Trigonometry-based motion blur parameter estimation algorithm

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Abstract: Restoration of blurred images requires information about the blurring function, which is generally unknown in practical applications. Identification of blur parameters is essential for yielding blurring function. This paper proposes a technique for estimation of motion blur parameters by formulating trigonometric relationship between the spectral lines of the motion blurred image and the blur parameters. In majority of the existing motion blur parameter estimation approaches, length of motion blur is estimated by rotating the Fourier spectrum to estimated motion angle. This requires angle estimation to be done forehand. The proposed method estimates both, length and angle simultaneously by exploring the trigonometric relation between spectral lines, thereby eliminating the need of spectrum rotation for length estimation. The proposed technique is applied on Berkeley segmentation dataset, Pascal VOC 2007 and USC-SIPI image database. The simulation results prove that the proposed method exhibit better parameter estimation performance as compared to existing state-of-the-art techniques.

Keywords: image degradation; motion blur; parameter estimation; point spread function.

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1 Introduction

Image blurring is one of the serious image degradations which results in loss of information. Blurring may be caused due to movement between sensor or camera and the object, defocusing of the lens of the camera, atmospheric turbulence, etc. To restore an image, information about the blurring function must be known. In practical cases, this blurring function or the point spread function is unknown. Many existing approaches estimate the image and the blurring function simultaneously, while some estimate the point spread function first and then restore the image based on the estimated blurring function (Kundur and Hatzinakos, 1996). So, it is important to estimate the parameters of the blurring function, in order to restore an image.

Degradation due to motion blur depends on the amount of blur length and the angle at which the pixels are spread. There are various methods to estimate the blur parameters in literature. Majority of the techniques estimate the blur parameters in spectral or cepstral domain by inspecting the zero crossings. Gennery (1973) proposed estimation of the motion blur parameters by identification of the zeros in the spectral domain. While Cannon (1976) estimated the parameters based on the inspection of the power spectrum of the image. Chang et al. (1991) extended the work by inspecting the zeros in the power bispectrum. Rom (1975) and Fabian and Malah (1991) analysed the zeros in the cepstral domain for parameter estimation. Tanaka et al. (2007) detected the patterns of zeros in the spectrum domain by using correlation between the blurred image and a detecting function.

Techniques using Hough and Radon transform are used to estimate the angle of the motion blur. Once the angle is estimated, the spectrum is rotated according to the detected angle for estimation of length of motion blur. Wu et al. (2007) and Lokhande et al. (2006) identify the blur angle using Hough transform and then estimate length in the cepstral domain. The Radon transform-based angle estimation is presented in Dobeš et al. (2010), Moghaddam and Jamzad (2006), Krahmer et al. (2006) and Deshpande and Patnaik (2014). Dobeš et al. (2010) proposes computation of power spectrum of image gradient in the spectral domain, while Moghaddam and Jamzad (2006) identifies blur direction and length using Radon transform and fuzzy set concepts. Krahmer et al. (2006) estimates the blur parameters in the cepstral domain using Radon transform and Deshpande and Patnaik (2014) estimates the blur angle by applying Radon transform to the fourth-bit plane of the log spectrum of the blurred image.

This paper proposes a method to accurately estimate the motion blur angle and length using trigonometric form in the spectral domain. Majority of the motion blur parameter estimation techniques in literature find the blur angle first using either Hough or Radon transform and then estimate the blur length by rotating the blur spectrum by the estimated angle. The proposed method estimates the blur angle and length independently. It does not require rotating the spectrum for estimating the length. This makes the motion blur parameter estimation task less complicated.

The paper is organised as follows: Section 2 gives background theory of image degradation and discusses the mathematical model of motion blur. The proposed technique for estimation of motion blur parameters is described in Section 3. Section 4 provides the experimental results and comparisons of the proposed technique with other methods and Section 5 gives the conclusion.

2 Background theory

The image degradation model and the process of image motion blurring is mathematically described below.

2.1 Image degradation

An image is degraded when it gets convolved by a linear spatially invariant blur. It can be modelled as:

$$g(x, y) = f(x, y) * h(x, y) + n(x, y), \tag{1}$$

where $g(x, y)$ is the degraded image, $f(x, y)$ is the original image, $h(x, y)$ is the degradation function, also known as point spread function (PSF), and $n(x, y)$ is the additive noise. In frequency domain, equation (1) can be expressed as:

$$G(u, v) = F(u, v) \,.\, H(u, v) + N(u, v), \tag{2}$$

where $G(u, v)$, $F(u, v)$, $H(u, v)$ and $N(u, v)$ are the frequency response of degraded image, original image, degradation function and noise, respectively. In absence of noise, equation (2) can be written as:

$$G(u, v) = F(u, v) \,.\, H(u, v). \tag{3}$$

2.2 Motion blur model

Motion blur is caused when there is relative motion between the camera and the object. The point spread function of motion blur of length L and angular displacement θ can be modelled as:

$$h(x,y)=\begin{cases}\dfrac{1}{L} & \text{if } \sqrt{x^2+y^2}\le\dfrac{L}{2} \text{ and } \dfrac{x}{y}=-\tan\theta \\ 0 & \text{elsewhere}\end{cases}. \tag{4}$$

The Fourier spectrum of motion blur along the motion direction is a sinc function having periodic zeros at $\omega = \pm 1/L, \pm 2/L, \ldots$

$$H(u,v)=\frac{\sin(\pi L\omega)}{\pi L\omega}=\mathrm{sinc}(\pi L\omega). \tag{5}$$

The zeros of motion blur appear along lines perpendicular to the blur direction. If distance between two consecutive zeros is d, then, the length of the blur can be found as

$$L = N/d, \tag{6}$$

where N is the size of the image. Figure 1 shows the examples of image blurred with motion PSF and the corresponding log of their Fourier spectrum.

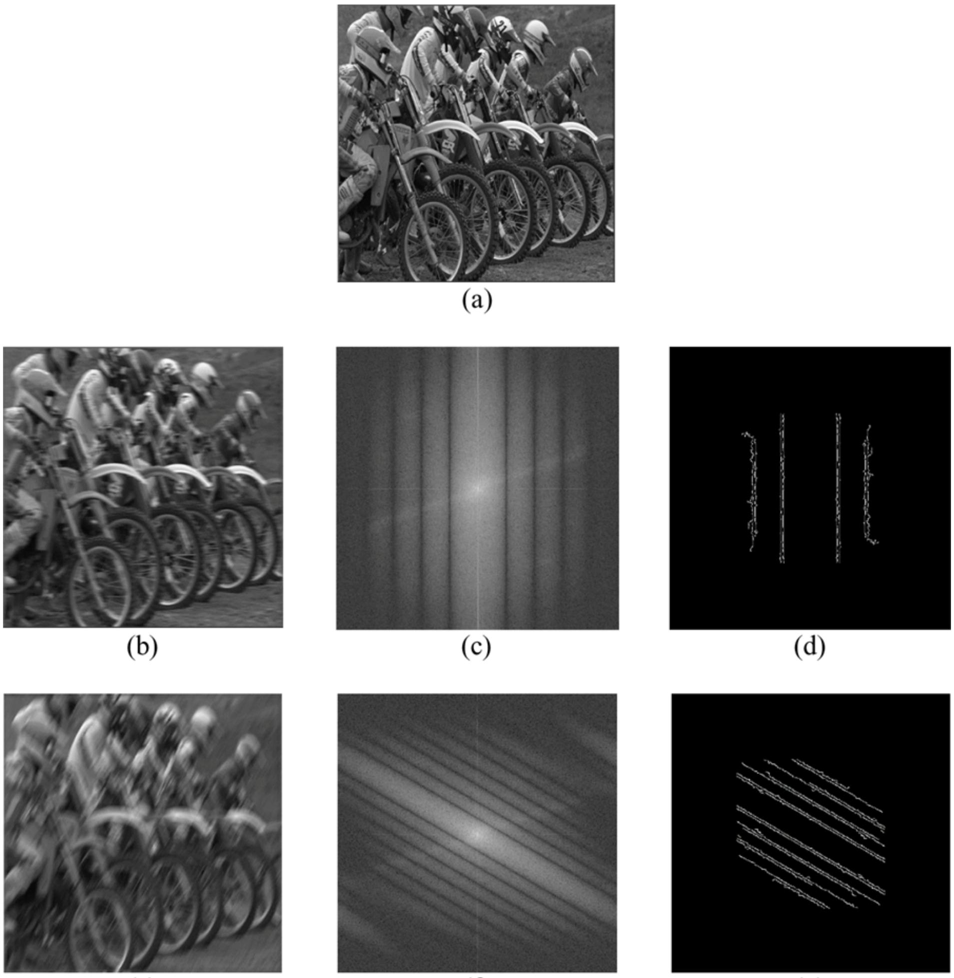
3 Estimation of motion blur parameters

Motion blur parameters estimation methods in literature working in spectral or cepstral domain, estimate the blur angle using Radon transform (Dobeš et al., 2010; Moghaddam and Jamzad, 2006; Krahmer et al., 2006; Deshpande and Patnaik, 2014) or Hough transform (Wu et al., 2007; Lokhande et al., 2006). Once the angle is estimated, the spectrum is rotated by the estimated angle in the opposite direction such that the spectral lines now become perpendicular to the x-axis. Length of the blur is now determined from this rotated spectrum.

Lokhande et al. (2006) estimates the length as the first negative in the real part of inverse Fourier transform of the 1D spectrum of the motion blurred image. Moghaddam and Jamzad (2006) finds the distance between the valleys adjacent to the central peak in the spectrum and using equation (6) estimates the length of the motion blur. While, Deshpande and Patnaik (2014) determines the length by finding the distance between the central peak and the larger peak on either of its side in the modified spectrum of the blurred image.

A method to estimate the motion blur parameters in spectrum domain is proposed in this paper. The proposed method does not require Radon or Hough transform for estimation of the angle, thereby eliminating the need for spectrum rotation to determine the length. In this method, both, motion blur angle and length are determined by the inference of the trigonometric relation between the spectral lines. A mathematical formulation of this trigonometric relation of the spectral lines of the blurred image and the blurred parameters is derived in this paper. The mathematical modelling and the proposed algorithm depending on the different angles and length is discussed as under:

Figure 1 (a) Original image (Franzen, 1999); (b) motion blurred image with length = 10 and angle = 0; (c) log of Fourier of image in (b); (d) Canny edge detection on (c); (e) motion blurred image with length = 20 and angle = 60; (f) log of Fourier of image in (e) and (g) Canny edge detection on (f)



(a)

(b) (c) (d)

(e) (f) (g)

3.1 Mathematical modelling of motion blur parameters

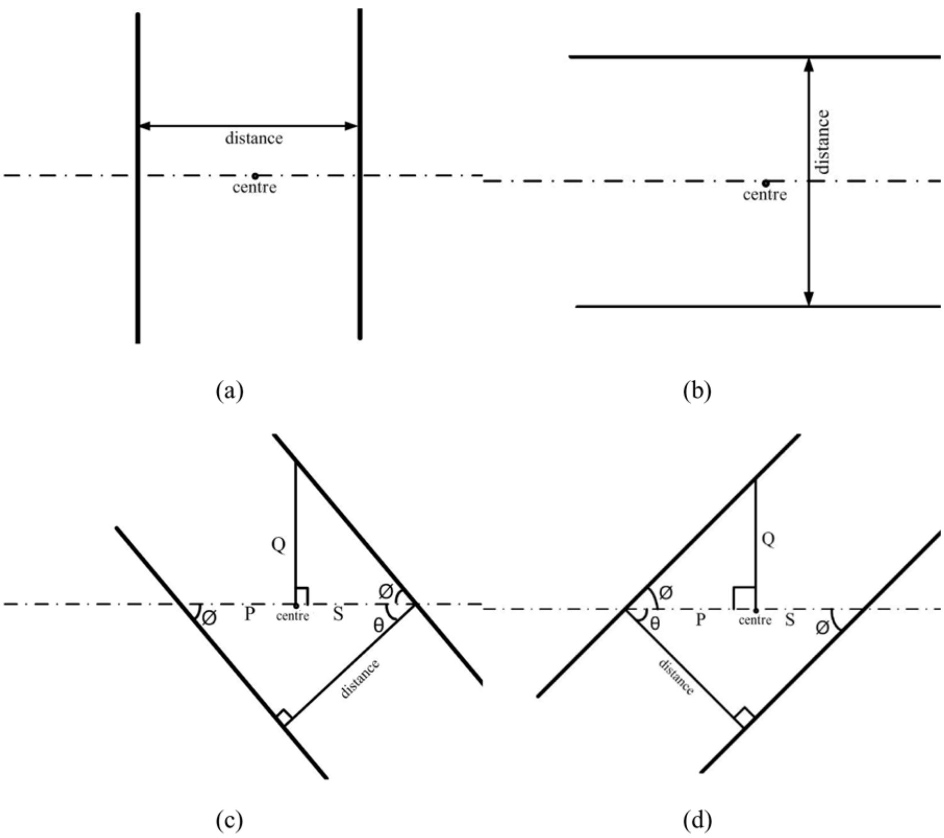
The log of FFT of motion blurred image displays parallel lines inclined at an angle proportional to the blur angle and the length is inversely proportional to the distance between the two parallel lines from the centre, as discussed in Section 2.

In this paper, the blur angle and length are determined by analysing the spectrum from trigonometric point of view. The proposed method formulates the trigonometric relationship between the spectral lines of the blurred image and motion blur parameters. For this, edge detection is applied to the log of FFT of the blurred image. Figure 2 shows the edge detection of log spectrum of motion blurred image for blur angles 0°, 90°, angle greater than 90° and less than 90°. The log spectrum is masked from all the sides. Here, the blur angle and length are mathematically modelled by considering the spectrum of motion blurred image as an image and analysing it by following four cases of varying angles.

3.1.1 Case 1: angle = 0°

When the blur angle is 0°, the spectral lines are parallel to each other making an angle of 90° with the central horizontal axis as shown in Figure 2(a). Hence, the spectrum will be empty in the central column. To estimate the angle, check for the spectral lines from the centre of the spectrum towards the top and bottom. Repeat the same with a difference of few pixels from the centre. If the spectrum is empty, the angle is estimated as 0°. The length now can be calculated by using equation (6), where *d* will be the distance between the two lines from the centre.

Figure 2 Edge detected log spectrum of motion blurred bikes image at length 20 and (a) angle = 0º; (b) angle = 90º; (c) angle <90º and (d) angle >90º



3.1.2 Case 2: angle = 90°

When the blur angle is 90°, the spectral lines are parallel to the central horizontal axis and the spectrum will be empty in the central row, Figure 2(b). So, to estimate the angle, check for the spectral lines from the in the central row and few rows above and below it.

If the spectrum is empty, the angle is estimated as 90°. The length is estimated by finding the distance between the first spectral line on either side from the central row. The distance is then fed to equation (6) and length is estimated.

3.1.3 Case 3: angle <90°

If the blur angle is less than 90°, the spectral lines will make an obtuse angle with the central horizontal axis in the positive direction. The precise blur angle can be estimated as shown in Figure 2(c).

$$\varnothing = \tan^{-1}\left(\frac{Q}{S}\right)$$

$$\text{distance}(d) = h.\sin(\varnothing), \tag{7}$$

where $h = S + P$

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right)$$

$$\text{angle} = 90 - \theta, \tag{8}$$

The length can be determined by placing the distance value in equation (6).

3.1.4 Case 4: angle >90°

If the blur angle is greater than 90°, the spectral lines are inclined at an acute angle in the direction positive with the central horizontal axis. The exact blur angle can be found from Figure 2(d) as:

$$\varnothing = \tan^{-1}\left(\frac{Q}{P}\right)$$

$$\text{angle} = 90 - \varnothing,$$

$$\text{distance}(d) = h.\sin(\varnothing), \tag{9}$$

where $h = P + S$

$$\text{angle} = 180 - \theta. \tag{10}$$

The length can be determined by placing the distance value in equation (6).

3.2 Proposed technique

The proposed method of motion blur parameter estimation detects the angle and length based on the trigonometric relations of the spectral lines. Algorithm 1 illustrates this technique. FFT is first computed on the input blurred image and the spectrum is shifted to the centre. Then log of its absolute values is taken, Figure 1(c) and (f). Now, this log of FFT spectrum is considered as an image and Canny edge detector is applied on it. Edge detection is applied in order to enhance the spectral nulls. The orientation and spacing between these spectral lines depends on the amount of motion blur, i.e., angle and length of motion. As the spectrum is shifted to the centre, the main focus is at the centre of the FFT spectrum. Hence we are masking the edges of the spectrum, such that the spectral lines concentrated near the centre are distinctly visible as seen in Figure 1(d) and (g). The spectral lines thus obtained would be resembling the cases described in Figure 2.

Algorithm 1 Motion blur parameter estimation

1. Input motion blurred image g(x, y)
2. Compute FFT G(u, v) of g(x, y)
3. Convert G(u, v) to log spectrum
4. Apply canny edge detection on the log spectrum of G(u, v)
5. Determine the case under which the log spectrum of the blurred image will fall
6. Compute the length and the accurate angle based on the corresponding case, by the above mentioned method
7. Output angle and length

The next step is to determine the motion blur parameters from this spectrum. For angle determination, if the spectral lines are parallel to the central horizontal axis, the angle is 90° and angle is 0°, if they are perpendicular to the central horizontal axis. For detecting whether the angle is <90° or >90°, the direction of inclination of the spectral lines is to be determined. For this, two subsequent columns from the right-hand side of the spectrum, near the centre and left hand side of the spectrum are taken. If the blur angle would be less than 90°, the spectral line in the leftmost column will appear first as compared to the column after that. This would be true for the central two columns as well as the rightmost two columns. It would be the reverse for angle greater than 90°. Thus, the angle of motion blur can be sorted as either being 0° or 90° or angle greater than 90° or less than 90° and length can be determined by the corresponding case from the above section.

If the angle is detected as 0°, length of blur can be estimated by the simply determining horizontal distance between the spectral lines from the centre as indicated in case 1. Similarly, of the angle is detected as 90°, length can be determined by finding vertical distance between spectral lines from the centre as mentioned in case 2. If the angle is detected as less than 90° or greater than 90°, the exact angle and length of motion blur can be calculated by using the formulation described in case 3 or case 4, respectively. Thus, the actual angle and length of motion blur can be determined by using the proposed formulation based on the trigonometric relationship between the spectral lines. The important criteria are the identification of the case under which the blur angle falls. If the case of angle of motion blur is incorrectly detected, then it would result into an erroneous output. Hence it is equally important to correctly identify the category of the angle of motion blur.

4 Experimental results

The proposed technique was applied to images from Berkeley Segmentation dataset (Arbelaez et al., 2007), Pascal VOC dataset (Everingham et al., 2008) and USC-SIPI database (SIPI, 2005). 200 images from Berkeley Segmentation dataset and Pascal VOC dataset were taken and 100 images from USC-SIPI were considered and blurred with varying motion blur parameters. All the images were resized to 512×512 and some were also converted to grey scale. Also standard test images like Lena, cameraman, peppers and images from Kodak colour image suite (Franzen, 1999) were considered for testing the proposed technique. The images were blurred with motion angle in the range of 0–170 in steps of 10 and length from 10 to 55 in steps of 5, giving in total 180 combinations of length and angle.

Parameter estimation using the proposed technique was done for all these combinations of length and angle. Table 1 shows the parameters estimated by the proposed technique for few of the combinations of motion blur length and angle on the bikes image of Figure 1(a). The estimation results display 100% accuracy for angles 0° and 90°, while the accuracy up to 97% for all other angles.

The images were restored using the estimated motion blur parameters by Lucy Richardson algorithm (Gonzalez, 1992) with 50 iterations. Figure 3 shows the restoration outputs when the deconvolution was done by using the motion blur parameters estimated by the proposed technique on grey scale images. The evaluation of restoration by the estimate length and angle was done by calculating the PSNR values, which is indicated in Table 1.

Table 1 Parameter estimation results for bikes image

Actual		*Estimated*		*PSNR*
Length	*Angle*	*Length*	*Angle*	(dB)
10	30	10.03	30.11	36.75
15	110	15.81	109.50	33.73
20	130	21.39	129.03	30.68
25	170	24.83	169.01	32.59
30	40	30.39	40.03	31.02
35	90	36.57	90.00	28.36
40	160	39.85	156.77	17.51
45	0	46.54	0.00	27.37
50	70	50.35	68.90	28.95
55	20	57.23	19.17	24.49

The proposed estimation technique works well in determining the motion blur length and angle for both colour and grey images. Figure 4 gives the output images after restoring the coloured building image blurred with length 40 and angle 20 with the estimated length 40.93 and angle 20.55. The image is restored with Lucy Richardson algorithm with 50 iterations, giving a PSNR of 36.21 dB. The quality of the restored image can further be improved by post-processing and ringing removal techniques.

The performance of the proposed technique is also evaluated in terms of maximum error in PSF parameters estimation. Table 2 shows comparison of the proposed method with existing motion blur parameter estimation methods available in literature. Method of Moghaddam and Jamzad (2006), outperforms all the other techniques in terms of error in estimation of both angle and length. The maximum error by the proposed method in angle estimation is 3.23 and in length estimation is 2.23, almost equivalent to the estimation of Deshpande and Patnaik (2014) and better as compared to Lokhande et al. (2006).

Figure 3 Restoration outputs by motion blur parameters estimated from the proposed technique: (a) image blurred with motion blur of length 10 and angle 30; (b) image (a) restored with estimated length 10.03 and angle 30.11; (c) image blurred with motion blur of length 20 and angle 130 and (d) image (c) restored with estimated length 21.39 and angle 129.03

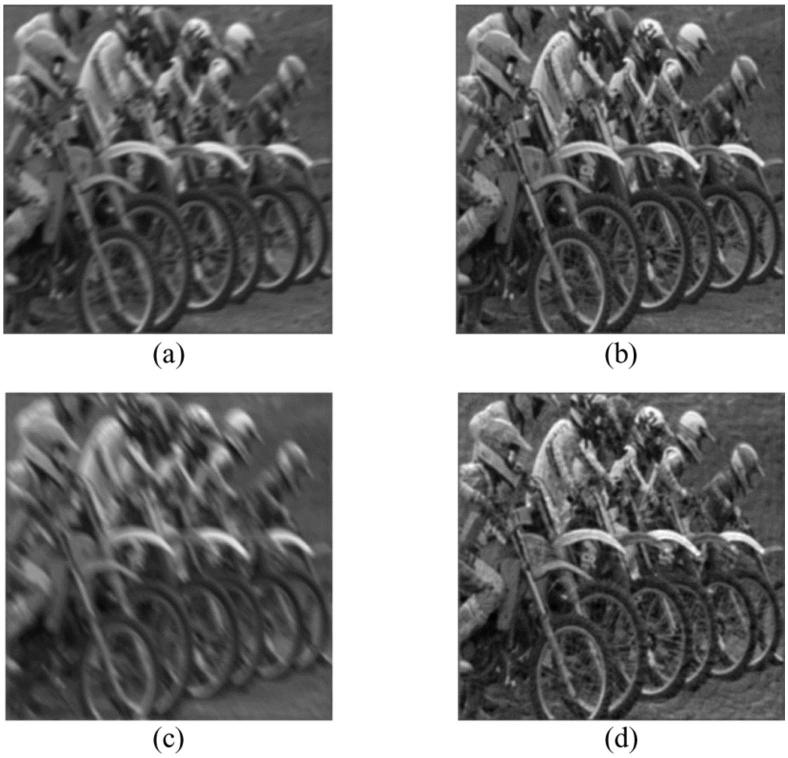


Figure 4 Restoration outputs for colour image by motion blur parameters estimated from the proposed technique: (a) original image; (b) image blurred with motion blur of length 40 and angle 20 and (c) image (b) restored with estimated length 40.93 and angle 20.55 (PSNR = 26.2134 dB) (see online version for colours)

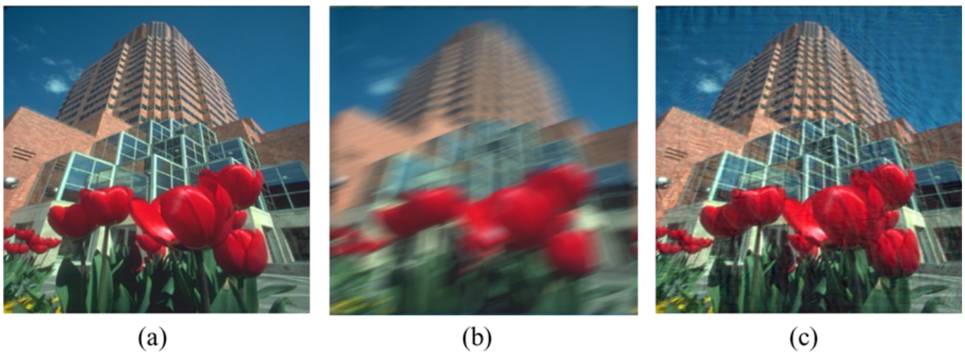


Table 2 Comparison of parameter estimation with other existing methods

	Test range		*Max error in estimation*	
Method	*Angle*	*Length*	*Angle*	*Length*
Deshpande and Patnaik (2014)	10–170	5–70	3	2
Moghaddam and Jamzad (2006)	0–180	10–50	2	1.9
Lokhande et al. (2006)	0–60	15–60	5	7
Proposed	0–170	10–55	3.23	2.23

5 Conclusion

A method to estimate the length and angle parameters in an image degraded with motion blur is proposed in this paper. The trigonometric relation between the spectral lines of the motion blurred image, in terms of the blur angle and the length is established and formulated. The proposed technique estimates the blur parameters without using any angle estimation technique such as Hough or Radon transform and thereby removing the need to determine the angle first for length estimation. The angle and length estimation results are compared with other available techniques and it can be seen that the proposed technique performs at par with them.

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