsewhere. The present paper, therefore, makes an attempt to extend existing literature further to include partial backordering with a controllable backorder rate and backorder price discount.

Thus, in this paper, a periodic review (R, T) mixture inventory model with backorder price discount is developed under fuzzy random environment where the annual customer demand is taken to be a normally distributed fuzzy random variable with associated fuzzy pdf. The aim of this model is to determine the optimal values of the review period, the target inventory level and the backorder price discount so that the crisp equivalent of the total expected annual cost is minimised.

The rest of the paper is constructed as follows: Section 2 presents a brief view of the preliminary concepts used to develop the model. The methodology is developed in Section 3. The proposed methodology is illustrated by way of a numerical example in Section 4. The numerical examples are developed under three scenarios – no price discount, fixed price discount and controllable price discount. Some concluding remarks and also scope for future research are made in Section 5.

2 Model development

2.1 Fuzzy random variable and its expectation

Kwakernaak (1978) introduced the concept of a fuzzy random variable. Since then several others have also put forward their versions of the same. For instance, Liu and Liu (2003) described a fuzzy random variable as $\tilde{D}(\omega) = (D(\omega) - \Delta_1, D(\omega), D(\omega) + \Delta_2)$, where $\omega \in \Omega$ and (Ω, B, P) is a probability space. Δ_1 and Δ_2 are the left and right spreads respectively, satisfying $0 < \Delta_1 < D(\omega)$ and $\Delta_2 > 0$, for all $\omega \in \Omega$ and $D(\omega)$ follows some continuous distribution. Also, the expectation of a fuzzy random variable is a unique fuzzy number and is defined by $E\tilde{X}(\omega) = \int_\Omega \tilde{X}(\omega)dP = [\int_\Omega X_\alpha^-(\omega)dP, \int_\Omega X_\alpha^+(\omega)dP]$, $0 \le \alpha \le 1$, and $\omega \in \Omega$. That is, if $\tilde{X}$ is a fuzzy random variable with associated probability density function $\tilde{f}(\tilde{x})$, then the expectation of $\tilde{X}$ is given by $E\tilde{X} = \int_{-\infty}^{\infty} \tilde{x}\tilde{f}(\tilde{x})dx$.

2.2 Notations

The following notations are used in this paper:

T	optimal period of review (control parameter)
R	target inventory level (control parameter)
s	backorder price discount from the supplier (control parameter)
A	reviewing cost plus ordering cost
L	lead-time (constant)
s_0	cost of lost demand per unit
β	backorder rate
β_0	upper bound of the backorder rate
h	unit holding cost of item
m	mean of the normal distribution
σ	standard deviation of the normal distribution
$\tilde{D}$	annual demand (continuous fuzzy random variable)
$\tilde{D}_L$	lead-time demand (continuous fuzzy random variable)
$\tilde{D}_{L+T}$	lead-time plus one period's demand (continuous fuzzy random variable)
$\tilde{f}(\tilde{D})$	fuzzy probability density function of $\tilde{D}$
Δ_1, Δ_2	left and right spread of $\tilde{D}$ respectively
SS	safety stock
$\overline{M}(\tilde{D}_{L+T} - R)^+$	expected shortage.

2.3 Assumptions of the model

- A periodic review (R, T) inventory model is considered.
- Shortages are allowed and assumed to be a mixture of backorder and lost sales. The backorder rate β is variable and is proportional to the price discount s per unit offered by the supplier. The backorder rate is defined as $\beta = \frac{s\beta_0}{s_0}$ where, $0 \le \beta_0 < 1$ and $0 \le s \le s_0$ (Pan and Hsiao, 2001). It is to be noted that, since

the price discount s is connected to the backorder rate β, therefore s is taken to be control parameter in place of β.

- The annual customer demand is taken to be a normally distributed continuous fuzzy random variable of the form $\tilde{D} = (D - \Delta_1, D, D + \Delta_2)$ with the associated fuzzy probability density function (pdf) $\tilde{f}(\tilde{D})$.
- Lead-time L is assumed to be constant.
- The lead-time demand and the lead-time plus one period's demand are connected to the annual demand through the duration of the constant lead-time and lead-time plus length of the period respectively, i.e., $\tilde{D}_L = \tilde{D} \times L$ and $\tilde{D}_{L+T} = \tilde{D} \times (L + T)$. So the lead-time demand and the lead-time plus one period's demand are also normally distributed continuous fuzzy random variables.

The aim of this model is to determine the optimal period of review T^*, the optimal target inventory level R^* and optimal price discount by the supplier s^* so as to minimise the crisp equivalent of the expected total annual cost ETC.

2.4 Normally distributed fuzzy random variable demand

The fuzzy random variable annual customer demand is $\tilde{D} = (D - \Delta_1, D, D + \Delta_2)$ with associated fuzzy pdf $\tilde{f}(\tilde{D})$ where the membership function of $\tilde{D}$ is:

$$\mu_{\tilde{D}}(x) = \begin{cases} \frac{x-(D-\Delta_1)}{\Delta_1}, & D - \Delta_1 \le x \le D \\ \frac{(D+\Delta_2)-x}{\Delta_2}, & D \le x \le D + \Delta_2 \end{cases} \tag{1}$$

Here, Δ_1 and Δ_2 are the left and right spread of $\tilde{D}$ respectively.

Now, from the probability density function of the normal distribution

$$y = f(x) = \frac{e^{-(x-m)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}, \quad -\infty < x < \infty$$

we get,

$$x = m \pm (-2\sigma^2 \ln \sqrt{2\pi}\sigma y)^{\frac{1}{2}} \tag{2}$$

where σ (>0) and m are the standard deviation and mean of the normal distribution respectively.

Then from equations (1) and (2) (Khan and Dey, 2017),

$$\begin{aligned} &D - \Delta_1 \le x \le D \Rightarrow D - \Delta_1 \le m \pm (-2\sigma^2 \ln \sqrt{2\pi}\sigma y)^{\frac{1}{2}} \le D \\ &\Rightarrow \frac{(D - \Delta_1 - m)^2}{-2\sigma^2} \ge \ln \sqrt{2\pi}\sigma y \ge \frac{(D-m)^2}{-2\sigma^2} \\ &\Rightarrow \frac{e^{-(D-m-\Delta_1)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} \ge y \ge \frac{e^{-(D-m)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}. \end{aligned}$$

Similarly, $\frac{e^{-(D-m)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} \ge y \ge \frac{e^{-(D-m+\Delta_2)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$.

Therefore, $\tilde{f}(\tilde{D}) = (\frac{e^{-(D-m+\Delta_2)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}, \frac{e^{-(D-m)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} \frac{e^{-(D-m-\Delta_1)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}) = (f(\underline{D}), f(D), f(\overline{D}))$ where,

$$f(\underline{D}) = \frac{e^{-(D-m+\Delta_2)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}, f(D) = \frac{e^{-(D-m)^2/2\sigma^2}}{\sigma\sqrt{2\pi}},$$
$$f(\overline{D}) = \frac{e^{-(D-m-\Delta_1)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} \tag{3}$$

Therefore, the fuzzy probability density function of $\tilde{D}$ is $\tilde{f}(\tilde{D})$ (Khan and Dey, 2017). It is to be noted that by setting, $\Delta_1 = \Delta_2 = 0$, the fuzzy pdf $\tilde{f}(\tilde{D}) = (f(\underline{D}, f(D), f(\overline{D}))$ reduces to the crisp probability density function $f(D) = \frac{e^{-(D-m)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$.

2.4.1 Expected lead-time plus one period's demand

The expected lead-time plus one period's demand $\tilde{D}_{L+T}$ is obtained of the form $(\underline{D}_{L+T}, D_{L+T}, \overline{D}_{L+T})$. Now, $\underline{D}_{L+T}$, D_{L+T} and, $\overline{D}_{L+T}$ are calculated as follows (Khan and Dey, 2017):

$$\begin{aligned}\underline{D}_{L+T} = DL(D_{Lower}) &= \int_{-\infty}^{\infty} ((D-\Delta_1)(L+T))f(\underline{D})dD \\ &= \int_{-\infty}^{\infty} ((D-\Delta_1)(L+T)) \left(\frac{e^{-(D-m+\Delta_2)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}\right) dD \\ &= (m-\Delta_1-\Delta_2)(L+T)\end{aligned}$$

Likewise,

$$\begin{aligned}D_{L+T} = DM(D_{Middle}) &= \int_{-\infty}^{\infty} ((D(L+T))f(D)dD \\ &= \int_{-\infty}^{\infty} ((D(L+T)) \left(\frac{e^{-(D-m)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}\right) dD = m(L+T)\end{aligned}$$

and,

$$\begin{aligned}\overline{D}_{L+T} &= DU(D_{Upper}) \\ &= \int_{-\infty}^{\infty} ((D+\Delta_2)(L+T)f(\overline{D})dD \\ &= \int_{-\infty}^{\infty} ((D+\Delta_2)(L+T) \left(\frac{e^{-(D-m-\Delta_1)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}\right) dD \\ &= (m+\Delta_1+\Delta_2)(L+T)\end{aligned}$$

Therefore, the expected lead-time plus one period's demand is of the form:

$$\begin{aligned}&(DL, DM, DU) = (\underline{D}_{L+T}, D_{L+T}, \overline{D}_{L+T}) \\ &= ((m-\Delta_1-\Delta_2)(L+T), m(L+T), (m+\Delta_1+\Delta_2)(L+T))\end{aligned} \tag{4}$$

Again, by setting $\Delta_1 = 0 = \Delta_2$, the fuzzy expected lead-time plus one period's demand reduces to the crisp equivalent $D_{L+T} = m(L+T)$

2.5 Determination of the expected shortage

If the expected shortage exceeds the target inventory level R then shortage appears. Now to investigate the expected shortage, two different cases arise depending according as the position of $R \in [DL, DU]$ subject to the criterion that the safety-stock will be non-negative (Dey and Chakraborty, 2009).

Figure 1 When R lies in $[DL, DM]$

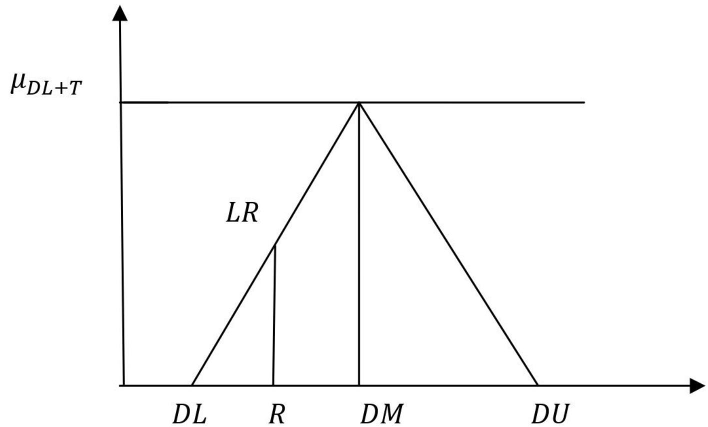
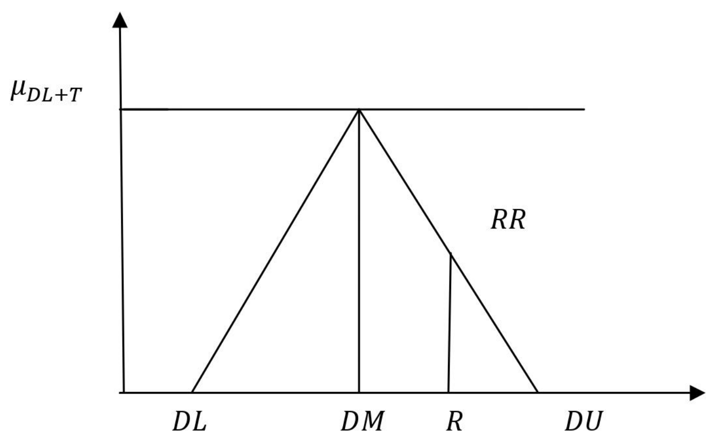


Figure 2 When R lies in $[DM, DU]$



LR and RR are the left and right shape functions of $\tilde{D}_{L+T}$ respectively.

That is,

$$LR = \frac{(R - DL)}{(DM - DL)} \text{ and, } RR = \frac{(DU - R)}{(DU - DM)}$$

Then using equation (4), LR and RR can be obtained as described by (Khan and Dey, 2017):

$$LR = \frac{R - (L + T)(m - \Delta_1 - \Delta_2)}{m(L + T) - (L + T)(m - \Delta_1 - \Delta_2)} \tag{5}$$

$$RR = \frac{-R + (L+T)(m + \Delta_1 + \Delta_2)}{-m(L+T) + (L+T)(m + \Delta_1 + \Delta_2)} \tag{6}$$

Then the expected shortage $\overline{b}(r)$ is obtained as below (Khan and Dey, 2017):

Case 1 When $R \in [DL, DM]$

$$\overline{b}(r) = \frac{DU}{2} - \frac{(DU - DM)}{3} + \frac{DL}{2} - \frac{(LR)^2 DL}{2} + \frac{(DM - DL)}{3} - \frac{(LR)^3 (DM - DL)}{3} - R\left(1 - \frac{(LR)^2}{2}\right) \tag{7}$$

Case 2 When $R \in [DM, DU]$

$$\overline{b}(r) = \frac{(RR)^2 DU}{2} - \frac{(RR)^3 (DU - DM)}{3} - \frac{(RR)^2 R}{2} \tag{8}$$

The total cost is derived in the next subsection.

2.5.1 Determination of the total cost

The total annual cost is the sum of the ordering cost, holding cost and stock out cost for the mixture periodic review system (Ouyang et al., 2003). That is

$$\widetilde{TC} = \frac{A}{T} + h\left[R - \tilde{D}L - \frac{DT}{2} + (1-\beta)\overline{M}(\tilde{D}_{L+T} - R)^+\right] + \frac{1}{T}[s\beta + (1-\beta)s_0]\overline{M}(\tilde{D}_{L+T} - R)^+ \tag{9}$$

where annual customer demand is $\tilde{D} = (D - \Delta_1, D, D + \Delta_2)$ with associated pdf $\tilde{f}(\tilde{D}) = (f(\underline{D}), f(D), f(\overline{D}))$.

Therefore expected total annual cost is given by

$$\widetilde{ETC} = \frac{A}{T} + h\left[R - \int_{-\infty}^{\infty} \tilde{D}\tilde{f}(\tilde{D})L dD - \int_{-\infty}^{\infty} \frac{\tilde{D}\tilde{f}(\tilde{D})T}{2} dD + (1-\beta)\overline{M}(\tilde{D}_{L+T} - R)^+\right] + \frac{1}{T}[s\beta + (1-\beta)s_0]\overline{M}(\tilde{D}_{L+T} - R)^+ \tag{10}$$

using $\int_{-\infty}^{\infty} f(\tilde{D})dD = 1$ (Khan and Dey, 2017).

The expected cost obtained above is further reduced to its crisp equivalent by means of the possibilistic mean of a fuzzy number (Carlsson and Fuller, 2001). In this vein, the possibilistic mean of the expected demand, obtained in equation (10), is derived below (Khan and Dey, 2017):

$$
\begin{aligned}
\bar{M}\left(\int_{-\infty}^{\infty}\tilde{D}\tilde{f}(\tilde{D})dD\right) &= \int_{-\infty}^{\infty}\bar{M}(\tilde{D}\tilde{f}(\tilde{D})dD) \\
&= \int_{-\infty}^{\infty}\left(\frac{\underline{\underline{Df}}+\overline{\overline{Df}}}{6}+\frac{2Df}{3}\right)dD \\
&= \frac{1}{6}\left[\int_{-\infty}^{\infty}\left((D-\Delta_1)\left(\frac{e^{-(D-m+\Delta_2)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}\right)\right)dD\right. \\
&\quad + \left.\int_{-\infty}^{\infty}\left((D+\Delta_2)\left(\frac{e^{-(D-m-\Delta_1)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}\right)\right)dD\right] \\
&\quad + \frac{2}{3}\left[\int_{-\infty}^{\infty}D(\frac{e^{-(D-m)^2/2\sigma^2}}{\sigma\sqrt{2\pi}})dD\right] \\
&= \frac{1}{6}[(m-\Delta_1-\Delta_2)+(m+\Delta_1+\Delta_2)]+\frac{2}{3}m = m
\end{aligned}
$$

Therefore the crisp equivalent of the expected total annual cost is obtained as

$$
\begin{aligned}
ETC = \frac{A}{T} &+ h[R - mL - \frac{mT}{2} + (1-\beta)\overline{M}(\tilde{D}_{L+T}-R)^+] \\
&+ \frac{1}{T}[\beta S + (1-\beta)s_0]\overline{M}(\tilde{D}_{L+T}-R)^+
\end{aligned}
$$

where $\overline{M}(\tilde{D}_{L+T}-R)^+$ is obtained from either of the following two cases:

Case 1 When R lies between DL and DM

$$
\begin{aligned}
ETC = \frac{A}{T} &+ h\left[R - mL - \frac{mT}{2} + (1-\beta)G(x)\right] \\
&+ \frac{1}{T}\left[s\beta + (1-\beta)s_0\right]G(x) \qquad (11)
\end{aligned}
$$

where $G(x) = \frac{DU}{2} - \frac{(DU-DM)}{3} + \frac{DL}{2} - \frac{(LR)^2DL}{2} + \frac{(DM-DL)}{3} - \frac{(LR)^3(DM-DL)}{3} - R(1-\frac{(LR)^2}{2})$.

Case 2 When R lies between DM and DU

$$
\begin{aligned}
ETC = \frac{A}{T} &+ h\left[R - mL - \frac{mT}{2} + (1-\beta)H(x)\right] \\
&+ \frac{1}{T}[s\beta + (1-\beta)s_0]H(x) \qquad (12)
\end{aligned}
$$

where $H(x) = \frac{(RR)^2DU}{2} - \frac{(RR)^3(DU-DM)}{3} - \frac{(RR)^2R}{2}$.

2.6 Derivation of the optimal solution

Situation 1: When R lies between DL and DM.

Here the total cost function is given by the equation (11) as

$$ETC = \frac{A}{T} + h[R - mL - \frac{mT}{2} + (1-\beta)G(x)] + \frac{1}{T}[s\beta + (1-\beta)s_0]G(x)$$

Since β_0 is the upper bound of the backorder rate β, so by substituting $\beta = \frac{\beta_0 s}{s_0}$ (Pan and Hsiao, 2001) we get the cost function as

$$ETC = \frac{A}{T} + h\left[R - mL - \frac{mT}{2} + (1 - \frac{\beta_0 s}{s_0})G(x)\right] + \frac{1}{T}\left[\frac{\beta_0 s^2}{s_0} + s_0 + \beta_0 s\right]G(x) \tag{13}$$

where $G(x) = \frac{s}{T}(\frac{DU}{2} - \frac{(DU-DM)}{3} + \frac{DL}{2} - \frac{(LR)^2 DL}{2} + \frac{(DM-DL)}{3} - \frac{(LR)^3(DM-DL)}{3} - R(1 - \frac{(LR)^2}{2})$.

The problem is therefore to find the optimal values of T, R and s, such that ETC in equation (13) is minimised. As mentioned in Khan and Dey (2017), the total cost function for the periodic review system is not necessarily convex in the control parameters. So, the following method is adopted to obtain the optimal values of the control parameters so that the cost is minimised. By taking the first partial derivatives of ETC with respect to R and s, respectively, we obtain

$$\frac{\partial ETC}{\partial R} = h\left[1 - \left(1 - \frac{\beta_0 s}{s_0}\right)\left(1 - \frac{(LR)^2}{2}\right)\right] - \frac{1}{T}\left[\frac{\beta_0 s^2}{s_0} + s_0 + \beta_0 s\right]\left(1 - \frac{(LR)^2}{2}\right) \tag{14}$$

and,

$$\frac{\partial ETC}{\partial s} = -\frac{\beta_0 h}{s_0}G(x) + \frac{1}{T}\left[\frac{2\beta_0 s}{s_0} + \beta_0\right]G(x) \tag{15}$$

By setting equations (14) and (15) equal to zero and solving the equations simultaneously, the values of R and s can be obtained for some fixed value of T.

Situation 2: When R lies between DM and DU.

Here the total cost function is given by the equation (12) as

$$ETC = \frac{A}{T} + h\left[R - mL - \frac{mT}{2} + (1-\beta)H(x)\right] + \frac{1}{T}[s\beta + (1-\beta)s_0]H(x)$$

Again, by substituting $\beta = \frac{\beta_0 s}{s_0}$ we get:

$$ETC = \frac{A}{T} + h\left[R - mL - \frac{mT}{2} + \left(1 - \frac{\beta_0 s}{s_0}\right)H(x)\right] + \frac{1}{T}\left[\frac{\beta_0 s^2}{s_0} + s_0 + \beta_0 s\right]H(x) \tag{16}$$

where $H(x) = \frac{(RR)^2 DU}{2} - \frac{(RR)^3 (DU-DM)}{3} - \frac{(RR)^2 R}{2}$.

As earlier, taking the first partial derivatives of ETC with respect to R and s, respectively, we get

$$\frac{\partial ETC}{\partial R} = h\left[1 - \left(1 - \frac{\beta_0 s}{s_0}\right)\frac{(RR)^2}{2}\right] - \frac{1}{T}\left[\frac{\beta_0 s^2}{s_0} + s_0 + \beta_0 s\right]\frac{(RR)^2}{2} \tag{17}$$

and,

$$\frac{\partial ETC}{\partial s} = -\frac{\beta_0 h}{s_0}H(x) + \frac{1}{T}\left[\frac{2\beta_0 s}{s_0} + \beta_0\right]H(x) \tag{18}$$

The process followed is described below:

For a fixed value of T, the values of R and s are evaluated using suitable equations. These values are then used to evaluate ETC. The value of T is then increased by a predetermined step-size and the process repeated. As in existing literature on periodic review models, it is observed here also that, as the value of T increases, the value of ETC first decreases and then starts increasing thereby yielding a minimum. This process is encapsulated in the form of an algorithm in the next section.

2.7 Algorithm

Step 1 Set $i = 0$, $T = T_0$ and $ETC^* = \infty$.

Step 2 Calculate the value of DL,DM and DU for T_0 from equation (4).

Step 3 Depending on the position of R in (DL, DM, DU), find R_0 and s (initial value of s) from the equations (14), (15) or (17), (18).

Step 4 Determine the total cost ETC_0 for R_0 and s (initial value of s) using equations either equations (13) or (16). Also find SS. Set $ETC^* = ETC_0$.

Step 5 If $SS_0 > 0$, go to next step 6, otherwise put $T_0 = 2T_0$ and go back to step 2.

Step 6 Set index $i = i + 1$.

Step 7 Assume $T_i = T_{i-1} + k$, where k is the size of the iteration.

Step 8 Determine DL,DM and DU, then calculate R_i and s_i using equations (14), (15) or (17), (18). Also find ETC_i for the value of R_i, s_i and T_i as in step 4.

Step 9 If $ETC_i < ETC^*$, then go back to the step 6, otherwise go to the next step 10.

Step 10 Set $T^* = T_{i-1}$, $R^* = R_{i-1}$, $s^* = s_{i-1}$ and $ETC^* = ETC_{i-1}$.

Step 11 If $|T_i - T^*|<\delta$, where δ is the preassigned positive value considered by the decision maker.

Then T^*, R^* and s^* are the optimal values of the period of review and the target inventory level respectively. Otherwise, set $T_0 = T_{i-2}$ and $\delta = \delta^*, \delta>\delta^*$, and go to the step 2.

3 Numerical example

Numerical examples are developed under the following three cases:

- no backorder price discount
- backorder price discount is fixed
- backorder price discount is a control parameter

The following data have been used for numerical example: A = 150, h = 30, s_0 = 15, L = 10/365, m = 200, σ = 50, Δ_1 = 50, Δ_2 = 75, β_0 = 75/100.

Case 1 When there is no price discount (i.e., $\beta = \beta_0$).

Table 1 Derivation of the optimal solution: no price discount

T	R	*Setup cost*	*Holding cost*	*Shortage cost*	TC
0.15	42.58	1000.00	698.06	115.88	1,813.95
0.20	52.42	750.00	869.52	153.48	1,773.00
0.25	61.84	600.00	1035.26	188.74	1,824.00
0.17	46.58	882.35	767.52	131.17	1,781.05
0.18	48.54	833.33	801.78	138.69	1,773.81
0.19	*50.49*	*789.47*	*835.77*	*146.13*	*1,771.38*
0.20	52.42	750.00	869.52	153.48	1,773.00
0.21	54.33	714.27	903.04	160.73	1,778.06

Therefore T^* = 0.19, R^* = 50.49 and minimum total annual cost is ETC^* = Rs.1,771.38.

Case 2 When the backorder price discount is fixed (first consider s = Rs.9 and then s = Rs.12).

Table 2 Derivation of the optimal solution: fixed price discount

T	R	s	*Setup cost*	*Holding cost*	*Shortage cost*	TC
0.19	47.30	9	789.47	732.05	186.11	1,707.63
0.20	48.97	9	750.00	757.57	196.56	1,704.14
0.21	*50.62*	*9*	*714.29*	*782.67*	*206.99*	*1,703.94*
0.22	52.24	9	681.82	807.37	217.36	1,706.55
0.23	53.86	9	652.17	831.71	227.68	1,711.56
0.19	47.33	12	789.47	719.98	198.85	1,708.31
0.20	48.97	12	750.00	743.19	210.95	1,704.14
0.21	*50.59*	*12*	*714.29*	*765.81*	*223.08*	*1,703.18*
0.22	52.17	12	681.82	787.88	235.25	1,704.95
0.23	53.74	12	652.17	809.43	247.45	1,709.65

Therefore, for s = 9, T^* = 0.21, R^* = 50.62 and minimum total annual cost is ETC^* = Rs.1703.94. And, for s = 12, T^* = 0.21, R^* = 50.59 and minimum total annual cost is ETC^* = Rs.1,703.18.

Case 3 When the backorder price discount is a control parameter.

Table 3 Derivation of the optimal solution: price discount rate s is a control parameter

T	R	s	*Setup cost*	*Holding cost*	*Shortage cost*	TC
0.15	40.33	9.75	1000.00	622.64	146.14	1,768.78
0.20	48.88	10.50	750.00	748.17	204.16	1,702.33
0.25	56.76	11.25	600.00	856.16	266.19	1,722.35
0.19	47.23	10.35	789.47	724.52	192.24	1,706.25
0.20	48.88	10.50	750.00	748.17	204.16	1,702.33
0.21	*50.50*	*10.65*	*714.29*	*771.12*	*216.23*	*1,701.64*
0.22	52.11	10.80	681.82	793.38	228.47	1,703.67
0.23	53.68	10.95	652.17	814.97	240.86	1,708.01

Therefore, $T^* = 0.21$, $R^* = 50.5$, $s^* = 10.65$ and minimum total annual cost $ETC^* =$ Rs.1,701.64.

Table 4 Effect of the parameters

Parameters	*Values*	T	R	s	TC
m	150	0.24	41.86	11.10	1,533.34
	200	0.21	50.50	10.65	1,701.64
	250	0.19	58.10	10.35	1,848.75
L	5/365	0.21	47.59	10.65	1,681.02
	10/365	0.21	50.50	10.65	1,701.64
	15/365	0.21	53.42	10.65	1,722.26
$\Delta_1(\Delta_2 = 75)$	40	0.21	50.27	10.65	1,673.05
	50	0.21	50.50	10.65	1,701.64
	60	0.21	50.75	10.65	1,730.22
$\Delta_2(\Delta_1 = 50)$	65	0.21	50.27	10.65	1,673.05
	75	0.21	50.50	10.65	1,701.64
	85	0.21	50.75	10.65	1,730.22

3.1 Analysis of the numerical result

It can be concluded from Tables 1, 2, 3 that, as the review period T increases, the target inventory level R and the backorder price discount rate s (when price discount is available) increases as well. This tendency of the result is practically correct because increasing T implies that the number of orders placed decreases, which implies that the target inventory level R needs to be higher to optimise the inventory system. Also, increasing of T implies more amount of backorders and hence the higher price discount from a supplier may result. This also implies that the ordering cost or setup cost decreases (as the number of orders decreases) while the other inventory costs like inventory holding cost and shortage cost both show an increase. Then finally the total annual cost ETC decreases. But, after a critical period of review, the total cost ETC starts increasing again. This critical period of review is the optimal period of review T^* and this total annual cost is the optimal inventory cost ETC^*.

It is also observed that, among the three considerations, the 3rd case i.e., when backorder price discount is a control parameter, gives the minimum total cost. Therefore the best result is obtained among the three cases when price discount is a control parameter.

From Table 4, it is observed that as expected demand m increases, the optimal cost also increases. This is intuitively correct since, an increase in mean demand implies a shorter period of review, which in turn implies lesser amount of backorders. Also, increasing m gives a higher target inventory level, as obtained numerically, which leads to a higher ETC incurred. Also it is observed that an increase in the lead-time L results in an increase in ETC incurred, which is as expected since, an increase in lead-time makes increase in the lead-time plus one period‘s demand. This in turn increases the shortage incurred which brings up the ETC incurred by the model. Further, increasing the value of Δ_1 (when Δ_2 fixed), and vice versa, results in an increase in the total cost incurred.

4 Concluding remarks and scope for future research

In this paper, a fuzzy random periodic review inventory model with controllable backorder rate is developed. It is assumed that, in case of shortage, the supplier provides backorder price discount to encourage customers to wait for their orders to arrive rather than take their orders elsewhere. The model is developed in the mixed fuzzy random framework with the annual customer demand taken to be a normally distributed fuzzy random variable with the associated probability density function taken to be fuzzy as well. An algorithm is developed to determine the optimal period of review, the optimal target inventory level and the optimal price discount by the supplier so as to minimise the crisp equivalent of the expected total annual cost. The model may be easily reduced to existing stochastic models by setting the spreads of the fuzzy random demand as zero. It may also be reduced to existing fuzzy models. Thus the model provides a more generalised framework where both fuzziness and randomness are incorporated simultaneously. It also allows the decision maker to incorporate his own subjective evaluations to provide efficient decision support. As a scope for future research, lead-time may considered to be a control parameter. Another possible extension of this research is to consider the backorder rate to be dependent on the length of protection interval, etc.

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