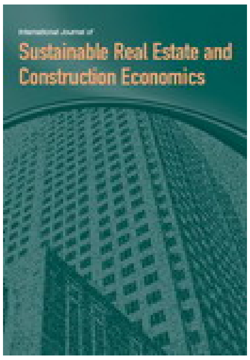

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Advanced excel programming of time value of money in the context of real estate financing

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Abstract: The research objective of this paper is to develop a practical and functional software tool to serve the needs of both practitioners and researchers in the mortgage industry. The software development capitalises on published research available in the literature, as documented by the cited references. The spreadsheet tool presented in this paper carries out computational and graphical analyses of loan-based investments at the intersection of real estate and economics. The effort is thereby designed to referee the impact of the time value of money on real estate finance from a capital investment perspective. Based on specific data entries, the utilised methodology leverages basic and advanced modules to generate comprehensive information for the assessment of mortgage repayments or the corporate financial strategy related to real estate financing scenarios. The paper has practical implications, particularly in the light of real estate transactions: The computational spreadsheet program modules, calculations and visualisations can pertinently guide mortgage situations and provide a fertile toolbox for financial decisions made by investors. Amid the recent substantial interest rate risings and increase in real estate finance cost, the presented methodology with dashboard-style elements is designed to boost transparency in loan-based property financing.

Keywords: time value of money; real-estate investment; loan amortisation; corporate financial strategy; equity break-even point.

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1 Introduction

It is often said that charity begins at home. By reverse analogical saying, the best home financial practice begins in the corporate world. This vice-versal relationship benefits both personal finance as well as corporate financial transactions. Capitalising on conventional corporate economic computations, Badiru (1986) developed a spreadsheet tool called Graphical Analysis of Mortgage Payments (GAMPS). The methodological knowledge gained from that initial software led to the need to extend the approach to general corporate instalment payments. There are plenty scholarly approaches to frame the mathematical collection that is under the hood of financial investment analysis in the context of real estate transactions. We consider, i.e., Arefaine (1988), Kurtz (1995), Randle and Johnson (1996), Badiru (2016a) and Badiru (2016b) to be part of the theoretical basis. Nonetheless, an identified research challenge is observable in a 'one size fits all' approach, which incorporates most relevant factors along the spectrum of real estate financing issues in a common spreadsheet program to create maximum financial transparency. Practical implications of the scholarly examination arise from offering application opportunities for specific data entries, tailored to actual situations of the loan-financing decision the investor faces, respectively. In addition to that, we endorse the academic aspiration to include certain pertinent *advanced* computations, considering the time value of money with regard to miscellaneous aspects of loan amortisation, such as refinancing. Not least a profound visual analysis of the mathematical framework completes the comprehensive analytical approach. The modified spreadsheet program we derived and applied is named *XGAMPS* (for Extended Xcell-based GAMPS). Although the narratives and examples use mortgage payments as the tool basis, the methodology presented in this paper is equally applicable to general corporate finance and amortisation schedules.

The originality of this research is affirmed by the original and novel application of the strong mathematical foundation that connects real estate and economic analysis. The underlying calculations enable a multifaceted question-resolving approach to deal with various real estate financing questions from a capital investment perspective. Additional originality of the research stems from the advancement and upgrade of the former GAMPS software tool (Badiru, 1986) into a modern MS EXCEL-based spreadsheet program. Its performance is comprehensively demonstrated by applying original mortgage repayment scenarios.

The capabilities of the program in terms of computational and graphical evaluations include:

- Calculation of basic loan information: equal monthly payment rate, total loan costs and total interest amount, interest charge, equity break-even point.
- Calculation of specific loan information on an arbitrary instalment payment: remaining unpaid balance, equity portion, interest charge, cumulative principal payment, cumulative interest payment and cumulative total payment.
- Advanced computation of the affordable size loan for a disposable monthly payment rate.
- Advanced computations regarding the impacts of continuous monthly extra payments: changed payoff period, saved time periods, saved interest charge, alternative investment.
- Advanced computations on a refinancing option: monthly payment rate, total loan costs, overall payoff period, break-even period of the refinancing loan, present value of the cumulative cost savings of the refinancing loan.
- Numerous graphical analyses with linkage to the computations, e.g., cumulative equity compared to the unpaid balance over the loan term (with break-even point), unpaid balance after a certain number of payments compared to the total principal amount, cumulative interest compared to cumulative equity, graphical evaluations on a specific payment.

1.1 Research background

Key interest rates have been vigorously raised by central banks all over the world in 2022 with an upward trend, including the current interest rate of the Federal Reserve in the USA. Yields on 10-year US Treasury bonds have more than doubled so far this year. These are relevant factors that undeniably influence the nationwide development of mortgage interest rates. The numbers may seem abstract and distant. Nevertheless, the loan environment for borrowers investing in real estate is about to become more challenging, perhaps even obstructive. We can demonstrate some practical impacts of the recent trend for mortgagors as well as for prospective buyers of debt-financed home property by applying different features of the computational tool.

As of Mid-September 2022, the USA weekly average for 30-year interest rates on a fixed-rate basis has exceeded the 6%-threshold for the first time since 2008 (Cook, 2022; Freddie Mac, 2022). By the end of October 2022, 30-year fixed even reached more than 7% (Cook, 2022) – a peak in more than two decades and at the same time a perplexity-fomenting status without any signs of poising to ease. Average 15-year mortgage rates have soared to more than 5.2% as of Mid-September 2022, (Cook, 2022; Freddie Mac, 2022), by the end of September 2022 then further skyrocketed to almost 6% (Orton, 2022). In comparison, the average 30-year rate was about 3.22% in the first week of the year 2022, and the 15-year rate at 2.43% (Freddie Mac, 2022). Merely to add further numbers to illustrate the trend, the year 2021 started with historically low rates at about 2.65% for a 30-year fixed-rate mortgage and 2.16% on a 15-year basis – whereas the 15-year value was undercut by a small margin later in 2021, e.g., 2.1% in late July 2021 (Freddie Mac, 2022).

Prior to the phenomenon of sharply rising interest rates, prices for home property soared over the last years. This development aggravates the precarious situation property investors already find themselves in. Enhanced usage of remote work opportunities during the COVID-19 plays an essential role in the remarkable surge in home pricing – US home prices, i.e., increased by more than 20% from November 2019 to November 2021, and apart from that, more than half of the price jump is linked with working from home during the Coronavirus pandemic (Barrabi, 2022). Own property that provides flexible, respectively, save work site has gained in value for employees. According to McMillin (2022) and Freddie Mac (2022), home property price growth in the USA is still estimated at more than 10% in 2022, despite the interest rate shock environment.

This recent development reinforces the motivation to create an appropriate instrument for maximum transparency of the financial decision behind loan-based real estate investments. How can an investor get comprehensive, dashboard-like information on the respective financial decision linked with loan financing at every relevant angle and with respect to the actual data input in the situation to be faced, respectively? On these grounds, we apply the methodology of a redesigned and extended spreadsheet tool, whose functionalities and outputs are part of the following part of the scientific approach. The premise of this paper is thereby not so much as identifying a research gap. Rather, the gap is the lack of a practical software tool to aid the decision process and policy development of practitioners, who may not be fully familiar with the intricacies of how time value of money works in the mortgage payment management context.

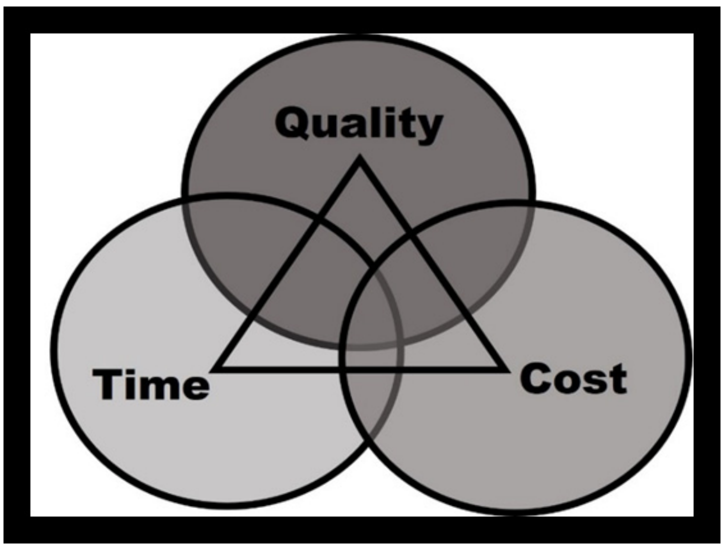
2 Methodology: computational tool

2.1 Quality-time-cost trade-off

Financial management begins the following essence: *Quality*, *time* and *cost* are the three momentous parameters, the nucleus, to assess the feasibility as well as efficiency of a financial decision to be carried out (see Figure 1), from a Quality-Time-Cost Trade-off Investment Space. To apply these parameters to a mortgage loan situation and to provide a pertinent set of information on the time value of invested money, it is crucial to reach the essence of this capital investment case. Loan-based home property purchases require capital to be employed through the home investment, by either accumulating equity and avoiding rental payments, or generating revenues. Besides the typical loan costs, such as interest charges, also the opportunity costs should be taken into account. The analysis of the advantageousness of extra payments is related to this matter, among others. Additional payments could likewise be invested alternatively, yielding returns, such as savings plans or treasuries. In the light of the compound interest effect, the time value dimension is addressed. Apart from that, refinancing considerations also cover the aspect of the time value of money. The savings from lower interest rates, i.e., are to be considered at their present value when deciding on possible refinancing options.

Suppose the *quality* dimension of a mortgage case mainly involves the investment value, which is usually determined by the major real estate features. These include, i.e., the room number, year of construction, location, condition, substance, special components or equipment of the real estate property. Since this is a loan-financed investment, the expression of each quality attribute affects the mortgage affordability and thus the **cost** dimension. Basically, the extent of the parameters ramps up or drops the value and, therefore, triggers the quality parameter. However, the market price of residential properties can also be driven by irrational aspects such as market surcharges due to speculations, or risk discounts on account of collapsing real estate markets. Quality surcharges or discounts have an impact on cost. The *time* parameter, in turn, is particularly influenced by the term of the loan. A stretched repayment plan might help enhance principal and thus finance property of higher quality. But likewise, an extended amortisation schedule means that the overall costs of the loan skyrocket. Extra payments, on the contrary, are a proven way to shorten the loan repayment period. They imply plunging total loan costs. Consequently, there are multifarious interrelations between the three investment parameters. They are all necessary for the computational and graphical evaluations of mortgage payments in an investment decision setting.

Figure 1 Quality-time-cost trade-off investment space



2.2 Methodical approach

To achieve the purpose of boosted transparency on the decision of loan-based financing in light of a real estate investment case, we have chosen an MS EXCEL-based approach. The program *XGAMPS* allows to enter user-specific data set. It is the calculation tool to evaluate the financial impacts, respectively, the time value of money of a particular mortgage payment plan based on according data entries. With the help of mathematical equations, the entered data set is transferred to data output. Besides a basic sphere, the tool includes an advanced sphere, which is an optional part and designed to address certain additional circumstances that may occur in the respective real estate investment situation. *XGAMPS* primarily includes the data entry and data output sheet, while the corresponding tabulations, calculations and graphical visualisations are derived and processed in the background on additional data sheets (see Figure 2). We examine data entry examples that are suitable to illustrate the informational effects from an application of the spreadsheet tool. Data entries are to be based on actual loan conditions in the respective practical case.

Figure 2 MS-excel-based computational tool – sheets overview

2.3 Basic computations

2.3.1 Data input sheet

Data entries are required to generate computational and graphical analyses of the input. The data input cells are arranged in a logical order and obligatorily involve a set of general loan information.

Figure 3 extracts the necessary entries on the data input sheet: besides the principal *P* in US-$ (cell F3), also the annual interest rate *i* in % (F5), the loan term in *n* years (F7) and the start date of the mortgage term (F13) are required. F13 falls back to the current date inasmuch as no entry is made. F9 in turn is completed automatically. As the tool is based on a monthly payment basis, its content reflects the loan term *n* in months (m). Finally, F15 is needed for the evaluation of specific details of an arbitrary payment period *t* within *n*. For *n* = 360 (m), *t* may take any value from 1 to 360.

Figure 3 Data input extract – required basic loan details

	A	B–E	F
1	**Basic Loan Details (Data Input Only)**		
2			
3	1	Enter **loan amount (*P*)** of the mortgage:	$500,000.00
4			
5	2	Enter **annual interest rate (*i*)** in percentage:	8.00%
6			
7	3	Enter the **term of the loan (*n*)** in years:	30
8			
9	**NOTE:** Calculation/data output will be carried out in **months.** Term of the loan in months (***Automatic cell completion***):		360
12			
13	4	Enter the **start date** of the mortgage (format MM/DD/YYYY): **NOTE:** If no entry is made, today's date will be applied.	1/1/2023
14			
15	5	Enter an arbitrary **payment period (t)** for specific data output:	**180**

Important assumptions for the mortgage calculations are:

- The principal *P* is borrowed on a fixed-rate basis.
- The annual interest rate *i* remains constant over the whole maturity.
- No other loan costs than the interest charge, e.g., insurance or taxes, are anticipated.
- The mortgage is amortised by a series of equal end-of-period monthly instalments.

2.3.2 Data output sheet

The basic data output section applies the data input to create general mortgage information. They are reproduced on the data output sheet (see Figure 4) and entail the constant monthly payment rate (F3), the scheduled number of payments (F5), at this point excluding eventual extra payments, the start date (F7), the last month of the mortgage payments (F9), the total loan costs (F11), the total interest charge (F13), respectively, the percentage of the interest amount in the total mortgage cost (F15) and the break-even period in which the remaining unpaid loan balance equals the cumulative equity (F17). The break-even point is computed by an application of the analytical, mathematical, and graphical frameworks of Badiru (1986) and Badiru (2016a, 2016b).

Figure 4 Data output extract – basic mortgage information (time and cost dimension)

	A	B–E	F
1	**Basic Loan Information (Data Output Only)**		
2			
3	1	Equal payments for each month (without extra payments):	**$3,668.82**
4			
5	2	Scheduled total number of equal payments is:	360
6			
7	4	Actual start date of the loan:	1/1/2023
8			
9	5	Last payment due in the following month:	Dec-2045
10			
11	6	Total costs of the loan (total payment made on the loan):	**$1,094,164.74**
12			
13	7	Total interest amount:	$594,164.74
14			
15	8	Percentage of the interest charge of total loan costs:	54.30%
16			
17	9	**Break-even point** (period in time when the **unpaid balance = the cumulative principal payment**) at:	**193.99**

A further part of the basic data output section incorporates specific loan information on instalment payment *t*. The computational view on this matter is shown in Figure 5. According equations from Badiru and Omitaomu (2011) and Badiru (2016a, 2016b) are applied for this endeavour. In the course of this, F21 represents the unpaid balance after *t* mortgage payments. F23/F24 includes the equity portion at *t* both as a number and as a percentage. The interest charge, both as a number and percentage, is shown in F26/F27. In addition, cell F29 contains the cumulative total payments made up to instalment *t*. In case *n* is entered for *t*, this cell reveals the total loan cost. Cells F31/F32 comprise the cumulative principal payments after *t* periods. The cumulative interest payment after *t* instalments is shown in cell F34 as a number, respectively, in cell F35 as a percentage.

Figure 5 Data output extract – specific loan information on instalment payment *t*

	A	B	C	D	E	F
19	**Loan Information for Installment Payment *t* (Data Output Only)**					
20						
21	10a	**Unpaid balance** after ***t*** installment payments:				$280,096.33
22						
23	10b	**Principal amount (equity portion)** of installment payment ***t:***				$2,087.60
24						52.60%
25						
26	10c	**Interest charge** in installment payment ***t:***				$1,881.23
27						47.40%
28						
29	10d	**Cumulatative total payment** after ***t*** periods:				$714,388.12
30						
31	10e	**Cumulative principal payment** (**equity**) after ***t*** periods (contained in the cumulative total payment):				$219,903.67
32						30.78%
33						
34	10f	**Cumulative interest payment** after ***t*** periods (contained in the cumulative total payment):				$494,484.45
35						69.22%

2.3.3 Graphical analysis

The following description of the graphical analysis of the basic loan data is divided into two parts – while the first part covers the graphical analysis of the basic mortgage information in the narrow sense, the second part deals with the specific information particularly provided for the payment *t*.

2.3.4 Graphical analysis of the basic loan information

Figure 6 is based on a pie chart and illustrates the cumulative interest portion of the whole loan cost. In particular, this graphic represents the percentage ratio between the total interest costs and the total loan costs. The percentage corresponds to the data output value of F15. In the example shown below (see Figure 6), interest payments represent more than 50% of the total mortgage costs.

The line chart of Figure 7 contains two curves: the unpaid balance and the equity curve. Whereas the unpaid balance drops with a rising number of payments, the cumulative principal amount increasingly accumulates. Both curve slopes simultaneously depend on the loan term n and in particular on the interest rate i. The higher i, the stronger the compound interest effect, leading to a steeper slope of both curves. The length of n moreover stretches the curve. Initially, the principal portion is lower than in later payments. The proportion and thereby the cumulative equity soars with every payment, ever faster with more instalments. Incidentally, the cumulative equity curve arrives at the principal amount P with the last payment. This is the point of full loan amortisation with the unpaid balance reaching 0$. The intersection of the curves is the break-even period, where the cumulative equity equals the unpaid balance. Figure 7 graphically highlights this point in time when the remaining unpaid balance of a loan is balanced by enough accumulated equity, so that the loan balance could be paid off by the cash equivalent of the equity (Badiru, 1986).

Figure 6 Graphical analysis – percentage interest charge of total mortgage costs (see online version for colours)

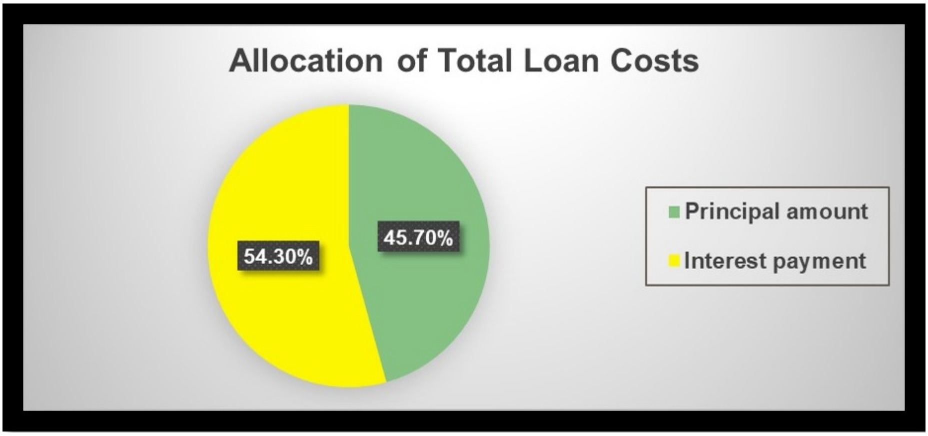
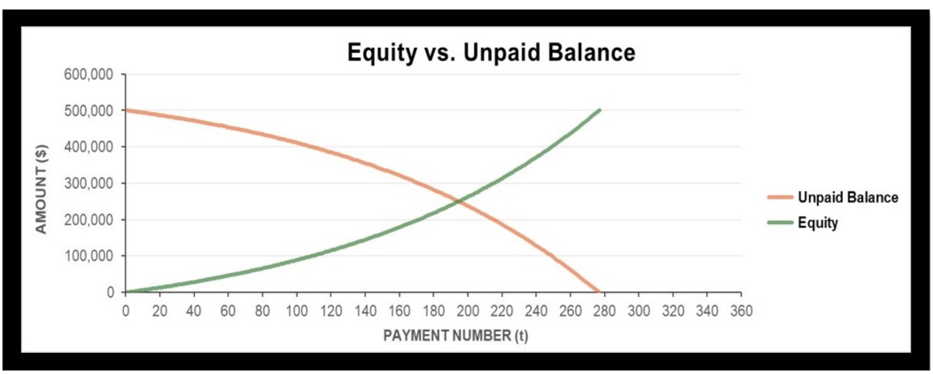


Figure 7 Graphical analysis – development of the cumulative equity compared to the unpaid balance over the term of the mortgage (with break-even period) (see online version for colours)



The effect of surging principal portions and of plunging interest portions is revealed by the column chart of Figure 8. It represents the complementary relation between the interest and the principal payment in each instalment over the loan term *n*. Its intersection is not equivalent to the break-even period. Instead, the principal amount per payment equals the interest charge at this point.

2.3.5 Graphical analysis of the specific loan information (instalment payment t)

The pie bar chart shown in Figure 9 captures the percentage ratio between the principal and the interest portion in instalment *t*. The ratio depends on the interest rate, the loan maturity, and the progress of repayment – the nearer *t* to the actual last payment, the higher the principal amount.

Figure 8 Graphical analysis – development of the principal portion compared to the interest portion per payment rate over the term of the mortgage (see online version for colours)

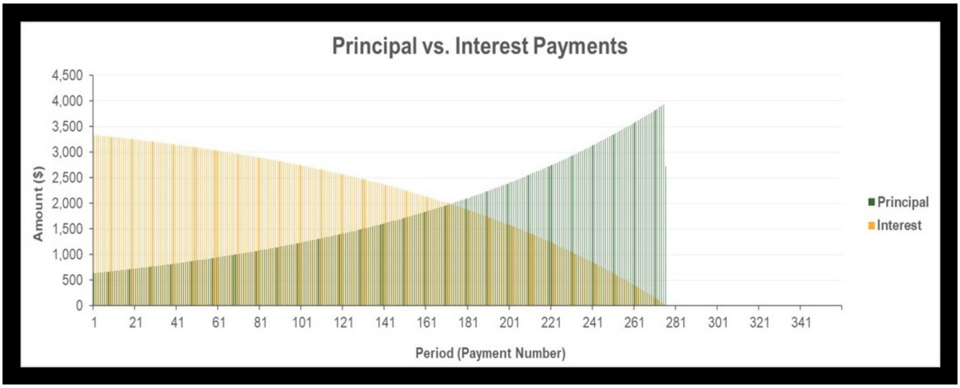
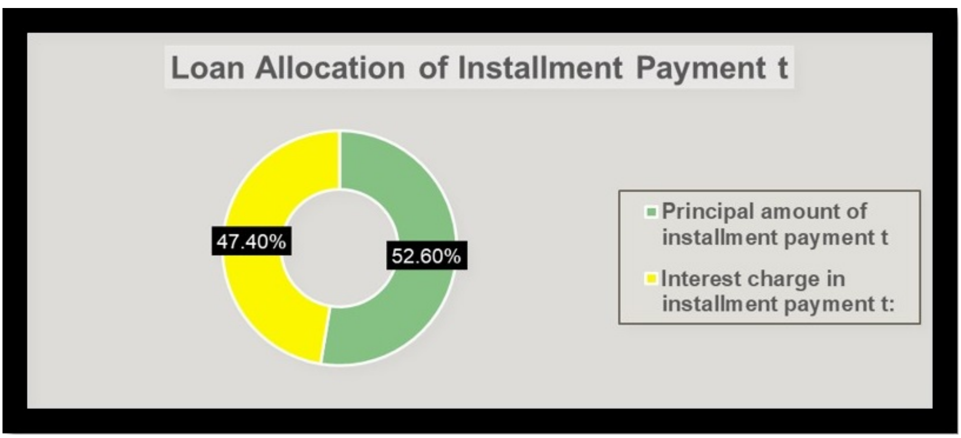
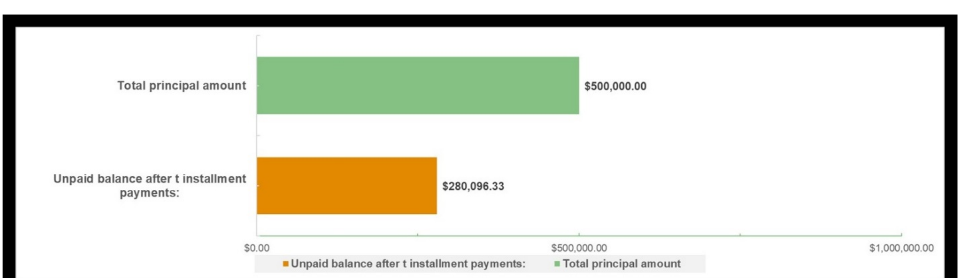


Figure 9 Graphical analysis – principal and interest portion (percentage) in payment *t* (see online version for colours)



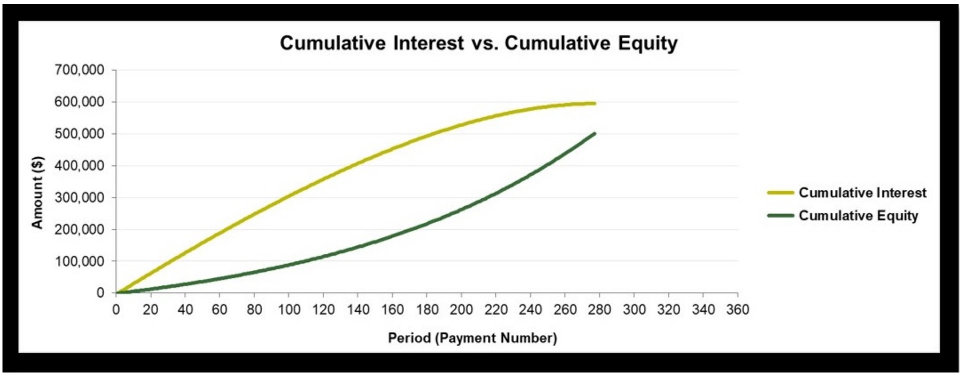
The bar chart in Figure 10 expresses the remaining unpaid balance at the stage of instalment payment *t* – the higher the selected value for ***t***, the lower the remaining unpaid balance at this point.

Figure 10 Graphical analysis – unpaid balance after *t* payments vs. total principal amount (see online version for colours)



The graphical representation of the progression of the cumulative interest and the cumulative equity are also part of the graphical analysis in the data output sheet. An example is illustrated by the line chart in Figure 11. The respective slope depends on the loan period and the interest rate.

Figure 11 Graphical analysis – cumulative interest vs. cumulative equity (see online version for colours)



2.3.6 *Computation of scenarios and evaluation*

Evaluation of the basic loan section: The following two tables sum up the software tool-provided data output for basic loan information of different mortgages. They have different loan terms (n in years (y)) and they are as well based on different interest rate (i) scenarios, but assumingly all have a consistent principal amount P of \$500,000. Table 1 shows the data output information for i = 5%, respectively, for the spectrum of different loan terms between n = 15 and n = 30 in years.

Table 1 Basic mortgage data output scenarios 1–4 for P = \$500,000 and i = 5%

	Scenario 1: i=5%, n=15 (y)	*Scenario 2: i=5%, n=20 (y)*	*Scenario 3: i=5%, n=25 (y)*	*Scenario 4: i=5%, n=30 (y)*
Monthly payment rate	\$3,953.97	\$3,299.78	\$2,922.95	\$2,684.11
Number of payments	180	240	300	360
Total interest charge	\$211,714.26	\$291,946.89	\$376,885.06	\$466,278.92
Total loan cost	\$711,714.26	\$791,946.89	\$876,885.06	\$966,278.92
Percentage of interest charge	29.75%	36.86%	42.98%	48.26%
Break-even period	106.46	148.77	194.03	241.88

Figure 12 depicts the increase of cumulative equity compared to the falling unpaid balance for scenarios 1 and 4. The curve paths in both scenarios are comparatively flat. Less than one-third of the total costs in the first scenario and less than half in scenario 4 are cumulative interest charges.

Figure 12 Graphical evaluation of scenarios 1 and 4 – equity versus (vs.) unpaid balance, percentage of interest charge of the total loan cost (see online version for colours)

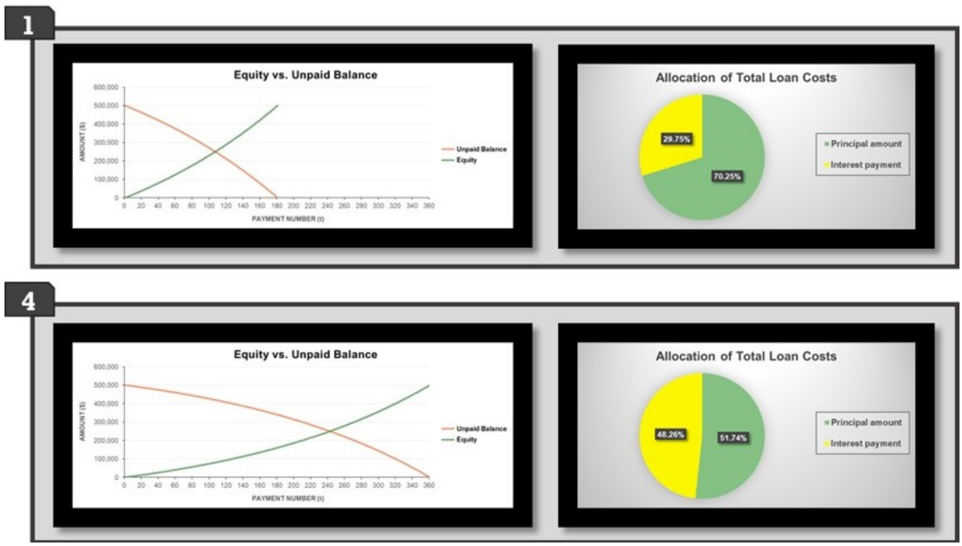


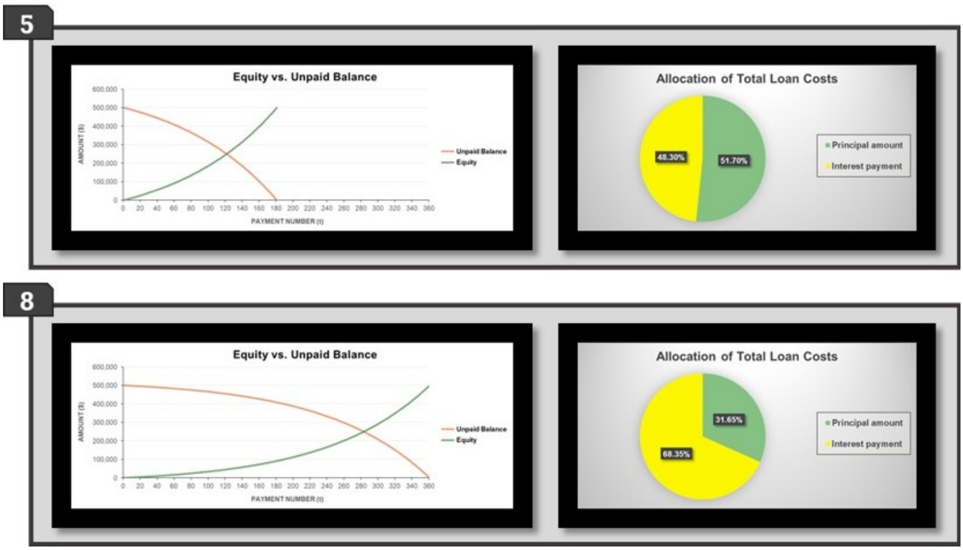
Table 2 includes four data output scenarios for *i* = 10%, i.e., an interest rate twice as high as in Table 1. It is as well based on different time spans lying between *n* = 15 and *n* = 30 (y). The graphical evaluation for two of these scenarios, namely, scenarios 5 and 8 is revealed in Figure 13.

Table 2 Basic mortgage data output scenarios 5–8 for *P* = $500,000 and *i* = 10%

	Scenario 5: i=10%, n=15 (y)	*Scenario 6: i=10%, n=20 (y)*	*Scenario 7: i=5%, n=25 (y)*	*Scenario 8: i=10%, n=30 (y)*
Monthly payment rate	$5,373.03	$4,825.11	$4,543.50	$4,387.86
Number of payments	*180*	*240*	*300*	*360*
Total interest charge	$467,144.61	$658,025.97	$863,051.12	$1,079,628.83
Total loan cost	$967,144.61	$1,158,025.97	$1,363,051.12	$1,579,628.83
Percentage of interest charge	48.30%	56.82%	63.32%	68.35%
Break-even period	120.88	171.89	226.08	282.40

The next graphic illustrates the trajectory of cumulative equity vs. unpaid balance over time for scenarios 5 and 8 (see Figure 13). It includes the percentage ratio of cumulative principal vs. interest payments of the mortgage. Owing the raised interest rate, the curves run considerably more steeply than those in the previous illustration of scenarios 1 and 4. The interest charge percentages of the total mortgage expenses range from almost 50% to more than 68%, i.e., more than two-thirds.

Figure 13 Graphical evaluation of scenarios 5 and 8 – equity vs. unpaid balance, percentage of interest charge of the total loan cost (see online version for colours)



The total loan costs of scenario 5 are higher than those of scenario 4. It appears remarkable that it is still more cost-efficient to borrow mortgage money for 30 years at an interest rate of i = 5% than it is for n = 15 (y) at an interest rate twice as high. And this phenomenon occurs even though the monthly payments in scenario 4 only are only about half of those of scenario 5, albeit for the double period in time. Scenarios 2 and 3, compared with 6 and 7, highlight considerably lower cumulative interest charges and monthly instalment rates at a comparative loan period. The break-even period of scenario 7 is 32 months, i.e., almost three years later than that of scenario 3, which is a major divergence. Other than that, the difference in the break-even point between the 5% on the one hand and the 10% interest rate scenarios, on the other hand, increases with a higher n (y).

2.3.7 Evaluation of the specific loan information for payment period t

The specific loan information section can be used for particular analyses of loan compositions in a certain phase of the mortgage term (see Figure 5). If applied, its composition as well counts for the impacts caused by extra payments, e.g., on the equity or interest portion, at a certain period in time. Consequently, the maximum value of t that can be evaluated at this point is the actual total number of repayments – considering the eventual entry of additional payments.

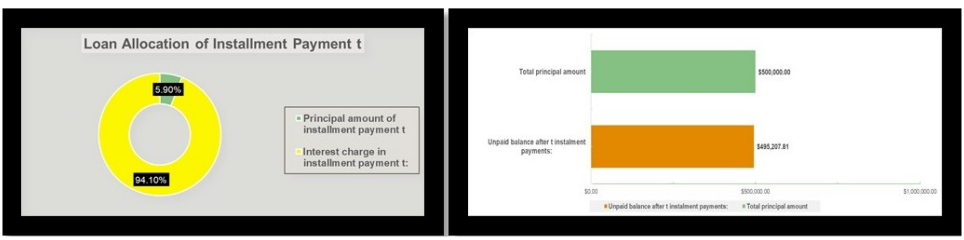
To demonstrate the opportunities the specific evaluations provide we assume the following loan conditions: the principal P is \$500,000, the interest rate i = 10% and the term is n = 30 (y), which means n = 360 (m). Devoid of any extra payments, the equal monthly payment rate is \$4,387.86.

After t_1 = 20 instalments, the specific evaluation yields the following data output on the loan:

- The unpaid balance amounts to \$495,207.81 at this point in time;
- Cumulatively, \$87,757.16 have been paid in equal monthly payment rates after t_1 periods;
- The equity portion of the instalment payment $t_1 = 20$ is \$258.97 or 5.90%;
- On the other hand, the interest charge in instalment payment t_1 is \$4,128.89 (94.10%);
- The cumulative equity reaches \$4,792.19, which is 5.46% of the cumulative total payment, while the cumulative interest payment is \$82,964.97 (94.54%) after t_1 periods.

Figure 14 depicts the principal-interest ratio of payment t_1 and the remaining unpaid balance.

Figure 14 Graphical evaluation of the loan allocation – principal vs. Interest portion – of instalment payment t_1 and remaining unpaid balance after t_1 instalment payments (P = \$500,000; i = 10%, and n = 30 (y); t_1 = 20) (see online version for colours)

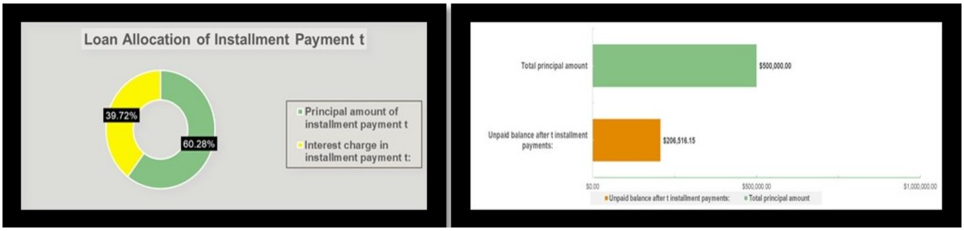


To illustrate the specific loan information at an advanced time period of repayment, in particular after having exceeded the break-even period, for the second example we explore a specific evaluation for t_2 = 300 instalment payments for the above-mentioned mortgage conditions. Using the computational tool, the following specific data output is generated for t_2:

- The unpaid balance in this period is \$206,516.15;
- The cumulative total payment after t_2 periods has reached \$1,316,357.36;
- The principal amount of instalment payment t_2 amounts to \$2,644.85 (60.28%);
- \$1,743.01 (39.72%) of interest charge is included in payment t_2;
- The cumulative principal payment is at \$293,483.85 after 300 instalment payments, i.e. a percentage of 22.30% of the cumulative total payment;
- The cumulative interest payment has reached \$1,022,873.50, which corresponds to 77.70% of the cumulative total payment.

Supplementary, the computed impacts of the advanced repayment of the loan, resulting from the advanced time-lapse and boosted by extra payments, are graphically revealed in Figure 15.

Figure 15 Graphical evaluation of the loan allocation – principal vs. interest portion – of instalment payment t_2 and remaining unpaid balance after t_2 instalment payments (P = \$500,000; i = 10% and n = 30 (y); t_2 = 300) (see online version for colours)



2.4 *Advanced 'What If' – calculations*

2.4.1 *Data input sheet*

Besides the basic loan information, the XGAMPS tool offers several advanced computation capabilities aiming at a guidance in terms of particular questions and occasions that may arise in the loan-based financial management of real estate endeavours.

The details that can be calibrated in the data input sheet to create advanced data output are depicted in the extract below (see Figure 16). They consist of the following three spheres:

- Extra payments, including an alternative investment view,
- Affordable size loan and
- Refinancing.

As illustrated in Figure 16, the respective part is framed by one of the three subheadings in the data entry section for advanced 'what if'– Calculations.

If the question for continuous extra payments is affirmed (F21), the monthly amount is to be entered (F23). F25 requires the interest rate of an alternative investment in the form of a savings plan. The background is an investment view taken on the extra payments. It is relevant to discover whether the continuous investment, providing additional principal into the mortgage, is beneficial for the real estate investor with respect to the generated interest charge savings. The effect of extra payments is to boost the cumulative equity and to save interest costs. Every additional dollar means 'pure' principal payment, reducing the unpaid balance and the total interest charge. Nonetheless, the invested capital could likewise be beneficially put on an account yielding returns.

Cell F23 allows to enter an available monthly payment rate to calculate an affordable size of the principal, assuming the basic data for i and n remain constant.

A further optional data entry section addresses refinancing. Especially in times of declining interest rates, monitoring refinancing options for their eventual advantageousness can become reasonable. If the calculation is required, 'YES' needs to be selected in F33. Traditionally, a refinancing option entails a lower interest charge than the original loan. On the other hand, refinancing implicates opportunity costs in the form of certain refinancing charges, such as prepayment penalties, closing costs or fees. Both factors are considered as data entries: F39 for the interest rate of the new mortgage and F41 for the overall costs of the refinancing option. Further required data input is the point in time of refinancing, namely the number of already paid instalments for the previous loan in months (F35). In addition, the term of the refinancing loan needs to be entered in cell F37.

Figure 16 Data input extract – advanced 'What If'– Data entries

	A	B	C	D	E	F
17	**Advanced "What if"-Loan Calculations (Data Input Only)**					
18						
19	EXTRA PAYMENTS					
20						
21	6	Are continous **extra payments** planned?				YES
22						
23	6a	Enter the monthly amount of extra payments:				$300.00
24						
25	6b	Beneficial to do the extra payments? Enter the interest rate of an alternative savings plan:				*5.00%*
26						
27	AFFORDABLE SIZE LOAN					
28						
29	7	For the **affordable size loan**, enter the disposable payment rate:				$1,500.00
30						
31	REFINANCING OPTION					
32						
33	8	Is **refinancing** of the mortgage intended?				YES
34						
35	8a	Enter the number of payments made on the old mortgage (last payment number of the old loan):				40
36						
37	8b	Enter the term of the new loan in years:				30
38						
39	8c	Enter the interest rate of the refinancing mortgage:				*7.00%*
40						
41	8d	Enter the overall costs of refinancing (incl. prepayment penalties, closing costs and all charges/fees):				$25,000.00

2.4.2 Data output sheet

Figure 17 depicts the data output calculated for the first two advanced calculation spheres. The first one encompasses 'extra payments', headed by the corresponding subtitle within. It involves the financial impacts of extra payments on the loan structure as well as the alternative investment approach. The second part of the depicted data output is the affordable size loan computation.

Figure 17 Data output extract – advanced computations on the impacts of extra payments and the affordable size loan

	A	B–E	F
37	**Advanced "What if"-Loan Calculations (Data Output Only)**		
38			
39	EXTRA PAYMENTS		
40			
41	**11a**	**Mortgage payoff period** = actual total number of equal payments (changes depending on extra payments):	*276 (23.0 years)*
42			
43	**11b**	**Saved months (time periods)** of payments due to the extra payments:	*84*
44			
45	**11c**	**Saved interest charge**/total loan cost due to the extra payments:	*$226,611.49*
46			
47	**11d**	**Alternative investment** with the extra payments (for the total time span of ***n***) yields:	*$249,677.59*
48			
49	**11e**	Difference between the interest savings of the extra payments & the alternative investment:	*$23,066.10*
50			
51	**11f**	Total savings in case of a deposit of the loan payments to the savings plan after the early payoff:	*$647,864.24*
52			
53	AFFORDABLE SIZE LOAN		
54			
55	**12**	**Affordable size loan** (principal amount) on the disposable payment rate:	**$204,425.24**

2.4.3 Data output for extra payments

As expressed in Figure 17, cell F41 reveals the actual loan payoff period regarding the term-shortening effect of continuous extra payments. The total number of payments differs from ***n***, because the cumulative principal rises faster, the unpaid balance is diminished more quickly, and thus, the mortgage is repaid earlier. F43 includes the saved periods in months. An essential output for the consideration of extra payments is incorporated in F45: the saved interest charge in comparison to a regular payment scheme. At the same time, the amount is equivalent to the total loan cost savings. F47 processes the interest rate for an alternative investment to calculate, respectively, to illustrate the earnings this alternative investment yields under consideration of the time value of money. To simplify the computation, the alternative investment is incorporated in the form of a very secure investment form – a savings plan with fixed, constant and equal interest revenues over the entire term. Cell F49 answers the question of whether the alternative investment is beneficial in the first place, by showing the difference between the earnable interest revenues and the amount of loan cost savings. A positive result means the savings plan appears favourable subject to the entered conditions. A negative result, however, indicates the extra payments to be the more efficient investment. F51 takes the idea further: it shows the total savings including an assumed continuous deposit of payments for the saved time span in view of the extra payments. For example, if 50 months of payment are saved through the consideration of additional payments, this 'what if'-computation includes a theoretical investment of the regular instalments for these 50 months into the savings plan because this money potentially occurs at an additional disposal.

2.4.4 Data output on the affordable size loan

As shown in the second part of the data output section (see Figure 17), F55 returns the value of the affordable size loan for the disposable monthly payments. This computation involves a reverse view of the loan payments. It completes the overall picture by providing insight into the principal amount the investor could conveniently afford. Referring to the introduction, financial decisions in a mortgage case are categorised by three different dimensions – 'quality' is one of them. The affordable size loan phenomenon accentuates the virtually maximum loan volume, strengthens awareness of money affordability and supports guiding the extent of quality features of property.

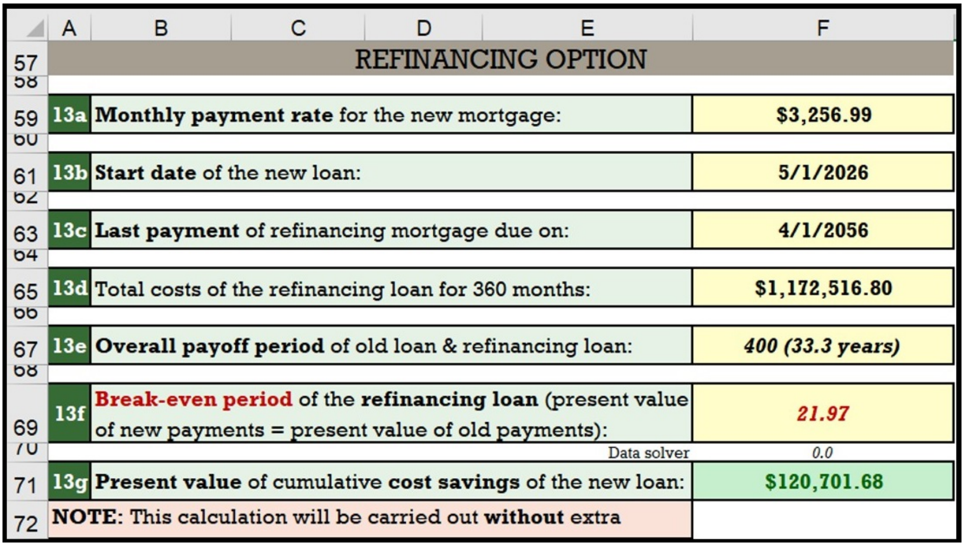
2.4.5 Data output on the refinancing option

It is vital to incorporate the aspect of the time value of money, as the nucleus of the whole approach, into the refinancing decision. Simplified considerations such as a ratio of refinancing costs vs. the difference of instalment payments between the old and the refinancing loan disclose the obvious weakness of neglecting the importance of the time value of money (Chen, 1997). Refinancing costs and future savings of payments, occurring at different points in time are not directly comparable and have to be converted to the present value (Randle and Johnson, 1996; Chen, 1997). This is remedied by applying the discounted cash flow method, which takes into account the present values of the net cash inflows and outflows of the refinancing option. The computation is adapted from the pertinent scholarly approaches to programming mortgage refinancing as applied by Randle and Johnson (1996); Chen (1997) and Johnson and Randle (2003). Another relevant publication with a similar focus is offered by Arefaine (1988).

Figure 18 displays an extract of the data output on refinancing decision analysis. The calculation is carried out without taking into account eventual extra payments since this is an independent advanced section. F59 represents the continuous monthly payment rate of the new mortgage, on principle well below the monthly burden of the original loan. With the entered data for the actual number of already carried out payments on the old loan, F61 – the start date of the refinancing loan – is calculated. Based on the entered term, cell F63 shows the last payment's due date. The calculated total costs of the new mortgage are displayed in F65. F67 reflects the total payoff period, inclusively the already executed payments on the previous mortgage both in months and years.

The essence of the refinancing analysis is reflected in the cells F69 and F71. F71 represents the present value of net cash flows at the end of the term of the new loan, particularly the discounted cumulative cost savings vs. the costs of refinancing. The present value is computed based on the overall costs of refinancing, the payment savings through the new loan, and the difference in the unpaid balance between the two mortgages over time, as applied from Chen (1997). The interest rate of the new mortgage thereby is processed as the discount rate. A positive present value generally means the new loan is financially beneficial. Necessarily, a point in time when the refinancing costs are amortised by the cumulative cost savings is required to complete the analyses. This aspect is processed in F69. It is an optimisation approach, intending to find the break-even period at which the present values of the cash flows of both loans are equal. The computation is based on the 'data solver'-methodology, elaborated by Johnson and Randle (2003). Thereby, the data solver function as an optimisation tool in MS Excel is applied in F70, a so called change cell (Johnson and Randle, 2003). At a value of *0*, the actual break-even period is reflected in cell F69.

Figure 18 Data output extract – advanced computations on a refinancing decision



	A	B–E	F
57		REFINANCING OPTION	
58			
59	13a	**Monthly payment rate** for the new mortgage:	**$3,256.99**
60			
61	13b	**Start date** of the new loan:	**5/1/2026**
62			
63	13c	**Last payment** of refinancing mortgage due on:	**4/1/2056**
64			
65	13d	Total costs of the refinancing loan for 360 months:	**$1,172,516.80**
66			
67	13e	**Overall payoff period** of old loan & refinancing loan:	***400 (33.3 years)***
68			
69	13f	**Break-even period** of the **refinancing loan** (present value of new payments = present value of old payments):	***21.97***
70		Data solver	*0.0*
71	13g	**Present value** of cumulative **cost savings** of the new loan:	**$120,701.68**
72		**NOTE**: This calculation will be carried out **without** extra	

2.4.6 Scenario-based computation, graphical analysis and evaluation

In this section, the advanced features and applications of the XGAMPS tool are experienced.

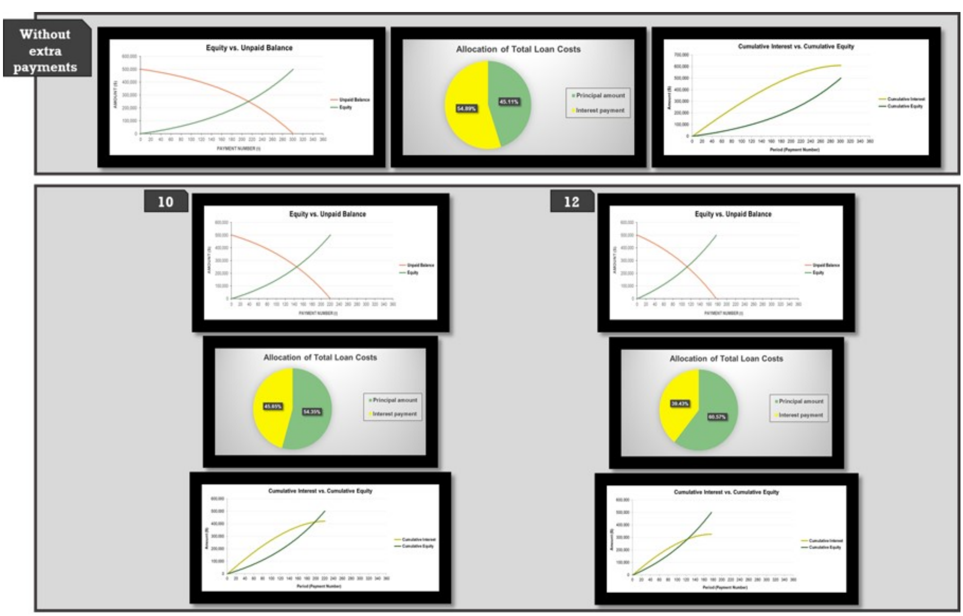
Extra payments: The table below (see Table 3) comprises a scenario-based approach to demonstrate the implications of extra payments to a regular repayment schedule. Suppose a mortgage for n = 25 (y) is borrowed, with a principal P = $500,000 at an interest rate of i = 7.5%. The tool-calculated basic output data reveals the following information: the equal monthly payment rate is $3,694.96 throughout the payoff period of n = 300 (m), whereas the total loan cost amount is $1,108,486.77 with a cumulative interest charge of $608,486.77, which means a percentage of 54.89%. The calculated break-even point lies at 211.77 periods. Table 3 is designed to capture the implications of different monthly extra payment amounts (expressed by the value of a) to these basic figures. Note that we assume in this analysis that no early repayment fees or comparable costs are due.

Table 3 Advanced data output scenarios 9–12 for P = \$500,000, n = 25 (y), i = 7.5% and for different additional monthly payment amounts (a)

	Scenario 9: a = \$300	*Scenario 10: a = \$500*	*Scenario 11: a = \$700*	*Scenario 12: a = \$1000*
Actual mortgage payoff period	*245 (20.4 years)*	*220 (18.3 years)*	*200 (16.7 years)*	*176 (14.7 years)*
Saved time (m)	55	80	100	124
Interest cost savings	\$131,091.51	\$188,593.67	\$232,762.19	\$283,028.04
Percentage of interest charge	48.84%	45.65%	42.90%	39.43%
Break-even period	165.31	145.07	129.32	111.00

The previous tabulation highlights some significant outcomes derived from extra payments. Owing the scenario 9 the amount of \$300 monthly, which is less than 10% of the regular instalments, suffices to cut back interest charges at about 25% and to prune the payoff period by more than four and a half years. At a = \$700 (scenario 11) the percentage of the cumulative interest charge plunges by more than 10% and 100 months of payment are saved. In scenario 12, computed with an extra payment rate of less than 30% of the regular instalment payment amount, the break-even period noticeably plummets by 100 periods, the savings of interest charge in total reach almost 50%. Moreover, the investor would face a strong reduction of the amortisation period by more than 10 years. Overall, these figures demonstrate the immense financial indication resulting from extra payments and highlight the relevance of even apparently marginal amounts of additional deposits.

Figure 19 Graphical evaluation of scenarios 10 and 12 compared to basic loan information – equity vs. unpaid balance, percentage interest charge, and cumulative equity vs. interest (see online version for colours)



Supplementary, two scenarios are evaluated graphically in Figure 19. Its dashboard-like graphics, illustrate the impacts of extra payments, particularly of \$500 (scenario 10) and \$1000 monthly (scenario 12) on the curve slopes of cumulative equity, unpaid balance and of cumulative interest. The curves are increasingly compressed, as equity soars faster and cumulative interest – as well as the unpaid balance – drops more intensively. The respective break-even period shifts to the left. As the pie charts show, the cumulative interest portion of the total loan is significantly diminished by extra payments. Last but not least, the cumulative equity and cumulative interest curve intersect earlier, while in the example devoid of any extra payments they do not intersect at all.

Alternative investment evaluation of extra payments: To reflect the preceding 'what-if' dimension in a balanced way, it is important to consider the alternative investment perspective and to prove appropriately whether the extra payment deposit is a beneficial investment. Therefore, we assume the following mortgage conditions: P = \$500,000, n = 30 (y) and i = 7.5%. We further assume a = \$300 monthly extra payments are disposable. Table 4 sums up the alternative investment view on the amount of extra payments and answers the question, at which interest rates of savings plans it may become favourable to invest the money alternatively. The scenarios are based on a simplified computation and assumingly would have to be complemented by other deductions from the earnings such as income tax or fees.

Table 4 Advanced data output scenarios 13–16 for P = \$500,000, n = 30 (y), i = 7.5%, a = \$300 and for different interest rates i_a of alternative investments

	Scenario 13: i_a = 2%	*Scenario 14:* i_a = 3%	*Scenario 15:* i_a = 4%	*Scenario 16:* i_a = 5%
Yield of alternative investment with the extra payments at i_a.	\$147,817.62	\$174,821.07	\$208,214.82	\$249,677.59
Difference between Interest savings & yield of alternative investment:	–\$55,000.79	–\$27,997.34	\$5,396.42	\$46,859.19
Savings including deposit of loan payments after mortgage payoff.	\$476,727.96	\$515,177.99	\$560,539.40	\$614,517.18

The computations clarify that, at the above-mentioned loan conditions, interest rates of 2% (scenario 13) or 3% (scenario 14) for an alternative savings plan have no comparative advantage in relation to the investment of extra payments. The consideration of accumulating the money in a savings account turns beneficial for an alternative interest rate of 4% or more (scenarios 15 and 16). Naturally, the calculation yields unequivocal positive results, if not only the extra payments are saved, but also the instalment payments for the saved time period of the underlying mortgage. For instance, at $\boldsymbol{i_a}$ = 3% the total savings inclusively deposits of loan payments in the saved time period yield \$515,177.99, which is more than the borrowed principal amount of the mortgage.

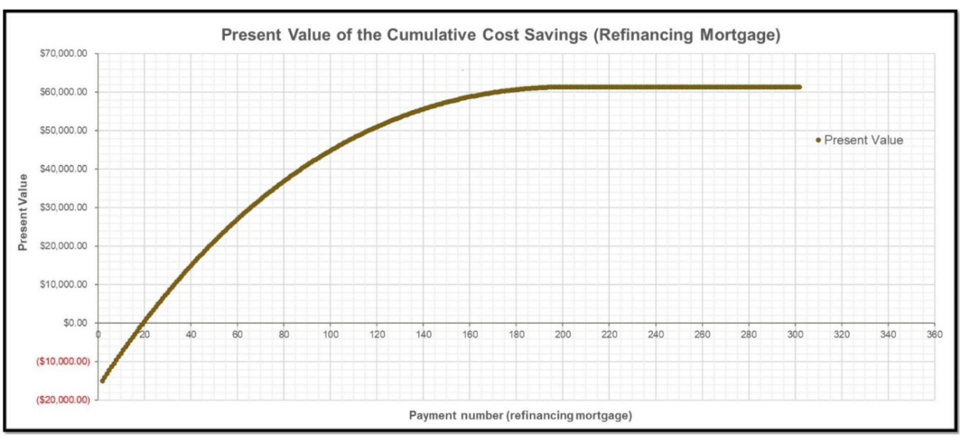
Affordable size loan: The following scenario-based analysis focuses on the affordable size loan at certain mortgage conditions. Therefore, Table 5 relates different disposable monthly instalment amounts within a matrix to interest rates i from 5% to 10% and loan terms from n = 15 to n = 30 years. The tabulation illustrates the affordable at the computed loan conditions, respectively. A glaring example of the effect of rising interest charge is the comparison between i = 7.5%, monthly payment of \$2000 and i = 10%, monthly payment of $2500 for n = 30 (y), scenario 20: despite having $500 more monthly disposable payments, this higher interest rate entails a lower affordable loan amount.

Table 5 Advanced data output scenarios 17–20: Respective affordable size loan for disposable monthly payment amounts and various i and n (y)

	Scenario 17: n = 15 (y)	*Scenario 18: n = 20 (y)*	*Scenario 19: n = 25 (y)*	*Scenario 20: n = 30 (y)*
i = 5%, $1000	$126,455.24	$151,525.31	$171,060.05	$186,281.62
i = 6%, $1500	$177,755.27	$209,371.16	$232,810.30	$250,187.42
i = 7%, $3000	$333,767.87	$386,947.52	$424,460.71	$450,922.70
i = 7.5%, $2000	$215,746.85	$248,264.26	$270,639.23	$286,035.25
i = 8%, $1500	$156,960.89	$179,331.44	$194,346.78	$204,425.24
i = 10%, $2500	$232,643.60	$259,061.55	$275,118.08	$284,877.05

Refinancing option: The present value curve in terms of the cumulative cost savings resulting from the alternative mortgage consideration is shown in the graphic of Figure 20. The curve intersects the payment number axis at the break-even period of the refinancing loan which is calculated by the mentioned Excel optimisation approach. Hence, the graphic supplements the break-even point computation approach and demonstrates the point at which the net present value of the cash flows turns positive.

Figure 20 Graphical analysis – present value of the cumulative cost savings



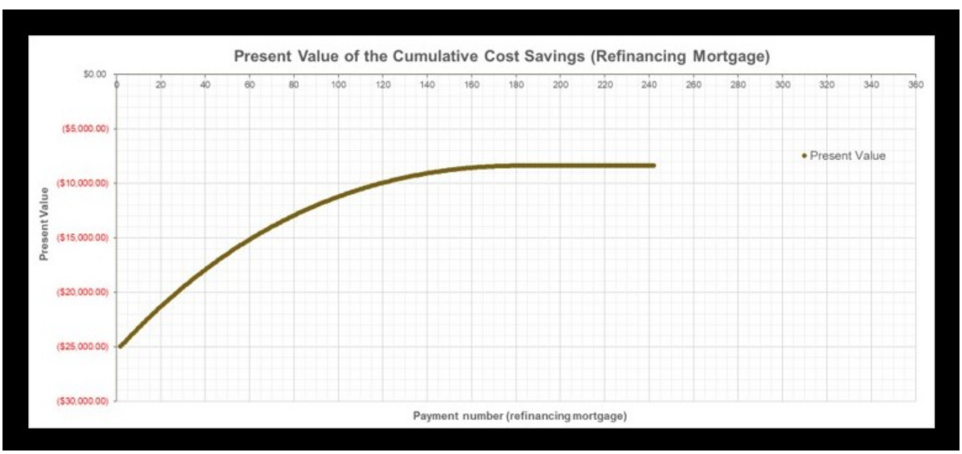
Regarding the graphical analysis, the course of the course slope is at first soaring significantly, with a then ever declining rise up to a certain point, from which it remains remarkably flat. This point depends on the number of payments already made on the previous loan because from this particular moment within the loan term the advantage of the monthly payment difference is eliminated. Besides the application of present values, this is a further important factor resulting from the time value of money. In other words, the eventual advantageousness of the refinancing mortgage competes with the old loan in several domains of consideration:

- Amount of the overall refinancing costs;
- The interest rate difference and the consequential aspects consisting of the unpaid balance difference and the monthly payment difference;
- The term of both mortgages and the potential time period to amortise the extra charge;
- Last but not least as mentioned, the number of instalments paid on the previous loan.

For instance, high refinancing expenses can be easier paid off over a longer term with the trend of a surging present value of the cumulative cost savings. To mention different loan terms, the advantageousness is impeded by a shorter time span of the refinancing option. The reason lies in the accordingly lower monthly payment difference and a different composition of the unpaid principal balance difference. The impact of a longer repayment term on the new mortgage appears vice versa. Apart from that, another trend can be derived from the number of payments already made on the previous mortgage: the higher the number, the less profitable tends a switch to the new loan to be, because of a shorter term for the amortisation of the refinancing costs.

To demonstrate the ramifications of the consideration of refinancing mortgages at a certain period in time of an in Figure 21.

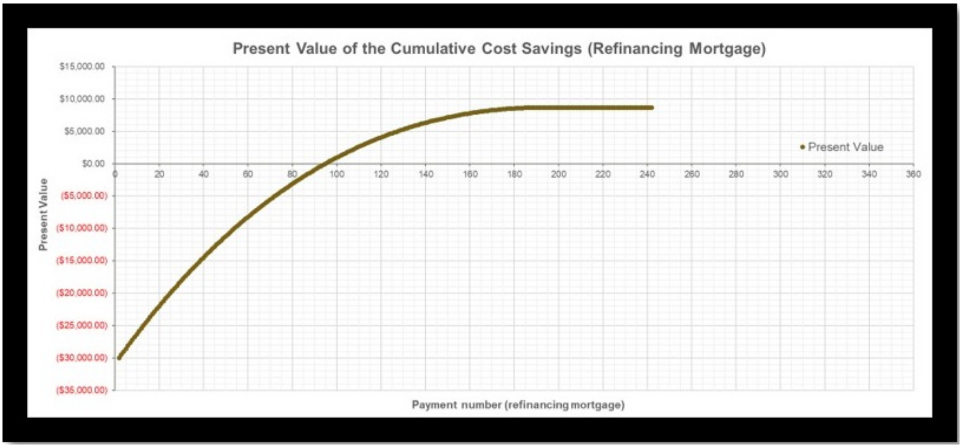
Figure 21 Graphical evaluation of refinancing scenario 1 – present value curve of the cumulative cost savings of the refinancing mortgage



The scenario 1 curve does not reach a positive present value within the loan term. This refinancing option does not imply a break-even period at all. The present value of the total cost savings of the alternative loan is negative and arrives at –$8,399.93. To describe it in other words: despite a more moderate interest charge and lower total costs of the new loan, the discounted cash flows of the cumulative cost savings do not consider the refinancing option to turn beneficial.

The conditions of the basis loan in refinancing scenario 2 are different: with ***P*** = $250,000, the interest rate amounts to ***i*** = 9.5%, whereas the time span is ***n*** = 20 (y). The refinancing option begins after 50 instalment payments on the previous loan. This mortgage option is available at an interest rate of $\boldsymbol{i}_\mathbf{r}$ = 7%, with refinancing costs of \$30,000 for the loan term of $\boldsymbol{n}_\mathbf{r}$ = 20 (y), thereby.

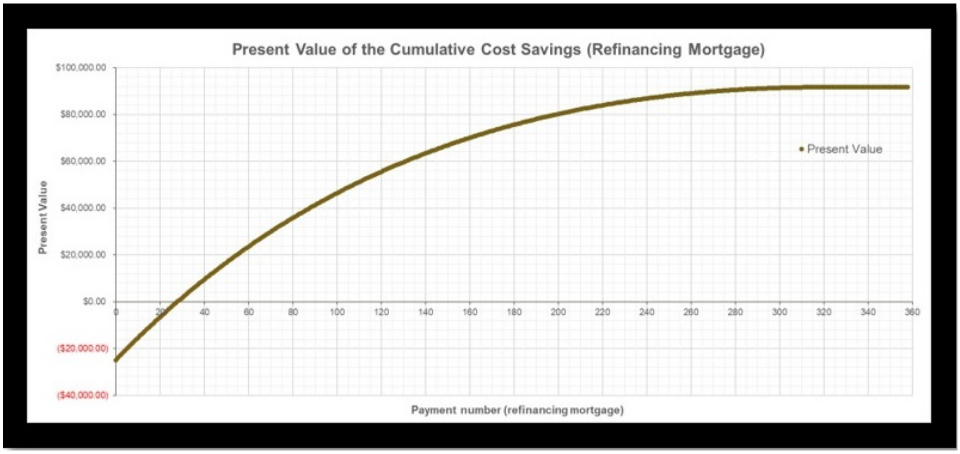
Figure 22 Graphical evaluation of refinancing scenario 2 – present value curve of the cumulative cost savings of the refinancing mortgage (with break-even point)



The refinancing option in scenario 2 implies a positive present value of the cumulative cost savings and amounts to \$8,624.26 after 20 years. Nonetheless, the result is merely partly encouraging, since it takes more than seven and a half years to amortise the financial decision. The computed break-even period of refinancing is at 93.21 periods, additionally visualised in Figure 22. Chen (1997) suggested that the refinancing option is desirable if the present value arrives to be positive at the last period the real estate owner plans to stay in this home. It cannot be precluded that such a refinancing decision will be reconsidered in view of the relatively long break-even length of time of more than 93 months. Probably, the borrower will hold steady in this hypothetical case.

In the third and last refinancing scenario, we assume the principal of the original loan is P = \$400,000, the interest rate is i = 10% and the time span is n = 30 (y). The refinancing option assumingly starts after 40 instalments have been carried out. The refinancing option includes an interest rate of i_r = 7%, a loan term of n_r = 30 (y) as well as overall refinancing costs of \$30,000. With regard to the computational and graphical evaluation (see Figure 23), this refinancing option appears favourable. The present value of the cumulative cost savings after 30 years considerably grows to the amount of \$91,561.34. Not least regarding the comparatively short refinancing break-even period of 28 months, the decision for the refinancing conditions of this option is encouraging. In spite of the refinancing costs, the borrower saves over \$900 in monthly payments. The decisive factors for the advantageousness of scenario 3 are the loan term of 30 years, the relatively short period of usage of the previous mortgage – 40 periods or 3 years and 4 months – and the significant interest rate differential amounting to 3%. The graphic of Figure 23 reflects the described pathway.

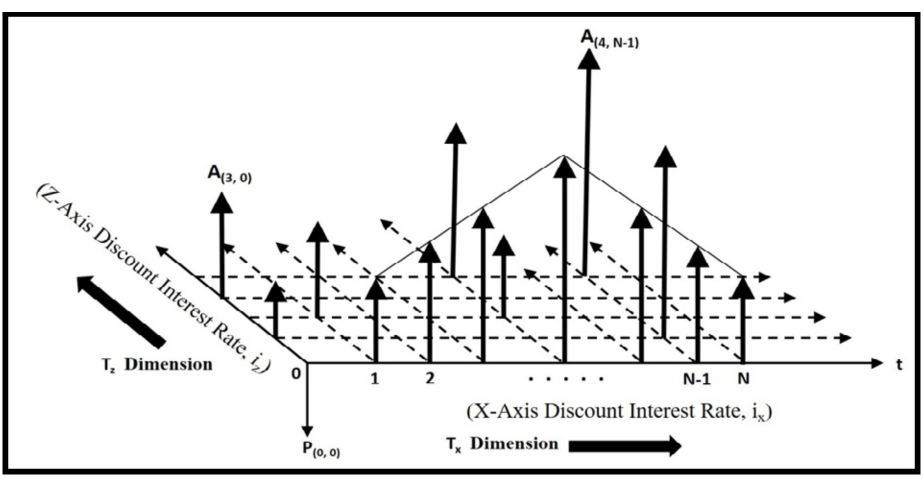
Figure 23 Graphical evaluation of refinancing scenario 3 – present value curve of the cumulative cost savings of the refinancing mortgage (with break-even point)



2.5 Advanced features foresight

Beyond the basic time-value-of-money computations, the introduced *XGAMPS* software tool incorporates advanced features such as of refinancing or extra payments. It is from these advanced features that we anticipate that other researchers will come up with additional modelling and incorporation of complex features that may fit untypical needs in mortgage analysis and or corporate investment strategies. For example, we envision multidimensional cash flow profiles similar to Figure 24, in which different levels of interest rates over different investment durations are subject to non-uniform payment profiles. Further work on this case-based idea and other unique considerations is left to the creative imaginations of readers and researchers.

Figure 24 Multidimensional combination of interest rates in multi-investment decisions



3 What's under the hood?

The backbone (internal structure) of the calculations embedded in the *XGAMPS* spreadsheet tool is derived from Badiru and Omitaomu (2011) and Kurtz (1995). This section presents a collection of the pertinent equations forming the computational horsepower of the software. It is from this set that other researchers can formulate new advancements and enhancements to customise the software for other unique corporate or personal applications.

1 Given a principal amount, *P*, a periodic interest rate, *i* (in decimals) and a discrete time span of *n* periods, the uniform series of equal end-of-period payments needed to amortise *P* is computed as

$$A = \frac{P\left[i(1+i)^n\right]}{(1+i)-1}.$$

It is assumed that the loan is to be repaid in equal monthly payments. Thus, $A(t) = A$, for each period *t* throughout the life of the loan.

2 The unpaid balance after making *t* instalment payments is given by

$$U(t) = \frac{A\left[1-(1+i)^{(t-n)}\right]}{i}.$$

3 The amount of equity or principal amount paid with instalment payment number *t* is given by

$$E(t) = A(1+i)^{t-n-1}.$$

4 The amount of interest charge contained in instalment payment number *t* is derived to be

$$I(t) = A\left[1-(1+i)^{t-n-1}\right]$$

where $A = E(t) + I(t)$.

5 The cumulative total payment made after t periods is denoted by

$$C(t) = \sum_{k=1}^{t} A(k) = \sum_{k=1}^{t} A = (A)(t).$$

6 The cumulative interest payment after *t* periods is given by

$$Q(t) = \sum_{x=1}^{t} I(x).$$

7 The cumulative principal payment after t periods is computed as

$$S(t)=\sum_{k=1}^{t}E(k)$$
$$=A\sum_{k=1}^{t}(1+i)^{-(n-k+1)}$$
$$=A\left[\frac{(1+i)^{t}-1}{i(1+i)^{n}}\right]$$

where

$$\sum_{n=1}^{t}x^{n}=\frac{x^{x+1}-x}{x-1}.$$

8 The percentage of interest charge contained in instalment payment number t is

$$f(t)=\frac{I(t)}{A}(100\%).$$

9 The percentage of cumulative interest charge contained in the cumulative total payment up to and including payment number t is

$$F(t)=\frac{Q(t)}{C(t)}(100\%).$$

10 The percentage of cumulative principal payment contained in the cumulative total payment up to and including payment number t is

$$H(t)=\frac{S(t)}{C(t)}$$
$$=\frac{C(t)-Q(t)}{C(t)}$$
$$=1-\frac{Q(t)}{C(t)}$$
$$=1-F(t).$$

11 Equity break-even expression

$$x=\frac{\ln\left[0.5(1+i)^{n}+0.5\right]}{\ln(1+i)}$$

where

ln is the natural log function,

n is the number of periods in the life of the loan, and

i is the interest rate per period.

12 The affordable size loan is computed as the present value of annuity, expressed as

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right].$$

where

P is the present value of an annuity of n payments of R US-$ each,

R is the constant monthly payment rate, paid at the end of each period,

n is the number of conversion periods (months) in the life of the loan and

i is the annual interest rate.

13 The future value of an alternative investment in the form of a savings plan is computed as

$$S = R\left[\frac{(1+i)^{n}-1}{i}\right].$$

where

S is the future value of an annuity of n payments of R US-$ each,

R is the constant monthly payment rate, paid at the end of each investment period,

n is the number of conversion periods (months) in the life of the savings plan and

i is the annual interest rate.

4 Concluding discussions

The purpose of our approach was to derive, apply and provide a pertinent instrument aiming at the enhancement of transparency in loan-based real estate investment situation. Oriented towards the appearance of a dashboard, the end-product yields computational and visual results to the investor's data entries and creates substantial informational value for the loan-based investment decision environment. The calculations are carried out in the background of the spreadsheet tool and are based on multifaceted mathematical functions in the context of loan amortisation. Although we consider a mortgage case as the basis for the examinations, the principles are equally applicable to corporate financial strategies which are related to real estate investments. We anticipate practical implications from the explored dimensions of the underlying financial strategy in terms of opportunities to assess financial impacts in actual loan repayment situations.

The spreadsheet program *XGAMPS* was the chosen means of our scientific endeavour. We have presented various mortgage phenomena in conjunction with miscellaneous calibrations of exemplified data input. The scenario-based application of formula and the visualisation allows the comprehensive performance of a whole bundle of examinations, particularly on

- the monthly payment charge,
- the interest and equity portion of a certain payment within the loan term,

- the cumulative interest charge and cumulative equity,
- the total loan costs,
- the break-even point in time when the equity equals the unpaid balance and when 'true equity' starts to develop (Badiru, 1986),
- percentage evaluations of an arbitrary instalment within the loan term,
- the affordable size loan,
- impacts from extra payments,
- the evaluation of an advantageousness of extra payments vs. alternative investments, respectively, and
- the consideration of refinancing options.

This methodology paves the way for a comprehensive and balanced view of the key elements of the financial situation that a prospective borrower deals with referring to the capital investment in the shape of a mortgage.

For *future research* is the topic of expectedly falling property prices as interest rates escalate steadily. This is what is currently being experienced in the USA property market as interest rates have gone up above 7%, the highest in over 20 years. There are already existing remarkable scholarly discussions in this potential field of research. Lee and Park (2022) deal with the interrelation between rising interest rates, demand for housing and real estate market pricing and thereby emphasise the negative effect of interest rates on housing prices. With the application of a time-varying parameter vector autoregressive model, this scholarly approach analyses the impact of interest rate shocks on housing prices and concludes for the Korean market a dramatically increased impact since the global financial crisis, with particular regard to the increased dependence on underlying loans for real estate purchases (Lee and Park, 2022). Yu and Chen (2018) examined aspects influencing soaring housing prices in Taipei. Their empirical results reveal low mortgage rates to be the most relevant factor for rising housing prices (Yu and Chen, 2018). Additionally, it is assumed – on the basis of the empirical data of Taipei – that there is an inverse key linkage between rising financing rates and declining house prices: a 1% growth in mortgage interest rate is estimated to result in dropping housing prices by 5% to 17% (Yu and Chen, 2018).

We envision a mathematical postulation similar to the equity breakeven point, whereby we can study the contrived demand-and-supply curve. As interest rates rise exponentially, prices must fall in order to encourage prospective buyers. There will need to be sweet point of 'price meets in rate' to make the market insensitive to interest rate hikes. Unfortunately, space limitation does not allow us to include that extension in this present version of the software and 'writeware'. We encourage readers and researchers to explore this new idea further.

The following segments highlight and recapitulate the meaningful aspects, related to the time value of money, that are recommendable to consider in a financial decision in a mortgage payment context. It is a composition of arguments stressing that it appears crucial to keep a watchful eye on the financial dimension of a mortgage decision in all the different nuances along the three main perspectives 'quality', 'time' and 'cost'. Being empowered with balanced overall information on the different factors is a useful, dynamic tool for real estate investors.

4.1 Monthly payments, interest charge and total costs

It is of most significance to deal with the financial burden of a mortgage, mainly expressed by its costs. Therefore, the basic analysis includes essential components such as the monthly repayment rate, the interest charge, and the total loan costs. It falls short to concentrate on the monthly instalments, not only because the financial dimension of a real estate acquisition typically includes other continuous expenses, e.g., insurance, utilities, maintenance, or repairs. An interest charge for instance of more than 50% of the total loan costs as in some of the computed scenarios can become a severe financial stress test for the borrower. This transparency is necessary. As this money is not part of the equity, the investor will never see this money again. The higher the interest charge, the slower the principal is paid down and vice-versa the equity built up.

4.2 Break-even period cumulative equity vs. unpaid balance

The specific loan information for any arbitrary payment number within the loan term illustrates the composition of a particular instalment. Its equity and interest portion, the remaining unpaid balance at this point in time as well as the cumulative equity and cumulative interest charge can be pertinently analysed both in numbers and as a percentage. From the break-even period, where the cumulative equity equals the unpaid balance, the equity increasingly clearly exceeds the money that is owed to the lender with every additional instalment payment. In the context of building up 'true' equity, the amount of real estate that is virtually owned by the debtor not only grows. Even in the hypothetical case, the remaining balance is repaid in one fell swoop, the amount of 'true' equity could be retained, subject to an accordingly offsetting market value of the property.

4.3 Affordable size loan

The reverse approach to evaluate the financial decision of future home property owners is to calculate the affordable size of the potential loan. Based on the respective disposable monthly payment rate, the borrower obtains an orientation on the financial corridor the affordable size of a potential mortgage is aligned with. An advantage of the provided calculation is for instance the fact that borrowers can enmesh monthly savings to build up financial reserves for unforeseen expenses such as repairs or maintenance into the consideration of disposable monthly instalments.

4.4 Extra payments and alternative investment

Extra payments beyond the calculated basic monthly payment rate are undoubtedly a pervasive means to repay the principal amount faster and to build equity more effectively. The assessments in this paper stress the financial outcomes of extra money applied to the principal. Nonetheless, with regard to an investment view, it does not appear recommendable to lose sight of the fact that the additional payment could as well be invested alternatively. Depending on the interest rate, e.g., of a savings plan, such an alternative option may offer an unambiguously beneficial way to handle the investor's disposable extra money. Particularly amid rising key interest rates, improving revenues from alternative investments such as savings plans, or treasury bonds can be expected.

4.5 Refinancing option

Last but not least, an enormous financial burden such as a mortgage requires consistent and serious efforts of market trend monitoring in times of rebounding interest rates, to opt for an eventually advantageous refinancing option. It is crucial to keep a watchful eye on the development of interest rates and as well for any refinancing expenses, like, e.g., closing costs, so that a favourable refinancing option can where appropriate be considered. In the context of the time value of money, this procedure can help to save interest charge substantially. It may be helpful to relate a positive present value of refinancing to the length of time the property is planned to be resided.

4.6 General recommendations

Finally, further notable points of consideration in terms of a mortgage are worth being named. The first aspect concerns the quality dimension: a possible instrument of conservation of costs includes a clear and honest consideration of home property requirements. This approach implies focusing on needs, pruning desires, and leaving out negligible home features. It is a switch of perspective to scrutinise which basic needs actually exist, and which requirements would meet these needs, in order to prevent 'over-supply'. An existing property that has to be remodelled or rebuilt instead of a pricier new construction may fully cover all requirements. Secondly, once the decision on a certain home property has been made, it proves fallacious to set the principal amount too low. The scenario of disregarded or uncovered costs such as the design of the outdoor property can lead to the risk of additional financing, purportedly at higher loan costs. Moreover, full financing of the future home should be avoided, as the interest rate can be higher than with disposable equity that is deposited to purchase the object, for instance, a rate of 20% of the real estate price. Thereby, all expenses including the ancillary purchase costs should be appropriately included in the overall calculation as addressed in the second recommendation. In times of rising interest rates, it sounds enticing to save interest costs by choosing shorter payoff periods. The last recommendation involves an insight into precisely this issue: a fixed-rate loan with a term that allows repaying a significant or at least the major part of the principal appears essential. According to McMillin (2022) and HMDA 2021 data (FFIEC, 2022), about 70% of home loans are financed on a 30-year fixed-rate basis. Other than that, a follow-up financing situation could go along with an enormous and unexpected financial risk of additional interest costs.

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