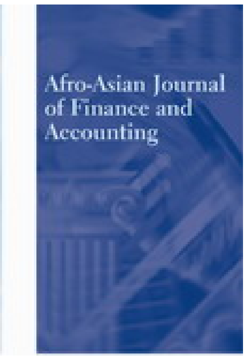


Afro-Asian J. of Finance and Accounting

ISSN online: 1751-6455 - ISSN print: 1751-6447
https://www.inderscience.com/aajfa

Explanatory power of realised moments

Seema Rehman, Saqib Sharif, Wali Ullah

DOI: 10.1504/AAJFA.2024.10061684

Article History:

Received:	19 March 2021
Last revised:	18 October 2021
Accepted:	15 November 2021
Published online:	18 January 2024

Explanatory power of realised moments

Seema Rehman*

Faculty of Management Sciences,
Indus University,
ST-2D, Block-17, Gulshan-e-Iqbal, Karachi, Pakistan
Email: seemarehman2014@yahoo.com
*Corresponding author

Saqib Sharif

Department of Finance,
Institute of Business Administration (IBA),
University Road, Karachi, 75270, Pakistan
Email: ssharif@iba.edu.pk

Wali Ullah

Department of Economics,
Institute of Business Administration (IBA),
University Road, Karachi, 75270, Pakistan
Email: waliullah@iba.edu.pk

Abstract: This study decomposes realised moments into high and low components and examines if the high minus low realised moment factors are helpful in explaining future stock returns. Realised moment factors are incorporated as extensions to basic asset pricing models. Evidence from this paper suggests the role of realised moments in enhancing the step wise model development. Such as there is risk premium at Pakistan Stock Exchange (PSX) for investing in stocks having volatile, more skewed return distributions with excess kurtosis. This study may help investors and fund managers to employ best strategies to gain maximum return on their investment. By including third and fourth moments within coherent framework acknowledges risk from asymmetries and fat tails and helps investors in constructing smart portfolios to earn higher returns. To the best of authors' knowledge, this is the first study to analyse the role of realised moments in explaining stock returns, using high frequency data in the emerging stock market of Pakistan.

Keywords: equity returns; emerging market; intraday data; realised volatility; skewness; kurtosis.

JEL codes: G11, G12.

Reference to this paper should be made as follows: Rehman, S., Sharif, S. and Ullah, W. (2024) 'Explanatory power of realised moments', *Afro-Asian J. Finance and Accounting*, Vol. 14, No. 1, pp.22–42.

Biographical notes: Seema Rehman is a PhD scholar at the Shaheed Zulfiqar Ali Bhutto Institute of Science and Technology, and has been working in asset pricing using tick by tick data for the first time on emerging stock market of Pakistan. She has publications in internationally renowned journals, including *Romanian Journal of Economic Forecasting*. She is working as a Lecturer at the Department of Business Administration, Faculty of Management Sciences, Indus University, ST-2D, Block-17, Gulshan-e-Iqbal Adjacent to National Stadium, Karachi, Pakistan.

Saqib Sharif holds a PhD in Finance from the Massey University, New Zealand. He has more than ten years of central banking experience prior to his PhD and more than eight years of teaching experience. He has published in internationally respected journals, including *Accounting & Finance* and *International Journal of Managerial Finance*. He is currently ranked in the top 10% of authors on the international working paper website SSRN (based on downloads of his papers). His research interests are asset pricing, financial regulation, microstructure data and Islamic finance. His personal research goal is to conduct and publish quality academic research. He is working as an Associate Professor at the Department of Finance, School of Business Studies, Institute of Business Administration (IBA), University Road, Karachi, Pakistan.

Wali Ullah holds a PhD in Economics from the Tohoku University, Sendai, Japan. He has more than 18-year of teaching experience. He has publications in internationally renowned journals, including *Journal of Applied Statistics* and *Journal of Forecasting*. His research interests are asset pricing, capital structure and monetary policy. He is working as a Professor at the Department of Economics, Institute of Business Administration (IBA), University Road, Karachi, Pakistan.

1 Introduction

1.1 Background

Sharpe (1964), Lintner (1965) and Mossin (1966) follow the foot prints of Markowitz (1959) in formulating the first mean variance-based capital asset pricing model (CAPM). The original CAPM as proposed by Sharpe (1964), Lintner (1965) and Mossin (1966) is based on hypotheses concerning investors and the opportunity framework. The CAPM faced a lot of critique due to the limitation of the empirical testing. For example, CAPM is unable to justify the return on a portfolio build-up of numerous securities. Ward and Muller (2013) argue that the standard one factor CAPM is insufficient in explaining cross-sectional variations in expected stock returns. Moreover, the accuracy of CAPM in return prediction is doubtful (e.g., Berk and Van Binsbergen, 2016). CAPM's assumptions are also considered as problematic; especially the concept of market portfolio is referred to as God's portfolio by some researchers because of containing all assets of the world. Due to the mis-specified model and limits imposed by these pivotal assumptions, numerous researchers struggled to form more generalised asset pricing models that provide relaxation on the assumptions and tested the implications empirically (e.g., Harvey and Siddique, 2000; Ang et al., 2006). Furthermore, the model faced criticism by the researchers on the widely employed mean variance criterion for selecting portfolios (e.g., Kraus and Litzenberger, 1976; Hwang and Satchell, 1999). They argue that the impact of higher moments should also be incorporated in asset pricing models. Chaudhary et al. (2020) finds that incorporating higher moments in two moment model provide symmetry in the up and down markets.

The incompetency of the conventional CAPM spurred financial analysts and researchers to design modified adaptations of CAPM and one such remarkable work is accredited to Black et al. (1972). In their model, efficient portfolio comprises of a zero beta portfolio and a market portfolio with the condition that there is no correlation between the returns on these portfolios. Roll (1977) marks the investigations of CAPM as invalid because of employing inefficient benchmark portfolio whereas an efficient benchmark is the primary requirement for valid test of CAPM. Ross (1976) develops arbitrage pricing theory (APT) as a substitute for CAPM. It is a single period model in which all investors believe that the stochastic characteristics of asset returns are in accordant with a factor framework. Merton (1973) develops the intertemporal CAPM having stochastic investing opportunities by relaxing the single period assumption, asserting that within a continuous time period context, the expected return on any security is derived from a multiple beta version of CAPM.

Banz (1981) paper captures attention for finding out the anomaly that companies having low market capitalisation tend to perform better than businesses with high market capitalisation. In finance, anomaly relates to a situation when the performance of a single stock or portfolio is deviant from the primary assumptions of efficient market hypothesis (Tversky and Kahneman, 1986). Fama and French (1992) show the role played by two factors, market equity (ME) and the book equity to market equity (BE/ME) ratio in explaining most of the cross-section of stock returns. Systematic differences in stock returns are because of differences in risk after rational pricing of stocks. Thus, if stocks are priced rationally, size variable of ME (stock price times shares outstanding) and BE/ME are proxies for sensitivity to common risk components in stock returns. Fama and French (1993) verified that, portfolios constructed mimicking risk components linked to size (ME) and BE/ME contributed largely in explaining the variations in stock returns by a market portfolio. Iqbal and Brooks (2007) check CAPM and Fama and French (1993) three factor model in Pakistani stock market and discuss the significance of the explanatory power of risk factors in Fama and French (1993) model as compared to CAPM. Mirza and Shahid (2008) deploy multivariate technique for testing the validity of Fama and French (1993) three factor model by also including financial firms' equities and find supporting evidence for the three factor model. Recent study of Ali et al. (2018) provide supportive evidence for the existence of SMB and HML factors at Pakistan Stock Exchange (PSX) and their role in forecasting the economic growth of the country in terms of GDP.

This research adds to the existing literature in several ways:

1 It analyses the time-series properties of realised variance, skewness and kurtosis of equity returns in the emerging stock market of Pakistan.

2 Checks whether the realised moments are informative in explaining stock market returns by calculating realised higher moments from high frequency data to ensure the effectiveness of the measurement of asymmetry and fat tails.

3 Compares the performance of CAPM, Fama and French (1993) three factor model with augmented four, five, six factor models, formed by adding realised moment factors stepwise to these asset pricing models for four sets of six portfolios constructed by double sorts on size and BE/ME, size and realised volatility (RV), size and realised skewness (RS) and size and realised kurtosis (RK).

The findings of this study exhibit that:

1 RV, skewness and kurtosis factors exist at PSX along with SMB and HML. Such as, investors are compensated with risk premium for investing in high volatile, more skewed stocks and firms having excess kurtosis. Also small firms earn higher returns as compared to big firms and stocks with high book to market (btm) ratio (value firms) earn higher returns in comparison to stocks with low btm ratio (growth firms).

2 The adjusted R-square improves meaningfully after adding RV factor to Fama and French (1993) model for four sets of six portfolios constructed on the basis of market value and btm ratio/realised moments. Similarly, the addition of RS factor to four factor model for four sets of six portfolios resulted in improved average R-square and the addition of RK factor to five factor model shows improvement in model characteristics verifying that the six factor model explains the variations in average stock returns.

3 The presence of realised moment premium at PSX is evidenced by the significant coefficients on RV, RS and RK factors. However, every emerging market has some unique environments, PSX is also different in market structure, legal environment and investors' profile, such as narrow price bands, but highly volatile.

Thus, the results of this study may be interpreted with caution and might not be extendable to other emerging markets.

The remaining paper is structured in the following manner. Section 2 reviews literature, Section 3 explains data and methodology, Section 4 discusses findings, and Section 5 concludes the study.

2 Literature review

Contradictory to the assumptions of modern portfolio theory (MPT), numerous research studies conducted using data from various markets indicate the non-normality of financial asset returns. Chaudhary et al. (2020) find that returns from all portfolios are asymmetric and leptokurtic in Indian stock market. Mensi et al. (2021) obtained data of 16 stock market indices and shows that the skewness and kurtosis values of stock returns of all markets are highly divergent from normal distributions. Similar patterns are observed by Wu et al. (2020). Aggarwal et al. (1989) suggest the presence of skewness and kurtosis in Japanese stock market. The presence of leptokurtosis in time series data is evidenced by Lux and Marchesi (2000). They find that volatility clustering is positively related to the fourth moment, i.e., kurtosis. Incorporating third and fourth order moment makes portfolio sort, a non-convex and non-smooth optimisation issue presented by different clashing and contending objectives, for example, maximisation of expected return and skewness (positive) and minimisation of volatility and kurtosis. Naqvi et al. (2017) employ polynomial goal programming (PGP) based on multi-objective technique introduced by Lai et al. (2006) for dealing with this problem. The multi-objective method has a capability of incorporating investors' preferences and at the same time identifying an optimum solution relying on numerous criteria.

The presence (or absence) of conditional and unconditional symmetries play an important role in situations within economic and statistical contexts. Skewness is vital from a financial point of view as it could itself be regarded as a scale to measure risk. Other things being constant, right skewed portfolios are preferred over left skewed portfolios by investors as implied by Arrow Pratt view of risk aversion. Kim and White (2004) assert that investors' preference for right skewed portfolios implies an expectation for skew premium to urge investors for investing in left skewed portfolios. Thus, securities that lower skewness of a portfolio by making the portfolio returns more skewed to the left are less demanded and should require higher expected returns. Similarly, securities that add to skewness of a portfolio should withhold lower returns. Chunhachinda et al. (1997) show considerable changes in expected return, if higher order moments are incorporated in the selection of optimal portfolio. Similarly, Jondeau and Rockinger (2004) measure benefits of employing strategy that accounts for skewness and kurtosis.

Measuring volatility and understanding its dynamics play a crucial role in dealing with many fundamental issues in the field of finance. As a basic gauge of risk in modern financial practices, volatility is an underlying factor while constructing optimal portfolios, in pricing options and other derivative instruments or determining the exposure of a firm to various risks and its expectation to earn for compensating from those risk exposures. It is also critical in finding new trading and investment opportunities that may offer appealing risk return trade-off (Ait-Sahalia and Yu, 2009). While there is a possibility of measuring actual returns with minimum measurement error and assessing them directly through usual time series techniques, modelling volatility conventionally relies on more complicated econometric methods with the aim of encompassing the innate latent characteristic of volatility. The RV concept helps in reversing this view effectively. RV is a measure of the ex-post variance of stock prices over a fixed time interval (Barndorff-Nielsen et al., 2010).

The importance of characterising the magnitude and patterns in time series variance of volatility to determine the suitable stylised facts that could be useful in evaluating the asset pricing models, cannot be ignored. Paye (2012) discovers the countercyclical behaviour of stock return volatility. He explains the positive skewness and leptokurtosis of aggregate stock return volatility as a partial outcome of numerous extreme episodes of stock return volatility that also includes Oct. 1987 market crash and the histrionic plunge in stock prices attributable to 2008 financial crisis. He uses linear approach to predict volatility taking ordinary least squares (OLS). OLS technique may be inferior to nonlinear measures if the regression errors are not normal and have fat tails. However, an approximately Gaussian sample can be generated if the natural log of RV is taken as documented in Andersen et al. (2001). Higher stock return volatility is observed through recessions as compared to expansions. Schwert (1989) runs a regression by taking volatility as a dependent variable and a dummy variable as an independent variable which is given the value of one across recessions of National Bureau of Economic Research (NBER) and discover high volatility through such periods.

As Merton (1980) notices that increasing sampling frequency leads to an accurate way of measuring volatility arbitrarily. Later research on RV applied his discernment for measuring time varying volatility by constructing daily measures of RV calculated using intraday squared returns. Based on the now long standing conception of RV, Amaya et al. (2015) compute RS and kurtosis using intraday cubed and quartic returns and show that relying on continuous time specificity of stock price dynamics that accounts for stochastic component and jumps, the realised moments convene to true moments validating that Merton's (1980) discernment also relates to higher moments.

3 Methodology

3.1 Data

Tick by tick data of stock prices of listed companies from July 2008 to August 2018, obtained from PSX is used to compute realised moments. To maintain sufficient liquidity, only those firms are included in the sample that have at least 80 transactions on a single trading day. Moreover, to avoid large returns, firms having stock price of Rs. 5 and more are considered, reducing the sample size to 306 firms. Data of daily prices, volume, number of shares outstanding and market value has also been provided by PSX. Book values of individual firms are obtained from Thomson Reuters Data Stream.

3.2 Modelling realised moments

Five minutes prices are extracted from tick by tick data by using nearest neighbour interpolation technique, such as if there is no price in some time slot, the value in the last slot is utilised. The trading time at PSX is incorporated for better results (for Monday till Thursday, from 9:30 AM to 3:30 PM and 9:15 AM to 4:30 PM for Friday). Friday breaks from 12:00 PM to 2:30 PM are also addressed, that resulted in 57 observations for Fridays and 72 observations for all other trading days. The returns are calculated for five minutes prices as follows:

$$R_t = \ln\left(P_t / P_{t-1}\right) \times 100 \tag{1}$$

where R_t is the return at period t, P_t is the price at period t, and P_{t-1} is the previous five minutes price. Five minutes returns are squared and then aggregated to obtain daily RV estimates (Andersen et al., 2003).

$$RDVar_t = \sum_{i=1}^{N} r_{t,i}^2 \tag{2}$$

Daily RS is computed by summing cubic returns of five minutes prices.

$$RDSkew_t = \frac{\sqrt{N}\sum_{i=1}^{N} r_{t,i}^3}{RDVar_t^{3/2}}. \tag{3}$$

Next, daily RK estimates are acquired by taking sum of quartic returns. For standardising purpose, both of these measures are scaled by dividing with RV.

$$RDKurt_t = \frac{N\sum_{i=1}^{N} r_{t,i}^4}{RDVar_t^2} \tag{4}$$

Following Amaya et al. (2015), weekly realised moments are acquired by taking average of daily measures. To avoid calendar related anomalies the week is taken from Wednesday till Tuesday.

$$RVol_t = \left(\frac{252}{5} \sum_{i=0}^{4} RDVar_{t-i} \right)^{1/2} \tag{5}$$

$$RSkew_t = \frac{1}{5} \sum_{i=0}^{4} RDSkew_{t-i} \tag{6}$$

$$RKurt_t = \frac{1}{5} \sum_{i=0}^{4} RDKurt_{t-i}. \tag{7}$$

To make interpretation easier, RV is annualised by multiplying with 252.

3.3 Model specification

First CAPM is run:

$$E(R_i) - R_f = \alpha_i + \beta_i \left[E(R_m) - R_f \right] + \varepsilon_i \tag{8}$$

Then, Fama and French (1993) three factor model:

$$E(R_i) - R_f = \alpha_i + \beta_i \left[E(R_m) - R_f \right] + c_i SMB1 + h_i HML + \varepsilon_i \tag{9}$$

Four factor model is developed by adding RV factor to Fama and French (1993) three factor model:

$$E(R_i) - R_f = \alpha_i + \beta_i \left[E(R_m) - R_f \right] + c_i SMB2 + h_i HML + rv_i RV + \varepsilon_i \tag{10}$$

Five factor model is obtained by adding RS factor to four factor model:

$$E(R_i) - R_f = \alpha_i + \beta_i \left[E(R_m) - R_f \right] + c_i SMB3 + h_i HML + rv_i RV + rs_i RS + \varepsilon_i \tag{11}$$

Finally, adding RK factor to the five factor model results in six factor model:

$$\begin{aligned} E(R_i) - R_f = {} & \alpha_i + \beta_i \left[E(R_m) - R_f \right] + c_i SMB4 + h_i HML + rv_i RV + rs_i RS \\ & + rk_i RK + \varepsilon_i \end{aligned} \tag{12}$$

where $E(R_i) - R_f$ is the excess return of portfolio i, α_i is the intercept of the regression i, presenting return via mechanisms other than the market. $E(R_m) - R_f$ presents excess return on market portfolio. SMB and HML stand for size and value factors. RV, RS and RK factors are formed by taking difference of excess return of stocks having high RV, skewness or kurtosis and stocks having low RV, skewness or kurtosis, respectively. β_i, c_i, h_i, rv_i, rs_i and rk_i represent sensitivities of portfolio i to market, size, value, RV, RS and RK factors, respectively. ε_i indicates the return constituent resulting from random events, specific to a portfolio.

3.4 Dependent variable

Dependent variables of the asset pricing model are constructed by dividing the excess returns of individual firms into two parts (i.e., small and big) by ranking on their market value and then into three parts at 30th and 70th percentiles (i.e., growth, neutral, value) ranked on their btm ratio and following Fama and French (1993), six value weighted portfolios are constructed at the intersections of these 2 × 3 portfolios. Same procedure is followed for the three realised moments, such as there are four sets of six portfolios, six size-btm, six size-rv, six size-rs and six size-rk portfolios and each is considered individually for running five of the models, explained in the preceding section, resulting in 4 × 6 × 5 = 120 regressions.

3.5 Independent variables

The independent variables of five asset pricing models are market, size, value, RV, RS and RK factors. Market factor is formed by subtracting risk-free rate from market portfolio, SMBs and HML are computed as follows:

$$SMB = 1/3(SV + SN + SG) - 1/3(BV + BN + BG) \tag{13}$$

where small-value (*SV*), small-neutral (*SN*), small-growth (*SG*), big-value (BV), big-neutral (*BN*) and big-growth (*BG*) are formed at the intersection of two portfolios (small, big) based on size (dividing stocks into two parts at median point) and three portfolio (growth, neutral, value) ranked on BE/ME (splitting stocks at 30th and 70th percentile).

$$HML = 1/2(SV + BV) - 1/2(SG + BG) \tag{14}$$

where small-value (*SV*), big-value (*BV*), small-growth (*SG*) and big-growth (*BG*) are formed at the intersection of two portfolios (small, big) based on size (dividing stocks into two parts at median point) and value and growth portfolios represent first 30% and last 30% stocks ranked on BE/ME. All portfolios are value weighted.

3.5.1 Realised moments' factors

Realised moments' factors are formed as follows:

$$RV = 1/2 \times (SHRV + BHRV) - 1/2 \times (SLRV + BLRV) \tag{15}$$

where small-high realised volatility (*SHRV*), big-high realised volatility (*BHRV*), small-low realised volatility (*SLRV*) and big-low realised volatility (*BLRV*) stand for portfolios formed at the intersection of two portfolios (small, big) formed by splitting stocks at median point ranked on their market value and low and high RV portfolios containing first 30% and last 30% stocks ranked on their RV. Similarly:

$$RS = 1/2 \times (SHRS + BHRS) - 1/2 \times (SLRS + BLRS) \tag{16}$$

where small-high realised skewness (*SHRS*), big-high realised skewness (*BHRS*), small-low realised skewness (*SLRS*) and big-low realised skewness (*BLRS*) stand for portfolios formed at the intersection of two portfolios (small, big) formed by splitting stocks at median point ranked on their market value and low and high RS portfolios containing first 30% and last 30% stocks ranked on their RS.

$$RK = 1/2 \times (SHRK + BHRK) - 1/2 \times (SLRK + BLRK) \quad (17)$$

where small-high realised kurtosis (*SHRK*), big-high realised kurtosis (*BHRK*), small-low realised kurtosis (*SLRK*) and big-low realised kurtosis (*BLRK*) stand for portfolios formed at the intersection of two portfolios (small, big) formed by splitting stocks at median point ranked on their market value and low and high RK portfolios containing first 30% and last 30% stocks ranked on their RK.

4 Data analysis and findings

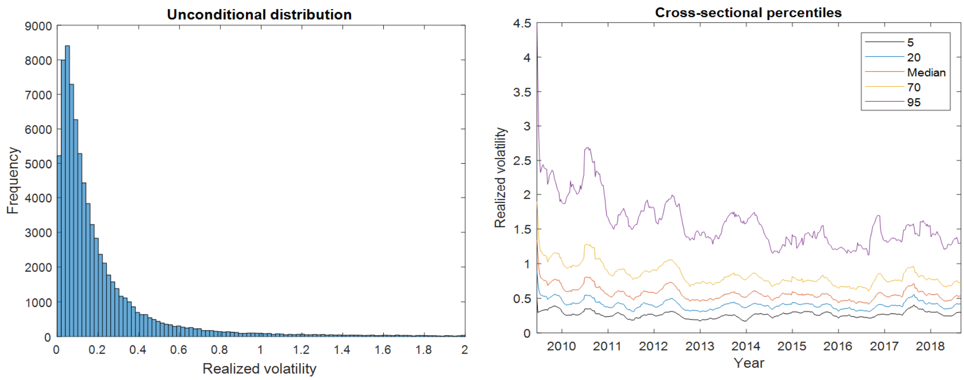
157,000 firm-week observations are utilised to compute realised moments for the period from July 2008 till August 2018. Left panel of Figure 1 depicts the unconditional distribution of the three realised moments. The distribution of RV and RK are clearly lognormal, implying their ability to help in return generating process. RS distribution is centred around mean, characterised by extreme positive or negative returns. Right panel of Figure 1 shows plots of the three month moving averages of the cross-sectional 5th, 20th, median, 70th and 95th quantiles of the three realised moments. It is evident from the cross-sectional quantiles of RV that the spread has decreased through-out the sample period under consideration, showing enhanced stability in stock returns in the emerging stock market of Pakistan. Similarly, time variations are evident in the quantiles of RS and RK.

Table 1 Descriptive statistics of value-weighted weekly average returns (in percentage) of dependent variables

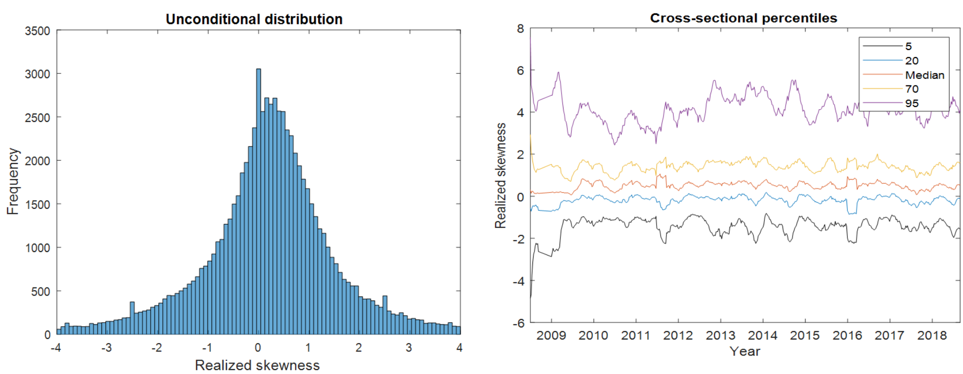
Panel A: six size-btm portfolios				*Panel B: six size-rv portfolios*			
	Growth	*Neutral*	*Value*		*Low vol.*	*Med. vol*	*High vol.*
Small size	0.18	0.08	0.32	Small size	–0.7	–0.48	1.22
	(4.17)	(2.98)	(3.81)		(3.72)	(3.46)	(5.11)
Big size	–0.06	0.06	0.18	Big size	–0.14	0.42	1.43
	(2.9)	(3)	(3.42)		(2.43)	(3.46)	(5.16)
Panel C: six size-rs portfolios				*Panel D: six size-rk portfolios*			
	Low skew.	*Med. skew.*	*High skew.*		*Low kur.*	*Med. kur.*	*High kur.*
Small size	–0.48	0.11	1.56	Small size	0.85	0.66	0.05
	(3.97)	(4.53)	(4.5)		(6.91)	(4.41)	(3.58)
Big size	–0.44	0.16	0.88	Big size	0.12	0.23	0.35
	(3.19)	(2.98)	(3.53)		(2.93)	(3.21)	(3.34)

Note: Mean and standard deviations (in parenthesis) for six size-btm, six size-rv, six size-rs and six size-rk portfolios.

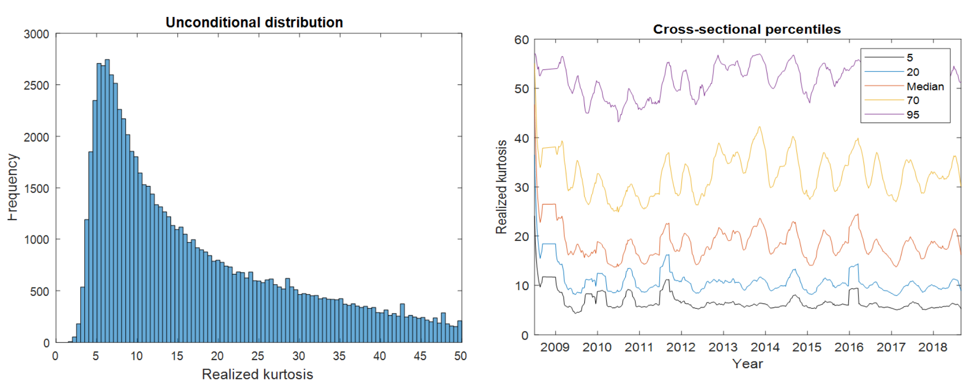
Figure 1 Histograms and quantiles (three month moving averages) of RV: Panel (a), RS: Panel (b) and RK: Panel (c) for the period between July 2008 and August 2018, based on 157,000 firm week observation (see online version for colours)



(a)



(b)



(c)

Summary statistics of four sets of six portfolios are presented in Table 1. Similar to the results of Ali et al. (2018), who report positive monthly returns for six size-btm portfolios for sampling period 2002–2015, this study finds positive weekly returns for six size-btm portfolios with one exception as shown by Panel A of Table 1. Panel B of Table 1 shows that high volatile stocks earn higher return and have higher standard deviations. Higher average returns and high volatility for high skewed firms are evident in Panel C of Table 1. Panel D reports the weekly average returns for six portfolios based on size and RK. The highest return is observed for small firms having low kurtosis.

The weekly average returns are positive for all explanatory variables with one exception. The positive and significant return on HML provides evidence for higher performance of value stocks as compared to growth stocks. Statistics on realised factors clearly depict higher returns at PSX with additional risk.

Table 2 Descriptive statistics of value-weighted weekly factor returns (in percentage)

	$R_m - R_f$	*SMB*	*HML*	*RV*	*RS*	*RK*
Mean	0.137	0.138	0.189	1.753	1.676	–0.284
Std.	2.673	2.464	2.167	3.726	2.56	3.566
t-statistics	1.16	1.27	1.97	10.66	14.83	–1.8

Notes: SMB is computed by subtracting average weekly return on portfolios having big firms (BV, BN, BG) from portfolios having small firms (SV, SN, SG). HML, RV, RS and RK are formed by subtracting bottom 30% average weekly return of portfolios ranked on BE/ME ratios, RV, skewness and kurtosis from top 30% average weekly return of portfolios ranked on BE/ME ratios, RV, skewness and kurtosis, respectively. Mean and standard deviations of independent variables are reported along with their t-statistics.

Correlation is checked to detect any high correlation among predictor variables. Results in Table 3 show that this research does not face any multi-collinearity issue while running regressions. The correlation values justified that all factors are independent of each other and could be considered as separate variables.

Table 3 Correlation matrix of independent variables

	$R_m - R_f$	*SMB*	*HML*	*RV*	*RS*	*RK*
$R_m - R_f$	1					
SMB	–0.09	1				
HML	0.05	0.05	1			
RV	–0	0.21	0.08	1		
RS	–0.02	0.09	–0.02	0.29	1	
RK	0.05	–0.05	–0.23	–0.05	0.04	

4.1 Regression results for CAPM model

Table 4 reports the time series regression results of CAPM model for each of the four sets of six portfolios. Panel A presents results of CAPM model for six size-btm portfolios, Panel B for six size-rv portfolios, Panel C for six size-rs portfolios and Panel D for six size-rk portfolios. The average adjusted R-sq.s for CAPM model are 2.22%, 2.43%, 1.89% and 2.28% for six size-btm, six size-rv portfolios, six size-rs portfolios and six size-rk portfolios, respectively, showing the low fit of the model at PSX. CAPM model is reported for comparison purpose later with three factor model. This technique helps in identifying the role of each factor in model development for the period from July 2008 to Aug. 2018.

Table 4 CAPM model

$R_i - R_f$	A	B	t(a)	t(β)	R-sq.	Adj. R-sq.	√SSE
Panel A: CAPM for six size-btm portfolios							
SG	0.166	0.119	0.9	1.73	0.0059	0.0039	94
SN	0.066	0.12	0.5	2.45	0.0116	0.0097	67.11
SV	0.3	0.18	1.79	2.88	0.0159	0.014	85.53
BG	–0.092	0.209	–0.73	4.43	0.037	0.0351	64.46
BN	0.024	0.232	0.19	4.77	0.0426	0.0407	66.42
BV	0.147	0.227	0.99	4.08	0.0316	0.0297	76.2
Panel B: CAPM for six size-rv portfolios							
SL	–0.717	0.104	–4.37	1.7	0.0056	0.0037	83.97
SM	–0.51	0.22	–3.39	3.91	0.0291	0.0272	77.03
SH	1.208	0.091	5.35	1.08	0.0023	0.0003	115.42
BL	–0.174	0.243	–1.69	6.28	0.0716	0.0697	52.89
BM	0.386	0.233	2.56	4.14	0.0325	0.0306	77.05
BH	1.4	0.245	6.18	2.9	0.0161	0.0142	115.76
Panel C: CAPM for six size-rs portfolios							
SL	–0.497	0.098	–2.84	1.5	0.0044	0.0024	89.52
SM	0.078	0.204	0.39	2.74	0.0144	0.0125	101.73
SH	1.545	0.111	7.78	1.49	0.0043	0.0024	101.57
BL	–0.473	0.248	–3.42	4.8	0.0432	0.0413	70.63
BM	0.136	0.21	1.05	4.33	0.0355	0.0336	66.24
BH	0.851	0.201	5.52	3.48	0.0232	0.0213	78.86
Panel D: CAPM for six size-rk portfolios							
SL	0.837	0.116	2.74	1.01	0.002	0.0001	156.17
SM	0.645	0.14	3.32	1.93	0.0072	0.0053	99.5
SH	0.028	0.177	0.18	3.02	0.0176	0.0157	80.17
BL	0.088	0.215	0.7	4.51	0.0383	0.0364	65.05
BM	0.205	0.213	1.47	4.06	0.0313	0.0294	71.56
BH	0.311	0.283	2.17	5.27	0.0516	0.0498	73.48

14 out of 24 intercepts are statistically significant. The values for R-sq. are lower for small firms already indicating the presence of size premium at PSX.

4.2 Regression results for Fama and French (1993) model

Table 5 shows results of Fama and French (1993) model for each of the four sets of six portfolios. The average values of adjusted R-sq. of 25.31%, 7.91%, 9.34% and 9.98% for six size-btm, six size-rv portfolios, six size-rs portfolios and six size-rk portfolios respectively show considerable improvement as compared to CAPM model. The average adjusted R-sq. for small firms in Panel A increases from 0.92% to 34.2% providing evidence that Fama and French (1993) model is better in explaining stock returns at PSX.

This is intuitive as these six portfolios are formed by double sorts on size and btm ratio, therefore SMB and HML can explain their return more effectively. Fama and French (1992) report that stocks having high btm ratio (value stocks) earn higher returns as compared to stocks having low btm ratio (growth stocks).

4.3 Regression results for four factor model

Next, the findings of four factor models for six portfolios sorted on size and btm ratio and each of the realised moments are reported. Table 6, Panel A presents results of four factor model for six size-btm portfolios, Panel B for six size-rv portfolios, Panel C for six size-rs portfolios and Panel D for six size-rk portfolios. Adding RV factor results into average R-sq. values of 40.98%, 31.51%, 30.34% and 29.75% for six size-btm portfolios, six size-rv portfolios, six size-rs portfolios and six size-rk portfolios, respectively. The meaningful improvement in model characteristics provides enough evidence for the explanatory power of RV at emerging stock market of Pakistan. Massive improvement is detected with the addition of RV factor, such as investors are compensated for investing in highly risky small firms. RV anomaly has been priced at PSX. Thus, four factor models is a better fit for PSX.

4.4 Regression results for five factor model

Table 7 reports findings of five factor model by adding RS factor. Enhancement in model characteristics is detected for six size-btm portfolios, six size-rv portfolios, six size-rs portfolios and six size-rk portfolios with the average R-sq. of 41.16%, 31.76%, 37.27% and 29.92%, suggesting the role of RS factor in model development.

4.5 Regression results for six factor model

Six factor model results are presented in Table 8. Improvement is seen after addition of RK factor for six size-btm portfolios, six size-rv portfolios, six size-rs portfolios and six size-rk portfolios as the average standard deviation of error term reduced to 56.58, 68.01, 65.18 and 65.84 from 57.93, 69.43, 67.19 and 77.06, respectively.

All intercept values decrease with addition of factors, assertive of satisfactory performance of augmented factor models in explaining expected stock returns. The standard deviation of error term as reported in the last column of all tables is a measure of unexplained variation of the model. Gradual reduction in these values with each additional factor further confirms the performance of augmented models. Almost all factor loadings for $R_m - R_f$ have significant t-values, reflecting a positive sensitivity to the market. The coefficients of SMBs are positive and significant for small firms and negative and significant for big firms (with a few exceptions), consistent with Hassan (2018). There is adequate evidence to support the existence of size factor at PSX. The loadings on HML are positive for value stocks and negative for growth stocks. This result is consistent with Ali et al. (2018). Thus, the evidence suggests presence of value premium at PSX. Factor loadings on RV factor for four sets of six portfolios are positive and significant except for small firms having low volatility, asserting the role of RV factor in explaining stock returns at PSX, confirming that riskier firms earn higher profit at PSX.

Table 5 Fama and French (1993) model

$R_i - R_f$	*a*	*b*	*S*	*h*	*t(a)*	*t(b)*	*t(s)*	*t(h)*	*R-sq.*	*Adj. R-sq.*	$\sqrt{SSE}$
Panel A: Fama and French (1993) model for six size-btm portfolios											
SG	0.068	0.218	1.042	−0.32	0.48	4.05	17.85	−4.84	0.3999	0.3963	73.04
SN	−0.038	0.153	0.479	0.176	−0.32	3.42	9.85	3.19	0.1883	0.1835	60.82
SV	0.05	0.212	0.743	0.755	0.4	4.51	14.53	13.02	0.4496	0.4463	63.97
BG	0.003	0.188	−0.359	−0.223	0.02	4.24	−7.46	−4.09	0.1618	0.1568	60.14
BN	0.048	0.2	−0.318	0.132	0.38	4.26	−6.23	2.28	0.1167	0.1115	63.8
BV	0.027	0.195	−0.059	0.702	0.21	3.9	−1.09	11.39	0.2285	0.224	68.01
Panel B: Fama and French (1993) model for six size-rv portfolios											
SL	−0.808	0.115	0.264	0.279	−5.04	1.92	4.06	3.78	0.0652	0.0597	81.42
SM	−0.594	0.227	0.209	0.282	−4.03	4.12	3.5	4.17	0.0852	0.0798	74.77
SH	1.024	0.133	0.711	0.423	4.9	1.71	8.39	4.4	0.1574	0.1524	106.07
BL	−0.155	0.223	−0.204	0.06	−1.53	5.88	−4.94	1.29	0.1158	0.1106	51.62
BM	0.386	0.21	−0.207	0.168	2.58	3.75	−3.42	2.45	0.0635	0.058	75.8
BH	1.374	0.239	−0.01	0.148	6.04	2.8	−0.11	1.42	0.02	0.0142	115.53
Panel C: Fama and French (1993) model for six size-rs portfolios											
SL	−0.631	0.116	0.406	0.4	−3.81	1.88	6.04	5.24	0.1208	0.1156	84.13
SM	−0.037	0.233	0.468	0.247	−0.19	3.24	6.01	2.8	0.0957	0.0904	97.45
SH	1.421	0.148	0.548	0.226	7.51	2.08	7.13	2.6	0.1088	0.1035	96.09
BL	−0.416	0.226	−0.296	−0.068	−3.08	4.48	−5.39	−1.09	0.0981	0.0927	68.58
BM	0.167	0.179	−0.331	0.097	1.34	3.82	−6.51	1.68	0.1126	0.1074	63.53
BH	0.882	0.178	−0.26	0.045	5.78	3.11	−4.19	0.63	0.0561	0.0506	77.52
Panel D: Fama and French (1993) model for six size-rk portfolios											
SL	0.604	0.144	0.674	0.718	2.09	1.33	5.73	5.39	0.1153	0.1101	147.04
SM	0.511	0.161	0.425	0.386	2.74	2.31	5.63	4.5	0.1035	0.0983	94.55
SH	−0.086	0.204	0.446	0.254	−0.58	3.7	7.43	3.74	0.1398	0.1347	75.02
BL	0.087	0.181	−0.287	0.243	0.71	3.96	−5.78	4.32	0.1237	0.1185	62.09
BM	0.23	0.185	−0.283	0.095	1.68	3.61	−5.09	1.5	0.0808	0.0754	69.7
BH	0.346	0.272	−0.161	−0.058	2.41	5.07	−2.76	−0.88	0.0675	0.062	72.87

Table 6 Four factor model

$R_i - R_f$	*a*	*b*	*s*	*H*	*rv*	*t(a)*	*t(b)*	*t(s)*	*t(h)*	*t(rv)*	*R-sq.*	*Adj. R-sq.*	√*SSE*
Panel A: four factor model for six size-btm portfolios													
SG	−0.558	0.212	0.923	−0.363	0.372	−3.87	4.33	17	−6.02	10.38	0.5048	0.5009	66.34
SN	−0.622	0.147	0.368	0.136	0.346	−5.33	3.71	8.37	2.78	11.93	0.366	0.361	53.75
SV	−0.536	0.206	0.632	0.714	0.348	−4.32	4.89	13.49	13.73	11.27	0.5597	0.5562	57.21
BG	−0.582	0.182	−0.47	−0.263	0.348	−5.07	4.66	−10.84	−5.47	12.16	0.3507	0.3456	52.93
BN	−0.54	0.194	−0.429	0.091	0.349	−4.36	4.62	−9.21	1.76	11.33	0.295	0.2894	57
BV	−0.596	0.189	−0.178	0.659	0.37	−4.51	4.21	−3.57	11.92	11.27	0.3828	0.378	60.83
Panel B: four factor model for six size-rv portfolios													
SL	−0.557	0.118	0.312	0.296	−0.149	−3.18	1.99	4.74	4.05	−3.44	0.0864	0.0792	80.49
SM	−1.219	0.22	0.09	0.239	0.372	−8.23	4.38	1.62	3.86	10.08	0.2376	0.2316	68.26
SH	−0.498	0.118	0.422	0.318	0.903	−3.03	2.11	6.8	4.62	22.08	0.5699	0.5666	75.78
BL	−0.367	0.221	−0.244	0.046	0.126	−3.34	5.93	−5.89	1	4.6	0.1512	0.1445	50.57
BM	−0.315	0.202	−0.34	0.12	0.416	−2.14	4.06	−6.14	1.95	11.38	0.2537	0.2478	67.67
BH	−0.434	0.22	−0.354	0.024	1.074	−2.68	4	−5.8	0.36	26.66	0.5915	0.5883	74.59
Panel C: four factor model for six size-rs portfolios													
SL	−1.275	0.11	0.284	0.356	0.383	−7.52	1.91	4.44	5.01	9.08	0.2435	0.2375	78.03
SM	−1.059	0.222	0.274	0.177	0.607	−5.83	3.6	3.99	2.33	13.42	0.3324	0.3272	83.73
SH	0.348	0.137	0.344	0.152	0.637	1.99	2.3	5.21	2.08	14.64	0.3732	0.3683	80.59
BL	−1.05	0.22	−0.416	−0.111	0.376	−7.9	4.86	−8.3	−1.99	11.38	0.2813	0.2757	61.22
BM	−0.351	0.173	−0.429	0.061	0.308	−2.77	4.03	−9	1.16	9.78	0.2532	0.2473	58.28
BH	0.015	0.169	−0.425	−0.015	0.515	0.11	3.52	−7.98	−0.25	14.66	0.3368	0.3316	64.98
Panel D: four factor model for six size-rk portfolios													
SL	−0.587	0.131	0.447	0.636	0.707	−2	1.32	4.04	5.18	9.69	0.2534	0.2475	135.08
SM	−0.49	0.15	0.235	0.317	0.595	−2.79	2.52	3.55	4.31	13.6	0.3427	0.3376	80.96
SH	−0.831	0.197	0.304	0.203	0.443	−5.84	4.06	5.66	3.41	12.49	0.342	0.3368	65.61
BL	−0.307	0.177	−0.361	0.216	0.234	−2.39	4.07	−7.48	4.03	7.33	0.2074	0.2012	59.05
BM	−0.503	0.178	−0.423	0.045	0.435	−3.87	4.03	−8.63	0.82	13.46	0.3225	0.3172	59.84
BH	−0.427	0.264	−0.307	−0.111	0.459	−3.15	5.75	−6.02	−1.96	13.63	0.3172	0.3118	62.35

Table 7 Five factor model

$R_i - R_f$	*a*	*b*	*s*	*h*	*Rv*	*rs*	*t(a)*	*t(b)*	*t(s)*	*t(h)*	*t(rv)*	*t(rs)*	*R-sq.*	*Adj. R-sq.*	*√SSE*
Panel A: five factor model for six size-btm portfolios															
SG	–0.544	0.212	0.923	–0.364	0.374	–0.01	–3.38	4.32	16.98	–6.02	10.01	–0.2	0.5049	0.5	66.34
SN	–0.69	0.148	0.367	0.139	0.336	0.051	–5.3	3.73	8.34	2.83	11.13	1.18	0.3677	0.3615	53.68
SV	–0.638	0.207	0.63	0.718	0.333	0.076	–4.62	4.92	13.46	13.82	10.37	1.67	0.5621	0.5577	57.06
BG	–0.684	0.183	–0.472	–0.259	0.333	0.076	–5.35	4.69	–10.91	–5.39	11.19	1.79	0.3548	0.3484	52.76
BN	–0.611	0.195	–0.431	0.094	0.338	0.053	–4.43	4.63	–9.24	1.82	10.56	1.17	0.2969	0.2899	56.92
BV	–0.58	0.189	–0.178	0.658	0.373	–0.012	–3.93	4.2	–3.56	11.88	10.88	–0.25	0.3829	0.3768	60.82
Panel B: five factor model for six size-rv portfolios															
SL	–0.55	0.118	0.312	0.296	–0.148	–0.005	–2.82	1.98	4.73	4.04	–3.27	–0.07	0.0864	0.0774	80.49
SM	–1.231	0.22	0.09	0.24	0.37	0.009	–7.44	4.37	1.61	3.86	9.62	0.16	0.2377	0.2302	68.25
SH	–0.653	0.119	0.419	0.325	0.881	0.116	–3.57	2.14	6.77	4.72	20.71	1.91	0.573	0.5688	75.5
BL	–0.442	0.222	–0.245	0.049	0.115	0.056	–3.61	5.96	–5.93	1.06	4.03	1.38	0.1544	0.146	50.48
BM	–0.481	0.204	–0.343	0.127	0.391	0.124	–2.95	4.11	–6.22	2.06	10.32	2.3	0.2614	0.2541	67.32
BH	–0.35	0.219	–0.352	0.021	1.086	–0.063	–1.94	3.98	–5.77	0.31	25.88	–1.05	0.5924	0.5883	74.51
Panel C: five factor model for six size-rs portfolios															
SL	–0.527	0.102	0.298	0.326	0.493	–0.559	–3.03	1.94	5.08	4.99	12.21	–9.73	0.3624	0.3561	71.64
SM	–1.028	0.222	0.274	0.176	0.612	–0.023	–5.07	3.59	4	2.31	12.97	–0.35	0.3326	0.326	83.72
SH	–0.608	0.146	0.326	0.191	0.497	0.714	–3.57	2.81	5.66	2.98	12.56	12.68	0.5242	0.5195	70.21
BL	–0.632	0.216	–0.408	–0.128	0.438	–0.313	–4.44	4.97	–8.48	–2.39	13.24	–6.64	0.3388	0.3323	58.72
BM	–0.477	0.175	–0.432	0.066	0.289	0.094	–3.39	4.07	–9.07	1.25	8.85	2.02	0.2592	0.2519	58.05
BH	–0.537	0.174	–0.435	0.007	0.433	0.413	–3.65	3.88	–8.73	0.13	12.65	8.47	0.419	0.4132	60.82
Panel D: five factor model for six size-rk portfolios															
SL	–0.43	0.13	0.45	0.63	0.73	–0.117	–1.32	1.3	4.07	5.12	9.61	–1.08	0.2551	0.2478	134.92
SM	–0.597	0.151	0.233	0.321	0.579	0.08	–3.05	2.54	3.52	4.36	12.71	1.23	0.3447	0.3382	80.84
SH	–0.87	0.197	0.303	0.205	0.437	0.029	–5.47	4.07	5.64	3.43	11.83	0.54	0.3423	0.3359	65.59
BL	–0.378	0.178	–0.363	0.219	0.223	0.054	–2.65	4.09	–7.5	4.08	6.72	1.13	0.2094	0.2016	58.98
BM	–0.583	0.179	–0.424	0.048	0.423	0.06	–4.03	4.05	–8.66	0.88	12.58	1.26	0.3247	0.318	59.75
BH	–0.502	0.265	–0.309	–0.108	0.448	0.056	–3.33	5.77	–6.05	–1.91	12.77	1.13	0.3189	0.3122	62.27

Table 8 Six factor model

$R_i - R_f$	*a*	*b*	*s*	*h*	*rv*	*Rs*	*rk*	*t(a)*	*t(b)*	*t(s)*	*t(h)*	*t(rv)*	*t(rs)*	*t(rk)*	*R-sq.*	*Adj. R-sq.*	$\sqrt{SSE}$
Panel A: six factor model for six size-btm portfolios																	
SG	−0.594	0.225	0.915	−0.43	0.368	0.003	−0.177	−3.76	4.69	17.19	−7.08	10.05	0.05	−4.8	0.5265	0.5209	64.88
SN	−0.733	0.16	0.36	0.081	0.331	0.062	−0.154	−5.76	4.13	8.38	1.64	11.22	1.48	−5.18	0.3996	0.3924	52.31
SV	−0.679	0.219	0.623	0.664	0.328	0.087	−0.146	−5	5.28	13.57	12.67	10.41	1.94	−4.58	0.5795	0.5745	55.91
BG	−0.723	0.194	−0.478	−0.312	0.328	0.086	−0.139	−5.76	5.06	−11.29	−6.44	11.26	2.08	−4.75	0.3823	0.375	51.62
BN	−0.658	0.208	−0.439	0.031	0.333	0.066	−0.167	−4.88	5.06	−9.64	0.6	10.64	1.47	−5.3	0.3338	0.3259	55.41
BV	−0.628	0.202	−0.186	0.594	0.367	0	−0.171	−4.35	4.59	−3.81	10.69	10.96	0.01	−5.06	0.4127	0.4057	59.34
Panel B: six factor model for six size-rv portfolios																	
SL	−0.589	0.129	0.306	0.244	−0.153	0.006	−0.138	−3.04	2.18	4.67	3.27	−3.4	0.09	−3.05	0.1029	0.0922	79.76
SM	−1.3	0.239	0.079	0.148	0.362	0.027	−0.245	−8.16	4.93	1.47	2.41	9.78	0.51	−6.57	0.2976	0.2892	65.52
SH	−0.724	0.139	0.407	0.229	0.872	0.135	−0.254	−4.09	2.57	6.81	3.36	21.22	2.3	−6.13	0.6025	0.5978	72.85
BL	−0.478	0.232	−0.251	0	0.11	0.066	−0.13	−3.98	6.34	−6.19	0	3.95	1.65	−4.61	0.1884	0.1788	49.45
BM	−0.529	0.217	−0.351	0.063	0.386	0.137	−0.169	−3.29	4.44	−6.48	1.02	10.36	2.58	−4.52	0.29	0.2816	66
BH	−0.354	0.22	−0.353	0.016	1.086	−0.062	−0.014	−1.95	3.99	−5.77	0.22	25.84	−1.03	−0.33	0.5925	0.5876	74.5
Panel C: six factor model for six size-rs portfolios																	
SL	−0.555	0.11	0.293	0.288	0.489	−0.551	−0.101	−3.21	2.09	5.02	4.32	12.18	−9.63	−2.49	0.3701	0.3627	71.2
SM	−1.152	0.256	0.254	0.01	0.596	0.009	−0.441	−6.21	4.53	4.05	0.14	13.86	0.15	−10.16	0.4456	0.4391	76.3
SH	−0.644	0.156	0.32	0.143	0.492	0.723	−0.126	−3.81	3.02	5.6	2.2	12.55	12.95	−3.19	0.5336	0.528	69.52
BL	−0.668	0.225	−0.414	−0.176	0.433	−0.303	−0.128	−4.75	5.26	−8.72	−3.25	13.28	−6.52	−3.9	0.3581	0.3505	57.85
BM	−0.533	0.19	−0.441	−0.009	0.283	0.109	−0.199	−3.92	4.59	−9.6	−0.16	8.96	2.42	−6.26	0.3124	0.3042	55.93
BH	−0.566	0.182	−0.44	−0.032	0.43	0.421	−0.103	−3.86	4.08	−8.89	−0.56	12.64	8.68	−3	0.4292	0.4224	60.29
Panel D: six factor model for six size-rk portfolios																	
SL	−0.825	0.238	0.385	0.102	0.682	−0.013	−1.405	−4.33	4.11	5.98	1.38	15.43	−0.2	−31.51	0.7486	0.7456	78.39
SM	−0.663	0.169	0.222	0.234	0.571	0.097	−0.233	−3.46	2.9	3.44	3.17	12.85	1.53	−5.2	0.3779	0.3706	78.76
SH	−0.841	0.189	0.308	0.244	0.441	0.021	0.104	−5.31	3.92	5.77	4	12	0.4	2.8	0.3524	0.3447	65.09
BL	−0.455	0.199	−0.375	0.116	0.214	0.074	−0.273	−3.41	4.89	−8.32	2.26	6.89	1.67	−8.75	0.3133	0.3052	54.97
BM	−0.636	0.193	−0.433	−0.023	0.417	0.074	−0.188	−4.52	4.5	−9.11	−0.42	12.76	1.6	−5.7	0.3654	0.3579	57.91
BH	−0.441	0.248	−0.299	−0.026	0.455	0.04	0.218	−3.03	5.6	−6.08	−0.47	13.48	0.84	6.41	0.3701	0.3626	59.89

Coefficients for RS factor are mostly positive. On the contrary, kurtosis factor have negative loadings, significant at 1% level. Investors are compensated for investing in highly skewed (either positive or negative) firms and firms having excess kurtosis at PSX. Loadings on RS factor are negative and significant for both small and big firms having low or negative skewness for six size-rs portfolios, which suggests that investors are compensated with negative risk premium by taking left-tailed skewness risk. However, positive and significant for both small and big firms having high or positive skewness, implying higher profit for positively skewed stocks. There is adequate evidence for risk premium for realised moment factors at PSX.

Emerging markets all over the world, e.g., China, Iran, Bangladesh, etc. have different market structures, legal environment, investors' knowledge, etc. PSX exhibits peculiar characteristics of an emerging market, such as enforcement of narrow scrip-wise circuit breakers/price limits at 7.5% and rightly skewed distribution of stocks based on size and volume. Hence, the results of this study should be interpreted with caution, as the evidence may not be generalisable to other emerging markets.

5 Conclusions

This study computes realised moments based on methodology introduced by Andersen et al. (2001) by taking five-minute prices extracted from tick by tick data and followed gradual model development by adding size and value factor to CAPM model and then each realised moment factor to Fama and French (1993) model to analyse whether realised moments explain expected stock market returns. Following Fama and French (1993), six portfolios are formed by bivariate sorts on size and btm ratio. Similarly, six portfolios are formed on size and each of the realised moments. Such as 24 portfolios are formed and thus five models are computed for each of the portfolios, resulting into 120 regressions.

The four factor model highly explains the equity returns at PSX after adding RV factor to Fama and French (1993) model (the adjusted R-square improves meaningfully). Similarly, the addition of RS factor to four factor model and the addition of RK factor to five factor model shows improvement in model characteristics verifying that the six factor model explains the variations in average stock returns at PSX. Evidence suggests that investors are compensated by their exposure to skewness risk because investors want to invest in securities having high positive skewness to earn from extreme returns (e.g., Harvey and Siddique, 2000). Ghysels et al. (2016) report analogous findings for emerging stock markets. Thus, this study finds that market risk premium, SMB, HML, RV and RK factors are important for explaining average stock returns at PSX, with slightly less contribution from RS factor. The departure from normality is primarily driven by kurtosis (Chen et al., 2019).

Investors can make informed investment decisions by following a disciplined framework and utilising diversification to earn relatively high returns across different asset classes even in the worst of times. Investors and portfolio managers can take long position in high volatile stocks and a short position in low volatile stocks to earn superior returns. In addition, fund managers can construct smart portfolios by adopting strategy of going long on high volatile stocks having high (positive) skewness and going short on low volatile stocks having low (negative) skewness, to get higher returns at the emerging stock market of Pakistan. Incorporating kurtosis factor, further improves the performance of a portfolio, i.e., the long-short positions that go long on high volatile stocks having low kurtosis values and go short on low volatile stocks having high kurtosis provide additional benefits to investors and portfolio managers. Lastly, this paper examines the impact of realised higher moments in explaining stock returns. However, future research can check the role of realised measures to predict the stock market volatility.

Acknowledgements

We are thankful to the management of PSX for providing a firm level tick by tick data.

References

Aggarwal, R., Rao, R.P. and Hiraki, T. (1989) 'Skewness and kurtosis in Japanese equity returns: empirical evidence', *The Journal of Financial Research*, Vol. 12, No. 3, pp.253–260.

Ait-Sahalia, Y. and Yu, J. (2009) 'High frequency market microstructure noise estimates and liquidity measures', *The Annals of Applied Statistics*, Vol. 3, No. 1, pp.422–457.

Ali, F., He, R. and Jiang, Y. (2018) 'Size, value and business cycle variables. The three-factor model and future economic growth: evidence from an emerging market', *Economies*, Vol. 6, No. 1, p.14.

Amaya, D., Christoffersen, P., Jacobs, K. and Vasquez, A. (2015) 'Does realized skewness and kurtosis predict the cross-section of equity returns?', *Journal of Financial Economics*, Vol. 118, No. 1, pp.135–167.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Ebens, H. (2001) 'The distribution of realized stock return volatility', *Journal of Financial Economics*, Vol. 61, No. 1, pp.43–76.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Labys, P. (2003) 'Modeling and forecasting realized volatility', *Econometrica*, Vol. 71, No. 2, pp.579–625.

Ang, A., Hodrick, R.J., Xing, Y. and Zhang, X. (2006) 'The cross-section of volatility and expected returns', *Journal of Finance*, Vol. 61, No. 1, pp.259–299.

Banz, R.W. (1981) 'The relationship between return and market value of common stocks', *Journal of Financial Economics*, Vol. 9, No. 1, pp.3–18.

Barndorff-Nielsen, O.E., Kinnebrock, S. and Shephard, N. (2010) 'Measuring downside risk: realised semivariance', in Bollerslev, T., Russell, J. and Watson, M. (Eds.): *Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle*, Oxford University Press, Oxford.

Berk, J.B. and Van Binsbergen, J.H. (2016) 'Assessing asset pricing models using revealed preference', *Journal of Financial Economics*, Vol. 119, No. 1, pp.1–23.

Black, F., Jensen, M.C. and Scholes, M. (1972) 'The capital asset pricing model: some empirical tests', in *Studies in the Theory of Capital Markets*, pp.79–121, Paeger Publishers Inc., New York.

Chaudhary, R., Misra, D. and Bakhshi, P. (2020) 'Conditional relation between return and co-moments – an empirical study for emerging Indian stock market', *Investment Management & Financial Innovations*, Vol. 17, No. 2, p.308.

Chen, G., Ding, J. and Zhao, X. (2019) 'A lottery-preference-based explanation of realized kurtosis puzzle in Chinese stock market', *Applied Economics*, Vol. 51, No. 50, pp.5466–5481.

Chunhachinda, P., Dandapani, K., Hamid, S. and Prakash, A.J. (1997) 'Portfolio selection and skewness: evidence from international stock markets', *Journal of Banking and Finance*, Vol. 21, No. 2, pp.143–167.

Fama, E.F. and French, K.R. (1992) 'The cross-section of expected stock returns', *Journal of Finance*, Vol. 47, No. 2, pp.427–465.

Fama, E.F. and French, K.R. (1993) 'Common risk factors in the returns on stocks and bonds', *Journal of Financial Economics*, Vol. 33, No. 1, pp.3–56.

Ghysels, E., Plazzi, A. and Valkanov, R. (2016) 'Why invest in emerging markets? The role of conditional return asymmetry', *The Journal of Finance*, Vol. 71, No. 5, pp.2145–2192.

Harvey, C.R. and Siddique, A. (2000) 'Conditional skewness in asset pricing tests', *Journal of Finance*, Vol. 55, No. 3, pp.1263–1295.

Hassan, W. (2018) *Agency-based Asset Pricing Model: Empirical Evidence from Pakistan Stock Exchange*, Doctoral dissertation, Capital University [online] https://thesis.cust.edu.pk/UploadedFiles/Wasif%20Hassan-MM141019.pdf (accessed 2 May 2019).

Hwang, S. and Satchell, S.E. (1999) 'Modeling emerging market risk premia using higher moments', *International Journal of Finance and Economics*, Vol. 4, No. 4, pp.271–296.

Iqbal, J. and Brooks, R. (2007) 'Alternative beta risk estimators and asset pricing tests in emerging markets: the case of Pakistan', *Journal of Multinational Financial Management*, Vol. 17, No. 1, pp.75–93.

Jondeau, E. and Rockinger, M. (2004) 'Conditional volatility, skewness and kurtosis: existence, persistence, and co-movements', *Journal of Economics Dynamics and Control*, Vol. 27, No. 10, pp.1699–1737.

Kim, T.H. and White, A. (2004) 'On more robust estimation of skewness and kurtosis: simulation and application to the S&P 500 index', *Finance Research Letters*, Vol. 1, No. 1, pp.56–70.

Kraus, A. and Litzenberger, R. (1976) 'Skewness preference and the valuation of risk assets', *Journal of Finance*, Vol. 31, No. 4, pp.1085–1100.

Lai, K.K., Yu, L. and Wang, S. (2006) 'Mean-variance-skewness-kurtosis-based portfolio optimization', in *First International Multi-Symposiums on Computer and Computational Sciences (IMSCCS'06)*, IEEE, June, Vol. 2, pp.292–297.

Lintner, J. (1965) 'The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets', *The Review of Economics and Statistics*, Vol. 47, No. 1, pp.13–37.

Lux, T. and Marchesi, M. (2000) 'Volatility clustering in financial markets: a micro simulation of interacting agents', *International Journal of Theoretical and Applied Finance*, Vol. 3, No. 4, pp.675–702.

Markowitz, H. (1959) *Portfolio Selection: Efficient Diversification of Investments*, Wiley, New York.

Mensi, W., Maitra, D., Vo, X.V. and Kang, S.H. (2021) 'Asymmetric volatility connectedness among main international stock markets: a high frequency analysis', *Borsa Istanbul Review*, Vol. 21, No. 3, pp.291–306.

Merton, R. (1973) 'An intertemporal capital asset pricing model', *Econometrica*, Vol. 41, No. 5, pp.867–887.

Merton, R. (1980) 'On estimating the expected return on the market: an exploratory investigation', *Journal of Financial Economics*, Vol. 8, No. 4, pp.323–361.

Mirza, N. and Shahid, S. (2008) 'Size and value premium in Karachi Stock Exchange', *Lahore Journal of Economics*, Vol. 13, No. 2, pp.1–26.

Mossin, J. (1966) 'Equilibrium in a capital asset market', *Econometrica*, Vol. 34, No. 4, pp.768–783.

Naqvi, B., Mirza, N., Naqvi, W. and Rizvi, S. (2017) 'Portfolio optimization with higher moments of risk at the Pakistan Stock Exchange', *Economic Research –Ekonomska Istrazivanja*, Vol. 30, No. 1, pp.1594–1610.

Paye, B. (2012) 'Déjà vol: predictive regressions for aggregate stock market volatility using macroeconomic variables', *Journal of Financial Economics*, Vol. 106, No. 3, pp.527–546.

Roll, R. (1977) 'A critique of the asset pricing theory's tests Part I: on past and potential testability of the theory', *Journal of Financial Economics*, Vol. 4, No. 2, pp.129–176.

Ross, S.A. (1976) 'The arbitrage theory of capital asset pricing', *Journal of Economics Theory*, Vol. 13, No. 3, pp.341–360.

Schwert, G.W. (1989) 'Why does stock market volatility change over time?', *Journal of Finance*, Vol. 44, No. 5, pp.1115–1153.

Sharpe, W.F. (1964) 'Capital asset prices: a theory of market equilibrium under conditions of risk', *Journal of Finance*, Vol. 19, No. 3, pp.425–442.

Tversky, A. and Kahneman, D. (1986) 'Rational choice and the framing of decisions', *The Journal of Business*, Vol. 59, No. 4, pp.251–272.

Ward, M. and Muller, C. (2013) 'Empirical testing of the CAPM on the JSE', *Investment Analysts Journal*, Vol. 76, No. 1, pp.1–12.

Wu, X., Xia, M. and Zhang, H. (2020) 'Forecasting VaR using realized EGARCH model with skewness and kurtosis', *Finance Research Letters*, Vol. 32, No. 1, p.101090.