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## A new edge weight-based measure for k-shell influential node identification in complex networks

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**Abstract:** There are mainly two problems with traditional k-shell centrality in complex networks. First, the traditional k-shell centrality divides many nodes into the same shell layer, which cannot accurately distinguish the propagation ability of nodes. Secondly, the network's local attributes and global perspective cannot be effectively combined into the k-shell centrality, and most of the methods ignore the role of edge weight. Because of these problems, a new edge weight is introduced based on traditional k-shell centrality. The edge weight between any two nodes is defined from the local degree centrality and the global k-shell centrality. From the dynamics of information propagation, a new edge weight-based measure for k-shell centrality is put forward. The simulation results indicate that this improved centrality based on edge weight is good at ranking the key nodes in a complex network, and the influential spreaders identified by this method can obtain better performance in the susceptible-infected (SI) model and susceptible-infected-recovered (SIR) model of infectious diseases.

**Keywords:** influential spreaders; edge weight; centrality; k-shell decomposition.

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## 1 Introduction

Mining and detecting influential nodes in complex networks has become a basic way to investigate the structure and dynamics of complex networks, so how to efficiently identify influential nodes has been one of the popular topics in complex networks. It has produced practical value in infectious disease control, information spreading in networks, market advertising, social network leaders, academic citation networks, etc. For example, in an industrial network, spreading information can be accelerated by identifying some key influential nodes, which can minimise the cost. On the other hand, cascading disasters in industrial networks can also be prevented by inhibiting (immunising) some specific nodes (Wang et al., 2022). Existing studies can be broadly classified into two parts: some works that analyse nodes based on the physical properties of the network and then rank the influence of the nodes based on their measure values, and other studies that focus on choosing a set of seeds to help maximise the spreading of influence from a complex network (Jiang et al., 2019).

Degree centrality (*degree*), betweenness centrality (*betweenness*), and closeness centrality (*closeness*) offer a foundation for the measure of influential nodes. Degree centrality (Bonacich, 1972; Zhao et al., 2020) uses connectivity to evaluate the nodes' spreading influence, which applies to a star-shaped central node with many neighbours but ignores the global structure of complex networks. The paper (Shi, 2023) utilised the degree of correlation to investigate the controllability of component failure in complex networks. Betweenness centrality (Newman, 2005) and closeness centrality (Sabidussi, 1966) overcome this shortcoming of degree centrality, but these cannot be applied to large-scale networks due to their time complexity. From a global perspective, the paper (Zhao et al., 2021) proposed a hybrid mathematical model for measuring influential nodes by considering not only the local degree centrality of the nodes but also introducing the closeness centrality to other nodes in the network. The paper (Lv et al., 2019) proposed an improved betweenness centrality named mean shortest path centrality. The results demonstrated that this method outperformed the conventional measures, such as degree centrality, betweenness centrality, and closeness centrality.

Recently, k-shell centrality (*ks*) (Kitsak et al., 2010) methods have been widely studied. As a global measure, many nodes will be split into various groups based on their network locations. The whole nodes in the same shell are shared the same *ks*-index. The most influential nodes are identified as those nodes in the core layer with the highest *ks* value. The disadvantage is that it is likely that a large number of nodes will be classified into the same shell, and thus, it is assumed that they all have the same spreading capacity. However, practical findings show that nodes in the same shell do not hold identical propagation capabilities. Given the coarse-grained aspect of using k-shell centrality for measuring node influence, many studies have introduced edge weights into the process of k-shell decomposition,

which allowed for classifying nodes at a fine-grained level. The paper (Garas et al., 2012) proposed a k-shell decomposition technique for weighted networks, which considered both the degrees and the edge weights of nodes, filling the gap in this area for identifying the influential nodes in weighted networks. The weighted LeaderRank algorithm (Li et al., 2014) was applied to directed weighted graphs, and experiments showed that the presented method could exactly and effectively identify the influential spreaders as long as they possess better robustness compared to degree centrality. The paper (Wei et al., 2015) used the degrees of two end nodes of an edge in unweighted networks to define the weight of the edge and improved the conventional k-shell decomposition method. This method not only takes the degree but also the edge weight into account during k-shell decomposition. The paper (Yang et al., 2017) proposed a novel centrality method with a second-order neighbourhood by introducing the local neighbourhood structure of nodes based on k-shell centrality, which integrated the global attributes of k-shell centrality and the local attributes of a second-order neighbourhood but did not take the edge weight into account. The paper (Yang et al., 2021) analysed a novel neighbourhood coreness method using path diversity based on information entropy to identify the influential spreaders.

In summary, as a global measure, k-shell centrality has significant advantages in identifying influential nodes. To improve the accuracy of this method, integrating the local and global attributes of networks is very necessary. The paper (Zhang et al., 2022) proposed a novel gravity centrality that combines a node's local and global information to properly describe the interaction between nodes. This work will propose a new edge weight-based k-shell centrality ( $k_s^{ew}$ ). This method takes the local attributes (degree) and global attributes (*ks*) to define the weight of an edge between any two network nodes, allowing for measuring and ranking node influences in a fine-grained way. The proposed method is also available for large-scale complex networks due to a low linear time complexity in degree centrality and k-shell decomposition.

The rest of the paper is organised as follows. Section 2 reviews the related works. Some related centrality methods are discussed in this section. The details about the new edge weight-based k-shell centrality are followed in Section 3. The dataset description and experimental simulation in real networks are shown in Section 4. Finally, the paper is concluded in Section 5.

## 2 Related works

To address the shortcoming of the conventional k-shell centrality methods that only focus on intra-shell node connectivity but ignore extra-shell node connectivity, the paper (Zeng and Zhang, 2013) proposed a mixing degree decomposition algorithm for node influence identification. Let  $k_r$  be the remaining connectivity of node  $i$ ,  $k_e$  be the connectivity between node  $i$  and the removed extra-shell

nodes, and  $\lambda$  be an adjustable parameter; the mixing degree km of node  $i$  is defined as follows:

$$k_m = k_r + \lambda k_e \quad (1)$$

When  $\lambda = 0$ , this formula is the conventional k-shell decomposition process; when  $\lambda = 1$ , this formula is the conventional degree centrality measure.

The paper (Liu et al., 2013) used the distance to the nodes in the largest shell to optimise the traditional k-shell centrality method. It proposed that the closer the distance to the core layer, the greater the importance. The k-shell distance centrality can discriminate the nodes' influence from the same shell.  $ks^d(v_i)$  represents the influence of node  $i$  in the  $ks$  shell.  $ks_{\max}$  denotes the largest shell of the network.  $\Gamma$  stands for all nodes in the largest shell  $ks_{\max}$ , and  $d_{ij}$  indicates the shortest distance between node  $i$  and node  $j$ . The larger  $ks^d(v_i)$  represents, the longer distance from node  $i$  to the core layer and the smaller influence of  $i$ . The k-shell distance centrality of any node  $i$  is calculated as in equation (2):

$$ks^d(v_i) = (ks_{\max} - ks + 1) \sum_{v_j \in \Gamma} d_{ij} \quad (2)$$

The paper (Bae and Kim, 2014) proposed a neighbourhood coreness ( $nc$ ) measure based on the assumption that a node with more neighbours in the core layer has a more influential capacity.  $\Gamma_i$  denotes the neighbours of any node  $i$ . The k-shell index of the neighbour node  $j$  of  $i$  is denoted as  $ks(v_j)$ , then the neighbourhood coreness measure  $ks^{nc}(v_i)$  is calculated as in equation (3):

$$ks^{nc}(v_i) = \sum_{v_j \in \Gamma_i} ks(v_j) \quad (3)$$

The paper (Wei et al., 2015) argued that most existing studies ignored the role of edges in measuring influential nodes and supposed that all edges hold the same weights in unweighted networks. To improve the performance, he proposed a weighted k-shell decomposition method using the degrees of the two end nodes for an edge to define the edge weight in unweighted networks. The weight  $w_{ij}$  of any edge  $e_{ij}$  is defined as shown in equation (4):

$$w_{ij} = k(v_i) + k(v_j) \quad (4)$$

where  $k(v_i)$  and  $k(v_j)$  denote the connectivity values of the two end nodes  $v_i$  and  $v_j$  of edge  $e_{ij}$ , respectively. Once the weights of all edges are computed, the value of the weighted k-shell influence can be calculated as in equation (5), where  $\alpha \in [0, 1]$  is an adjustable parameter and  $\Gamma_i$  is the neighbourhood set of the node  $i$ .

$$ks^w(v_i) = \alpha k(v_i) + (1 - \alpha) \sum_{v_j \in \Gamma_i} w_{ij} \quad (5)$$

2021 a hybrid measure (Zhao et al., 2021) for influential nodes is proposed. They suggested that a node's influence depends on its degree and is closely determined by the neighbours' influence. The proposed hybrid measure consisted of the degree  $k(v_i)$  of node  $i$  itself, the degree  $k(v_j)$  of the neighbour  $j$ , and the distance  $d_{ij}$  between  $i$  and  $j$ . The

formula for the hybrid measure  $GIN(v_i)$  of node  $v_i$  is as follows:

$$GIN(v_i) = e^{\frac{1}{n} * k(v_i) * \alpha} \times \sum_{v_j \in \Gamma_i} \frac{k(v_j) * \delta}{d_{ij}} \quad (6)$$

where  $\alpha$  and  $\delta$  are adjustable parameters. When  $\alpha$  increases, the model is more biased towards the degree of the node itself. When  $\delta$  increases, the model is more biased towards the degrees of the neighbours and the distance between two nodes.

From those mentioned above, it can be seen that most works only employ the degree centrality or k-shell centrality to define the weight of an edge to identify the influential nodes. The paper's contributions are summarised as follows:

- A new edge weight combining the degree centrality and k-shell centrality is proposed. Instead, it can be well applied in non-connected networks (loose networks) and fully-connected networks (dense networks).
- Using the SI and SIR epidemic model evaluation, the similar ranking and indefinite correlation are improved with a high monotonicity value. The proposed method is compared with other state-of-the-art techniques.
- There is a low linear time complexity in degree centrality and k-shell centrality. The proposed method can be well suited for the time requirements in large-scale complex networks.

### 3 The new edge weight-based k-shell centrality

Degree centrality typically exhibits good measurement performance in non-connected graphs and loose networks, as this method only calculates the number of neighbours and excludes neighbours' connections. The paper (Kitsak et al., 2010) demonstrated that nodes with a large degree of centrality located at the periphery of networks are not necessarily influential spreaders, and nodes located at the core layer after k-shell decomposition in densely connected networks have a significant impact. It was proposed that core-layer nodes typically have a large degree of centrality, while nodes with a large degree of centrality located at the periphery of networks are not necessarily located at the core. Notably, when a network significantly collapses, and large-scale non-connected areas are generated, the effectiveness of k-shell centrality would degrade drastically. The degree centrality measure is highly suitable for such cases.

The edge weights play a key role in the identification of influential nodes. For instance, roads connecting two metropolitan areas tend to be more important than roads connecting two small cities in transportation networks (Du et al., 2020); meanwhile, nodes connected by backbone transportation lines indicate that the city itself has influenced. Hence, introducing edge weight into the measure of node influences is of great significance.

To address the drawbacks of the previous works, this study introduces both local attributes (*degree centrality*) and global attributes (*k-shell centrality*) of nodes to define the edge weights between two nodes. In our proposed method, the edges in the network have varying importance. Both degree centrality and k-shell centrality are introduced into the influence measurement, in which both non-connected networks (loose networks) and fully-connected networks (dense networks) are compatible. Unlike previous studies limited by high time complexity where the measure of global influence was achieved based on the distance between two nodes (Lv et al., 2019), there is a low linear time complexity in degree centrality and k-shell centrality. Therefore, the proposed method can satisfy the time requirements in large-scale complex networks. The weight  $w_{ij}$  of edge  $e_{ij}$  between nodes  $i$  and  $j$  is defined by:

$$w_{ij} = \alpha * k(v_i) \times \delta * ks(v_i) + \alpha * k(v_j) \times \delta * ks(v_j) \quad (7)$$

where  $\alpha, \delta \in (0, 1]$  are tunable parameters that regulate the importance of degree centrality or k-shell centrality, respectively. The value named edge weight-based k-shell centrality can be achieved with the degree ( $k$ ) and k-shell index ( $ks$ ) of the two end nodes of an edge. The degree represents a node's local impact, and  $ks$  compensates for the global influence. When  $\alpha$  increases, the model is more biased towards the local impact of the node; when  $\delta$  increases, the model is more biased towards the global influence of the connected nodes. For the convenience of calculation,  $\alpha$  and  $\delta$  are set to 1 in this experiment.  $k(v_i)$  and  $k(v_j)$  indicate the connectivity of the end nodes  $v_i$  and  $v_j$ , respectively. The k-shell centrality of  $v_i$  and  $v_j$  is represented as  $ks(v_i)$  and  $ks(v_j)$ , respectively.  $ks^{ew}$  is calculated as in equation (8), where  $\Gamma_i$  is the neighbourhood of node  $i$ .

$$ks^{ew}(v_i) = \sum_{v_j \in \Gamma_i} w_{ij} \quad (8)$$

It is shown that the degree centrality (*degree*), k-shell centrality ( $ks$ ), k-shell distance centrality ( $ks^d$ ), weighted k-shell centrality ( $ks^w$ ), and  $ks^{ew}$  are used to rank node influences in a toy network in Figure 1. Table 1 shows the values of the influential nodes obtained by different methods. As observed, the conventional methods of k-shell centrality ( $ks$ ) and k-shell distance centrality ( $ks^d$ ) cannot distinguish the nodes  $a, b, c$ , and  $d$  in the highest shell in terms of influence. Both nodes  $a$  and  $b$  have two extra-shell neighbour nodes outside the highest shell ( $ks_{\max} = 3$ ); hence, their influences differ from those of the intra-shell nodes  $c$  and  $d$ . Although degree centrality (*degree*) could distinguish the nodes  $a, b, c$ , and  $d$  to a certain extent, it still assumes the same influence between the nodes  $e, f, g$ , and  $h$ . It shows that the nodes  $e, f, g$ , and  $h$  are outside the highest shell ( $ks_{\max} = 3$ ). According to the single node spreading principle proposed in the susceptible-infectious-removed (*SIR*) model of infectious diseases (Newman, 2002), since the nodes  $a, e$ , and  $f$  are closely connected to form a cluster structure (Kartun-Giles and Bianconi, 2019), even if the nodes have only one chance (in the *SIR* model, immunisation rate  $\gamma = 1$ ) to infect its neighbours, the individual nodes  $e$  or  $f$  are also

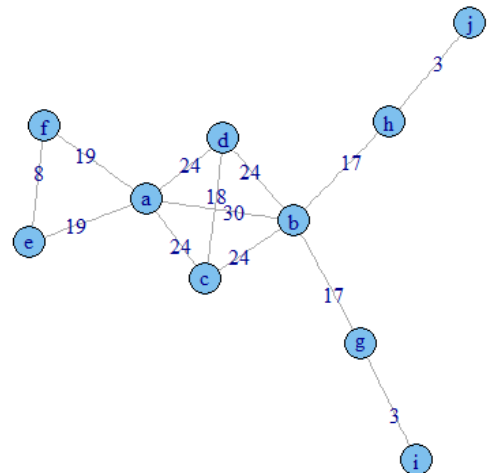
more likely to spread information through the cluster ( $ks = 2$ ) into the highest shell ( $ks_{\max} = 3$ ) and spread the information throughout the whole network. Similarly, node  $g$  or  $h$  only attempts to infect node  $b$  through the edges  $e_{bg}$  or  $e_{bh}$ . Once the infection fails, node  $g$  or  $h$  loses the opportunity to spread the information throughout the network. Hence, nodes  $e$  and  $f$  are significantly more influential than  $g$  and  $h$ . Fortunately, the k-shell centrality and k-shell distance centrality can distinguish.

As shown in Table 1, weighted k-shell centrality ( $ks^w$ ) and  $ks^{ew}$  exhibit an effective ranking. Since the weighted k-shell centrality ( $ks^w$ ) only considers the local attributes of end nodes (degree centrality) in calculating edge weights, nodes  $a$  and  $b$  have the same influence. Unlike the  $ks^w$ ,  $ks^{ew}$  considers both the local attributes (*degree*) and the global attributes (*k-shell*) of end nodes in calculating edge weights, allowing for a more fine-grained ranking of  $a$  and  $b$ . Although nodes  $a$  and  $b$  have two neighbours outside the highest shell ( $ks_{\max} = 3$ ) and the degree centrality of nodes  $e, f, g$ , and  $h$  is the same, the cluster structure of the above-mentioned makes the nodes  $e$  and  $f$  more influential than  $g$  and  $h$ . Thus,  $ks^{ew}$  suggests that the node  $a$  is more influential than the node  $b$ .

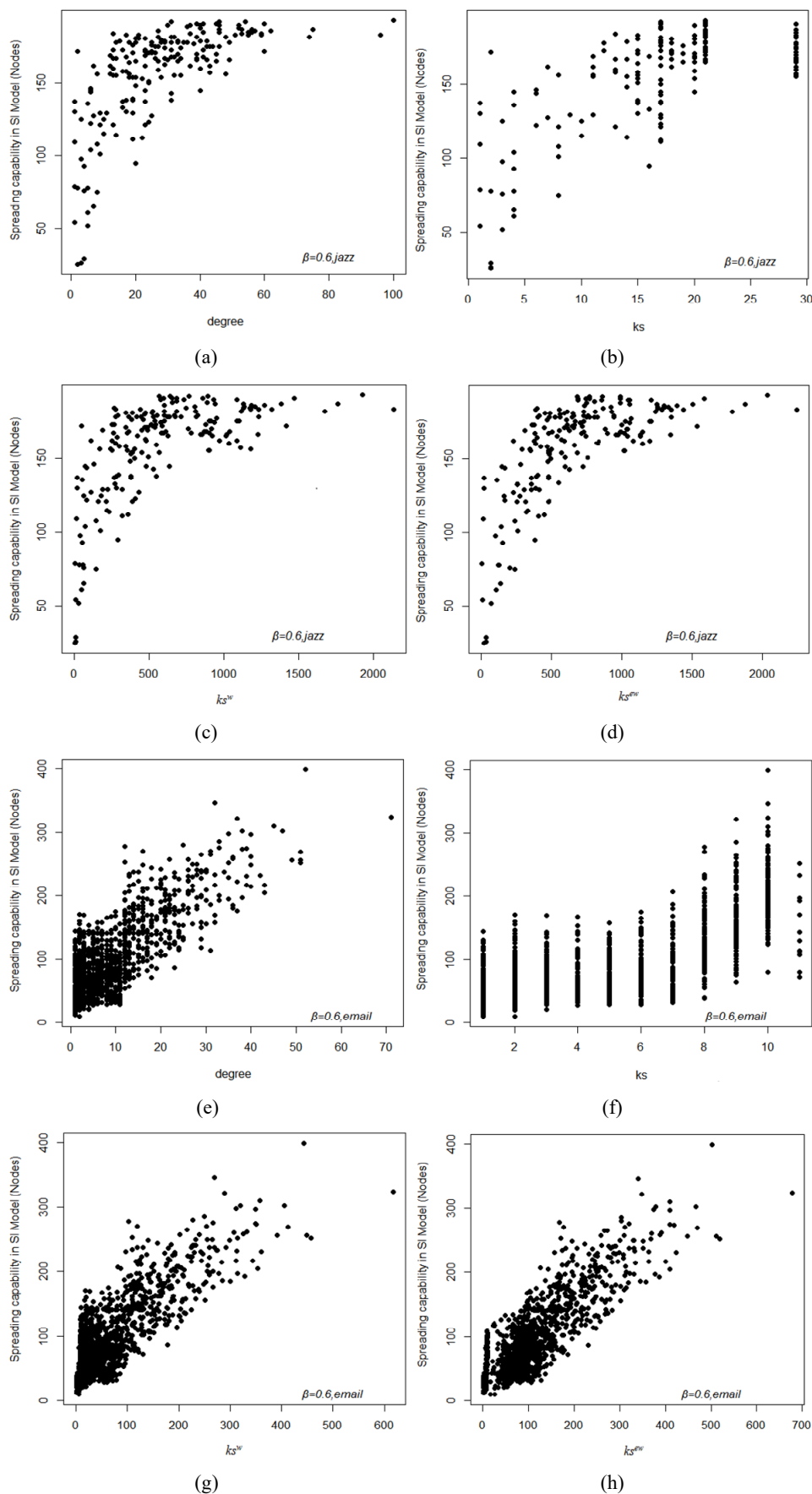
**Table 1** Measure values of influential nodes for different methods

Node	Degree	$ks$	$ks^d$	$ks^w$ ( $\alpha = 0$ )	$ks^{ew}$
$a$	5	3	3	40	116
$b$	5	3	3	40	112
$c$	3	3	3	22	66
$d$	3	3	3	22	66
$e$	2	2	14	11	27
$f$	2	2	14	11	27
$g$	2	1	21	10	20
$h$	2	1	21	10	20
$i$	1	1	33	3	3
$j$	1	1	33	3	3

**Figure 1** An example network with the edge weight-based k-shell measure (see online version for colours)



**Figure 2** Relationship between the methods and the real spreading capacities of nodes, (a) degree, Jazz network (b)  $k^s$ , Jazz network (c)  $ks^w$ , Jazz network (d)  $ks^{ew}$ , Jazz network (e) degree, e-mail network (f)  $k^s$ , e-mail network (g)  $ks^w$ , e-mail network (h)  $ks^{ew}$ , e-mail network



**Figure 3** The spreading influence of the methods in the *SI* model and *SIR* model for the *e-mail network*, (a)  $k_S^{ew}$  vs.  $k^s$ , *SI* model (b)  $k_S^{ew}$  vs. *degree*, *SI* model (c)  $k_S^{ew}$  vs.  $k_S^w$ , *SI* model (d)  $k_S^{ew}$  vs.  $k^s$ , *SIR* model (e)  $k_S^{ew}$  vs. *degree*, *SIR* model (f)  $k_S^{ew}$  vs.  $k_S^w$ , *SIR* model (see online version for colours)

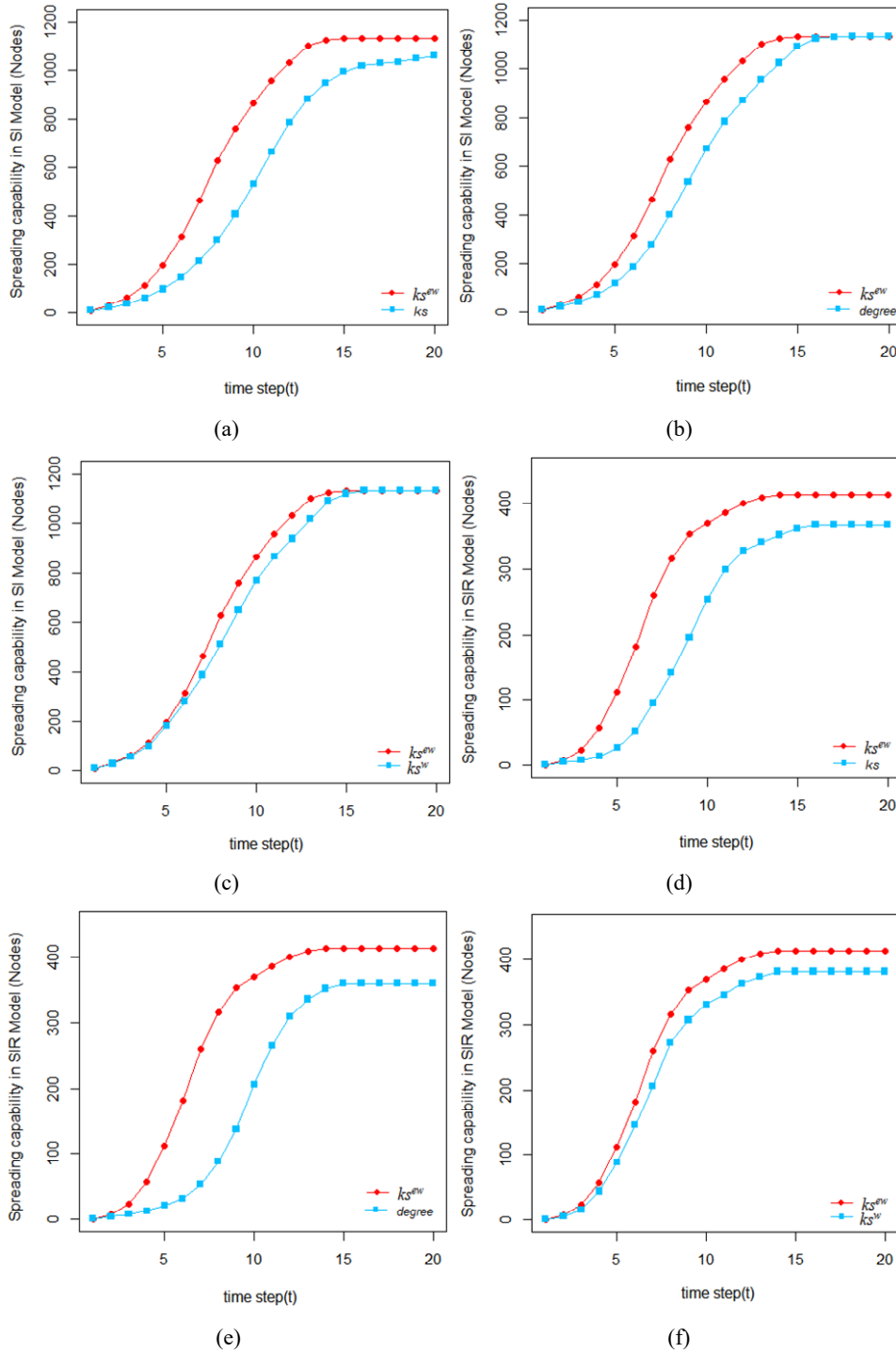


Table 2 shows the nodes ranking with centrality values calculated by different methods. As observed, there are so many nodes with the same rank in degree centrality (*degree*), k-shell centrality ( $k_S$ ), and k-shell distance centrality ( $k_S^d$ ). It is not easy to distinguish the nodes' influence. Compared to the weighted k-shell centrality ( $k_S^w$ ),  $k_S^{ew}$  can obtain a more accurate ranking of influential nodes.

Table 2 discussion shows that each classical centrality is highly coarse-grained, resulting in many indistinguishable nodes with the same centrality value. Therefore, it is very necessary to distinguish the role of nodes with the new edge weight-based k-shell centrality at a fine-grained level.

**Table 2** Ranking of influential nodes by measure values for different methods

Rank	Degree	$ks$	$ks^d$	$ks^w$ ( $\alpha = 0$ )	$ks^{ew}$
1	<i>a, b</i>	<i>a, b, c, d</i>	<i>a, b, c, d</i>	<i>a, b</i>	<i>a</i>
2	<i>c, d</i>	<i>e, f</i>	<i>e, f</i>	<i>c, d</i>	<i>b</i>
3	<i>e, f, g, h</i>	<i>g, h, i, j</i>	<i>g, h</i>	<i>e, f</i>	<i>c, d</i>
4	<i>i, j</i>		<i>i, j</i>	<i>g, h</i>	<i>e, f</i>
5				<i>i, j</i>	<i>g, h</i>
6					<i>i, j</i>

## 4 Experimental simulation

### 4.1 Datasets

The Jazz network (Gleiser and Danon, 2003) and e-mail network (Guimera et al., 2003) datasets are used in this experiment. *E-mail* is a popular dataset used in social networks and widely applied in CRAN R-package. *Jazz* is the classical small-world network in musical networks and is usually used to study the dynamics of small-world networks due to its small size. The topological parameters of the two networks are shown in Table 3. The number of nodes is defined as  $V$ , and the number of edges is marked as  $E$ . The network clustering coefficient is  $C$ .  $k_{max}$  is the maximum degree,  $\langle k \rangle$  is the mean network degree, and  $ks_{max}$  is the maximum number of shells during k-shell decomposition.

**Table 3** Topological parameters

Network	$V$	$E$	$C$	$k_{max}$	$\langle k \rangle$	$ks_{max}$
Jazz	198	2,742	0.63	100	27.7	29
E-mail	1,133	5,451	0.22	71	9	12

It is shown that the number of nodes in the *e-mail* network is nearly six times that of the *Jazz* network. However, the number of edges is only twice that of the *Jazz* network. The clustering coefficient  $C$  and the maximum number of shells  $ks_{max}$  are smaller than the *Jazz* networks. This indicates that the nodes of the *Jazz* network are more clustered to each other, which could be explained by the social grouping phenomenon in the musical business.

### 4.2 Analysis

The susceptible-infectious (*SI*) model and *SIR* model are applied to analyse the results for identifying the influential nodes:

- 1 The *SI* model is applied to compute the practical capacity of each node in both networks to analyse the relationship between the methods and the real capacity of each node.
- 2 The experiments are compared by using the nodes with varying spreading capacities in different methods as the spreading source, in which the accuracy is evaluated separately in *SI* model and *SIR* model.

#### 4.2.1 Relationship between measure methods and spreading capacities of nodes

Since the *SI* model does not consider the immunisation mechanism, all network nodes will eventually get infected with saturated spreading, making it impossible to identify the spreading capacities of nodes. Hence, in this experiment, a specific time (e.g.,  $t = 5$ ) is used to sequentially determine the number of nodes each origin can spread to. Figure 2 shows the scatterplots of the four measures (*degree*,  $ks$ ,  $ks^w$ , and  $ks^{ew}$ ) and the practical spreading capacities of nodes in the case of infection probability  $\beta = 0.6$  in the *SI* model.

As shown in Figure 2(b) and 2(f), k-shell centrality ( $ks$ ) distributes the same value to many nodes with varying spreading capacities, making it unable to determine the real spreading influence of nodes. Figures 2(a) and 2(e) shows that although degree centrality addresses the shortcomings of k-shell centrality to some extent, differences between the neighbour nodes regarding spreading capacity are still large, especially in areas with small degree centrality. The spreading capacity of many nodes does not show a monotonically increasing trend with degree centrality. Figures 2(c) and 2(g) shows the weighted k-shell centrality measure ( $ks^w$ ) results. As can be observed, the spreading capacity of nodes shows an overall increasing trend with measure values. However, in areas with small  $ks^w$ , especially in areas with many overlapping nodes with varying spreading capacities, it is shown in Figure 2(g), and the scatterplot is relatively divergent. It is seen from Figure 2(d) and 2(h) that the  $ks^{ew}$  method scatterplot shows a monotonically increasing trend, the neighbours have similar spreading capacities, and the scatterplot is relatively clustered. This phenomenon reveals that our method can effectively measure and rank influential nodes.

#### 4.2.2 Analysis of the spreading performance

Given the limited scale of the *Jazz* network, this experiment uses the *SI* and *SIR* models to analyse the performance in the large-scale e-mail network to reflect the spreading dynamics better. Since the *SI* model does not consider the immunisation mechanism, all network nodes eventually get infected with saturated spreading. For this reason, the spread speed of influential nodes is typically evaluated in the *SI* model. In this experiment, the infection probability  $\beta$  is set to 0.6, and the spreading time  $t$  is set to 20 (for the convenience of observing the saturated infection of the network). Also, the proposed  $ks^{ew}$  is compared with degree centrality (*degree*), k-shell centrality ( $ks$ ), and weighted k-shell centrality ( $ks^w$ ). The nodes that appeared either in  $ks^{ew}$  or the other three methods among the top 20 are selected as the spreading sources in *SI*. Without the common nodes in ranking lists, the comparison can be clearly distinguished from these methods. The spreading speed of the selected influential nodes is then observed.

According to Figures 3(a), 3(b) and 3(c), the source nodes selected by the  $ks^{ew}$  spread information faster, and all nodes are infected with saturated spreading at  $t = 15$ . Figure 3(a) shows that the source nodes selected by the  $ks$



method only infect 93% of nodes at  $t = 20$ , which can be attributed to the fact that the source nodes selected by the  $ks$  method have overlapping spreading areas. Figures 3(b) and 3(c) reveals that the *degree* and  $ks^w$  methods infect all nodes at  $t = 18$  and  $t = 16$ , respectively, indicating that introducing the local attributes of nodes in multi-source spreading benefits the selection of dispersed nodes.

According to Figures 3(d), 3(e) and 3(f), the spreading performance in the *SIR* model is compared. The infection probability  $\beta$  is set to 0.1, and the immunity recovery probability  $\gamma$  is set to 1. It is indicated that any node has only one chance to infect neighbour nodes when  $\gamma = 1$ . Hence, it is meaningful to highlight the role of the node's location in selecting influential nodes by reducing the infection probability ( $\beta = 0.1$ ). It should be noted that this experiment only selects the first different node ranked in the top 20 nodes between two methods as the single origin, which can avoid the problem of overlapping areas of multiple source nodes. It will better reflect the accuracy of the methods. Figures 3(d), 3(e) and 3(f) shows that the  $ks^{ew}$  achieves the best spreading performance, indicating that this method combining local attributes and global attributes can generate a more accurate ranking for influential nodes. Notably, Figures 3(d) and 3(e) shows that the  $ks$  method outperforms the degree method in the case of single origin, which verifies the superiority of the k-shell centrality in the measure for a single influential node.

## 5 Conclusions

The  $ks^{ew}$  measure was proposed to mitigate the limitations of conventional methods for k-shell decomposition. The shortcomings of the k-shell centrality method that classifies many nodes into the same shell are addressed in this novel method. The method uses degree centrality (the local attributes) and k-shell centrality (the global attributes) to define the weight of the edge between any two nodes, greatly improving the accuracy of identifying influential spreaders. Compared to the degree centrality, k-shell centrality, and weighted k-shell centrality, the experimental results from two real networks demonstrated that the proposed method could rank the key nodes more effectively, and the influential nodes identified by this method had a greater spreading speed and propagation range in the *SI* model and *SIR* model.

The proposed method measures influential spreaders by merging the degree centrality and k-shell centrality for two end nodes of an edge to define the edge weight. Further work is required to extend the definition of edge weight to make it applicable to weighted networks.

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## References

- Bae, J. and Kim, S. (2014) 'Identifying and ranking influential spreaders in complex networks by neighborhood coreness', *Phys A*, Vol. 395, No. 2, pp.549–559.
- Bonacich, P. (1972) 'Factoring and weighting approaches to status scores and clique identification', *J. Math. Sociol.*, Vol. 2, No. 1, pp.113–120.
- Du, Z., Tang, J., Qi, Y., Wang, Y., Han, C. and Yang, Y. (2020) 'Identifying critical nodes in metro network considering topological potential: a case study in Shenzhen City, China', *Phys A*, Vol. 539, No. 2, p.122926.
- Garas, A., Schweitzer, F. and Havlin, S. (2012) 'A k-shell decomposition method for weighted networks', *New J. Phys.*, Vol. 14, No. 8, p.83030.
- Gleiser, P. and Danon, L. (2003) 'Community structure in Jazz', *Advances in Complex Systems*, Vol. 6, No. 4, pp.565–573.
- Guimera, R., Danon, L. and Diaz-Guilera, A. (2003) 'Self-similar community structure in a network of human interactions', *Phys. Rev. E*, Vol. 68, No. 2, p.65103.
- Jiang, L., Zhao, X., Ge, B., Xiao, W. and Ruan, Y. (2019) 'An efficient algorithm for mining a set of influential spreaders in complex networks', *Phys A*, Vol. 516, No. 2, pp.58–65.
- Kartun-Giles, A.P. and Bianconi, G. (2019) 'Beyond the clustering coefficient: a topological analysis of node neighbourhoods in complex networks', *Chaos Solitons Fractals*, Vol. 1, No. 2, p.100004.
- Kitsak, M., Gallos, L.K., Havlin, S., Liljeros, F., Muchnik, L., Stanley, H.E. et al. (2010) 'Identification of influential spreaders in complex networks', *Nat. Phys.*, Vol. 6, No. 11, pp.888–893.
- Li, Q., Zhou, T., Lü, L. and Chen, D.B. (2014) 'Identifying influential spreaders by weighted leader rank', *Phys A*, Vol. 404, No. 6, pp.47–55.
- Liu, J.G., Ren, Z.M. and Guo, Q. (2013) 'Ranking the spreading influence in complex networks', *Phys A*, Vol. 392, No. 18, pp.4154–4159.
- Lv, Z., Zhao, N., Xiong, F. and Chen, N. (2019) 'A novel measure of identifying influential nodes in complex networks', *Phys A*, Vol. 523, No. 6, pp.488–497.
- Newman, M.E. (2002) 'Spread of epidemic disease on networks', *Phys. Rev. E*, Vol. 66, No. 1, p.16128.
- Newman, M.E. (2005) 'A measure of betweenness centrality based on random walks', *Soc. Netw.*, Vol. 27, No. 1, pp.39–54.
- Sabidussi G. (1966) 'The centrality index of a graph', *Psychometrika*, Vol. 31, No. 4, pp.581–603.
- Shi, W. (2023) 'Research on system safety in the case of component failure based on degree correlation', *Int. J. of Secu. & Netw.*, Vol. 18, No. 2, pp.65–74.
- Wang, T., Zeng, P., Zhao, J., Liu, X. and Zhang, B. (2022) 'Identification of influential nodes in industrial networks based on structure analysis', *Symmetry*, Vol. 14, No. 2, pp.211–222.
- Wei, B., Liu, J., Wei, D., Gao, C. and Deng, Y. (2015) 'Weighted k-shell decomposition for complex networks based on potential edge weights', *Phys A*, Vol. 420, No. 2, pp.277–283.

- Yang, F., Zhang, R.S., Zhao, Y., Hu, R.J., Li, M.T. and Yuan, Y.N. (2017) 'Identifying the most influential spreaders in complex networks by an extended local k-shell sum', *Inter. J. Modern Phys. C*, Vol. 28, No. 1, p.1750014.
- Yang, X., Xie, G.Q. and Li, X.F. (2021) 'Identifying influential spreaders in complex networks using neighbourhood coreness and path diversity', *Int. J. of Secu. & Netw.*, Vol. 16, No. 3, pp.174–182.
- Zeng, A. and Zhang, C.J. (2013) 'Ranking spreaders by decomposing complex networks', *Phys. Lett. A*, Vol. 377, No. 14, pp.1031–1035.
- Zhang, Q.Y., Shuai, B. and Lv, M. (2022) 'A novel method to identify influential nodes in complex networks based on gravity centrality', *Inf. Sci.*, Vol. 618, No. 12, pp.98–117.
- Zhao, J., Mo, H. and Deng, Y. (2020) 'An efficient network method for time series forecasting based on the DC algorithm and visibility relation', *IEEE Access*, Vol. 8, No. 1, pp.7598–7608.
- Zhao, J., Wang, Y.C. and Deng, Y. (2021) 'Identifying influential nodes in complex networks from global perspective', *Chaos, Solitons & Fractals*, Vol. 133, No. 4, p.109637.