The *b*-chromatic number of Mycielskian of some graphs

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Abstract: A b-colouring of a graph G is a proper colouring of the vertices of G such that there exists a vertex in each colour class joined to at least one vertex in each other colour classes. The b-chromatic number of a graph G, denoted by $\varphi(G)$ is the largest integer k such that G has a b-colouring with k colours. The Mycielskian or Mycielski graph $\mu(H)$ of a graph H with vertex set $\{v_1, v_2, \ldots, v_n\}$ is a graph G obtained from H by adding $n + 1$ new vertices $\{u, u_1, u_2, \ldots, u_n\}$, joining u to each vertex u_i $(1 \leq i \leq n)$ and joining u_i to each neighbour of v_i in H. In this paper, we obtain the b-chromatic number of Mycielskian of paths, complete graphs, complete bipartite graphs and wheels.

Keywords: b-chromatic number; b-colouring; b-dominating set; Mycielskian.

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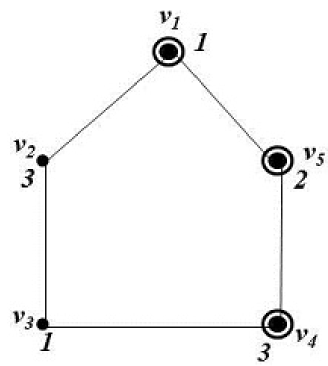
1 Introduction

The concept of b-chromatic number of a graph G is introduced by Irving and Manlove in 1999. They have defined the b-chromatic number $\varphi(G)$ of a graph G as the largest positive integer k such that G admits a proper k-colouring in which every colour class has a representative vertex which is adjacent to at least one vertex in each of the other colour classes and this representative vertex is known as the b-dominating vertex. It is clear that for a graph G to have a b-colouring of k colours, G must contain at least k vertices, each of degree at least $k - 1$. Bonomo et al. (2009) proved that P_4 – sparse graphs (and, in particular, cographs) are b-continuous and b-monotonic. Besides, they described a dynamic programming algorithm to compute the b-chromatic number in polynomial time within these graph classes. El Sahili and Kouider (2006) studied the b-chromatic number of a d-regular graph of girth 5. Also, Sergio and Marcove (2011) discussed the b-chromatic number of regular graphs according to their girth and diameter. Javadi and Omoomi (2009) studied the b-colouring of Kneser graphs $K(n; k)$ and determine the b-chromatic number of $K(n; k)$ for some values of n and k. Moreover, they proved that $K(n; 2)$ is b-continuous for $n \leq 17$. In 2007, Elghazel et al. discussed the applications of b-colouring in clustering. Vivin and Venkatachalam (2012) obtained the b-chromatic number of corona of two graphs with same number of vertices. In 2011, Eric et al. discussed the applications of corona of graphs in network design and analysis. In 2013, Venkatachalam and Vivin obtained the b-chromatic number of windmill graph. In 2014, Venkatachalam and Vivin discussed the b-chromatic number for the central graph, middle graph, total graph and line graph of double star graph $K_{1,n,n}$. Also, they have found a relationship between the b-chromatic number and three other colouring parameters, the equitable chromatic number, harmonious chromatic number and the achromatic number. In 2014, Vivin and Venkatachalam (2014a) obtained the b-chromatic number of middle and total graph of fan graph. In 2014, Vivin and Venkatachalam (2014b) discussed the b-chromatic number for the sun let graph S_n, line graph of sun let graph $L(S_n)$, middle graph of sun let graph $M(S_n)$, total graph of sun let graph $T(S_n)$, middle graph of wheel graph $M(W_n)$ and the total graph of wheel graph $T(W_n)$. In 2012, Vijayalakshmi and Thilagavathi (2012b) obtained the b-chromatic number of corona product of path, cycle and star graph with complete graph, the strong product of path with cycle and Cartesian product of cycles. In 2012, Vijayalakshmi and Thilagavathi (2012a) discussed the b-chromatic number of transformation graph G^{++-} for cycle, path and star graph. Also, they have discussed the b-chromatic number of corona product of path graph with cycle and path graph with complete graph along with its structural properties. The Mycielskian or Mycielski graph $\mu(H)$ of a graph H with vertex set $\{v_1, v_2, \ldots, v_n\}$ is a graph G obtained from H by adding $n + 1$ new vertices $\{u, u_1, u_2, \ldots, u_n\}$, joining u to each vertex u_i $(1 \leq i \leq n)$ and joining u_i to each neighbour of v_i in H. In 2001, Massimiliano et al. discussed the applications of Mycielski graphs in multiprocessor task scheduling problem. In this paper, we obtain the b-chromatic number of Mycielskian of paths, complete graphs, complete bipartite graphs and wheels.

Example 1.1:

Figure 1 A b-colouring of a graph with three colours



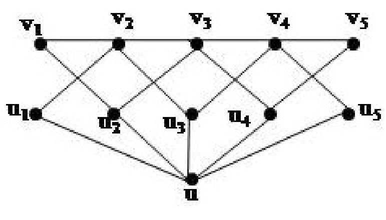
The Mycielskian or Mycielski graph of a graph G is an important concept in the area of graph theory. It is denoted by $\mu(G)$ and is defined as follows:

Definition 1.2 (Chartrand and Zhang, 2009): The Mycielskian or Mycielski graph $\mu(H)$ of a graph H with vertex set $\{v_1, v_2, \ldots, v_n\}$ is a graph G obtained from H by adding $n + 1$ new vertices $\{u, u_1, u_2, \ldots, u_n\}$, joining u to each vertex u_i $(1 \leq i \leq n)$ and joining u_i to each neighbour of v_i in H.

The Mycielski graph has an application in multiprocessor task scheduling problem. If we let M_1 be the Mycielski graph with two nodes and one single edge, M_i is recursively defined as $M_{i+1} = \mu(M_i)$. The minimum number of processors such that the intersection graph of the processor task requirements is the given Mycielski graph M_i is exactly $| E_i |$, where E_i is the edge set of M_i. Also, the minimum completion time $C^i_{\max}$ of the multiprocessor task max scheduling problem obtained from M_i is equal to $\chi(M_i)$, where $\chi(M_i)$ is the chromatic number of M_i (Massimiliano and Paolo, 2001).

Example 1.3:

Figure 2 Mycielskian of the path P_5



2 Main results

In this section, we discuss the b-chromatic number of Mycielskian of paths, complete graphs, complete bipartite graphs and wheels. These results are proved using the m-degree $m(G)$ of a graph G, which is an upper bound for the b-chromatic number $\varphi(G)$ as suggested in Irving and Manlove (1999).

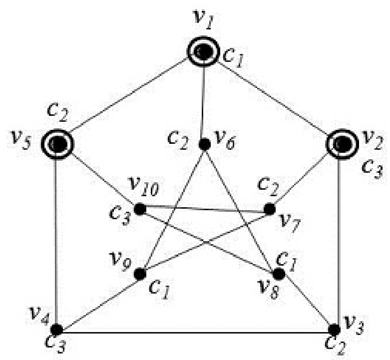
Definition 2.1 (Irving and Manlove, 1999): For a graph $G = (V, E)$, suppose that the vertices of G are ordered $v_1, v_2, \ldots, v_n$ so that $d(v_1) \geq d(v_2) \geq \ldots = d(v_n)$. The m-degree, $m(G)$, of G is defined by

$$m(G) = \max_i \{i : d(v_i) = i - 1\}.$$

Theorem 2.2 (Irving and Manlove, 1999): For any graph G, $\varphi(G) \leq m(G)$.

Example 2.3:

Figure 3 A b-colouring of Petersen graph with three colours



For a Petersen graph, $m(G) = 4$ and $\varphi(G) = 3$ and thus $\varphi(G) < m(G)$.

Next, we discuss the b-chromatic number of Mycielskian of paths, complete graphs, complete bipartite graphs and wheels.

Theorem 2.4: The b-chromatic number of Mycielskian of a path P_n is

$$\varphi(\mu(P_n)) = \begin{cases} 2; & \text{for } n = 1 \\ 3; & \text{for } n = 2, 3, 4 \\ 4; & \text{for } n = 5, 6, 7 \\ 5; & \text{for } n \geq 8 \end{cases}$$

Proof: Let the vertex set of P_n be $\{v_1, v_2, \ldots, v_n\}$ and that of $\mu(P_n)$ be $\{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\} \cup \{u\}$.

Case 1: $1 \leq n \leq 7$

- $n = 1$

  Here, $\mu(P_1)$ is a disconnected graph $K_2 \cup K_1$ with three vertices and one edge. Hence, it is clear that $\varphi(\mu(P_1)) = 2$.

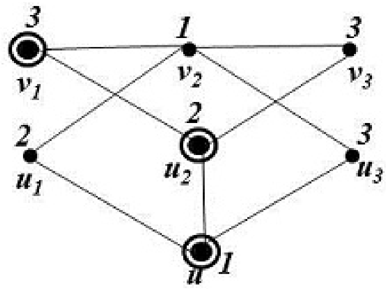
- $n = 2$

  $\mu(P_2)$ is a cycle with five vertices and $\varphi(C_5) = 3$ (Sergio and Marcove, 2011). Thus $\varphi(\mu(P_2)) = 3$.

- $n = 3$

  In this case, a b-colouring with four colours is not possible, because here we have only three vertices with degree at least 3. A b-colouring with three colours is given in Figure 4.

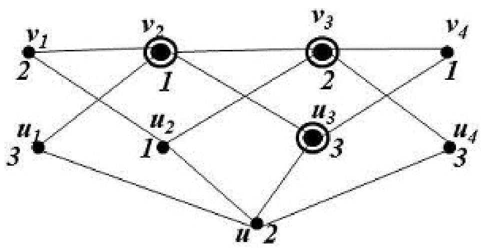
Figure 4 A b-colouring of $\mu(P_3)$ with three colours



- $n = 4$

  In this case also, we prove that $\varphi(\mu(P_4)) = 3$. On the contrary, assume that $\varphi(\mu(P_4)) = 4$. Here, we have exactly five vertices, v_2, v_3, u_2, u_3 and u each having degree at least 3. Among these five vertices, we can choose any of the four vertices as the b-dominating vertices. Suppose that the vertex u is included in the set of b-dominating vertices and let the colour of u be c. Since u is adjacent to all the u_i's, $1 \leq i \leq 4$, we cannot assign colour c to any of the u_i's, $1 \leq i \leq 4$. But to make the remaining three vertices b-dominating, we assign the colour c to some v_i's. But if we assign colour c to any of the v_i's; $1 \leq i \leq 4$, u_2 and u_3 will have two neighbours with same colour. But the degree of u_2 and u_4 is exactly 3. So to make these vertices b-dominating, the three neighbours should receive distinct colours. Thus if assign colour c to any of the v_i's; $1 \leq i \leq 4$, u_2 and u_3 will not become b-dominating. If u is not included in the set of b-dominating vertices, then to make u_2 and u_3 b-dominating we assign colour of v_2 or colour of v_3 to u. Hence in this case, either u_2 or u_3 will have two neighbours with same colour. So they will not become b-dominating. Thus a b-colouring with four colours is not possible here. A b-colouring with three colours is given in Figure 5.

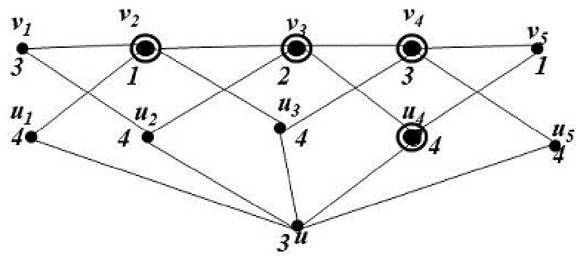
Figure 5 A b-colouring of $\mu(P_4)$ with three colours



- $n = 5$

  Here, we prove that $\varphi(\mu(P_5)) = 4$. On the contrary, assume that $\varphi(\mu(P_5)) = 5$. Then, there will be at least five vertices each with degree at least 4. But, here we have only four vertices each with degree at least 4. Hence, a b-colouring with five colours is not possible. A b-colouring with four colours is obtained in Figure 6.

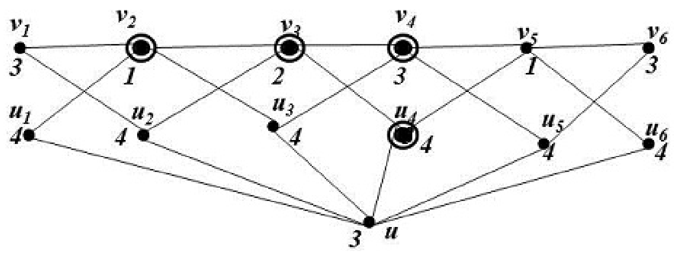
Figure 6 A b-colouring of $\mu(P_5)$ with four colours



- $n = 6$

  Here we prove that $\varphi(\mu(P_6)) = 4$. On the contrary assume that $\varphi(\mu(P_6)) = 5$. Here, we have exactly five vertices v_2, v_3, v_4, v_5 and u each with degree at least 4. So, we can assign five different colours to these vertices. Suppose that the colour of $u = c_1$ and colour of $v_{i+1} = c_{i+1}$; $1 \leq i \leq 4$. Since all the u_i's are adjacent to u, we cannot assign colour c_1 to any of the u_i's. So the vertices v_3 and v_4 are not adjacent to a vertex with colour c_1. Hence, these vertices will not become b-dominating. Thus, a b-colouring with five colours is not possible here. A b-colouring with four colours is obtained in Figure 7.

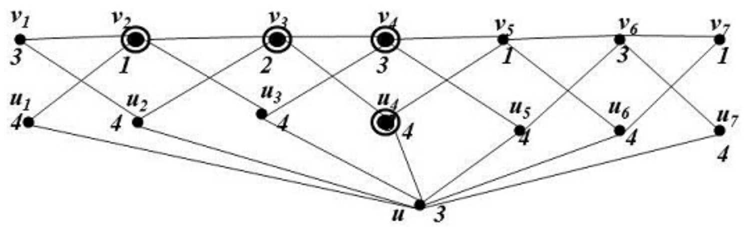
Figure 7 A b-colouring of $\mu(P_6)$ with four colours



- $n = 7$

  Here, we prove that $\varphi(\mu(P_7)) = 4$. On the contrary, assume that $\varphi(\mu(P_7)) = 5$. Here, we have six vertices v_2, v_3, v_4, v_5, v_6 and u each with degree at least 4. We suppose that u is not a b-dominating vertex. Then v_2, v_3, v_4, v_5 and v_6 will be the b-dominating vertices, each with degree exactly 4. We suppose that colours of these vertices are c_1, c_2, c_3, c_4 and c_5 respectively. Here, u_4 is a common neighbour of v_3 and v_5. So, we cannot assign colours c_2 and c_4 to u_4. If we assign any colour from the remaining colours, then either v_3 or v_5 will have two neighbours with same colour. Hence, they will not become b-dominating. Next, we suppose that u is included in the set of b-dominating vertices. Let c_1 be the colour of u. Since each u_i is adjacent to u, we cannot assign colour c_1 to any of the u_i's, $1 \leq i \leq 7$. The vertices v_1 and v_7 have degree 2. But, here we are constructing a b-colouring with five colours. So each b-dominating vertex should have degree at least 4. Hence the remaining four b-dominating vertices will be from the set $\{v_2, v_3, v_4, v_5, v_6\}$. To make these four vertices b-dominating, we should assign colour c_1 to at least one of their neighbour. If we assign colour c_1 to v_1, v_7 and v_4, the vertices v_2, v_6, v_3 and v_5 become adjacent to a vertex having colour c_1. Thus we will get four vertices, which are adjacent to the vertex with colour c_1. So, we select v_2, v_3, v_5 and v_6 as the remaining four b-dominating vertices and assign colours c_2, c_3, c_4 and c_5 respectively. Now, each b-dominating vertex is adjacent to a vertex with colour c_1. Now, we consider the vertex v_3. Colour of v_3 is c_3. We consider the neighbours of v_3. The colour of v_2 is c_2 and v_4 is c_1. Now, the options for the other neighbours u_2 and u_4 of v_3 are c_4 and c_5. In this case, v_5 will have two neighbours with colour c_5. Hence, v_5 will not become b-dominating. Thus, in either case, a b-colouring with five colours is not possible and a b-colouring with four colours is given in Figure 8.

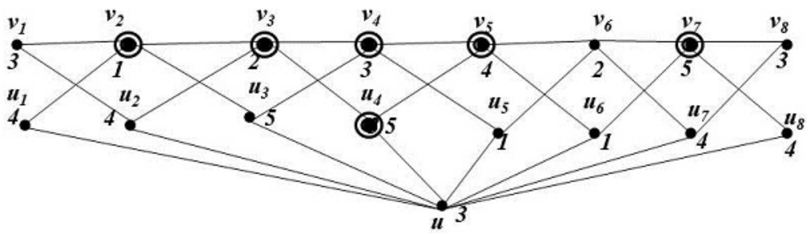
Figure 8 A b-colouring of $\mu(P_7)$ with four colours



Case 2: $n \geq 8$

In this case, we prove that $\varphi(\mu(P_n)) = 5$, $n \geq 8$. A b-colouring with six colours is not possible, because here we have only one vertex with degree more than 4. A b-colouring with five colours is obtained in Figure 9 for $\mu(P_8)$.

Figure 9 A b-colouring of $\mu(P_8)$ with five colours



For $n \geq 8$, we can choose the colouring which is used for $\mu(P_8)$ and colour the remaining uncoloured vertices can be coloured properly as follows.

- If n is an odd number

  We assign colour c_1 to $\{v_9, v_{11}, v_{13}, \ldots, v_n\}$ and $\{u_9, u_{11}, u_{13}, \ldots, u_n\}$ and colour c_2 to $\{v_{10}, v_{12}, v_{14}, \ldots, v_{n-1}\}$ and $\{u_{10}, u_{12}, u_{14}, \ldots, u_{n-1}\}$.

- If n is an even number

  We assign colour c_1 to $\{v_9, v_{11}, v_{13}, \ldots, v_{n-1}\}$ and $\{u_9, u_{11}, u_{13}, \ldots, u_{n-1}\}$and colour c_2 to $\{v_{10}, v_{12}, v_{14}, \ldots, v_n\}$ and $\{u_{10}, u_{12}, u_{14}, \ldots, u_n\}$.

Thus, we obtain a proper b-colouring with five colours. □

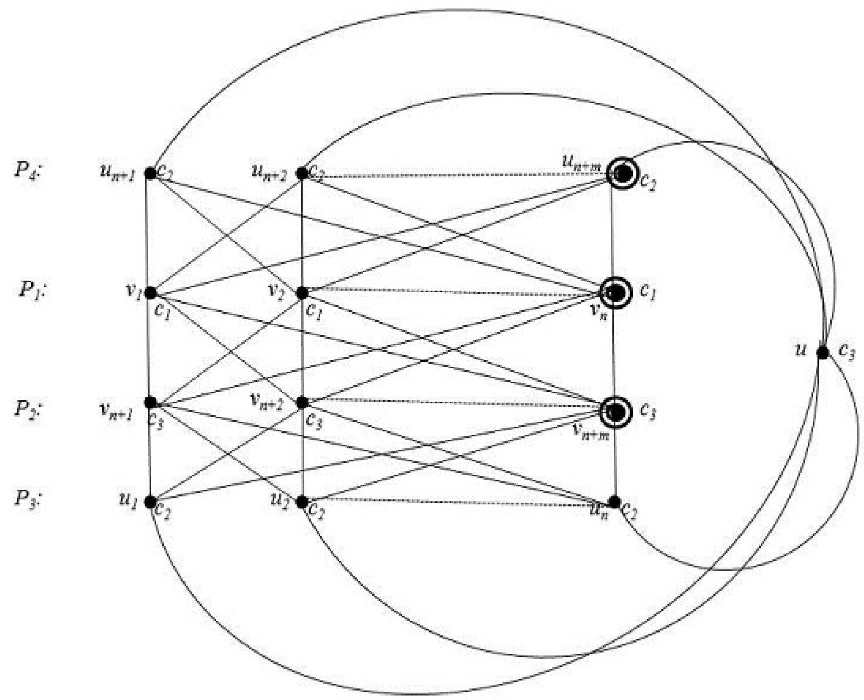
Theorem 2.5: The b-chromatic number of Mycielskian of a complete bipartite graph $K_{n,m}$ is

$$\varphi(\mu(K_{n,m})) = 3 \quad \forall n, m \geq 1.$$

Proof: Let the vertex set of $K_{n,m}$ be $\{v_1, v_2, v_3, \ldots, v_{n+m}\}$, where each of the v_i's are adjacent to all the v_j's; $1 \leq i \leq n$, $n + 1 \leq j \leq n + m$. Let the vertex set of Mycielskian of $K_{n,m}$ be $V(\mu(K_{n,m})) = V(K_{n,m}) \cup \{u, u_i; 1 \leq i \leq n + m\}$. Here, each u_i is adjacent to all the neighbours of v_i; $1 \leq i \leq n + m$ and u is adjacent to all the u_i's ($1 \leq i \leq n + m$). Now, a b-colouring with three colours can be obtained as given in Figure 10 by assigning colour c_1 to $v_1, v_2, \ldots, v_n$, colour c_2 to $u_1, u_2, \ldots, u_{n+m}$ and colour c_3 to $v_{n+1}, v_{n+2}, \ldots, v_{n+m}$ and u. Next, we prove that a b-colouring with more than four colours does not exist. On the contrary, we assume that there exists a b-colouring with four colours. Now, we partition the vertex set of $\mu(K_{n,m}) \setminus u$ into four parts, $P_1 = \{v_i; 1 \leq i \leq n\}$, $P_2 = \{v_j; n + 1 \leq j \leq n + m\}$, $P_3 = \{u_i; 1 \leq i \leq n\}$ and $P_4 = \{u_j; n + 1 \leq j \leq n + m\}$.

We assume that, we have selected two b-dominating vertices from a single partite set. Let it be v' and v'' with colours c_1 and c_2 respectively.

Figure 10 A b-colouring of $\mu(K_{n,m})$ with three colours



Since v' and v'' have same neighbours, we cannot assign colour c_1 to a neighbour of v'' and colour c_2 to a neighbour of v'. Thus, the vertex v' will not be adjacent to the colour c_2 and similarly the vertex v'' will not be adjacent to the colour c_1. Thus, these two vertices will not become b-dominating vertices. Thus the maximum number of b-dominating vertices that can be selected from each partite set is one. Next, we assume that there exists a b-colouring with four colours. Let B be the set of b-dominating vertices. Here either $u \in B$ or $u \notin B$. We assume that $u \in B$ and we assign colour c_1 to u. Thus, the remaining three b-dominating vertices will be from any of the four partite sets. But in this case, we can select only one b-dominating vertex from the partite sets P_1 and P_2. The reason is as follows. Suppose that we have selected two b-dominating vertices v_i and v_j from P_1 and P_2 respectively and assign colours c_2 and c_3 to v_i and v_j. To become a b-dominating vertex, these two vertices should be adjacent to a vertex with colour c_1. Since u is adjacent to all the vertices in P_3 and P_4, colour c_1 cannot be assigned to any of the vertices in these two partite sets. So, we assign colour c_1 to one of the vertices in P_1. Thus, the vertex v_j will be adjacent to a vertex with colour c_1. Now, we cannot assign colour c_1 to any of the remaining uncoloured vertices in $\mu(K_{n,m})$. Thus the vertex v_i will not be adjacent to a vertex with colour c_1 and hence it will not become a b-dominating vertex. Thus, from P_1 and P_2 we can select only one b-dominating vertex. Without loss of generality, we select the three b-dominating vertices from P_1, P_3 and P_4. Let it be v_i, u_k and u_j and assign colours c_2, c_3 and c_4 respectively to these vertices. We consider the vertex v_i having colour c_2. This vertex is adjacent to u_j having colour c_4. The vertex v_i become a b-dominating vertex if it is adjacent to vertices having colour c_1 and c_3. Since we cannot assign colour c_3 to any of the vertices in P_2, we assign colour c_3 to an uncoloured vertex in P_4. But now we cannot assign colour c_3 to any of the neighbours of u_j. Thus u_j will not become a b-dominating vertex. Hence in this case a b-colouring with four colours is not possible.

Next, we assume that $u \notin B$. If so, four b-dominating vertices will be from the four partite. Let v_i, v_j, u_k and u_l be the four b-dominating vertices from P_1, P_2, P_3 and P_4 respectively. We assign colours c_1, c_2, c_3 and c_4 to v_i, v_j, u_k and u_l respectively. We consider the vertex u_l. It is not adjacent to the vertex with colour c_2. Since all the vertices in P_1 is adjacent to v_j, we cannot assign colour c_2 to any of the vertices in P_2. So, we assign the colour c_2 to u. Next, we consider the vertex u_k. Here all the uncoloured neighbours of u_k are adjacent to v_i. So, we cannot assign colour c_1 to any of its neighbours and hence the vertex u_k is not adjacent to a vertex with colour c_1. Thus the vertex u_k will not become a b-dominating vertex. So in this case a b-colouring with four colours is not possible.

Corollary 2.6: The b-chromatic number of the Mycielskian of a star graph $K_{1,n}$ is

$$\varphi\left(\mu\left(K_{1,n}\right)\right) = 3 \text{ for all } n.$$

Theorem 2.7: The b-chromatic number of a Mycielskian of a complete graph K_n is $n + 1$.

Proof: Let the vertex set of the complete graph K_n is $\{v_1, v_2, \ldots, v_n\}$ and that of $\mu(K_n)$ be $V(K_n) \cup \{u_1, u_2, ..., u_n, u\}$, where each u_i neighbours of v_i, $1 \le i \le n$ and the vertex u is adjacent to all the u_i; $1 \le i \le n$. Thus, the Mycielskian of a complete graph K_n consists of $n + 1$ vertices with degree n and n vertices with degree $2n - 2$. A b-colouring with $n + 1$ colours can be obtained by assigning colour c_i to v_i; $1 \le i \le n$, c_{n+1} to u_i; $1 \le i \le n$, and c_1

to u. By Theorem 2.2, $\varphi(\mu(K_n)) \leq m(\mu(K_n)) \leq n + 1$. Since we have already established a b-colouring with $n + 1$ colours, $\varphi(\mu(K_n)) = n + 1$. □

Theorem 2.8: The b-chromatic number of the mycielskian of a wheel graph W_{n+1} is

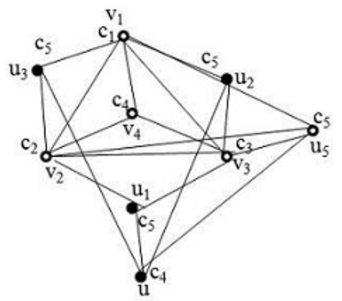
$$\varphi(\mu(W_{n+1})) = \begin{cases} 5; & \text{for } n = 3, 5 \\ 4; & \text{for } n = 4 \\ 6; & \text{for } n = 6, 8 \\ 7; & \text{for } n = 7 \text{ and } n \geq 9 \end{cases}$$

Proof: Let W_{n+1} be a wheel graph with $n + 1$ vertices, say $v_1, v_2, \ldots, v_{n+1}$ with v_{n+1} as the central vertex with $deg(v_{n+1}) = n$. Let $\mu(W_{n+1})$ denote the Mycielskian of W_{n+1} with vertex set $V(W_{n+1}) \cup \{u, u_1, u_2, \ldots, u_{n+1}\}$, where u is adjacent to all the u_i's; $1 \leq i \leq n + 1$. Hence, $\mu(W_{n+1})$ contains n vertices with degree 6, n vertices with degree 4, one vertex with degree $2n$ and two vertices with degree $n + 1$. Thus by Theorem 2.2, $\varphi(\mu(W_{n+1})) \leq 7$.

- $n = 3$

  Assign colour i to v_i, $1 \leq i \leq 4$ and colour 5 to u_i, $1 \leq i \leq 4$ and colour 4 to u. By Theorem 2.2, $\varphi(\mu(W_4)) = m(\mu(W_4)) = 5$. Since we have already established a b-colouring with five colours, $\varphi(\mu(W_4)) = 5$.

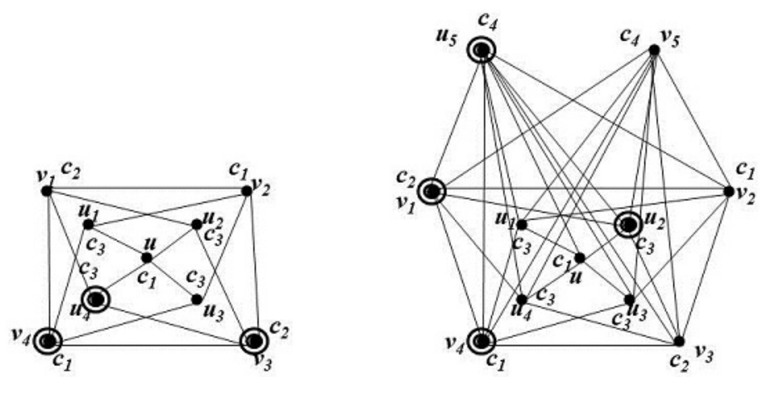
Figure 11 A b-colouring of $\mu(W_4)$ with five colours



- $n = 4$

  Here $\mu(W_5)\backslash\{v_5, u_5\} = \mu(C_4)$. So first we prove that $\varphi(\mu(C^4)) = 3$ and after that we find $\varphi(\mu(W_5))$. We consider $\mu(C_4)$. Let B be a set of b-dominating vertices. Let $N(v)$ denote the neighbourhood of v. Here $N(v_1) = N(v_3)$, $N(v_2) = N(v_4)$, $N(u_1) = N(u_3)$ and $N(u_2) = N(u_4)$. Thus, from each set $\{v_1, v_3\}$, $\{v_2, v_4\}$, $\{u_1, u_3\}$ and $\{u_2, u_4\}$ we can select at most one vertex to the set B. Here first we prove that $\varphi(\mu(C_4)) = 3$. On the contrary, we assume that $\varphi(\mu(C_4)) = 4$. Let $u \notin B$. Here we select one vertex from each of the set $\{v_1, v_3\}$, $\{v_2, v_4\}$, $\{u_1, u_3\}$ and $\{u_2, u_4\}$. Let it be v_i, v_{i+1}, u_j and u_{j+1}, where $i + k = i + k \bmod 4$ and $j + k = j + k \bmod 4$ for $i + k > 4$ and $j + k > 4$ $(1 \leq i, j, k \leq 4)$. Note that v_i is adjacent to u_j or u_{j+1} but not to both. Also note that v_i and v_{i+1} are adjacent to each other but these two vertices does not have any common neighbour. Without loss of generality we assume that v_i is adjacent to u_j and v_{i+1} is adjacent to u_{j+1}. Also we assign colours c_1, c_2, c_3 and c_4 to v_i, v_{i+1}, u_j and u_{j+1} respectively. Now we consider u_j and u_{j+1}. u_j is having colour c_3 and is adjacent to the vertex v_i with colour c_1. To become a b-dominating vertex it should be adjacent to the vertices having colour c_2 and c_4. The remaining uncoloured neighbours of u_j are v_{i+2} and u. Since v_{i+1} is adjacent to v_{i+2}, we cannot assign colour c_2 to v_{i+2}. So the only choice is to assign colour c_2 to u. Since u_{j+1} is having only three neighbours and we are constructing a b-colouring with four colours, the three neighbours of u_{j+1} should receive distinct colours. But if we assign colour c_2 to u, the vertex u_{j+1} will have two neighbours with colour c_2. Thus, the vertex u_{j+1} will not become a b-dominating vertex. Next, we assume that $u \in B$. Assume that we have selected a vertex u_j from the set $\{u_i; 1 \leq i \leq 4\}$ and two adjacent vertices v_i and v_{i+1} from the set $\{v_i; 1 \leq i \leq 4\}$. We assign colour c_1, c_2, c_3 and c_4 to u, u_j, v_i and v_{i+1} respectively. The vertex u_j will be adjacent to either v_i or v_{i+1} but not to both. Without loss of generality we assume that u_j is adjacent to v_i. Now, u_j is adjacent to vertices having colour c_1 and c_3. To become a b-dominating vertex it should be adjacent to a vertex having colour c_4. Now the only uncoloured neighbour of u_j is v_{i+2}. But since v_{i+1} is adjacent to v_{i+2}, we cannot assign colour c_4 to this vertex. Thus, in this case u_i will not become a b-dominating vertex. Next, we consider that we have selected two vertices u_j and $u_j + 1$ from the set $\{u_j; 1 \leq j \leq 4\}$ and one vertex vi from the set $\{v_i; 1 \leq i \leq 4\}$. As in the last case here also we cannot construct a b-colouring with four colours. A b-colouring with three colours can be obtained as shown in Figure 12(a). Thus the b-chromatic number of $\mu(C_4)$ is 3. Next, we consider $\mu(W_5) = \mu(C_4) \cup \{v_5, u_5\}$ such that the vertex v_5 is adjacent to u_i and v_i; $1 \leq i \leq 4$ and u_5 is adjacent to all the v_i's and u. Here we prove that $\mu(W_5) = 4$. So, we colour the u_i's, v_i's and the vertex u as in the case of $\mu(C_4)$ for $1 \leq i \leq 4$ and select the b-dominating vertices as in $\mu(C_4)$. Since our aim is to construct a b-colouring with four colours, we assign a new colour c_4 to v_5. We cannot assign a new colour to the vertex u_5, because it is not adjacent to any of the u_i's; $1 \leq i \leq 4$ and v_5. So we assign any colour from the list $\{c_1, c_2, c_3, c_4\}$ in a proper way. Thus we obtained a four b-colouring for $\mu(W_5)$.

Figure 12 A b-colouring of $\mu(C_4)$ and $\mu(W_5)$ with three and four colours



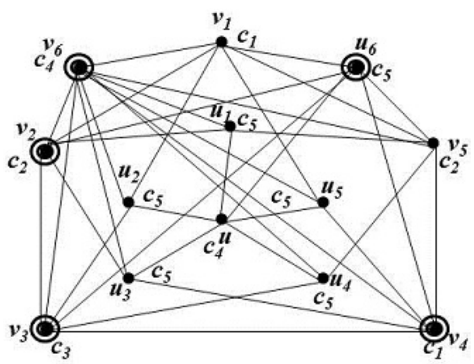
(a) (b)

- $n = 5$

In this case we prove that the b-chromatic number of $\mu(W_{n+1}) = 5$. On the contrary, we assume that $\varphi(\mu(W_{n+1})) = 7$. Here we have exactly eight vertices $\{u, u_6, v_i;$ $1 \leq i \leq 6\}$ with degree at least 6. From this set of eight vertices, we can select any of the seven vertices as the b-dominating vertices. Out of this seven vertices, at least four of them will be from the set $\{v_i; 1 \leq i \leq 5\}$. We assume that we have selected any four vertices from the set $\{v_i; 1 \leq i \leq 5\}$ and the vertices v_6, u_6, u as the b-dominating vertices. We assign seven distinct colours to these vertices. Since u is not adjacent to v_6, any of the neighbours of u should receive colour of v_6. But all the neighbours of u is adjacent to v_6. So u will not be adjacent to a vertex having colour of v_6. Thus the vertex u will not become b-dominating. Next, we assume that we have selected all the five vertices from the set $\{v_i; 1 \leq i \leq 5\}$ and any two vertices from the set $\{v_6, u_6, u\}$ as the b-dominating vertices. We assign colour c_i to v_i for $1 \leq i \leq 5$. Since v_6 and u_6 common neighbours of $\{v_i; 1 \leq i \leq 5\}$, we assign colour c_5 and c_6 to v_6 and u_6 respectively. The degree of each v_i; $1 \leq i \leq 5$ is exactly 6. So to become a b-dominating vertex, the six neighbours of these five vertices should receive distinct colours. We consider the vertex v_3. v_3 is adjacent to vertices having colours c_2, c_4, c_6 and c_7. To become a b-dominating vertex, v_3 should be adjacent to vertices having colours c_1 and c_5. Here u_2 and u_4 are the only uncoloured neighbours of v_3. Since u_2 is adjacent to v_1 and u_4 is adjacent to v_5, the only choice is to assign colour c_1 to u_4 and c_5 to u_2. But if we colour like this, the vertices v_1 and v_5 will have two neighbours with same colour. Thus they will not become b-dominating. Thus a b-colouring with seven colours is not possible here.

Next, we will check the existence of a b-colouring with six colours. Let B be a set of b-dominating vertices. We consider the set $\{u, u_6, v_6\}$. Out of these three vertices, only two belongs to B. On the contrary we assume that all the three vertices, u, u_6 and v_6 are included in B with colours c_1, c_2 and c_3 respectively. We consider the neighbours of u. We can see that all the neighbours of u are adjacent to v_6 except u_6. Since $u_6 \in B$, we cannot assign colour c_3 to any of the neighbours of u. Thus, u will not become a b-dominating vertex. Thus from the set $\{u, u_6, v_6\}$ at most two vertices belongs to B and the remaining b-dominating vertices will be from the set $\{v_i; 1 \leq i \leq 5\}$. We suppose that we have selected two vertices from the set $\{u, u_6, v_6\}$ and four vertices from the set $\{v_i; 1 \leq i \leq 5\}$. Without loss of generality we assume that the four b-dominating vertices from the set $\{v_i; 1 \leq i \leq 5\}$ are v_1, v_2, v_3 and v_4. The two b-dominating vertices from the set $\{u, u_6, v_6\}$ can be either u and u_6 or u and v_6 or u_6 and v_6. First, we assume that the two b-dominating vertices are v_6 and u_6 and we assign colours c_1 and c_2 respectively. For the remaining four b-dominating vertices v_1, v_2, v_3 and v_4, we assign colours c_3, c_4, c_5 and c_6 respectively. We consider the vertex v_1. v_1 is having colour c_3 and is adjacent to the colours c_1, c_2 and c_4. To become a b-dominating vertex, v_1 should be adjacent to the vertices having colours c_5 and c_6. So we assign colours c_5 and c_6 to v_5 and u_2 respectively. Next, we consider the vertex v_2. v_2 is having colour c_4 and is adjacent to the vertices with colours c_1, c_2, c_3 and c_5. To become a b-dominating vertex, v_2 should be adjacent to a vertex having colour c_6. So we assign colour c_6 to u_1. Next, the vertex v_3 is having colour c_5 and is adjacent to the vertices with colours c_1, c_2, c_4 and c_6. To become a b-dominating vertex, v_3 should be adjacent to a vertex having colour c_3. So we assign colour c_3 to u_4. Now we consider the vertex v_4. The vertex v_4 is having colour c_6 and is adjacent to the vertices with colours c_1, c_2 and c_5. To become a b-dominating vertex, v_4 should be adjacent to vertices having colours c_3 and c_4. So we assign colours c_3 and c_4 to u_3 and u_5 respectively. Now, we consider the vertex v_6. This vertex is not adjacent to the vertex with colour c_2. So we have to assign colour c_2 any of the uncoloured neighbours of v_6. But to make the vertices v_1, v_2, v_3 and v_4 b-dominating, we assign colours to all the uncoloured neighbours of v_6. Thus we cannot assign colour c_2 to any of the neighbours of v_6. Thus the vertex v_6 will not become a b-dominating vertex. Next we assume that the two b-dominating vertices from the set $\{u, u_6, v_6\}$ are u and v_6. We assign colours to v_i's and u_i's; $1 \leq i \leq 5$ as mentioned above. Also we assign colours c_1 and c_2 to v_6 and u respectively. Since u_6 is adjacent to u we cannot assign colour c_2 to u_6. So the vertices v_i; $1 \leq i \leq 4$ will not be adjacent to the vertices with colour c_2. Hence, they will not become b-dominating. Next we assume that the two b-dominating vertices from the set $\{u, u_6, v_6\}$ are u and u_6. We assign colours to v_i's and u_i's; $1 \leq i \leq 5$ as mentioned above and we assign colour c_1 and c_2 to u and u_6 respectively. But, here the vertex u is not adjacent to the vertex with colour c_5. Thus the vertex u will not become b-dominating. Next, we assume that we have selected all the vertices from $\{v_i; 1 \leq i \leq 5\}$ and one vertex from the set $\{u, u_6, v_6\}$ as the b-dominating vertices. We assign colour c_i to v_i; $1 \leq i \leq 5$. We consider the vertex v_3 having colour c_3. It is adjacent to the vertices v_2 and v_4 having colours c_2 and c_4 respectively. To become a b-dominating vertex, v_3 should be adjacent to vertices having colours c_1 and c_5. So we assign colours c_1 and c_5 to u_4 and u_2 respectively. Next, we consider the vertex v_4 having colour c_4. v_4 is adjacent to vertices having colours c_3 and c_5. But to become a b-dominating vertex, it should be adjacent to vertices with colour c_1 and c_2. The vertex u_5 is adjacent to v_4. So we can assign colour c_2 to u_5. Now, the remaining uncoloured neighbours of v_4 are adjacent to the vertex v_1. So, we cannot assign colour c_1 to any of the neighbours of v_4. Thus, the vertex v_4 will not become b-dominating. Thus a b-colouring with six colours is also not possible. A b-colouring with five colours can be obtained as given in Figure 13.

Figure 13 A b-colouring of $\mu(W_6)$ with five colours



- $n = 6$

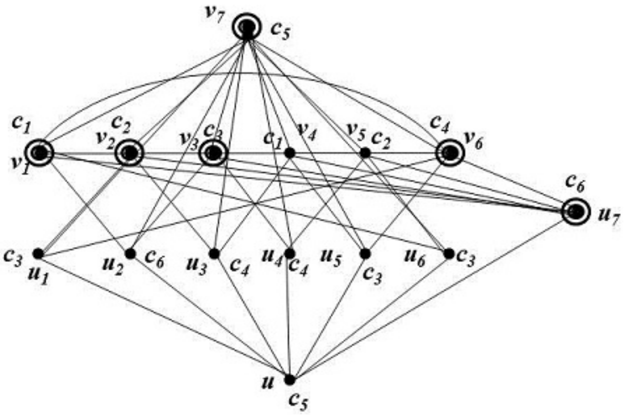
  If possible, we assume that there exists a b-colouring with seven colours. Here, we have nine vertices $\{u, u_7, v_i; 1 \leq i \leq 7\}$ with degree at least 6. Out of this nine vertices we can select any seven vertices as the b-dominating vertices and let B be the set of seven b-dominating vertices.

  We assume that $u \in B$. Note that, if $u \in B$, then either $v_7 \in B$ or $u_7 \in B$ but not both. On the contrary, we assume that u_7 and v_7 belongs to B. We assign colours c_1, c_2, c_3 to u, u_7, v_7 respectively. Here, the vertex u is not adjacent to v_7. So, we have to assign colour c_3 to any of the uncoloured neighbours of u. But, all the uncoloured neighbours of u are adjacent to v_7. Thus, the vertex u will not become b-dominating.

Thus, if $u \in B$, then either $u_7 \in B$ or $v_7 \in B$ but not both. Next, we assume that $u \in B$. So, either $u_7 \in B$ or $v_7 \in B$. Thus, the remaining five b-dominating vertices will be from the set $\{v_i; 1 \leq i \leq 6\}$. Note that the set $\{v_i; 1 \leq i \leq 6\}$ constitute a cycle with six vertices. But from this set of six vertices, we are selecting only five vertices as the b-dominating vertices. So there will be a vertex in this set, which is not in B. Let it be v_k, $k \in \{1, 2, \ldots, 6\}$. Now, $\{v_i; 1 \leq i \leq 6\} \backslash v_k$ forms a path on five vertices, say P. Let the vertex set of P be $\{v_a, v_b, v_c, v_d, v_e\}$, where $a, b, c, d, e \in \{1, 2, \ldots, 6\}$ and $a \neq b \neq c \neq d \neq e \neq k$. We assign colours $c_1, c_2, \ldots, c_5$ to $v_a, v_b, \ldots, v_e$ respectively. Since v_7 and u_7 are common neighbours of v_i; $1 \leq i \leq 6$, we assign colours c_6 and c_7 to them. Now, each v_i; $1 \leq i \leq 6$ has exactly four uncoloured neighbours. Since we are constructing a b-colouring with seven colours, each of these four uncoloured neighbours should receive distinct colours. We consider the vertex v_b. v_b is with colour c_2 and is adjacent to vertices having colours c_1, c_3, c_6 and c_7. To become a b-dominating vertex, v_b should be adjacent to the vertices with colours c_4 and c_5. The vertices u_a and u_c are the only uncoloured neighbours of v_b. Since u_c is adjacent to v_d, we cannot assign colour c_4 to u_c. So we assign colour c_4 to u_a and c_5 to u_c. Now the vertex v_d will have two neighbours with colour c_5. Thus the vertex v_d will not become a b-dominating vertex.

Next we assume that $u \notin B$. If $u \notin B$, then at least five b-dominating vertices will be from the set v_i; $1 \leq i \leq 6$. Thus as in the above case, here also we cannot construct a b-colouring with seven colours. A b-colouring with six colours can be obtained as given in Figure 14.

Figure 14 A b-colouring of $\mu(W_7)$ with six colours



- $n = 8$

We consider the graph $\mu(C_8) \backslash \{u, v_9, u_9\} = G'$.

First we prove that $\varphi(G') = 4$.

We assume that $\varphi(G') = 5$. Here the vertices with degree at least 4 are $\{v_i; 1 \leq i \leq 8\}$. All the other vertices have degree 2. So if the b-chromatic number of G' is 5, then all the five b-dominating vertices will be from the set $\{v_i; 1 \leq i \leq 8\}$. Thus, from a set of eight vertices we are selecting five vertices and that can be done in ${}^8C_5$ ways. In each case there will be two b-dominating vertices v_{i-1} and v_{i+1} such that v_{i-2} and v_{i+2} are also b-dominating or three or four consecutive b-dominating vertices.

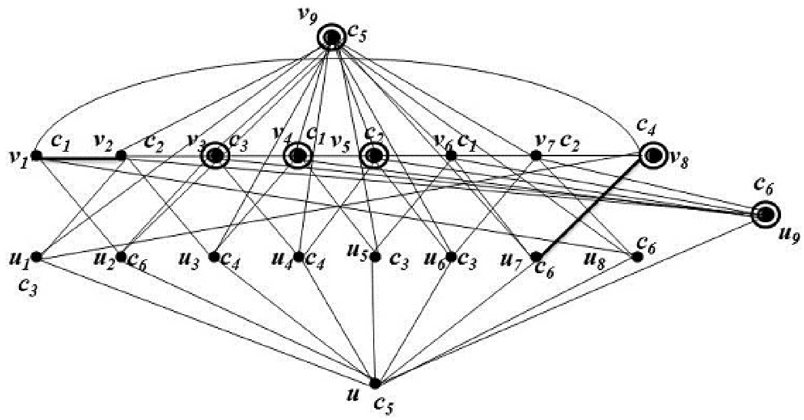
We consider the first case, that is there exist two b-dominating vertices v_{i-1} and v_{i+1} such that v_{i-2} and v_{i+2} are also b-dominating. Here the fifth b-dominating vertex will be v_{i+4}. Since the degree of each v_i; $1 \leq i \leq 8$ is 4 and we are constructing a b-colouring with five colours, the four neighbours of each of these five b-dominating vertices should receive distinct colours. We assign colours c_1, c_2, c_3 and c_4 to v_{i-2}, v_{i-1}, v_{i+1} and v_{i+2} respectively. The only one colour that can be assigned to the vertex v_i is c_5. So we assign colour c_5 to v_i. The vertex v_{i-1} is assumed to a b-dominating vertex with colour c_2 and is adjacent to the vertices v_{i-2} and v_i with colours c_1 and c_5 respectively. To become a b-dominating vertex, vi-1 should be adjacent to two vertices with colours c_3 and c_4. The only two uncoloured neighbours of v_{i-1} are u_i and u_{i-2}. Since the colour of v_{i+1} is c_3, we cannot assign colour c_3 to u_i. So the only option is to assign colour c_4 to u_i and c_3 to u_{i-2}. But if we do so, the vertex v_{i+1} will have two neighbours with same colour c_4. Thus the vertex v_{i+1} will not become b-dominating. Hence, in this case a b-colouring with five colours is not possible.

Next we consider the case that there are three consecutive b-dominating vertices. Let the three consecutive b-dominating vertices be v_i, v_{i+1} and v_{i+3}. Now the remaining two b-dominating vertices will be v_{i+4} and v_{i+6} (if $i + k > 8$, $i + k = i + k \bmod 8$ for $1 \leq k \leq 7$, $1 \leq i \leq 8$). We assign colours c_1, c_2, c_3, c_4 and c_5 to v_i, v_{i+1}, v_{i+2}, v_{i+4} and v_{i+6} respectively. Here also the four neighbours of each of the b-dominating vertices should receive distinct colours. We consider the vertices v_{i+3} and u_{i+3}. These vertices can receive either colour c_1 or colour c_5. So without loss of generality we assign colour c_1 to v_{i+3} and c_5 to u_{i+3}. Similarly, the vertices v_{i+5} and u_6 can receive either colour c_2 or colour c_3. So without loss of generality we assign the colour c_2 to v_{i+5} and c_3 to u_{i+5}. Now, we consider the vertex v_{i+6}. To become a b-dominating vertex, it should be adjacent to two vertices with colours c_4 and c_1. But v_{i+7} and u_{i+7} are the only two uncoloured neighbours of v_{i+6} and these two vertices are connected to the vertex v_i with colour c_1. Thus, here also we cannot have a b-colouring with five colours.

Next, we consider the case that there exists four consecutive b-dominating vertices. Let it be v_i, v_{i+1}, v_{i+2} and v_{i+3}. The remaining one b-dominating vertex can be either v_{i+5} or v_{i+6} (if $i + k > 8$, $i + k = i + k \bmod 8$ for $1 \leq k \leq 7$, $1 \leq i \leq 8$). Without loss of generality we assume that the fifth b-dominating vertex be v_{i+5}. We assign colours c_1, c_2, c_3, c_4 and c_5 to v_i, v_{i+1}, v_{i+2}, v_{i+3} and v_{i+5} respectively. Note that here all the four neighbours of each of the b-dominating vertices should receive distinct colours. Thus, the vertices v_{i+4} and u_{i+4} can receive either colour c_1 or colour c_2. So we assign colour c_1 to v_{i+4} and c_2 to u_{i+4}. To become a b-dominating vertex, v_{i+5} should be adjacent to two vertices with colour c_3 and c_4. So we assign colour c_3 to v_{i+5} and c_4 to u_{i+5}. Now, the only one colour that can be assigned to u_{i+1} is c_5. So we assign colour c_5 to u_{i+1}. Now, we consider the vertex v_i. To become a b-dominating vertex, v_i should be adjacent to two vertices with colours c_3 and c_4. But here v_{i+7} and u_{i+7} are the only uncoloured neighbours of v_1 and the vertex v_{i+7} is adjacent to two vertices with colours c_3 and c_4. Hence, we cannot assign c_3 or c_4 to v_{i+7}. Thus here a b-colouring with five colours is not possible. Thus $\varphi(G') \neq 5$. A b-colouring with four colours can be obtained by assigning colour c_1 to v_i, v_{i+2}, v_{i+5} and u_i, colour c_2 to v_{i+3}, u_{i+5}, u_{i+6} and u_{i+7}, colour c_3 to v_{i+1}, v_{i+4} and v_{i+6} and c_4 to v_{i+7}, u_{i+1}, u_{i+2}, u_{i+3} and u_{i+4}.

Next, we consider $\mu(W_9) = G' \cup \{u, v_9, u_9\}$ such that v_9 is adjacent to all the v_i's and u_i's; $1 \leq i \leq 8$, u is adjacent to all the u_i's; $1 \leq i \leq 9$ and u_9 is adjacent to all the v_i's; $1 \leq i \leq 8$. Assume that we have assigned three new colours c_5, c_6 and c_7 to u, u_9 and v_9 respectively. Here all the neighbours of u are adjacent to v_9 but they are not adjacent to u_9. Since we have already assigned colour c_6 to u_9, we cannot assign colour c_7 to any of the neighbours of u and thus u will not become a b-dominating vertex. So we cannot as-sign three new colours to u, v_9 and u_9. Thus a b-colouring with seven colours is not possible. A b-colouring with six colours can be obtained as given in Figure 15.

Figure 15 A b-colouring of $\mu(W_9)$ with six colours



- $n = 7$ and $n = 9$

Consider the case when $n = 7$. A b-colouring with seven colours can be obtained by assigning colour c_1 to v_2, u_5 and u_6, colour c_2 to v_3 and v_6, colour c_3 to v_1 and v_4, colour c_4 to v_5, u_1 and u_2, colour c_5 to v_7, u_3 and u_4, colour c_6 to v_8 and u and colour c_7 to u_7 and u_8.

Next, we consider the case when $n = 9$. A b-colouring with seven colours can be obtained by assigning colour c_1 to v_2, u_5 and u_6, colour c_2 to v_3, v_6 and v_9, colour c_3 to v_1, v_4 and v_8, colour c_4 to v_5, u_1, u_2 and u_8, colour c_5 to v_7, u_3, u_4 and u_7, colour c_6 to v_{10} and u and colour c_7 to u_{10} and u_9. For all the remaining value of n, we can assign this colouring and the remaining uncoloured vertices can be coloured as follows

a If n is an even number

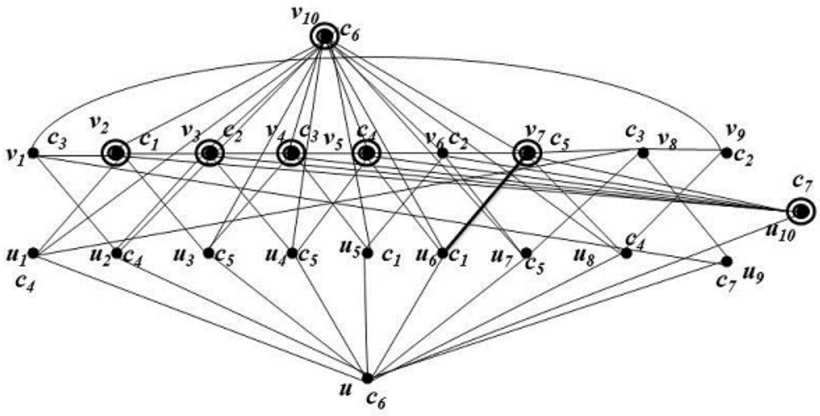
We assign colour c_1 to v_{10}, v_{12}, …, v_n and colour c_2 to v_{11}, v_{13}, …, v_{n-1}.

b If n is an odd number

We assign colour c_1 to v_{10}, v_{12}, …, v_{n-1} and colour c_2 to v_{11}, v_{13}, …, v_n.

And assign colour c_6 to v_{n+1} and u and colour c_7 to u_{n+1}.

Figure 16 A b-colouring of $\mu(W_{10})$ with seven colours



3 Conclusions

The b-colouring of a graph has application in clustering techniques (Elghazel et al., 2007). Mycielskian is a graph operation, which has application in multiprocessor task scheduling problem (Eric et al., 2011). In this paper, we obtained the b-chromatic number of Mycielskian of paths, complete graphs, complete bipartite graphs and wheels. The b-chromatic number of Mycielskian of a graph is used in clustering techniques.

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