THE PATH TO FAILURE: WHERE ARE BANKRUPTCY STUDIES AT NOW?

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Under existing modeling methodologies, a reliable model of bankruptcy prediction has eluded researchers. This has provided the impetus to search for either new modeling methodologies, or to use emerging technologies drawn from the artificial intelligence field and that of nonlinear dynamics to help model and explain the characteristics of financially distressed firms. This paper gives an update on some of this recent work and argues that the focus of the research ought to be on the distress continuum which requires the formulation of dynamic rather than static models of failure.

A major impetus for this work arose from the conviction that our insight into the process of firm failure can be enhanced by making more extensive and appropriate use of the internal financial histories of failed firms, along with the data that describes the external economic environment in which these firms operated. However, these data are often artifactual in nature and the notion of "failure" is, itself, subjective. At best, it can be measured on a qualitative or categorical scale as an event rather than as a process, given that failure is preceded by degrees of financial distress. Unfortunately, the data that we have are generally available for only a few discrete, and not completely regular, points in time and it is these data with which we are forced to work. We need new methodologies that better link our data with the realities of financial distress. Hence, it is argued that the financial distress prediction field as an area of research is still at a relatively formative stage, since no rigorous theoretically derived hypotheses have yet been formulated and tested within the existing methodology.

Despite the measurement problems referred to above, there has been strong and continued interest in the subject of bankruptcy prediction, because accurate instruments would benefit many interested parties, such as investors, lenders, auditors, management, employees and their unions. Using multivariate statistical techniques, early bankruptcy models have had varying degrees of success in classifying firms ex post, as bankrupt/nonbankrupt. These methods have usually employed an estimation sample that consisted of bankrupt versus solvent, usually strong, firms as the basis for discrimination.

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More recent research by Gilbert, Menon and Schwartz (1990) and by Flagg, Giroux and Wiggins (1991) have produced new insights into the finer distinction between distressed firms and failed firms.

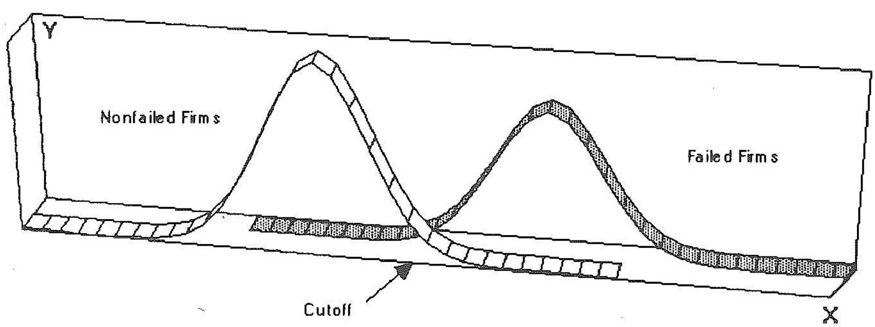
The work described here concentrates on (disequilibrium) dynamics and not state comparisons, because it is more likely that bankers and other resource suppliers need to assess the likelihood of bankruptcy for problem companies, not financially strong ones. What is needed are models that can either discriminate between "at risk" firms that survive and "at risk" firms that fail or that give an indication of which factors are involved when a firm that was surviving becomes one that is failing.

THE DISTRESS CONTINUUM

The major problem in bankruptcy research to date is highlighted in the exposition that follows:- that the nature of the dependent variable, "failure", is not a well-defined dichotomy as it should be for the types of modeling techniques that have traditionally been used to analyze it e.g. logit analysis or multiple discriminant analysis (MDA). Let us examine the basic bankruptcy problem as now embedded in the literature:

Discrimination on One Normally Distributed Variable

FIGURE 1



Consider a model developed on n variables to distinguish between two groups of failed and non-failed firms. This can be represented graphically on some (n-1)-dimensional hyperplane. For instance, and for ease of exposition, **Figure 1** shows the cut-off for discrimination between the two populations on only one normally distributed variable that has historically been shown to influence firm failure, as a point on the X axis. (Probability density shown on the vertical axis.)

In many studies of bankruptcy the reported misclassification rates have been relatively small, usually when the two groups of failed and non-failed firms are already well separated in multidimensional space, as in the illustrative diagram, **Figure 1**, but we are often interested in a much finer distinction. It is the area of overlap, or indecisive area which is most difficult to classify but which is also of most interest. The performance of a model is highly dependent on its potential to separate the groups in multidimensional space, i.e. to reduce this "grey" area to a minimum, which is in turn dependent on the sophistication of the modeling technique and whether the model is complete i.e. it includes the important explanatory variables and, more importantly, where the sampled firms lie on the success-failure continuum. It is not surprising that these model formulations are most successful when the data conforms to the expectation that the two groups are already well separated on the this continuum—i.e. a bankrupt group and a non-risky surviving group.

Gilbert, Menon and Schwartz (1990) provided evidence to support this logic by excluding obviously strong firms from the nonbankrupt portion of their estimation sample to conclude that ratio-based models perform poorly in identifying likely bankruptcies from a pool of problem companies. Flagg, Giroux, and Wiggins (1991) considered distressed firms exclusively and used four failure-related events in addition to standard financial ratios to predict which firms will ultimately go bankrupt. Both these studies used designs that represent a far more difficult predictive environment than previous studies but neither took into account the external influence of the macro-economy on the failure process. Neither did they veer from the traditional cross-sectional study, assuming stationary of failure models over time. What is needed are models designed to examine failure risk (and many attempt to predict it) that also incorporate appropriate lead-lag relationships among observed economic data series—a common feature of empirical economic research.

RECENT WORK USING EMERGING TECHNOLOGIES:

Artificial Neural Networks

When the distinction between survival and failure is a fine one, neural network technology, borrowed from the artificial intelligence field, may yet be a promising tool for solving the classification problem: the problem of classifying an entity into one of a finite collection of groups based on the attributes of that entity. Instead of a linear function, any response surface can be used to map the data, in this case for discrimination purposes, allowing the model to represent the complex relationships inherent in the interpretation of financial ratios. Hence, without a linear restriction, the model can be made to fit the data "like a glove." In addition, artificial neural networks can overcome the effect of autocorrelation which is often present in time series data and the technique tolerates data errors and missing values; problems not accounted for in multiple regression models.

Operation of a neural network occurs in two phases—learning and recall. The network is "trained" in the learning phase with numerous examples of entities as input; their attributes and, in the case of bankruptcy studies, their binary group membership as output. The method used for neural network prediction is called generalization (Dutta and Hekhar, 1988) in that once the network has been trained, new data is input for the network to predict the output.

De-Bodt, Cottrell and Levasseur (1995) provide an introduction to these new tools along with thirty applications in finance, including some in the domain of bankruptcy prediction. Predominantly, papers in this area have compared an Artificial Neural Network (ANN) with a published MDA model of failure prediction. (Odom and Sharda,1990; Koster, Sandak, and Bourbia, 1990; Cadden,1991; Coats and Fant, 1992; Lee, Han, and Kwon, 1996). Exceptions are Fletcher and Goss (1993), and Udo (1993), who compare a neural network with a logit model of bankruptcy. All show superior classification outcomes and have shown that, in essence, neural network models are easier to use, more robust, more flexible, and more responsive to change than a regression model. They appear, also, to be more robust on small sample sizes.

Until recently these models suffered from a lack of generalisability due to overfitting the ANN (as discussed by Koster, Sandak, and Bourbia (1990) and Fletcher and Goss(1993)), and were generally used to assist rather than become an alternative to traditional statistical and mathematical models. Yet, these authors appear now to have provided a principled mechanism for determining the optimal network architecture that mitigates against this problem, as their results have verified.

Hence, benefits from the implementation of this technology have aided researchers with the classification problem in bankruptcy studies but not with explanation of process. The most important problem is the "black box" nature of the ANN, i.e., we have no understanding or knowledge regarding how it solves a particular problem. The theory is highly mathematical and the application is essentially one of numerically solving sets of simultaneous equations, so no indications regarding theory are given to aid in the construction of hypotheses that can be tested to give meaning or explanation to this or any other area of research to which ANN is applied.

Chaos Theory

Another promising avenue of research that has shown some success in predicting firm failure comes from the field of non-linear dynamics. Non-linear dynamic models have proven quite successful in the prediction of certain endogenously determined catastrophic system failures, such as myocardial infarction and since a firm's principal investors and creditors would also consider firm bankruptcy to be a catastrophic event, then it should be possible to exploit the characteristics of chaotic behaviour in predicting this type of failure. Etheridge and Sriram (1993) argue that economics and finance researchers have already successfully used chaos theory to study systems such as the stock market (Peters, 1991), and that it is time for accounting researchers to begin using the methodology.

Lindsay and Campbell (1996) applied chaos theory to bankruptcy research with results that were consistent with Goldberger's hypothesis (1990) that healthy systems exhibit more chaos than unhealthy ones. They found that the returns of firms nearing bankruptcy indeed exhibited significantly less chaos than other, healthy firms. The amount of chaos was measured with Lyapunov exponents (Wolf et al, 1985) from which were constructed univariate and multivariate bankruptcy prediction models using industry match-paired data of 46 bankrupt and 46 non-bankrupt firms.

Dynamic Models

After Gilbert, Menon, and Schwartz (1990) provided the evidence that a bankruptcy prediction model developed along traditional lines would not be able to distinguish firms that fail from other financially distressed firms, a study by Hill, Perry and Andes (1996) employed a dynamic event history methodology to model this distinction. The event history analysis looks at the transitions to and from stable and financially distressed states and from these states to the bankrupt state (using longitudinal data). This contrasts with cross sectional analysis which uses a snap-shot focus assuming that each firm will remain in one state (since this type of analysis typically measures the financial state only once).

Secondly, the dynamic model also explicitly accounted for time-varying independent variables which may change over the observation period and included in their model a measure indicating whether a firm received a qualified account or unqualified audit opinion (one of the four events that Flagg, Giroux, and Wiggins (1991) used and postulated would signal that a firm is experiencing financial distress). The two macro-economic variables identified by Rose, Andrews, and Giroux (1982) as leading business failure, the prime rate and the unemployment rate, were also included in order to partly control for changes in the business environment. Since the model was successful in identifying significant explanatory variables that differ between financially distressed and bankrupt firms and also between industry classifications, the authors were able to conclude that the explanatory variables do, indeed, play a differential role in financial distress and bankruptcy as well as across industries

Another area that traditionally uses dynamic models is survival analysis in various health fields. The statistical techniques drawn from survival analysis lend themselves well to bankruptcy research knowing that the potential for a firm to fail always exists ex ante. Since these techniques require no specification of the distributions of the data set, models based on them overcome the problems of bias outlined by Zmijewski (1984). Many use the Cox (1972) proportional hazards model incorporating regression-like arguments into life-table analysis. A life table is a method of summarizing the results of a study by grouping the times to failure into time intervals. For each time interval the table records the number of firms which are still in the study at the start of the interval, and the number censored. The ability to use censored data (the time to failure is greater than the predetermined time horizon of the study) distinguishes this technique from other statistical methodology. The probability of interest is the conditional probability of an organization failing in the time interval, i.e., the probability of failure in the next period given survival to that period(the hazard rate or age-specific failure rate). Studies of financial distress incorporating these techniques include those of Crapp and Stevenson(1987), Eddey, Partington, and Stevenson (1989), who predict the timing and probability of takeover bid success, Kassab, McLeay, and Shani (1991), and Partington, Peat, and Stevenson(1991)—all concerned with financial distress.

Two other studies which focussed on the distress continuum are: the failing firm model of Flagg, Giroux, and Wiggins(1991) that included postulated events signaling that a firm is experiencing financial distress, viz., reductions in dividends, "going concern" qualified opinions, troubled debt restructurings, and violations of debt covenants; and the sequential dynamic failure prediction model of Thoedossiou (1991) using multivariate time-series which can detect the critical point at which a firm's financial variables shift from a "good performance" distribution to a "bad performance" distribution.

A new approach consists of the family of models, described below, that allows for a comparison of each firm with itself over time—from when the firm was surviving, or avoiding bankruptcy, to when it failed.

A NEW STUDY DESIGN AND A NEW DYNAMIC MODELLING METHODOLOGY

If we want to examine the process of failure and since most distressed firms do not fail (become bankrupt), a critical examination of those that do may provide additional insights into the process. So the immediate real question of interest is how firms transform from surviving or even successful ones into failed ones. Presented is a methodology for examining the process of failure of firms rather than testing specific hypotheses. It therefore represents a break from the past in that it moves away from models that attempt to discriminate between failed and non-failed firms. For this departure it has employed, within the overall methodology, a sample set composed entirely of firms which all fail: the firm's earlier financial statements from its surviving years providing the data to compare with the data in the firm's final financial statements before bankruptcy or liquidation is imminent.

Work in this area is extended here, both in the formulation of a radically different failure model design and by incorporating an extensive investigation of the macroeconomic environment within which the firms operated over the 20 year period of this study. Specific models are tested empirically here using all sixty failed service industry firms from the COMPUSTAT database for the years 1971-1991 and examining the dynamics of the same firms over time in the periods (the last four years of the firm's existence) prior to failure.

This shift from models based on a single equation to a multi-equation structure incorporates the time factor as well as allowing for continuous dependent variables or jointly dependent variables to become the focus of research attention. Both single-equation pooled-time models of failure risk incorporating lagged estimates of failure risk are estimated as well as multi-equation models that treat as endogenous, some of the significant financial ratios that are collinear as well as the risk of failure itself. These give some insight into how the important correlates with failure risk change over this four-year window.

This work investigates the distress continuum with a design which is an improvement over past studies in model specification for relevance over time and for its ability to investigate the pattern of failure over time for firms that have eventually failed. Of immense methodological significance is the fact that a discrimination model is built on an estimation sample consisting of failed firms only; the firm's earlier financial statements from its surviving years providing the data to compare with the data in the firm's final financial statements before bankruptcy or liquidation is imminent. Each firm acts as its own control; its own surviving years providing the best match on all the non-important variables for a typical "case-control" study of failure risk. Though, if a comparison is to be made with a survival study in the health field, here we are more interested in the progression mechanisms of the "disease" once it is established rather than examining healthy versus diseased firms—or the probability of contraction.

The Time Dimension

It is known that the rate of corporate failures rises sharply during economic recessions. (Lev, 1974, pp.134-139). This fact helps in explaining the lack of consistency both in the values of the coefficients reported and the relative importance of various financial ratios across different bankruptcy prediction studies. Besides using different sets of ratios in their models, researchers typically pool data across different years without considering the underlying economic events in those years. Mensah (1984) states that while multivariate models of bankruptcy may correctly identify the more common characteristics of failing companies which make them susceptible to bankruptcy, the actual occurrence and its timing will usually depend on the coupling of these characteristics with certain economic events (external to the firm). Wood and Piesse (1987) have also suggested that data instability due to changes in inflation, interest rates, and/or phases of the business cycle may be responsible for the differences in the classification rates from estimation (ex post) to forecast (ex ante) time periods in past models used to discriminate failed and non-failed firms.

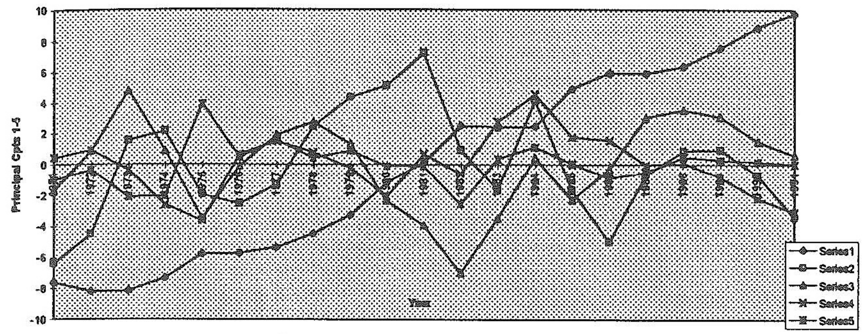
Since a purely ratio-based model may not show sufficient discrimination between the earlier "at risk" or distressed firm-years and the final bankrupt year, external factors relating to the macro-economy were included to help properly model both the external environment of the firm as well as the time dimension of the model. Johnson (1970, p.1166,1167) points out that "ratios to predict failure... do not contain information about the intervening economic conditions... the riskiness of a given value for (a) ratio changes with the business cycle." And Zavgren (1983) states that account needs to be taken of company soundness criteria, which will vary over the business cycle. In boom periods when failures are relatively rare, the empirical link between certain otherwise important indicators and the actual occurrence of failure will be weak.

In the research described here, the four years prior to the last audited statements of each failed firm were examined with respect to both the financial ratios deemed important in the financial accounting literature as well as various economic series taken from the *Abstracts of the USA*. (See Appendix A.) Since many of these series are serially correlated, they were first analyzed by the method of principal components which reduced the number of variables that represent the external environment of the firms from 43 economic series to only five orthogonal factors graphed over time in **Figure 2**, below. The five factors were identified, based on the series upon which each factor loaded as:

The Level of Activity/Demand or Growth Factor
The Cost of Capital Borrowing Factor
The Labor Market Tightness Factor
The Construction Factor
The Expenditure (Private, Public, Business) Factor

Figure 2

Principal Cpts of USA Economy 1971-1991



Since it is expected that the macroeconomic variables will lead failure risk, these and their lagged values for up to three years were included for each year of data on the failed firms along with the values of the 23 financial variables. (See **Appendix B** which shows the full explanatory variable set)

A FAMILY OF MODELS FOR FAILURE RISK

The conceptual framework for any failure model must reflect the fact that in any economic process, of which the financial dealings of a firm are a part, all variables are interdependent. Marschak (1950) commented that "economic data are generated by systems of relations that are in general stochastic, dynamic and simultaneous." Thus far bankruptcy prediction models have almost completely avoided coming to grips with this methodological problem. This further translates into the realization that economic data that are direct outcomes of an existing economic system should therefore be modeled as a system of simultaneous relations among the random economic variables, both current and lagged, internal and external to the firm. Marschak further noted that, although these properties of the data give rise to many unsolved problems in statistical inference, they constitute the basic ingredients underlying economic theory, and quantitative knowledge of them is needed for sound economic practice. What is intriguing about these comments is that they were written almost half a century ago, but such a vital problem as failure of firms has not yet been fully analyzed in any way approaching this manner.

Hence, critically, a modeling procedure is proposed where a family of models is explored—from the most general simultaneous set with non-linear form through the least general recursive system to single equation models of the simplest form. Only the last of these has, to date, been used exclusively in bankruptcy studies. As opposed to this, a model with simultaneity and timed variables may comprise a time series to predict a vector of values for several financial variables at time t from their previous values at time t-1, and any other important exogenous variables such as macroeconomic indicators. The results from various equations can feed the full set of explanatory variables into a logit model where the dependent variable is the outcome (survival=0, bankrupt=1) at time t. Each firm will provide a number of rows to the data matrix depending on the number of years, previous to bankruptcy, that its financial data are available. This will be an intrinsically non-linear, simultaneous equation system which is, in principle, testable.

Consider the completely specified linear model which, using information from k prior years to failure, is specified as:

$$\mathbf{X}_t = A_0\mathbf{X}_t + A_1\mathbf{X}_{t-1} + A_2\mathbf{X}_{t-2} + \cdots + A_k\mathbf{X}_{t-k} + B\,\mathbf{Z}_t + \mathbf{e}_t \qquad (1)$$

where $\mathbf{X}_t$ is an mx1 vector of internal variables for the firm at time t, and $\mathbf{Z}_t$ is an nx1 vector of exogenous variables including macroeconomic indicators for the period (which may also be lagged). The matrices A_i and B are made up of the structural parameters for the model and $\mathbf{e}_t$ is the error vector. The A_i's are mxm order matrices and B is mxn. Here, for simplicity, these parameters are assumed to be fixed for all firms. The model is cast in a linear or log-linear form here purely for exposition but clearly it is more general for it to be non-linear form. Note that the internal variables in model (1) may include the outcome variable Y which is a 0/1 variable or, alternatively, the probability of failure in the next year, or the logit transformation of this probability This is especially important as it has a significant impact upon model construction and upon the appropriateness(or lack thereof) of linear estimation techniques.

The reduced form of the structural model (1) is then:-

$$\mathbf{X}_t = (I-A_0)^{-1} A_1\mathbf{X}_{t-1} + \cdots + (I-A_0)^{-1} B\mathbf{Z}_t + (I-A_0)^{-1} \mathbf{e}_t \qquad (2)$$

or

$$\mathbf{X}_t = A^*_1\mathbf{X}_{t-1} + A^*_2\mathbf{X}_{t-2} + \cdots + B^*\mathbf{Z}_t + \epsilon_t \qquad (3)$$

where $A^*_i = (I-A_0)^{-1} A_i$, $B^* = (I-A_0)^{-1} B$, and $\epsilon_t = (I-A_0)^{-1} \mathbf{e}_t$.

The final model is a set of simultaneous equations which are linear in the parameters.

By ignoring the exogenous variables (assuming **Z** is constant) it is possible to reduce (3) to a first-order system in an augmented X-vector and examine the dynamic properties of the model, although this method has not yet been applied to financial distress models. The stability of the model can thus be determined by investigating the nature of the eigenvalues of the coefficient matrix of the reduced model. The theoretical aspects of the stability conditions can then be discussed in terms of the failed firm's internal dynamics.

Returning to the embedded logit form, if it proves difficult to incorporate the 1/0 dichotomous outcome directly into a limited dependent simultaneous model, as one of the elements of X, then the model can be recast, at least as a starting point, as a single equation such that the ratios vector X_t feeds recursively into a logit model of failure risk as in equations (4) and (5).

So for firm i:

$$p = F(X_t, X_{t-1}, \ldots, Z_t) \text{ or just} \quad (4)$$

$$= F(X_t, Z_t) \text{ if lagging X proves too cumbersome, or infeasible} \quad (5)$$

where p is the vector of risks p_t for the firm's risk of failure in the next year at time t, t-1, t-2 and so on, but is actually input as the event Y_t taking on values of 1 or 0, depending on whether t is respectively the year of failure outcome, or some prior year, and F is the logit link function (6):-.

$$Pr(Y_i = 1) = P_i = (1 + \exp\{-W_i\})^{-1} \quad (6)$$

where $W_i = b_0 + \sum_{j=1}^{m} b_j X_{ij}$ and

Y_i represents the outcome or dependent variable, for the ith firm in the next period: either

$Y_i = 0$ for the population of surviving firms, or

$Y_i = 1$ for the population of failed firms,

X_{ij} is an element of the matrix of explanatory variables - the value of the jth characteristic for the ith firm,

$b_0, b_1, b_2, \ldots b_m$ are constants of the model,

m = the number of explanatory variables, X_j, taken for each firm.

In the complete non-linear model, the probability of failure is embedded in the simultaneous form rather than recursive form ie. it occurs on both sides of the equation. This is important because financial distress can be expected to feed on itself, a form of positive feedback. An example would be the situation where the overall financial health of the firm, as measured by the probability of failure, will have an impact upon the ability to raise capital and so on.

The completely specified model may be empirically impractical to implement, e.g. through lack of appropriate data, and may need to be made less general, but the existence of the completely specified model gives a methodological basis for the more appropriate construction of smaller, more specific models. Thus, each smaller and less complete model can be fitted into an overall family of models. This also makes any weaknesses of the smaller, practical models more transparent.

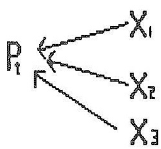
Specified next are a family of models, all of them related, for which implementable examples are given and for which the service industry data set served as a testing ground.

These models were used for estimating the risk of failure and are outlined here in order of increasing complexity and consequent difficulty of estimation technique. For ease of exposition, all error terms and residual terms have been omitted.

(A) Single Equation Logit (All Explanatory Variables are Exogenous)

$$\log [P_t / \{1 - P_t\}] = a_0 + a_1 X_{1t} + a_2 X_{2t} + a_3 X_{3t} \tag{7}$$

P_t is the relative frequency measure or other measure of risk of failure. The causal relationships can be represented as follows:

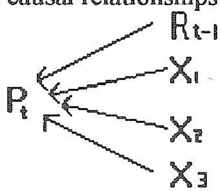


$X_{1,2,3}$ are the exogenous variables - three are used for exposition only, any number may be included. This model was estimated with the exogenous variables as the first five principal component factors and their one, two and three-year lags for the US economy incorporating economic series over a twenty-one year period. The model was estimated using observations on firms from all four lag periods before failure and matching the economic factors, X_{it}, on the appropriate year. The same model can be applied with the X's taking on the values of the internal financial ratios of the firm, assuming that these are all independent of each other and also independent of the risk of failure (ie. exogenous with no causal feedback). This model was estimated using a stepwise logit estimation of risk regressed on 23 ratios, and again pooling observations for all four lag periods. Finally, the more useful combined single-equation model using both types of variables, internal and external to the firm, was estimated and also falls into this category.

(B) Single Equation Logit (All Explanatory Variables are Exogenous or Lagged Endogenous).

$$\log [P_t / \{1 - P_t\}] = a_0 + a_1 X_{1t} + a_2 X_{2t} + a_3 X_{3t} + b_1 R_{t-1} \tag{8}$$

R is an endogenous variable that could be, for example, the risk of failure or a financial ratio which is influenced by risk of failure. All the X are assumed exogenous. The causal relationships can be represented as follows:



(C) Simultaneous Equations (All Explanatory Variables are Exogenous, Lagged Endogenous and Current Endogenous Variables).

(i) $\log [P_t / \{1 - P_t\}] = a_0 + a_1 X_{1t} + a_2 X_{2t} + a_3 X_{3t} + b_1 R_{t-1} + b_0 Q_t$

(9)

where, in addition, the following relationship exists:

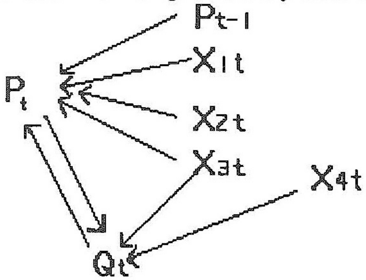
$$\text{(ii) } Q_t = \alpha_0 + \alpha_1 \log [P_t / \{1 - P_t\}] + \sum_{j=1}^{m} a_j X_{jt} \qquad \textbf{(10)}$$

or with a different functional relationship between Q_t and P_t:

$$\text{(iia) } Q_t = \alpha_0 + \alpha_1 P_t + \sum_{j=1}^{m} a_j X_{jt}$$

(11)

Q_t and P_t are current endogenous variables and the X_{jt} are exogenous (may be some in common with equation (i)). Q_t may be one of the internal ratios that has a two way (feedback) relationship with failure risk as in the diagram below. This model is linear in form as a simultaneous equation system. Equations (i) and (ii) cannot be estimated independently of each other. 2SLS cannot be used directly because equation (i) has to be estimated from original binary data so a logit form is required.



To perform the proper estimation of this simultaneous equation model firstly requires an estimation of the unrestricted reduced form equation for (i) providing a first-stage estimate of $\log [P_t / \{1 - P_t\}]$, or P_t as its inverse, using logit analysis, and secondly, an estimation of Q_t from the structural form equation for (ii) based upon the reduced form estimates. This eliminates the non-linearity of $\log [P_t / \{1 - P_t\}]$ in (ii). Then estimate the unrestricted reduced form for Q_t which can then be substituted into (i) and so on, as in 2SLS estimation.

(D) Seemingly Unrelated Regressions.

Consider the following equations, one for each lag time, one to four periods before failure:

$$\text{(i)}\ \log[P_t / \{1 - P_t\}] = \alpha_t + \sum_{j=1}^{m} a_{t,j} X_{j,t}$$

$$\text{(ii)}\ \log[P_{t-1} / \{1 - P_{t-1}\}] = \alpha_{t-1} + \sum_{j=1}^{m} a_{t-1,j} X_{j,t-1}$$

$$\text{(iii)}\ \log[P_{t-2} / \{1 - P_{t-2}\}] = \alpha_{t-2} + \sum_{j=1}^{m} a_{t-2,j} X_{j,t-2}$$

$$\text{(iv)}\ \log[P_{t-3} / \{1 - P_{t-3}\}] = \alpha_{t-3} + \sum_{j=1}^{m} a_{t-3,j} X_{j,t-3}$$

These equations are not simultaneous; each equation represents the same firms over different time periods but, since they are a combination of cross-sectional and time-series data for the same firms - a reasonable assumption is that the disturbances or error terms in different equations are correlated. Providing the initial estimates of the logit component are input into the algorithm to eliminate non-linearity, this set can be analyzed together using SUR estimation (incorporating generalized least squares.) [1]

The $X_{j,t}$ may also include lagged endogenous variables (which are deemed to be predetermined) and still be treated as "seemingly unrelated" since no simultaneity is present.

RESULTS

Results for the most simple single-equation model within the family of models-Equation (7), estimated using the service industry data set are summarized below :

The effects of the economy and the effects of the financial variables on failure risk were found to be mutually reinforcing.

The effects of the important macroeconomic variables are more significant, statistically, than the effects of the important internal financial ratios to the risk of failure, at least as judged by their order of entry into a forward stepwise selection logit model[2] (See **Appendix C** for the parameter estimates and their statistical significance to the model)

The liquidity ratio, working capital to total assets, is statistically the most significant ratio to failure risk. In addition, when combined with the external variables, it vies with the external variables for early entry into the stepwise model. This is consistent with past studies of financial distress across different research designs and time periods in that, as this ratio increases, failure risk decreases.

The effect of the "labor market tightness" factor, a complex external variable derived as a principal component, highly weighted (negatively) on the total number unemployed, and (positively) on the change in the size of the employed labor force, is the most significant variable ($p<.0001$), overall, to failure risk. Other external variables significant to failure risk ($p<.0005$) are the "private, public, and business expenditure" factor lagged one year (negatively, $p<.0001$), and the "cost of capital borrowing" factor lagged one year (positively, $p<.0004$).

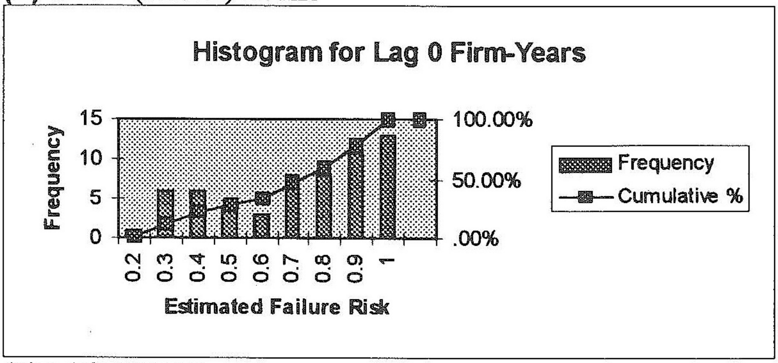
Tentative interpretations of these effects are offered. As the labor market tightens ie. there are fewer unemployed, unions are stronger and the cost of labor increases, one would expect the risk of failure to increase. As private, public and business expenditure increases, demand for products and services is higher so one expects the risk of failure to decrease. As the short-term cost of borrowings increases, the costs of production are higher to service borrowings, so one would expect the risk of failure to increase.

The logit model based on the variables statistically significant to failure risk used only a binary 0/1 input variable for the events "fails in the next year" / "survives the next year" and yet provided a good instrument for measuring failure risk at each period prior to failure. It not only has given a logical result that the risk of failure increases with each period but also that it increases quite markedly in the final three years before failure. This is illustrated in **Figure 3** showing the histograms for the estimated failure risk separately for each lag year. The shape of the cumulative frequency ogive gets flatter going from figures A to D as more of the distribution of failure risks are concentrated at the lower end in the earlier years before failure. There is also an impressive modal shift from the highest risk category to the lowest in the first lag year, and there it remains for all lags.

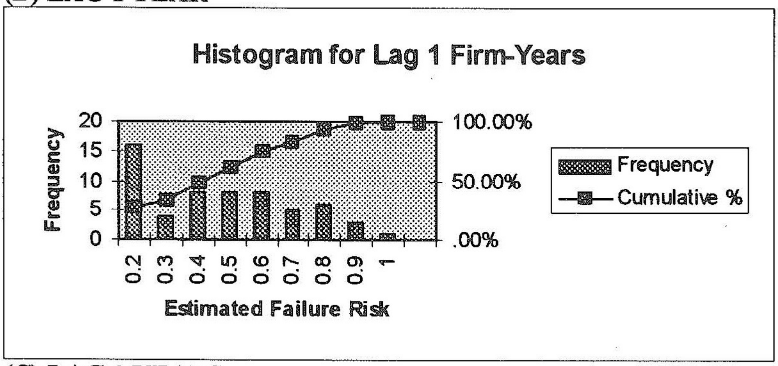
Figure 3

DISTRIBUTIONS OF ESTIMATED FAILURE RISK FOR EACH LAG YEAR

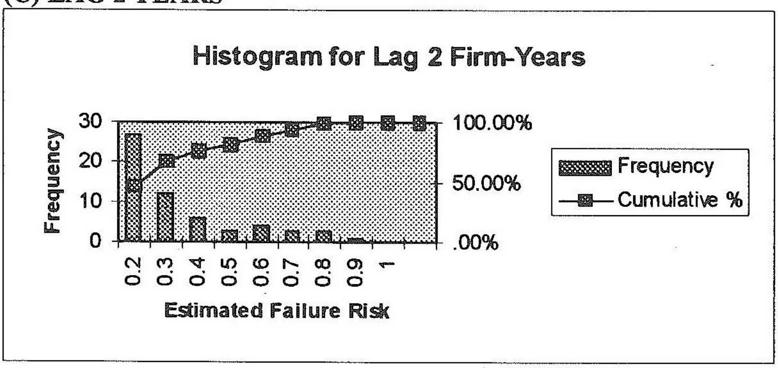
(A) LAG 0 (FINAL) YEAR



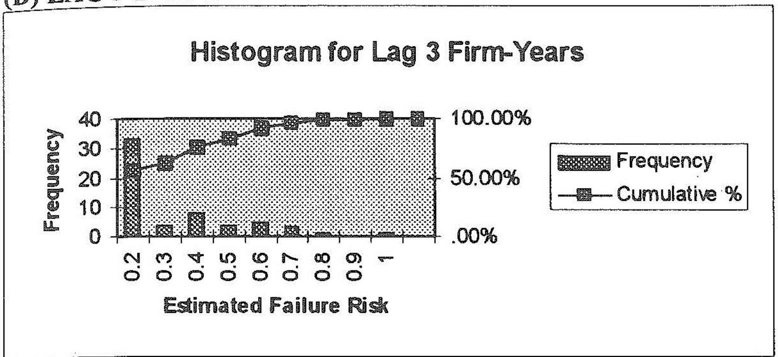
(B) LAG 1 YEAR



(C) LAG 2 YEARS



(D) LAG 3 YEARS



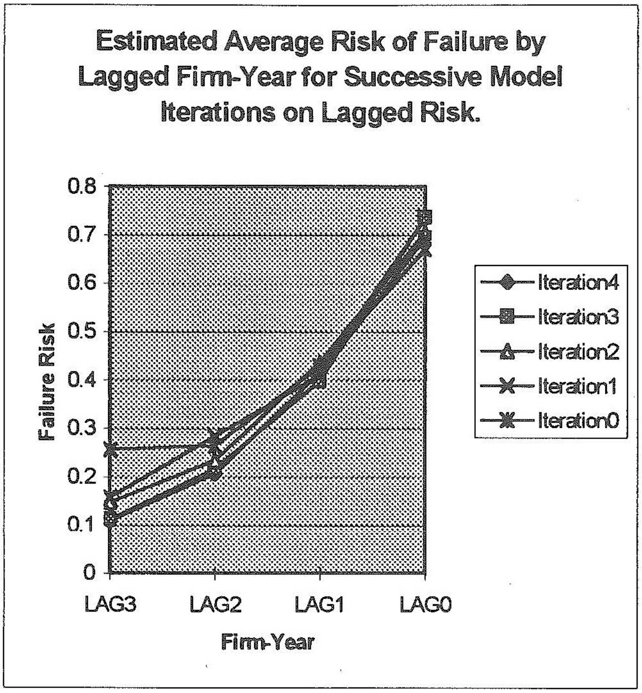
The risk is estimated at around one chance in four at three and four years prior, and increases to two chances in three at one year prior to failure—an absolute jump in risk of 41% within two years. This is a strong indication that a reorientation to process dynamics is very much overdue.

Incorporating a measure of lagged risk into the single-equation model of Equation (8) is the simplest way to add the dynamic element into the model specification, yet the improvement in model fit is good and the sequential increase in estimated failure risk from year to year is also more marked than in the model with no dynamic included. One may conclude that these increases coincide with a very rapid decline in financial stability for these firms as they

approach bankruptcy. This rapid decline is borne out by the increasing weight on the lagged risk when its value was consecutively iterated until stability was reached—indicating a distress feedback mechanism. The averages of the estimated risk for each lag period from models sequentially iterated on lagged risks are graphed in **Figure 4** below.

Results from the multiple equation models within the family of models tested on the service industry data follow the finding, that at certain lag years some variables have a much more significant effect on failure risk than they do at other lag years (or they may not be statistically significant in some years). This offers considerable justification for the multi-equation modeling of failure risk using time series as well as cross-sectional data for the analysis set. In these situations, the coefficient effects in the single-equation model are averaged out over time and the power of any significance tests applied to them are consequently reduced. The result is that these effects are usually deemed statistically insignificant under a single-equation failure model specification.

Figure 4



One such variable is "cash flow from operations to total liabilities" which appears to be an immediate trigger for bankruptcy in the final year before failure. As such it is important in the final year but not at other times. Consequently this variable is not useful as an early pointer since a drop in it's value is predictive of failure only at too short an advance period.

This may also be the reason for the controversy over cash flow variables in general as to whether they contain information value beyond that of accrual variables in bankruptcy prediction analyses. Thus, studies may reach very different conclusions with respect to this variable depending on the time of the measurement of variables as opposed to the point of time of failure.

By investigating the differences in parameter estimates and their significance to failure risk for each prior year before failure by the method of seemingly unrelated regressions, some insights into the patterns of failure over the final four years of a service industry firm's existence have been gained here. Firstly, an important result is that some variables not only remain highly significant over all four years prior to failure but also have consistent signs on the coefficients over those four years. This finding lends some weight to the validity of some single-equation failure model specifications with respect to these variables. As a consequence, these variables can be confidently used in these simple failure prediction models as their discriminatory powers are reasonably long term (at least of four years duration before failure.)

Because of this discriminatory robustness over time it was useful to examine the relative sizes of the coefficients of each of these significant variables over the four lag years. The variables that showed little difference in size from year to year were working capital to total assets, total liabilities to total assets, and interest coverage. Again, despite the comments in the point above, no stress is laid here on the strength of individual variables.

The variables that show larger differences from year to year are of even greater interest. The "labor market tightness" factor and the "expenditure (business, private and public)" factor—the latter measured both in the same year as the firm's final financial statements were submitted as well as in the previous year—all increased in their effects on failure risk as failure approached, having the most pronounced effect in the penultimate year before failure and this effect dropped only slightly in the final year prior. These results suggest that if the firm is already vulnerable to failure, low expenditure levels in the economy and a tight labor market at this time can have a devastating effect on the ultimate solvency of the firm.

Of the financial ratios, sales to net plant showed a considerably larger positive effect on failure risk at three years prior to failure. Particularly if this ratio was large, it was associated with an increased failure risk at this time. One possible reason for this effect is that the value of fixed assets was eroded at this time by selling off assets, rather than by sales increasing.

The three significant ratios to failure risk that exhibit high multi-collinearity in the data are the liquidity ratio: working capital to total assets, the leverage ratio: total liabilities to total assets, and the profitability ratio: net income to total assets. Including a second linear equation in a simultaneous equation set with working capital to total assets as a jointly dependent variable overcame the problem of an unexpected negative sign on the coefficient of total liabilities to total assets in the single-equation logit models of earlier chapters. The incorrect sign was concluded to be due to the multi-collinearity of supposedly "independent" predictors.

CONCLUSION

The proposed new modeling methodology outlined in this paper, with corresponding procedures for implementation, has the following advantages:

(a) At this stage of the development of the methodology, using only failed firms and making comparisons within failed firms in a panel design rather than between going-concerns and failed firms avoids the problem of overrepresentation of failed companies that has led to bias in past models of public company failure.

(b) Using only failed firms avoids confounding truly healthy surviving firms with those that are potential failures in the short term

(c) Each firm is essentially matched with its previous states as information from its failed year is used together with information from its surviving years.

(d) A time series formulation allows us to incorporate changes in macroeconomic conditions, and many allow us to translate vague statements such as, "A company which may have been able to survive during a boom period may not be able to survive in a slump" into more precise and useful forms.

Without exception, past models have used a single equation model that must necessarily be mis-specified, since the failures feed back into the firm itself causing a ripple effect. Nevertheless, it is a property of the methodology that it can accommodate these discrimination models within the family of models that it includes. In this context these models can be seen as partial or special case models so that their failings and/or special assumptions are made more explicit and apparent. This is not only shown to be the situation at the methodological level through the construction of a family of models, but the models examined within the methodological framework are empirically more successful than the partial, special case models.

NOTES

1. The first-stage estimates come from a "full" single equation model, where all lag years are pooled. It would not be possible to get the estimates for failure risk from these four equations analyzed separately as the Y values are constant for each; either 0 or 1. The SAS/ETS statistical package provides the procedure MODEL, which is able to fit these directly when they are written in their logistic function form.

2. The order of entry into the forward stepwise logit model was: the Labor Tightness factor, the Construction factor, the Public and Private Expenditure factor, the Cost of Capital Borrowing factor lagged one year, the Public and Private Expenditure factor lagged one year; then the ratios: Working Capital to Total Assets, Total Liabilities to Total Assets, Cash Flow from Operations to Total Current Assets, Interest Coverage after Tax (binary dummy variable), Sales to Net Plant- lowest values dummy variable, Sales to Net Plant-highest values dummy variable.

3. This number applies to the 1992 publication of "Abstracts of the U.S.A."

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APPENDIX A

The External Variables in the Model: Principal Components Analysis of the U.S. Economy 1970-1991

Using tables from *ABSRACTS OF THE U.S.* (1992) and matching the table names with earlier years' publications going back to 1970. Amounts are in Current dollars. Variables A-BX are defined below. X=not included in Principal Components Analysis due to many missing values. In other series, single missing values are interpolated or extrapolated.

	Table No.*	Table Name		
A	DNA	years covered 1970-1991 - not used in PCA		
B	449	All Governments - Debt outstanding (bil.$)		
C	491	Federal Budget Surplus or Deficit(-) (mill $)		
D		Gross Federal Debt as percent of GNP		
E	494	Federal Budget Outlays Total (mill $)		
F		Net Interest Payments (mill $)		
G	501	Total Funds Loaned in U.S. Credit Markets(bill $)		
H	525	National Defense Outlays (bil $)		
I	608	Labor Force - Total employed ('000)		
J		- Total Unemployed('000)		
K	X 623	Weekly Hours (*missing 1971-74, 1975-78)		
L	X 624	Total Self-employed Workers('000)(missing 1971-4,1976-9,1981-2)		
M	673	GDP-all in current $ (bil $)		
N		Personal Consumption Expenditures(bil $)		
O		Gross private Domestic Investment (bil $)		
P =ΔM	675	Average annual Percentage Change in GDP from former year		
Q =ΔN		Average annual Percentage Change in Personal Consumption Expenditure		
R*	683	Gross Savings (bil $)		
S*		Gross Investment (bil $) (*missing 1971)		
T	738	Consumer Price Indexes	(base av. 1982-84)	
U =ΔT	739	Percentage Change in CPI from former year		
V	746	Producer Price Indexes	Crude Materials	(base 1982)
W			Intermediate Materials	
X			Finished Goods	
Y			Capital Equipment	
Z*X	778	Insured Commercial Banks - Provisions for Loan Losses (bil $)		

AA*X (*missing 1970-79)Percentage of Banks Losing Money
AB* 789 Mortgage Debt Outstanding - Commercial (bil $)
(*missing 1981,1991)
AC 802 Money Stock and Liquid Assets M1 Total
AD (bil $) M2 Total

AE* M3 Total
(*missing 1971-72)
AF* 806 Money Market Interest Rates Commercial paper 3 mth
AG* Prime Rate charged by banks
AH* US Govt Securities 1yr Treasury bill
AI*X 808 Security Prices - Bond Prices -Dow Jones Yearly High
AJ*X (*missing 1971-74,1976-77,1982) Yearly Low
BB replace above two bond prices with Standard and Poor's Municipal
AK* 809 Bond and Stock Yields (%)-US Treasury constant maturities 3 -yr
AL* (*missing 1971-72) Replaced 72 with S&P's preferred 10 stocks 5-yr
AM*
10-yr
AN 862 Business Expenditure for New Plant and Equipment - All Industries (bil $)
AO* 864 Composite Indexes of Economic Cyclical Indicators-Leading Indicators
AP*(1982=100) Coincident Indicators
AQ* (*missing 1991) Lagging Indicators
AR* 866 Manufacturing and Trade Sales
AS* (*missing 1971) Inventories (NA 1990-91)
AT* 925 Fossil Fuel prices - Crude Oil (cents per million Btu)
(*missing 1971,1991)
AU 1204 Value of New Construction Put in Place - Total (mill $)
AV 1250 Manufacturers' Shipments
AW (bil $) Inventories
AX New Orders
AY 1315 US International Transactions -Balance on Current Acc (bil$)*
AZ 1407 Exchange rates (index of value relative to US $) Germany
BA (1982=100) Japan

The following series were added. They are the rates of change (denoted Δ) of many of the above series.

BC = ΔE 494	Federal Budget Outlays Total	
BD = ΔF	Net Interest Payments	
BE = ΔI 608	Labor Force - Total employed	
BF = ΔR 683	Gross Savings	
BG = ΔS	Gross Investment	
BH = ΔY 746	Producer Price Indexes	Capital Equipment
BI = ΔAC 802	Money Stock and Liquid Assets	M1 Total
BJ = ΔAD		M2 Total
BK = ΔAE		M3 Total
BL = ΔAF 806	Money Market Interest Rates	Commercial paper 3 mth
BM = ΔAG	Prime Rate charged by banks	
BN = ΔAH	US Govt Securities 1yr Treasury bill	
BO = ΔAK 809	Bond and Stock Yields (%)-US Treasury constant	maturities 3 - year
BP = ΔAL		5 - year
BQ = ΔAM		10- year
BR = ΔAN 862	Business Expenditure for New Plant and Equipment - All Industries	
BS = ΔAR 866	Manufacturing and Trade Sales	
BT = ΔAS		Inventories
BU = ΔAU 1204	Value of New Construction Put in Place - Total	
BV = ΔAV 1250	Manufacturers' Shipments	
BW = ΔAW		Inventories
BX = ΔAX		New Orders

1ST PRINCIPAL COMPONENT:
NAME: LEVEL OF ACTIVITY/DEMAND IN THE ECONOMY FACTOR

This factor is a surrogate for YEAR as it is highly correlated with YEAR (an increasing function -almost monotonic.)
PC1 (54% of variation explained) loads highest (**p<=0.0002**) on:-
(in descending order of correlation magnitude)
(all +ve unless stated as -ve)
E 494 Federal Budget Outlays Total (mill$)
T 738 Consumer Price Indexes
AS 866 Manufacturing and Trade Inventories
AE 802 Money Stock and Liquid Assets M3 Total
AD 802 Money Stock and Liquid Assets M2 Total
M 673 GDP -all in current $ (bil $) - GNP only available for early (7) years
N 673 Personal Consumption Expenditures(bil $)

2nd PRINCIPAL COMPONENT
NAME: COST OF CAPITAL BORROWING FACTOR

PC2 (17.6% of variation accounted for) loads highest (**p<=.0005**) on :
(in descending order of correlation magnitude)
AF 806 Money Market Interest Rates Commercial paper 3-mth
BP(ΔAL)809 Δ(Bond and Stock Yields (%)-US Treasury constant maturities 5 -yr)
AH 806 Money Market Interest Rates US Govt Securities 1yr Treasury bill
BO(ΔAK)809 Δ(Bond and Stock Yields (%)-US Treasury constant maturities 3 -yr)
BD(ΔF) 494 Δ(Net Interest Payments)
BQ(ΔAM)809 Δ(Bond and Stock Yields (%)-US Treasury constant maturities 10-yr)
AG 806 Money Market Interest Rates Prime Rate charged by banks
BM(ΔAG) 806 Δ(Money Market Interest Rates Prime Rate charged by banks)
AK 809 Bond and Stock Yields (%)-US Treasury constant maturities. 3-year
BN(ΔAH)806 Δ(Money Market Interest Rates US Govt Securities 1yr Treasury bill)
AL 809 Bond and Stock Yields (%)-US Treasury constant maturities 5-yr

3rd PRINCIPAL COMPONENT
NAME: LABOUR MARKET TIGHTNESS FACTOR

PC3 (12.4% of variation accounted for) loads highest **($p<=0.02$)** on :-
(in descending order of correlation magnitude)

J (-ve)	608	Labor Force - Total Unemployed ('000)
BB	808	Security Prices -Standard and Poor's Municipal
BE(ΔI)	608	Δ(Labor Force - Total employed)

4th PRINCIPAL COMPONENT
NAME: CONSTRUCTION ACTIVITY FACTOR

PC4 (accounting for 5.6% of the variation) loads highest **($p<0.03$)** on:
BU(ΔAU)1204 Δ(Value of New Construction Put in Place - Total)
BE(ΔI) 608 Δ(Labor Force - Total employed)
BH(ΔY)(-ve) 746 Δ(Producer Price Indexes Capital Equipment)

5th PRINCIPAL COMPONENT
NAME: EXPENDITURE FACTOR (PRIVATE, PUBLIC, BUSINESS)

PC5 (accounting for 2.7% of the variation) loads highest **($p<0.04$)** on:

BR ΔAN 862	Business Expenditure for New Plant and Equipment - All Industries
BC ΔE 494	Federal Budget Outlays Total
Q=ΔN 675	Average annual Percentage Change ,Δ(Personal Consumption Expenditure)