

Analysis of the Bullwhip Effect in Supply Chains

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A general Monte Carlo simulator for examining the nature of the bullwhip effect in supply chain management is presented here. It is a helpful pedagogical tool for demonstrating the presence of the bullwhip effect in classroom settings while avoiding the related and very difficult mathematical foundation of the effect. The computer program permits choosing customer demands that follow the uniform, binomial, Poisson and normal probability distributions, as well as demand that has linear growth with random fluctuations over time. Output of the simulator includes the mean and standard deviation of periodic order size at each supply chain level. The output consistently shows increasing order size standard deviations at successive supply chain levels. The simulator has sufficient flexibility to permit the user to input the desired parameter values, ordering rule and probability distributions.

INTRODUCTION

Supply chain management researchers have noted the presence of an interesting phenomenon known as the bullwhip effect. This is the increase in demand variability when moving sequentially through the supply chain from the retail customer. The phenomenon is typified by demand patterns back through the supply chain having a regular oscillatory pattern. There will be consecutive periods of zero or negative demand, followed by consecutive periods of positive demand, with the demand changes between periods being fairly smooth and consistent. The bullwhip effect can be demonstrated in a straightforward exercise. One of these is the noted Beer Game (Sternan, 1989). This game has recently been extended by Kaminski and Simchi-Levi (1998).

It is no longer possible for management to regard supply chain issues as being confined to the production function. There certainly are production-related issues in supply chain management. These include inventory control and forecasting problems, as well as production scheduling and sequencing. However, the importance of supply chain issues has grown to encompass many functional and behavioral management areas. It is clear that finance departments will have their own concerns, since cash flows will lag through the supply chain. Forecasting may be housed within the finance, production, or marketing functions. Production scheduling may well not be controlled solely within the production department, but may instead be directed under joint control from several departments. Personnel issues arise because dependent lumpy demand can cause employment swings. Management roles become more ambiguous as separate and distinct organizational entities are forced to act sequentially. The evolving organizational structure in a supply

chain atmosphere is portrayed by Gimeno and Woo (1996), Cooper, *et al.* (1997), and Galunic and Rodan (1998). The decreasing supply chain manager role clarity with the interdependence in a supply chain network is presented by Lambert, Emmelhentz and Gardner (1996).

Supply chains have been regarded as mechanisms by which competitiveness may be enhanced. This is easily observed in diverse ways such as quality enhancement, timely delivery, and cost containment. These considerations clearly exceed the narrow bounds of production or operations control. The advantages arise from developing seamless links between organizations as much as from refining operations within a single organization. Morgan (1996) analyzed the purchasing aspect of the interdependencies. Mabert and Venkataramanan provided an overview of the importance of the linkages (1998). Tracey and Vonderemse investigated how operational performance criteria are affected by supplier characteristics (2000). In all of these examinations, the area of investigation centered on the organization and organizational linkages.

The interdependence and shared assets in a supply chain have been addressed in a network context by Liebeskind, Oliver, Zucker and Brewer (1996) and by Argyris and Liebeskind (1999). Management issues in a supply chain network were recently presented by Novicevic, Buckley and Harvey (2000). They show that a successful supply chain manager must have a very broad skill set, including openness, flexibility in a dynamic environment, being outwardly focused and having an entrepreneurial nature. A supply chain manager must, in short, have the flexibility and background to continually solve problems as they arise. This requires rapid communication with others within the firm and also out in the supply chain. Consensus decisions in an environment of rapid change and limited information must be sought. There are some natural and recurring patterns in the supply chain. These patterns will certainly affect operations within any single supply chain entity, so they must be understood and anticipated. If that happens, some of the surprise in the dynamic supply chain atmosphere will be tempered.

One typical pattern that affects vertical links in the chain and also is felt horizontally in any single organization within the chain is the bullwhip effect. Increased fluctuations in order size through the supply chain give rise to undesired complexities in cash management, inventory control, production scheduling, personnel training, layoffs and forecasting, which quickly begin to affect the entire organization. The bullwhip effect is quite foreseeable and has a significant impact upon the entire organization, so it should be understood across departmental lines, as well as through the supply chain. It is properly of interest throughout the organization, not just within the narrow and technical realm of the production function.

The published works that have introduced the bullwhip effect have for the most part been quite recent. An early work was that of Kahn (1987), which related customer orders to production planning and scheduling. More currently, Metters derived the bullwhip effect quantitatively (1996). Lee, *et al.* (1997 a, b), provided a managerial overview of the

bullwhip effect They suggested that this effect might arise from forecasting methods, supply shortages, lead times, batch ordering, and price variations.

The newest work on the subject is by Chen, Drezner, Ryan and Simchi - Levi (2000). Their article provided the basis for this work. Because it will be referenced so frequently here, it will be referred to as CDRS, the initials of the authors' surnames. CDRS showed that the variance of demand at any stage of the supply chain will be at least as great as the variance of demand at the previous level. Their approach used a moving average forecasting technique to obtain the variance in the required order size when following a common "order up to" policy. They assumed that the retailer will apply a forecasting technique for customer demand that will yield errors that are independently, identically, and symmetrically distributed. It is a straightforward extrapolation that the nondecreasing variances will extend to all succeeding levels, because the order quantity at any given level becomes the demand at the next level back.

The CDRS work provides the seminal analysis in this area. We seek to extend that direction here by providing a mechanism whereby the bullwhip effect might be observed and estimated, regardless of the nature of the original demand process. Specifically, the bullwhip effect is shown to exist for customer demands that follow the uniform, Poisson, binomial, and normal probability distributions. Also, it is shown that the effect holds when customer demand shows linear growth over time.

The bullwhip effect is of pedagogical interest in the areas of finance, marketing, production, organization and interorganizational linkages because of its widespread existence and its impact upon all organizational functions. The work done here develops a simulator that portrays the existence of the bullwhip phenomenon. It can be introduced into classroom settings to demonstrate the effect in a context that does not require a difficult mathematical development or the limited and simplifying assumptions of a mathematical model. Similarly, it can be used in an organization contemplating the advantages of strengthening its supply chain relationships because the bullwhip effect is not intuitively obvious. There will be organizational stresses that arise because of the increasing order size variability found sequentially through the chain. These will be particularly difficult for the cash management and inventory control functions.

ANALYSIS

A Monte Carlo simulation of the supply chain that permits the use of any desired original customer demand process has been developed. The replenishment ordering mechanism is the common "order-up-to" rule. The model is designed to receive inventory replenishment at the beginning of a period. Reorder lead times may be set to as many periods as desired. If the reorder placed at the end of a period is received at the beginning of the following period then there is no waiting for the goods. The lead time parameter will be denoted as L , where L is the number of time periods in the future when the reorder will arrive and be added to inventory at the beginning of the period. Therefore, with no waiting, $L = 1$.

Several variables and parameters of the model must be defined. They are:

N = number of time periods to include

I = period number

IMAX = the order-up-to inventory target level

DEMAND(I) = customer demand in period I

ORDER(I) = size of supply replenishment ordered at the end of period I

L = number of periods ahead when a reorder will arrive

INVEN(I) = inventory on hand at the end of period I

NLEVEL = number of loops the program runs in order to represent the number of levels in the supply chain

TOTORD = amount of existing orders that have arisen over the L recent periods

If a reorder is placed at the end of period I, the goods will be available at the start of period I + L. NLEVEL is equal to one less than the number of levels in the supply chain. If NLEVEL = 1 the program will generate demands at the customer level and also calculate orders at the next level. Each increment in NLEVEL then introduces the next level back in the supply chain.

Following the order-up-to policy, the reorder rule is:

$$\text{ORDER}(I) = \text{IMAX} - \text{TOTORD} - \text{INVEN}(I) \quad (1)$$

This is comparable to the equation

$$q_t = y_t - y_{t-1} + D_{t-1}$$

from CDRS.

Note that ORDER(I) could be negative. CDRS point out that this is equivalent to the free return of excess inventory. This may not be possible, so the program resets ORDER(I) to zero if it is ever negative. This feature may be removed at the discretion of the user. CDRS also noted that the results are not significantly different either way.

The computer program is shown in Listing 1 (Appendix A). It is written in QBasic. The listing consists of a main program and two subroutines. The first subroutine begins at line 10000. It is used to generate the periodic demands. It presents the choice of demands following the uniform, binomial, Poisson and normal probability distributions, as well as demand growing linearly over time. The user can easily modify this to accommodate any desired demand process.

The periodic reorder is calculated in the subroutine that starts in line 20000. It uses the order-up-to rule. After these two subroutines are used to obtain demands and order sizes, the model is run for N time periods. The mean and standard deviation of periodic demands, order sizes and ending inventory are calculated and results are printed out.

RESULTS

Figure 1 shows the typical bullwhip pattern. The one illustrated there arose from periodic demands that followed the uniform probability distribution over [5, 15]. N was set at 15 time periods, L was set at 3 periods of reorder lag, and NLEVEL was equal to one. Note that customer demands fluctuate according to the uniform distribution, while periodic reorders conform to a bullwhip pattern with oscillating order magnitude. The standard deviation of customer demand is 2.250, while the standard deviation of periodic order quantity is 7.750. This illustrates in a small way that when the bullwhip effect occurs, the standard deviation or variance of order size will increase as the supply chain level sequentially increments from the retail customer.

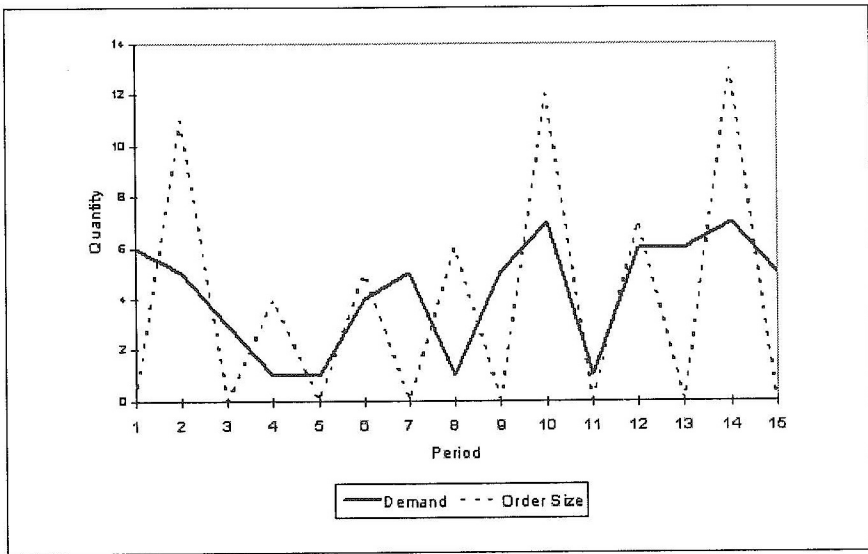


Figure 1. Typical Bullwhip Behavior. Original Periodic Demand is Integer Uniformly Distributed on [5,15]; L = 3 and NLEVEL = 1

The Monte Carlo simulation was run repeatedly to gather results showing the nature of the bullwhip effect. For each computer run a different pseudo-random number sequence was used. This is particularly helpful for testing the hypothesis that the results demonstrate the existence of the bullwhip effect, as will be done below. The value of N was always kept at 1000 and IMAX was set at 100. L was varied from 1 to 4 and NLEVEL was varied from 1 to 3. The order-up-to rule of (1) was used throughout. Customer demands at level 0 were generated from the uniform, binomial, Poisson and normal probability distributions. The integer uniform distribution used the interval [0,7]. The binomial distribution used N = 10 and P = .35. The Poisson distribution set lambda to 3.5. The normal distribution used mu = 4 (with an integer requirement) and sigma = 1. Thus, in all cases the expected periodic demand was 3.5.

Table 1 provides a summary of the Monte Carlo results from repeated computer runs. The table presents the means and standard deviations of order sizes per period for all levels, as well as the mean and standard deviation of original customer demand per period. It is quickly observed from table 1 that the standard deviation of the periodic order size never decreased as the number of levels in the supply chain increased. In a few cases the standard deviations were equal when moving from a level to the next sequential level, but most often the standard deviation of order size increased with the level. Most importantly, in every case the standard deviation of order size at level 1 exceeded the standard deviation of demand size at level 0. Hence, in all cases the order-up-to policy revealed the existence of the bullwhip effect.

The increasing magnitude of the standard deviation of order size as the level in the supply chain increments was subjected to statistical testing. The hypothesis to be tested is that the standard deviation of order size increases over consecutive levels of the supply chain. Let the standard deviation of order size at level i of the supply chain be S_i . The sample estimates of these parameters values are shown in table 1. Of course, S_0 is the standard deviation of original customer demand (level 0). The hypothesis will be formally tested in the following way. The null hypotheses H_0 and alternative hypotheses H_a are:

$$\begin{aligned} H_0: S_{i-1} &\leq S_i & i = 1, 2, \dots, \text{NLEVEL} \\ H_a: S_{i-1} &> S_i & i = 1, 2, \dots, \text{NLEVEL} \end{aligned}$$

If H_0 is rejected in any instance, the conclusion to be made is that the standard deviation of order size strictly increases as orders move further back the supply chain.

It is seen from table 1 that there was never an instance in all of the reported results where the estimated value of S_i was smaller than that of S_{i-1} . In table 1 there were actually three cases out of the 180 calculated standard deviations where the two consecutive values were equal.

The order size at any level i of the supply chain is dependent upon the order size at the previous level. This means that a straightforward test of the equality of standard deviations or variances cannot be done, since the observations will not be independent. Instead, the test of H_0 can be carried by reliance upon the central limit theorem. The simulations can be repeated many times to obtain averages of the standard deviations. These averages will tend toward following the normal probability distribution if the sample size (number of repetitions of the simulation) is sufficiently large. There is common agreement that a sample size of thirty observations is adequate. The 180 standard deviations of table 1 were each replicated 50 times for the purpose of the hypothesis testing. These replications were carried out with different pseudo-random number streams that were created by changing the seed of the generator. This is needed in order to have the 50 replications consist of independent samples.

Table 1
Monte Carlo Simulation Results

| Mean and Standard Deviation of Order Size Per Period (at level 0 the order size is original customer demand) Periodic Demand is Integer Uniform [0,7] | | | | | | | | |
|---|---------|-------|---------|-------|---------|-------|---------|-------|
| L | Level 0 | | Level 1 | | Level 2 | | Level 3 | |
| | mean | sd | mean | sd | mean | sd | mean | sd |
| NLEVEL = 1 | | | | | | | | |
| 1 | 3.428 | 2.322 | 3.425 | 3.748 | | | | |
| 2 | 3.395 | 2.280 | 3.394 | 3.656 | | | | |
| 3 | 3.490 | 2.265 | 3.489 | 3.685 | | | | |
| 4 | 3.414 | 2.243 | 3.411 | 3.628 | | | | |
| NLEVEL = 2 | | | | | | | | |
| 1 | 3.432 | 2.350 | 3.432 | 3.686 | 3.432 | 3.968 | | |
| 2 | 3.494 | 2.330 | 3.494 | 3.658 | 3.494 | 4.432 | | |
| 3 | 3.409 | 2.264 | 3.406 | 3.709 | 3.406 | 4.200 | | |
| 4 | 3.513 | 2.270 | 3.510 | 3.694 | 3.507 | 4.346 | | |
| NLEVEL = 3 | | | | | | | | |
| 1 | 3.507 | 2.278 | 3.500 | 3.824 | 3.500 | 4.042 | 3.500 | 4.089 |
| 2 | 3.398 | 2.277 | 3.392 | 3.617 | 3.391 | 4.282 | 3.391 | 4.506 |
| 3 | 3.503 | 2.252 | 3.502 | 3.782 | 3.501 | 4.250 | 3.500 | 4.409 |
| 4 | 3.508 | 2.229 | 3.547 | 3.888 | 3.546 | 4.509 | 3.545 | 4.688 |
| Periodic Demand is Poisson (lambda = 3.5) | | | | | | | | |
| L | Level 0 | | Level 1 | | Level 2 | | Level 3 | |
| | mean | sd | mean | sd | mean | sd | mean | sd |
| NLEVEL = 1 | | | | | | | | |
| 1 | 3.535 | 1.847 | 3.533 | 3.776 | | | | |
| 2 | 3.446 | 1.876 | 3.444 | 3.565 | | | | |
| 3 | 3.401 | 1.896 | 3.397 | 3.593 | | | | |
| 4 | 3.521 | 1.843 | 3.516 | 3.675 | | | | |
| NLEVEL = 2 | | | | | | | | |
| 1 | 3.489 | 1.867 | 3.489 | 3.674 | 3.483 | 3.848 | | |
| 2 | 3.602 | 1.906 | 3.601 | 3.753 | 3.601 | 4.449 | | |
| 3 | 3.481 | 1.917 | 3.478 | 3.696 | 3.478 | 4.052 | | |
| 4 | 3.409 | 1.839 | 3.409 | 3.582 | 3.409 | 4.094 | | |
| NLEVEL = 3 | | | | | | | | |
| 1 | 3.447 | 1.820 | 3.447 | 3.690 | 3.447 | 3.846 | 3.447 | 3.873 |
| 2 | 3.580 | 1.833 | 3.578 | 3.653 | 3.576 | 4.408 | 5.574 | 4.692 |
| 3 | 3.479 | 1.866 | 3.479 | 3.669 | 3.479 | 4.075 | 3.479 | 4.189 |
| 4 | 3.504 | 1.843 | 3.503 | 3.635 | 3.503 | 4.229 | 3.503 | 4.426 |

Table 1
Monte Carlo Simulation Results (continued)

| Periodic Demand is Binomial (N = 10, P = .35) | | | | | | | | | |
|---|---------|-------|---------|-------|---------|-------|---------|-------|----|
| L | Level 0 | | Level 1 | | Level 2 | | Level 3 | | sd |
| | mean | sd | mean | sd | mean | sd | mean | sd | |
| NLEVEL = 1 | | | | | | | | | |
| 1 | 3.560 | 1.499 | 3.559 | 3.758 | | | | | |
| 2 | 3.568 | 1.538 | 3.565 | 3.549 | | | | | |
| 3 | 3.540 | 1.528 | 3.536 | 3.727 | | | | | |
| 4 | 3.568 | 1.502 | 3.568 | 3.704 | | | | | |
| NLEVEL = 2 | | | | | | | | | |
| 1 | 3.444 | 1.530 | 3.440 | 3.601 | 3.440 | 3.735 | | | |
| 2 | 3.451 | 1.511 | 3.449 | 3.461 | 3.447 | 4.193 | | | |
| 3 | 3.565 | 1.536 | 3.565 | 3.735 | 3.565 | 3.945 | | | |
| 4 | 3.491 | 1.497 | 3.491 | 3.642 | 3.491 | 4.119 | | | |
| NLEVEL = 3 | | | | | | | | | |
| 1 | 3.417 | 1.536 | 3.412 | 3.616 | 3.412 | 3.718 | 3.412 | 3.724 | |
| 2 | 3.634 | 1.504 | 3.633 | 3.598 | 3.632 | 4.361 | 3.631 | 4.644 | |
| 3 | 3.451 | 1.550 | 3.451 | 3.679 | 3.451 | 3.811 | 3.451 | 3.818 | |
| 4 | 3.581 | 1.563 | 3.578 | 3.749 | 3.578 | 4.229 | 3.578 | 4.391 | |
| Periodic Demand is Integer Normal (mu = 4, sigma = 1) | | | | | | | | | |
| L | Level 0 | | Level 1 | | Level 2 | | Level 3 | | sd |
| | mean | sd | mean | sd | mean | sd | mean | sd | |
| NLEVEL = 1 | | | | | | | | | |
| 1 | 3.452 | .027 | .451 | .593 | | | | | |
| 2 | 3.449 | .034 | .443 | .473 | | | | | |
| 3 | 3.439 | .025 | .439 | .599 | | | | | |
| 4 | 3.500 | .027 | .500 | .572 | | | | | |
| NLEVEL = 2 | | | | | | | | | |
| 1 | 3.522 | .029 | .522 | .664 | .522 | .670 | | | |
| 2 | 3.530 | .050 | .526 | .510 | .526 | .258 | | | |
| 3 | 3.475 | .049 | .475 | .580 | .475 | .685 | | | |
| 4 | 3.496 | .011 | .490 | .616 | .490 | .011 | | | |
| NLEVEL = 3 | | | | | | | | | |
| 1 | 3.452 | .020 | .452 | .596 | .452 | .604 | .452 | .604 | |
| 2 | 3.485 | .049 | .481 | .491 | .478 | .186 | .478 | .411 | |
| 3 | 3.465 | .032 | .465 | .610 | .465 | .637 | .465 | .653 | |
| 4 | 3.524 | .016 | .523 | .581 | .523 | .011 | .523 | .167 | |

Table 1
Monte Carlo Simulation Results (continued)

| Periodic Demand has Stochastic Linear Increase | | | | | | | | | |
|--|---------|-------|---------|--------|---------|--------|---------|--------|--|
| L | Level 0 | | Level 1 | | Level 2 | | Level 3 | | |
| | mean | sd | mean | sd | mean | sd | mean | sd | |
| NLEVEL = 1 | | | | | | | | | |
| 1 | 19.574 | 5.820 | 19.574 | 21.226 | | | | | |
| 2 | 19.560 | 5.855 | 19.560 | 20.268 | | | | | |
| 3 | 19.574 | 5.828 | 19.574 | 21.231 | | | | | |
| 4 | 19.521 | 5.862 | 19.506 | 21.253 | | | | | |
| NLEVEL = 2 | | | | | | | | | |
| 1 | 19.543 | 5.850 | 19.543 | 21.214 | 19.543 | 21.214 | | | |
| 2 | 19.514 | 5.832 | 19.488 | 20.175 | 19.482 | 23.851 | | | |
| 3 | 19.543 | 5.894 | 19.543 | 21.233 | 19.543 | 21.254 | | | |
| 4 | 19.598 | 5.888 | 19.598 | 21.041 | 19.598 | 23.338 | | | |
| NLEVEL = 3 | | | | | | | | | |
| 1 | 19.538 | 5.866 | 19.538 | 21.217 | 19.538 | 21.218 | 19.538 | 21.218 | |
| 2 | 19.507 | 5.806 | 19.507 | 20.824 | 19.507 | 24.838 | 19.507 | 25.486 | |
| 3 | 19.584 | 5.796 | 19.584 | 21.222 | 19.584 | 21.240 | 19.584 | 21.251 | |
| 4 | 19.582 | 5.887 | 19.553 | 21.235 | 19.524 | 23.619 | 19.522 | 24.442 | |

Mean values across the 50 replications of these 180 standard deviations were obtained. Let the number of replications of the simulation be given as SAMP. For any standard deviation at supply level i , let its mean value be M_i , and its variance over the sample replications be V_i . The test statistic for H_0 then is:

$$Z_i = (M_i - M_{i-1}) / ((V_{i-1} + V_i) / SAMP)$$

An interesting question of the sensitivity of the order variance to the value of L arises. That question will not be pursued here, but the simulation computer program would provide the means by which the problem might be investigated. CDRS do show that the variance is an increasing function of L for the special case of forecasting variances having errors that are symmetrically distributed.

It is apparent from table 1 that the increases in the order size standard deviation decline in magnitude as orders progress back through the supply chain. This is seen further in table 2, where the Z -statistics decline in magnitude at successive levels of the supply chain. This suggests that the most prominent manifestation of the bullwhip effect happens between levels zero and one of the supply chain. Customer demand variability (level 0) gives rise to much greater order size variability for the retailer (level 1). Thereafter, however, the variability in order size continues growing, but only at diminishing magnitude.

Table 2
Z-Statistics on Successive Level Standard Deviations

| Periodic Demand has Stochastic Linear Increase | | | | | | |
|--|----------------|----------------|----------------|----------------|----------------|----------------|
| L | NLEVEL = 1 | | NLEVEL = 2 | | NLEVEL = 3 | |
| | Z ₁ | Z ₁ | Z ₂ | Z ₁ | Z ₂ | Z ₃ |
| Periodic Demand has Uniform Distribution | | | | | | |
| 1 | 135.49 | 123.28 | 17.17 | 122.04 | 17.62 | 4.37 |
| 2 | 105.70 | 115.98 | 38.27 | 122.68 | 41.47 | 12.94 |
| 3 | 140.98 | 114.75 | 34.59 | 124.37 | 35.01 | 12.02 |
| 4 | 104.22 | 103.56 | 34.34 | 119.05 | 37.61 | 13.79 |
| Periodic Demand has Poisson Distribution | | | | | | |
| 1 | 139.43 | 170.38 | 13.46 | 146.95 | 11.97 | 1.53* |
| 2 | 133.05 | 135.22 | 44.32 | 164.92 | 55.46 | 17.01 |
| 3 | 148.47 | 153.44 | 26.59 | 119.08 | 21.24 | 6.67 |
| 4 | 158.05 | 150.93 | 34.78 | 142.81 | 35.21 | 11.03 |
| Periodic Demand has Binomial Distribution | | | | | | |
| 1 | 1694.95 | 1536.58 | 0.13* | 1436.69 | 0.20* | 0.00* |
| 2 | 968.56 | 597.95 | 83.05 | 962.28 | 138.05 | 26.55 |
| 3 | 1585.53 | 513.58 | 3.19 | 1603.10 | 8.73 | 5.29 |
| 4 | 1474.93 | 1137.88 | 111.74 | 1093.74 | 102.76 | 27.65 |
| Periodic Demand has Normal Distribution | | | | | | |
| 1 | 412.53 | 469.39 | 2.61 | 467.09 | 2.81 | 0.10* |
| 2 | 370.45 | 361.25 | 78.36 | 381.41 | 85.23 | 24.07 |
| 3 | 431.42 | 444.77 | 6.83 | 410.96 | 7.02 | 2.27 |
| 4 | 432.71 | 440.20 | 60.89 | 415.91 | 57.80 | 18.75 |
| Periodic Demand has Stochastic Linear Increase | | | | | | |
| 1 | 2855.60 | 2835.44 | 0.05* | 2401.75 | 0.05* | 0.00* |
| 2 | 310.88 | 339.99 | 49.81 | 291.54 | 43.30 | 8.95 |
| 3 | 3057.98 | 2736.76 | 3.72 | 2572.58 | 3.58 | 2.19 |
| 4 | 599.88 | 530.35 | 47.58 | 650.25 | 57.81 | 15.52 |

* Z-statistic not significant at .05

The above analyses derive from independent demand probabilities that follow four well - known probability distributions: the uniform, binomial, Poisson and normal. These probability distributions all take the external demands at level zero to be independently and identically distributed. However, the Monte Carlo simulator is not limited by these artificial restrictions. It can accommodate demands that have any general pattern. As an illustration, the simulator will be used to show the existence of the bullwhip effect when demands at level zero are generally increasing over time. Specifically, let the actual demand in period I ($1 \leq I \leq N$) be generated as:

$$\text{DEMAND}(I) = 10 + (I/N)^*20 + .8*(-2*\text{LN}(\text{RND}))^5*\text{COS}(2\text{PI}*\text{RND})$$

Here, for period I the actual demand will be normally distributed about the expected value $10 + (I/N)*20$. Therefore, there will be observable secular growth in demand over time. The demand time series will be nonstationary AR(1), so a moving average forecasting procedure as used in CDRS will fail to yield residuals that satisfy their requirements of being independently and identically symmetrically distributed about zero. Because their assumptions are violated, their results do not apply. The Monte Carlo approach offered here, however, still clearly demonstrates the existence of the bullwhip effect. The bottom portions of tables 1 and 2 reveal the Monte Carlo results for this level zero demand pattern. Note from table 1 that in every case the standard deviation of order size increases with the supply chain level. Further, table 2 shows that in every case the magnitude of the Z_i statistic decreases over i . Hence, here again the bullwhip pattern is developed in the transition from level zero to level one, then does not continue to become much more pronounced when progressing further back in the supply chain.

CONCLUSION

This work offers a mechanism whereby the possible existence of the bullwhip effect of supply chains may be observed under quite general demand conditions at level zero, where the original customer periodic order size is set. A Monte Carlo simulation model has been used to compare the standard deviation of order size at each level of the supply chain.

The model showed that the order-up-to policy always brought about the bullwhip effect when original customer periodic demand follows the above four classical probability distributions. Further, it was also demonstrated that the bullwhip effect arises when there is secular growth in customer demand.

The recent CDRS paper has extended the bullwhip theory to situations where original periodic customer demands are stationary in the mean and yield forecasting errors that are identically, independently and symmetrically distributed. The Monte Carlo approach presented here consistently demonstrated the existence of the bullwhip effect under more general demand conditions.

In this computer model the user is free to specify the important parameters and conditions, including the number of replications, number of levels in the supply chain, the nature of the customer demand at level zero, and the ordering rule in effect. Output includes mean order size per period and its standard deviation. Further, variances of order size are compared at successive supply chain levels. As reported by CDRS, the magnitudes of the successive variances indicate the possible existence of the bullwhip effect.

This simulation model offers a clear demonstration of the existence and magnitude of the bullwhip effect under some general demand conditions. It avoids the complex and

difficult mathematical approach, which is restrictive both in confusing mathematical detail and narrow operational assumptions that make the problem solvable. It can be employed to show to any employee in the organization in an understandable way the working of the bullwhip effect. The same advantages exist when using this in business schools within the pedagogy of business functional areas and organizational management.

As with any Monte Carlo model, the results of the analysis do not prove the existence or nonexistence of the bullwhip effect in a supply chain. Statistical results are used to reject or fail to reject the existence of the bullwhip effect. The obvious benefit of this approach is that it easily provides a demonstration of the bullwhip effect under very general demand assumptions. This is important because the analytical theory has to this point extended only to some limited original periodic demand situations.

FURTHER RESEARCH

This simulator can be used to pursue further research issues. One of these is the unknown change the magnitude of the bullwhip effect when reorder lead-time L varies through the supply chain. Another is the nature of the cyclicity imposed upon cash flows for any firm in the chain. A third is the unknown supply chain effects upon the capacitated supply chain. That is, if order sizes exceed capacity then inventories are exhausted and final assemblies cannot proceed. This would be moderated by the existence of safety stocks. The related research question is to choose an adequate safety stock that is not excessive.

The simulator could quite easily be used to investigate another open problem. It was shown here that order sizes exhibit increasing variance through the supply chain. Nowhere has it been shown if the probability distribution of order size at any supply chain level remains that of the original customer demand, or if some kind of central tendency of periodic order size appears to arise at progressive supply chain levels. The simulator could be run to collect a large number of order sizes at each of the several supply chain levels, and then the resulting samples could be grouped in histograms and subjected to a goodness-of-fit test.

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APPENDIX A

Listing of the Computer Program

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REM PROGRAM BULLWHIP
REM EXAMINE THE BULLWHIP EFFECT OF SUPPLY CHAINS:
REM THE VARIANCE OF THE ORDER QUANTITY IS EXPECTED
REM TO INCREASE WITH THE SUPPLY CHAIN LEVEL.
REM
REM THIS BASIC PROGRAM IS A MONTE CARLO SIMULATION THAT SHOWS REM THE
INCREASED STANDARD DEVIATION IN ORDER QUANTITY AS THE
REM LEVEL MOVES UP TOWARD THE ORIGINAL SUPPLIER.
REM
REM
REM *****
REM THE USER MUST ENTER THE DESIRED VALUES OF SEVERAL
REM VARIABLES:
REM L = THE NUMBER OF PERIODS IN THE REORDER LEADTIME.
REM NEW INVENTORY ARRIVES AT THE BEGINNING OF A PERIOD.
REM AN ORDER FOR MORE INVENTORY IS PLACED AT THE END OF
REM A PERIOD.
REM THUS, IF THERE IS NO WAIT FOR THE ARRIVAL OF
REM NEW GOODS, L = 1.
REM
REM N = NUMBER OF PERIODS OVER WHICH THE SIMULATION IS TO RUN.
REM (THIS IS THE NUMBER OF REPLICATIONS FOR THE SIMULATION)
REM IMAX IS THE INTENDED OR DESIRED INVENTORY LEVEL.
REM
REM THE DESIRED VALUES OF L, N AND IMAX ARE ENTERED ON LINE
REM 100 BELOW.
REM
REM THE USER MUST ALSO DECLARE THE NUMBER OF LEVELS IN THE
REM SUPPLY CHAIN.
REM REM THIS IS SET BY VARIABLE NLEVEL. THE FIRST LEVEL OF THE REM PROGRAM
REM GENERATES INDEPENDENT CUSTOMER DEMANDS, THEN CALCULATES THE
REM NEEDED RETAILER ORDERS BACK TO ITS SUPPLIER.
REM THE NEXT LEVEL RECEIVES THESE ORDERS AS A CUSTOMER DEMAND,
REM THEN GENERATES THE REQUIRED ORDERS TO THE UPSTREAM VENDOR.
REM THE SAME PROCESS EXTENDS AS FAR UPSTREAM AS DESIRED.
REM
REM THE USER SPECIFIES THE NUMBER OF LEVELS IN LINE 200 WITH
REM THE VARIABLE NLEVEL.
REM
REM THE NUMBER OF LEVELS WILL BE NLEVEL + 1
REM FOR EXAMPLE, IF THE SUPPLY CHAIN HAS A CUSTOMER, A RETAILER
REM AND A PRODUCER, SET NLEVEL = 2. THEN, AT THE FIRST LEVEL
REM THERE ARE CUSTOMER DEMANDS AND RETAILER ORDERS. AT THE
REM SECOND LEVEL THE RETAILER ORDERS BECOME THE DEMANDS FROM THE
REM RETAILER (THE CUSTOMER AT THIS LEVEL) TO THE PRODUCER.
REM
REM *****
REM *****
REM

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REM THE DEMAND FOR EACH PERIOD IS SET BY THE PROGRAM IN A
REM SUBROUTINE
REM THAT STARTS AT LINE 10000. THE USER ENTERS THE DESIRED
REM PERIODIC
REM RANDOM DEMAND PATTERN OF INTEREST. THE PATTERNS PROVIDED REM THERE
REM NOW INCLUDE DEMANDS FOLLOWING THE UNIFORM, BINOMIAL, POISSON
REM AND NORMAL PROBABILITY DISTRIBUTIONS, AS WELL AS DEMANDS THAT REM
REM GROW OVER TIME.
REM
REM ALSO, THE USER SHOULD DESCRIBE IN LINE 500 THE NATURE OF THE
REM ORIGINAL INDEPENDENT DEMANDS.
REM
REM *****
CLS
RANDOMIZE TIMER
DIM DEMAND(2000), INVEN(2000), ORDER(2000)
REM *****
REM ENTER N, L AND IMAX HERE
100 N = 100: L = 1 : IMAX = 100
REM *****
REM
REM SET THE NUMBER OF LEVELS DESIRED IN THE SUPPLY CHAIN ANALYSIS
REM
200 NLEVEL = 1
REM
REM *****
NL = 0
FOR I = 1 TO N + L
GOSUB 10000
NEXT I
1500 NL = NL + 1
REM *****
REM IF IT IS DESIRED TO RESET ANY PARAMETER FOR THE NEW LEVEL IN
REM THE SUPPLY CHAIN, DO IT HERE.
REM EXAMPLES ARE:
REM IF NL = 2 THEN IMAX = 150
REM IF NL = 3 THEN IMAX = 200
REM *****
IF NL > NLEVEL GOTO 1600
INVEN(1) = IMAX - DEMAND(1)
ORDER(1) = DEMAND(1)
FOR I = 2 TO L
INVEN(I) = INVEN(I - 1) - DEMAND(I)
ORDER(I) = DEMAND(I)
NEXT I
FOR I = L + 1 TO N + L
INVEN(I) = INVEN(I - 1) + ORDER(I - L) - DEMAND(I)
GOSUB 20000
NEXT I
TD = 0: TI = 0: TORDER = 0
FOR I = L + 1 TO N + L
TD = TD + DEMAND(I)
TI = TI + INVEN(I)

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```

TORDER = TORDER + ORDER(I)
NEXT I
MD = TD / N
MI = TI / N
MORDER = TORDER / N
TD = 0 : TI = 0 : TORDER = 0
FOR I = L + 1 TO N + L
TD = TD + (DEMAND(I) - MD) ^ 2
TI = TI + (INVEN(I) - MI) ^ 2
TORDER = TORDER + (ORDER(I) - MORDER) ^ 2
NEXT I
FOR I = L + 1 TO N + L
REM *****
REM IF INDIVIDUAL PERIODIC DEMANDS AND ORDERS ARE TO BE
REM DISPLAYED, USE THE FOLLOWING PRINT STATEMENT
REM
REM PRINT I - L, DEMAND(I), ORDER(I)
REM
REM *****
NEXT I
SDD = SQR(TD / (N - 1))
SDI = SQR(TI / (N - 1))
SDORDER = SQR(TORDER / (N - 1))
PRINT: PRINT
PRINT "MONTE CARLO SIMULATION OF THE SUPPLY CHAIN BULLWHIP EFFECT"
500 PRINT "CUSTOMER DEMANDS ARE SET TO BE UNIFORMLY DISTRIBUTED OVER [5,15]"
PRINT "LEVEL OF THE SUPPLY CHAIN IS: "; NL
PRINT "SAMPLE SIZE IS "; N ; " PERIODS. ORDER LEAD TIME IS "; L ; " PERIODS"
PRINT "MEAN AND STANDARD DEVIATION OF PERIODIC DEMAND ARE "; MD ; " AND "; SDD
PRINT "MEAN AND STANDARD DEVIATION OF INVENTORY LEVEL ARE"; MI ; " AND"; SDI
PRINT "MEAN AND STANDARD DEVIATION OF ORDER QUANTITY ARE "; MORDER ; "
AND";SDO
PRINT "DESIRED INVENTORY LEVEL IS "; IMAX
FOR I = 1 TO N + L
DEMAND(I) = ORDER(I)
NEXT I
GOTO 1500
1600 END
REM
REM
10000 REM GENERATE A DEMAND FOR THE PERIOD
REM DEMAND IS UNIFORM ON [5,15]
DEMAND(I) = INT(5 + 11 * RND)
REM *****
REM
REM DEMAND IS POISSON
REM DIM P(100), PTOT(100)
REM *****
REM LM IS LAMBDA, THE MEAN PARAMETER OF THE POISSON.
REM THE USER MUST ENTER THE DESIRED VALUE OF LM.
REM THE PROGRAM WILL CALCULATE INDIVIDUAL POISSON
REM PROBABILITIES UNTIL THE CUMULATIVE PROBABILITY
REM EXCEEDS .999

```

```

REM *****
REM LM = 3
REM P(0) = EXP(-LM) : PTOT(0) = P(0)
REM FOR II = 1 TO 80
REM P(II) = P(II - 1) * LM / II
REM PTOT(II) = PTOT(II - 1) + P(II)
REM PMAX = II
REM IF PTOT(II) > .999 GOTO 10100
REM NEXT II
REM 10100 REM CALCULATE THE PERIODIC DEMANDS
REM R = RND
REM IF R < PTOT(0) THEN DEMAND(I) = 0
REM FOR II = 1 TO PMAX
REM IF R >= PTOT(II - 1) AND R < PTOT(II) THEN DEMAND(I) = II
REM NEXT II
REM IF R > .999 THEN DEMAND(I) = PMAX
REM
REM *****
REM DEMAND IS BINOMIAL
REM THE USER MUST ENTER THE POPULATION SIZE AND THE PROBABILITY
REM OF SUCCESS ON ANY TRIAL. THESE ARE DENOTED BELOW AS
REM NN = POPULATION SIZE AND PP = PROBABILITY OF SUCCESS IN A REM TRIAL
REM *****
REM
REM DIM P(100), PTOT(100)
REM PP = .9 : NN = 10
REM P(0) = (1 - PP) ^ NN : PTOT(0) = P(0)
REM FOR II = 1 TO NN
REM P(II) = P(II - 1) * PP * (NN - II + 1) / (II * (1 - PP))
REM PTOT(II) = PTOT(II - 1) + P(II)
REM NEXT II
REM 10100 R = RND
REM IF R < PTOT(0) THEN DEMAND(I) = 0
REM FOR II = 1 TO NN
REM IF R >= PTOT(II - 1) AND R < PTOT(II) THEN DEMAND(I) = II
REM NEXT II
REM
REM
REM *****
REM DEMAND FOLLOWS THE NORMAL PROBABILITY DISTRIBUTION
REM WITH MEAN MU AND STANDARD DEVIATION SS
REM THE USER MUST ENTER THE DESIRED VALUES OF MU AND SS BELOW.
REM *****
REM MU = 10 : SS = 3
REM 10050 Z = SQR(-2 * LOG(RND)) * COS(2 * 3.14159 * RND)
REM DEMAND(I) = INT(MU + Z * SS)
REM IF DEMAND(I) < 0 GOTO 10050
REM
REM
REM *****
REM MODEL WITH INCREASING DEMAND OVER TIME
REM 10250 DEMAND(I) = 10 + (I / N) * 20
REM DEMAND(I) = DEMAND(I) + .8 * (SQR(-2 * LOG(RND)) * COS( 2 * REM 3.14159 * RND)

```

```
REM DEMAND(I) = INT(DEMAND(I))
REM IF DEMAND(I) < 0 GOTO 10250
REM *****
REM
RETURN
REM
REM
REM
20000 REM CALCULATE THE PERIODIC ORDER QUANTITY
IF INVEN(I) >= IMAX GOTO 20010
TOTORD = 0
FOR IJ = 1 TO L
TOTORD = TOTORD + ORDER(I - IJ)
NEXT IJ
ORDER(I) = IMAX - INVEN(I) - TOTORD
GOTO 20100
20010
ORDER(I) = 0
20100 RETURN
```