Determining statistical parameters of ADC using probability-based histogram method with noise as stimuli

Bhawana Garg* and Deepak Kumar Mishra

Department of Electronics and Instrumentation,
SGSITS,
Indore, M.P., India
Email: garg.bhawana@gmail.com
Email: mishrad_k@hotmail.com
*Corresponding author

Abstract: This paper aims for the estimation of an analogue to digital converter (ADC) parameters using probability-based histogram method with white Gaussian noise as stimuli. ADC characterisation based on this methodology is described and error in computing the code transition level with suggested corrective action is reported. Uncertainty in the computation of effective number of bits (ENOB) is investigated and simulation results for an 8-bit ADC is presented and compared with other existing methods.

Keywords: differential nonlinearity; DNL; effective number of bits; ENOB; gain; histogram technique; integral nonlinearity; INL; white Gaussian noise.

Reference to this paper should be made as follows: Garg, B. and Mishra, D.K. (2018) 'Determining statistical parameters of ADC using probability-based histogram method with noise as stimuli', *Int. J. Instrumentation Technology*, Vol. 2, No. 1, pp.50–61.

Biographical notes: Bhawana Garg received her BE degree in Electronic Instrumentation & Control Engineering from the University of Rajasthan, Jaipur, India, in 1999, and MTech in Instrumentation Engineering from the Institute of Instrumentation, DAVV, Indore, India, in 2002. In 2002, she joined the Department of Electronics and Instrumentation Engineering, DAVV, as a Lecturer, and in 2006 Pad. Dr. D.Y. Patil Institute of Engineering and Technology, Pimpri, Pune as a Lecturer where she became an Assistant Professor in 2010. In 2015, she joined the Department of Instrumentation Engineering, Ramrao Adik Institute of Technology, Navi Mumbai as an Assistant Professor. She is currently pursuing PhD at Shri GSITS, Indore. Her current research interests include testing of mixed signal devices, embedded systems. She is a life member of the Indian Society of Instrumentation. Her research interests are testing of mixed signal devices and embedded systems.

Deepak Kumar Mishra is a Professor in the Electronics and Instrumentation Engineering Department at SGSITS, Indore, MP, India. He completed his BE in Electronics in 1979, ME in Applied Electronics and Servo Mechanism in 1984 and PhD Electronics and Telecommunication Engineering with title 'Some methods for dynamic testing of A/D converters', May 2000. His areas of interest are electronics and instrumentation, VLSI technology and low power analogue and mixed signal VLSI design and dynamic testing of A/D converters. He guided more than 50 PG and four PhD students. He has published more than 50 research papers in international and national journals and more than 25 papers in international and national conferences.

1 Introduction

With the new technological advancement in automation, digital instruments have replaced traditional analogue instruments. Physical signals measured from various sensors are analogous in nature. They must be converted into digital signals prior to the processing. Analogue to digital converter (ADC) play significant role in wide range of applications in the field of instrumentation, telecommunications, consumer and applied electronics. Characterisation and testing of this device is a crucial factor in determining the cost and accuracy of the system. ADC testing methods can be categorised as static and dynamic. Static method uses the input signal with low frequencies, nearly zero and determine parameters like gain error and offset error. These errors are rectifiable. Dynamic methods use variable input stimuli. ADC characteristics are estimated by calculating the code transition levels and derive figures of merit such as differential nonlinearity (DNL), integral nonlinearity (INL) and effective number of bits (ENOB) (Martins and Serra, 2000). Linear regression method is applied to calculate errors and uncertainty in the measurement of ADC parameters. Dynamic method uses the sine wave signal as a stimulus for determining the parameters of the ADC. Accuracy of the figures of merit are confined using this signal because its probability density function (pdf) depends on its geometry. Alternative stimuli for the dynamic method are the triangular and Gaussian noise.

Advantages of using Gaussian noise over other stimuli are as follows:

1 Since noise is mathematically modelled as multi-frequency sine wave signal, the device can be tested for the whole bandwidth in a single run.

2 It is easier to generate the noise signal as compared to the sine wave signal.

3 While using Gaussian distribution, any disturbance generated will only induce a gain and offset error that can be corrected.

4 In case the generated noise has normal distribution, any noise associated with the test assuming that it also possesses the Gaussian distribution, adds its variance to that of the generated noise.

These advantages have been identified in Martins and Serra (1999), Björsell and Händel (2004) and Moschitta et al. (2003). ADC characterisation methods based on the code density test have been described in Martins and Serra (2000), Belcher (2015) and frequency-domain-based test are reported in Flores et al. (2004) and Magstadt et al. (2016).

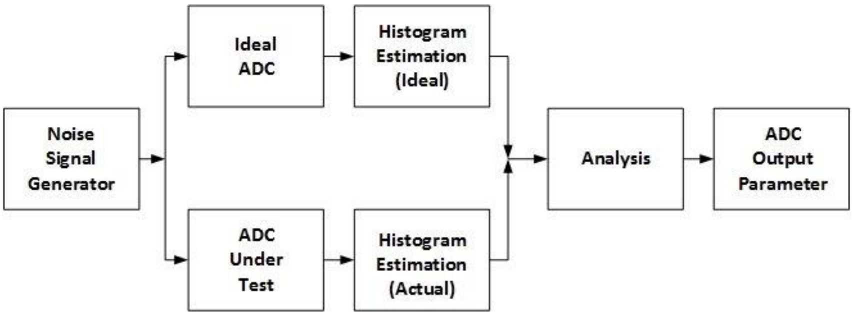
Histogram method using the Chebyshev technique to determine INL and DNL with noise for ADC testing was outlined by Flores et al. (2004). The code density test produces an array of samples obtained in each code bin width (Mishra and Gamad, 2007). This method is used to compute code transition levels that characterise and estimate the figures of merit of an ADC. A technique to compute ENOB using sine wave, based on histogram testing is presented in Mishra and Gamad (2007). Estimation of DNL and INL with Gaussian noise as an input has already been reported using the histogram method. This paper focuses on probability-based method to compute the DNL, INL, code transition level and ENOB for the Gaussian noise input.

Possessing uniform power spectral density, easy to generate and does not necessitate the use of complex sampling techniques, these features of Gaussian noise have attracted many researchers to use it as an input signal for the characterisation and testing of ADC. Although the test methods are simulated, these methods are equally suitable for testing the ADC in real life test conditions.

2 Histogram method

Actual ADC comprises of gain, offset and nonlinearities along with the quantisation error of an ideal quantiser. Histogram is measure of the number of times each individual code has occurred (Martins and Serra, 2000). It is an extraction procedure of the transfer function of an ADC by comparing pdf of the stimulus and its response. Zero frequency of occurrence is the indication of missing code. A change in slope of the ADC transfer curve causes a gain error (Blair, 1994).

Figure 1 Typical histogram technique



The typical histogram technique is presented in Figure 1. Estimation of the histogram is elicited by the application of a noise signal to both ideal and actual ADC.

A comparative analysis of both histograms is utilised to evaluate the parameters of the ADC. The histogram test returns the probabilities of occurrence of each code for a given stimulus signal in the form of vector P_i, with 2^n elements each being an estimator for the i^{th} code (Martins and Serra, 1999). Analytically it may be shown as:

$$P_i = V_i \le V_s \le V_{i+1} \tag{1}$$

$$P_i = \int_{V_i}^{V_{i+1}} f_d\left(V_s\right) dV_s \tag{2}$$

where f_d is the pdf of the input signal, V_s is the input stimulus signal, V_i and V_{i+1} are the lower and upper transition voltages for i^{th} and $(i + 1)^{th}$ code respectively. For normally distributed white Gaussian noise, pdf can be defined as:

$$f_d\left(V_s\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(V_s-\mu)^2}{2\sigma^2}} dV_s \tag{3}$$

where μ is the mean and σ is the standard deviation of the input signal.

2.1 Code transition level based on the computation of probability

Probability of occurrence is the ratio of samples presented in the ith code bin to the total number of samples.

$$P(i) = \frac{H(i)}{N} \tag{4}$$

where $H(i)$ is the number of times samples occur in code bin i, N is the total number of samples and $P(i)$ is a sample mean. Since the total number of samples is a constant, the mean probability in the whole population is:

$$\mu_p = E(P) = \frac{P(i)}{2^n} \tag{5}$$

Similarly, the variance of the probability is

$$\sigma_p^2 = \sigma_{\frac{H}{N}}^2 = \frac{P(1-P)}{N}. \tag{6}$$

Figure 2 Probability density for the ideal and the actual ADC (see online version for colours)

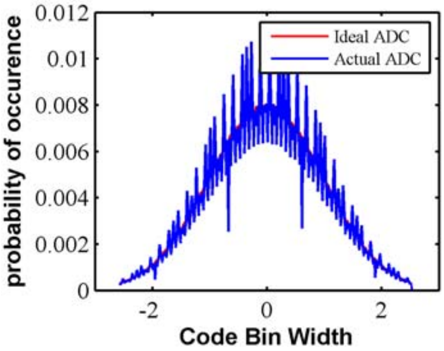


Figure 2 shows the graph between probability density and code bin width for the ideal and the actual ADC. The inserted nonlinearities are apparent in the figure. The graph is governed by binomial distribution due to its discrete nature. Since large number of samples are captured, it can be assumed as Gaussian distribution. The ideal probability of occurrence depends on quantisation, mean and standard deviation of the input stimulus. Estimated values of mean $\hat{\mu}$ and standard deviation $\hat{\sigma}$ can be computed from the obtained histogram.

The code transition level can be calculated by integrating the pdf (Martins and Serra, 2000).

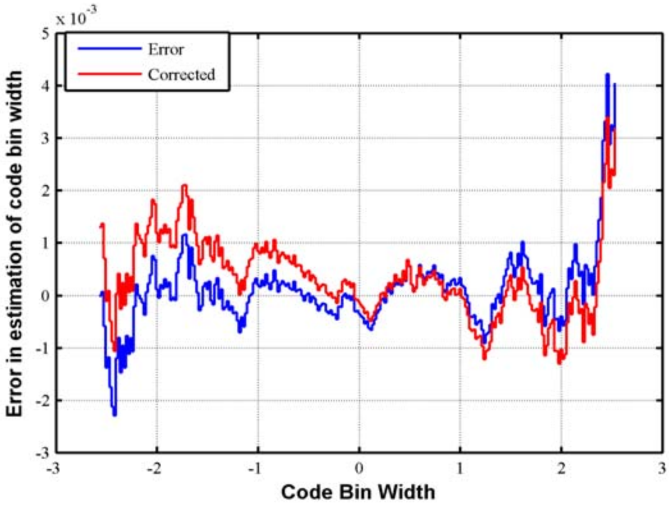
$$P(i) = \frac{1}{2}\left[erf\left(\frac{V_{i+1} - \hat{\mu}}{\hat{\sigma}\sqrt{2}}\right) - erf\left(\frac{V_i - \hat{\mu}}{\hat{\sigma}\sqrt{2}}\right)\right] \tag{7}$$

$$\hat{T}_{i+1} = \hat{\sigma}\sqrt{2}erf^{-1}\left[2P(i) + erf\frac{\left(\hat{T}_i - \hat{\mu}\right)}{\hat{\sigma}\sqrt{2}}\right] + \hat{\mu} \tag{8}$$

2.2 Correction in the estimation of the code transition level

The code transition level for an ideal ADC is estimated using equation (8). Computational error in the code transition level is determined and shown in Figure 3. It is minimised by applying square error minimisation technique.

Figure 3 Error and correction in the estimation of code transition level (see online version for colours)



Correction in the computation of code transition level has been done as per equation (9). The circumflex symbol (^) is used to represent estimated values.

$$\hat{T}_i^{corr} = Gain \times \hat{T}_i + offset \tag{9}$$

3 ADC figures of merit estimation

It has been assumed that the noise internal to the ADC and external noise caused by disturbances has Gaussian distribution. Hence, equivalent wideband noise will be a combination of both input white noise and disturbances (or noise other than input stimuli). Variance by disturbance will be added to the variance of the signal, which will only affect gain and offset.

3.1 Gain and offset error

Gain and offset error can be estimated using the linear regression method. Gain as per the terminal-based method is the ratio of the difference between the first and last code transition level from actual to ideal ADC. Since the first code transition level of the actual ADC is a random value, the parameters estimated with this will not offer an accurate estimation hence, the terminal-based method is unsuitable for noise stimuli. Independently-based ADC gain and offset error estimation is the slope and point of intersection with the axis of transition voltage of the best fit line respectively. The estimated gain error is the slope of the fitted straight line and the estimated offset error is the point of intersection of that straight line with the axis of the ideal transition voltages (vertical axis) (Alegria and Serra, 2009). A simple linear regression model is assumed to depict the linearity between the applied input voltage and corresponding output digital code. The response *I*(*i*) is related to the code transition level $T_{\min}(i)$

$$\hat{I}(i) = a_0 + a_1 T_i(i) \tag{10}$$

a_0 and a_1 are regression coefficients, $T_i(i)$ and $\hat{I}(i)$ are the code transition level and the most appropriate estimation of i^{th} code respectively. The residual error of the regression line is given by:

$$\varepsilon(i) = I(i) - \hat{I}(i) \tag{11}$$

Residual errors are used to estimate the amount of variation in the dependent variable and the variance of the error term. a_0 and a_1 are regression coefficients. The values of regression coefficients can be calculated as (Walpole and Myers, 2013):

$$a_1 = \frac{\sum_0^{n-1}\left(T(i)-\bar{T}\right)\left(I(i)-\bar{I}\right)}{\sum_0^{n-1}\left(T(i)-\bar{T}\right)^2} \tag{12}$$

$$a_0 = \frac{\sum_0^{n-1} I(i) - a_1 \sum_0^{n-1} T(i)}{n} \tag{13}$$

Estimated variance of the error term obtained:

$$\hat{\sigma}^2 = \frac{\sum_0^{n-1} \varepsilon^2(i)}{n-2} \tag{14}$$

Simplify the equation (14) by substituting the values from equations (12) and (13), it may be written as (Mishra, 2000):

$$\hat{\sigma}^2 = \sqrt{\frac{\sum_0^{n-1} I^2 - a_1 \sum_0^{n-1} I \times T(i) - a_0 \sum_0^{n-1} I}{n-2}} \tag{15}$$

3.2 *Differential and INL*

$$DNL(i) = \frac{\text{actual } P(i)}{\text{ideal } P(i)} - 1 \tag{16}$$

$$\text{actual } P(i) = \frac{H(i)}{N} \tag{17}$$

DNL is defined as the difference between a specific code bin width and the average code bin width, divided by the average code bin width. In terms of the histogram, DNL is the ratio of deviation between actual and ideal ADC to the ideal ADC. Mathematically, it can be expressed as (Mishra, 2000):

$$DNL(i) = \frac{Hist(i) - Hist_{ideal}(i)}{Hist_{ideal}(i)} \tag{18}$$

$Hist(i)$ and $Hist_{ideal}(i)$ is the histogram of actual ADC and ideal ADC for ith code.

INL is the maximum deviation of a transfer characteristic of an ADC from its ideal value. It is defined as the difference between the ideal and the measured code transition levels following correction of static gain and offset. For an Ideal ADC, nominal values of gain and offset are 1 and 0 respectively, deviation from these values indicates the degree to which the device varies from nominal performance. Analytically, it is defined as:

$$INL(i) = \frac{(i-1) * width + T(1) - offset - G * T(i)}{width} \tag{19}$$

T is the code transition level and *width* being the average code bin width.

3.3 ENOB

A noise signal is mathematically modelled as combination of infinite sine waves. ENOB determination method applicable to a single sine wave can also be extended to the noise signal. ENOB may be defined as the number of bits of a hypothetical ADC consisting quantisation error equal to the total RMS error from all sources in the actual ADC. Mathematically, it can be expressed as (Mishra and Gamad, 2007):

$$ENOB = N - \log_2 \left[\frac{rms\_error(actual)}{rms\_error(ideal)} \right] \tag{20}$$

RMS value of actual and ideal noise signal can be computed using the following equation:

$$rms\_noise(i) = \sqrt{\left[\frac{\int_{x(i)}^{x(i+1)} e^2 dx}{x(i-1) - x(i)} \right]} \tag{21}$$

where e is the error signal (conversion error) between the digital output code and the input of transfer characteristics and is expressed as a straight line passing through the points $x(i)$, $e(i)$ and $(x(i+1), e(i+1))$, $x(i)$ is the code transition level for ith code. Conversion error may be expressed as a straight line with slope m and intercept c.

$$e = mx + c \tag{22}$$

For ideal ADC, error (e) is a sawtooth waveform with zero mean and uniform distribution between ±lsb / 2, but for non-ideal ADC, the conversion error may be of different geometrical shapes spread over the entire dynamic range of the ADC, namely triangles or trapezoids (Wagdy and Awad, 1999). In an ideal case scenario, this error can be determined as the difference between the centre values of the best fit transition voltage.

For actual ADC, it can be calculated as the difference between the best fit value of the ideal and the actual code transition level.

$$\int_{x(i)}^{x(i+1)} e^2 dx = \frac{z^2}{3}\frac{\left((x(i+1))^3-(x(i))^3\right)}{w^2}+\frac{p^2}{w}+\frac{z*p}{w^2}\left((x(i+1))^2-(x(i))^2\right) \tag{23}$$

where

$$z = e(i) - e(i+1)$$

$$p = x(i) * e(i+1) - e(i) * x(i+1)$$

$$w = x(i) - x(i+1).$$

Residual errors are assumed to be distributed normally as per the given value of input voltage for multiple experiments. This assumption implies that I_0, I_1, …, I_n are also normally distributed with the probability distribution $N(I(i); a_0 + a_1 V_{\min}(i), \sigma_{\Box})$. If $\hat{a}_0$ and $\hat{a}_1$ are unbiased estimators of a_0 and a_1 respectively and are linear functions of independent normal variables then $\hat{a}_0$ and $\hat{a}_1$ are also distributed normally. Confidence interval 100(1 – α)% for the parameter a_1 in the regression line is (Walpole and Myers, 2013):

$$\hat{a}_1 - t_{\alpha/2}\frac{\hat{\sigma}}{\sqrt{\left(V_{\min}(i)-\bar{V}_{\min}\right)}} < a_1 < \hat{a}_1 + t_{\alpha/2}\frac{\hat{\sigma}}{\sqrt{\left(V_{\min}(i)-\bar{V}_{\min}\right)}} \tag{24}$$

where $t_{\alpha/2}$ is a value of t-distribution. Confidence interval 100(1 – α)% for the parameter a_0 in the regression line is

$$\hat{a}_0 - t_{\alpha/2}\frac{\hat{\sigma}\sqrt{\sum_0^{n-1} V_{\min}^2(i)}}{\sqrt{n\left(V_{\min}(i)-\bar{V}_{\min}\right)}} < a_1 < \hat{a}_0 + t_{\alpha/2}\frac{\hat{\sigma}\sqrt{\sum_0^{n-1} V_{\min}^2(i)}}{\sqrt{n\left(V_{\min}(i)-\bar{V}_{\min}\right)}} \tag{25}$$

Internal uncertainties in the estimation of ENOB can be expressed as:

$$\hat{I}(i) = a_0 \pm U_{a_0} + \left(a_1 \pm U_{a_1}\right)V_{\min}(i) \tag{26}$$

U_{a_0} and U_{a_1} are uncertainties in the measurement of a_0 and a_1 respectively as calculated in equations (16) and (17).

4 Simulation results and discussions

A practical sampling ADC, regardless of its architecture, contains many noise and distortion sources. The quantiser introduces quantisation noise, DNL and INL. In this research, only noise external to the ADC was considered and normally distributed white Gaussian noise was simulated and stimulated to the ideal and actual ADC. Transfer characteristics of 8-bit bipolar ADC with 5.12-volt reference voltage and 3 × 106 samples of input signal with zero mean and variance of 1.00 was simulated. Probability density of the output of the actual ADC is shown in Figure 3. As the resolution of ADC increases, the pdf of the ideal and the actual ADC coincides, this specifies the offset to be approximately zero.

Known arbitrary values of nonlinearities are inserted into the ideal ADC making it the actual ADC incorporating gain, offset and nonlinearity errors. The code transition level is equally spaced in the case of the ideal ADC, but apart from that the least significant aspect is the indication of a presence of nonlinearity. Probability of occurrence of each sample for the ideal and actual ADC can be estimated with this figure. DNL and error in code transition level have been estimated.

Figure 4 DNL estimation (see online version for colours)

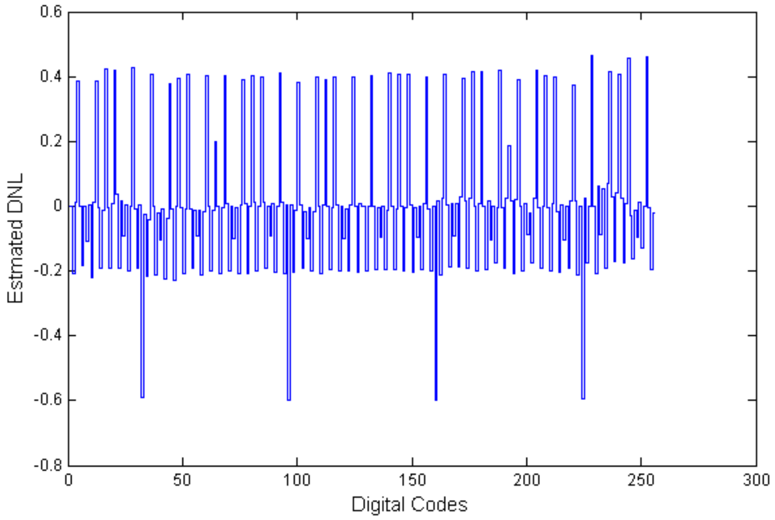
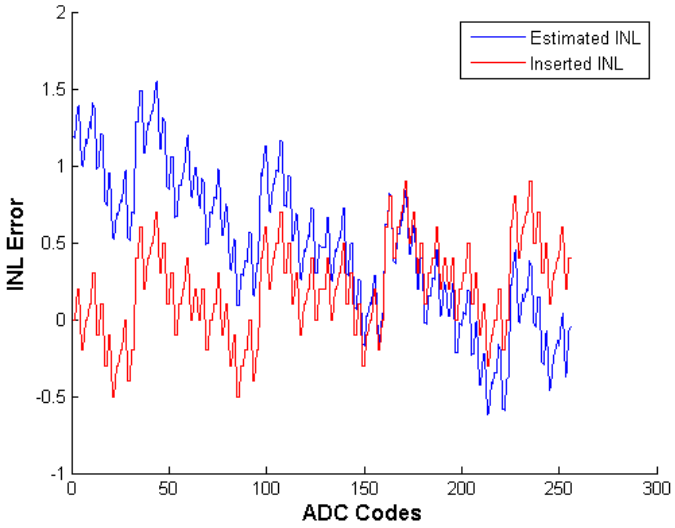


Figure 5 Comparison of inserted INL with estimated values from the probability method (see online version for colours)



Corrections have been made to eliminate the error in code transition level. A plot of DNL presented in each code is shown in Figure 4. Estimated INL follows the same pattern as the values inserted as shown in Figure 5. Maximum error in estimation occurs at the corners, where the number of samples is very few and the error function tends to infinity.

Errors in the measurement of the code transition level are observed to be maximum at edges. This result is reflected in Figure 5 as maximum INL error at the terminals due to less number of samples are obtained in the histogram at lower and higher code bins with noise signal. Figure 6 shows the conversion error for the 8-bit ADC.

Figure 6 Conversion error for 8-bit ADC (see online version for colours)

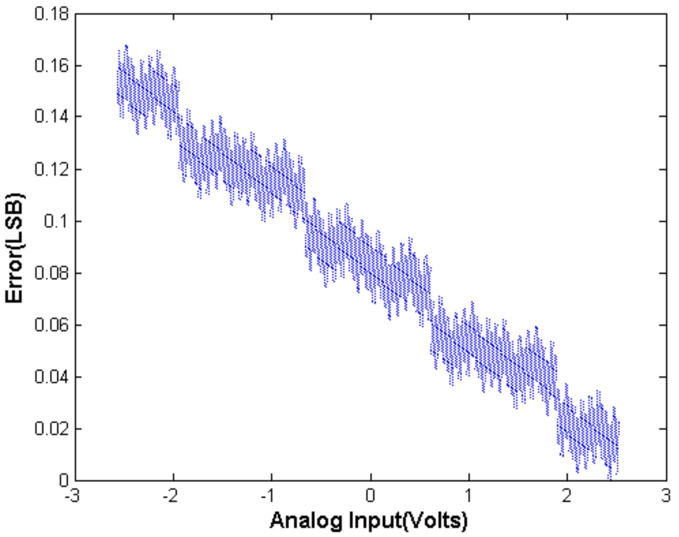


Table 1 display measured values of gain and offset for ideal and actual ADC with white Gaussian noise. Table 2 shows a comparison of estimated ENOB with Gaussian noise to sine signal and triangular signal (Mishra and Gamad, 2007; Belcher, 2015).

Table 1 Gain and offset estimation

ADC	*Method*	*Gain*	*Offset*
Ideal ADC	Probability method	1.0000	–.0144
Actual ADC	Probability method	.9379	–.0297

Table 2 Comparison of ENOB

Number of bits	*ENOB sine signal (N = 3 × 104)*	*ENOB Gaussian noise (N = 3 × 104)*
5	4.685989	4.7423
6	5.691857	5.8774
7	6.840638	6.9267
8	7.764216	7.9750
9	8.763451	8.8920

Table 3 Comparison of ENOB

ADC	*Number of samples*	*ENOB sine signal*	*ENOB triangular signal*	*ENOB Gaussian noise*	*ENOB DCF*
8-bit	30,000	7.76	7.66	7.9750	6.6

Under the same test conditions, white Gaussian noise is applied to determine the ENOB and RMS error for both the ideal and the actual ADC. Table 3 shows the comparative values of the ENOB with arbitrary input values and white Gaussian noise (Belcher, 2015; Mishra, 2000). The conversion error curve is shown in Figure 6.

5 Conclusions

Many methods have been proposed for testing ADC and a variety of input signals have been used to characterise it. Precise and well characterised excitation signals needed for this are difficult to generate. Noise signal as compared to other signals is easier to generate and is therefore a convenient signal for the purpose of system measurements and testing. This research has attempted to present a combination of the histogram and error minimisation technique in order to generate better results for the estimation of gain, offset, nonlinearities and ENOB. The novelty of this approach is to apply a corrective action on the actual ADC to remove the effect of gain and offset errors. Thus, the accuracy in the measurement of nonlinearities and ENOB improved. The objective was to evaluate the different stimuli so that the test procedure can be as efficient as possible to find an appropriate characterisation. The research work has derived an expression for uncertainties and confidence intervals in the estimation of ENOB and has contributed significantly for testing the ADC. Obtained results are validated with the simulated ADC. This methodology is equally useful for the designers and manufacturers of the high resolution ADC.

References

Alegria, F.C. and Serra, A.C. (2009) 'Precision of independently based gain and offset error of an ADC using the histogram method', *IEEE Transactions on Instrumentation and Measurement*, Vol. 58, No. 3, pp.512–521.

Belcher, R.A. (2015) 'ADC standard IEC 60748-4-3: precision measurement of alternative ENOB without a sine wave', *IEEE Transactions on Instrumentation and Measurement*, Vol. 64, No. 12, pp.3183–3200.

Blair, J. (1994) 'Histogram measurement of ADC nonlinearities using sine waves', *IEEE Transactions on Instrumentation and Measurement*, Vol. 43, No. 3, pp.373–383.

Björsell, N. and Händel, P. (2004) 'Benefits with truncated Gaussian noise in ADC histogram tests', *Proceedings of the 9th Workshop on ADC Modelling and Testing*, Vol. 2, pp.787–792.

Flores, M.D.G.C., Negreiros, M., Carro, L. and Susin, A.A. (2004) 'INL and DNL estimation based on noise for ADC test', *IEEE Transactions on Instrumentation and Measurement*, Vol. 53, No. 5, pp.1391–1395.

Magstadt, B., Zhuang, Y. and Chen, D. (2016) 'Accurate spectral testing with impure source and noncoherent sampling', *IEEE Transactions on Instrumentation and Measurement*, Vol. 65, No. 11, pp.2454–2463.

Martins, R.C. and Serra, A.C. (1999) 'Automated ADC characterization using the histogram test stimulated by Gaussian noise', *IEEE Transactions on Instrumentation and Measurement*, Vol. 48, No. 2, pp.471–474.

Martins, R.C. and Serra, A.C. (2000) 'ADC characterization by using the histogram test stimulated by Gaussian noise', *Measurement*, Vol. 27, No. 4, pp.291–300.

Martins, R.C. and Serra, A.C. (n.d.) 'The use of a noise stimulus in ADC characterization', *1998 IEEE International Conference on Electronics, Circuits and Systems. Surfing the Waves of Science and Technology (Cat. No.98EX196)*, pp.457–460.

Mishra, D.K. (2000) *Some Methods for Dynamic Testing of A/D Converters*, May, thesis, DAVV University Indore.

Mishra, D.K. and Gamad, R.S. (2007) 'Determination of nonlinearity and effective resolution of an A/D converter for arbitrary application input', *Proceedings of the World Congress on Engineering and Computer Science 2007 WCECS 2007*, 24–26 October, pp.220–223.

Mishra, D.K. and Gamad, R.S. (2009) 'Determination of error in nonlinearity estimation in an A/D converter with triangular wave input', *International Journal of Electronics*, Vol. 96, No. 12, pp.1237–1247.

Mishra, D.K., Chaudhari, N.S. and Shrivastava, M.C. (1997) 'ADC's effective number of bits determination by combination of histogram and least square error minimization techniques', *IETE Journal of Research*, Vol. 43, No. 4, pp.327–332.

Moschitta, A., Carbone, P. and Petri, D. (2003) 'Statistical performance of Gaussian ADC histogram test', *Proceedings of the 8th International Workshop on ADC Modelling and Testing*, September, pp.213–217.

Wagdy, M.F. and Awad, S. (1999) 'Determining ADC Effective number of bits via histogram testing', *IEEE Transactions on Instrumentation and Measurement*, Vol. 40, No. 4, pp.471–474.

Walpole, R.E. and Myers, R. (2013) *Probability and Statistics for Engineers & Scientists*, Pearson, Boston.