
A review on process models and controller design in proton-exchange membrane fuel cells

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Abstract: A fuel cell is a device which changes the chemical energy into electrical energy by using the hydrogen and oxygen. There are numerous types of fuel cells including proton-exchange membrane fuel cell (PEMFC) which uses hydrogen as a fuel as treated at anode where electrons are disconnected from protons on the surface of a platinum-based catalyst. Process models for PEMFCs have received significant attention in past decades due to potentials for PEMFC to produce zero emissions creates a great prospect for clean energy in the transport industry. This paper provides an overarching view on the process models developed and control systems utilised for optimising the energy generation in PEMFC technology. Several recommended control systems have been also reviewed.

Keywords: controller design; fuel cells; process modelling; mechatronics; hydrogen; energy; sustainability; robust controller design; mathematical models; optimal control; sliding mode control.

Reference to this paper should be made as follows: Özel, T. (2022) ‘A review on process models and controller design in proton-exchange membrane fuel cells’, *Int. J. Mechatronics and Manufacturing Systems*, Vol. 15, No. 1, pp.1–19.

Biographical notes: Tuğrul Özel is Full Professor and the Director of Manufacturing and Automation Research Laboratory (MARLAB) with the Industrial and Systems Engineering Department at Rutgers University. He authored over 200 peer-reviewed journal and conference papers, co-authored four books. He is the Founding Editor and the Editor-in-Chief of *International Journal of Mechatronics and Manufacturing Systems*, an Associate Editor for *Journal of Manufacturing Science and Engineering* and an editorial board member of several international journals. He is engaged with funding agencies including National Science Foundation, European Commission, US Departments of Commerce, Energy, Advanced Manufacturing, and Industry, advises several national and international funding agencies.

1 Introduction

A fuel cell is a ‘triode’: anode, membrane, and cathode. Depending on the type of the membrane, there are several types of fuel cells. In this proposal we will focus on the

proton-exchange membrane fuel cells (PEMFC). These are the most developed among all fuel cells and they can be used for both mobile (vehicles and portable devices) and stationary applications (residential and industrial electric power generation). They are relatively small, light, have low corrosion and long life, efficient (40–50%), work at low temperature of operation (50–80°C), have faster dynamic response than most of fuel cells, provide electric power up to 1 MW, and very importantly have a very high power density (~2000 Wh/kg comparing to acid batteries of ~400 Wh/kg). Another class of fuel cells, solid oxide fuel cells (SOFC) has similar mathematical models to PEMFC and hopefully with minor modifications the control strategies produced in the course of this project will be applicable to SOFC. Due to their increased electric power ~3 MW and high operating temperature 600–1000°C SOFC are convenient for stationary application providing both electricity and heat. In the following we first review mathematical models of PEMFC relevant to current state of research in this paper, then, summarise results obtain by control system researchers, and propose several new research topics on control of fuel cells.

A typical structure of a fuel cell power plant (FCPP) is shown in Figure 1. To satisfy the requirements of applying fuel cell to power generation industry, several subsystems are necessary (see Figure 2) for issues such as

- 1 operating stably and safely
- 2 minimising the hydrogen consumption
- 3 ensuring the quality of the generated electric power.

The *air/fuel processing unit* plays the role of compressing and humidifying the air/fuel before the gas being pumped into the fuel cell stack. Moreover, the unit is in control of the *hydrogen/oxygen ratio* that relative with the efficiency of the fuel cell (Hasikos et al., 2009) and are responsible for preventing the damage of the fuel cell from oxygen starvation problem (Zhang, 2008). Since hydrogen can easily leak from most containers in the gas form and requires costly refrigerating units to maintain its liquid form, using methane to reform hydrogen simultaneously while the fuel cell is operating is an acceptable alternative until the invention of a suitable storage method for hydrogen takes place. Thus a reformer is also needed to be considered when modelling the air/fuel processing units. The fuel cell stack is the unit to transform the chemical energy into the electric power. Water management and temperature control determine the efficiency and even the life-time of the stack. Finally, the fuel cell power plant output should follow the requirement of the IEEE standard for commercial applications, that is, the output voltage and frequency deviation must be controlled under the acceptable limits. The major challenge of reaching certain constraints is that numerous uncertainties may exist in the power plant. On one hand, the load demand varies randomly and frequently. On the other hand, each part in the power plant is modelled as *linear time invariant* (LTI) unit for simplicity but in reality, the parameters change with the deviation of temperature or different status of operation. In other words, because of the coupling structure and various *uncertainties*, the fuel cell systems have very sophisticated dynamics and face difficult control problems to be solved to become *safe and cost-effective power generators*.

Figure 1 A typical proton-exchange membrane fuel cell (PEMFC) assembly (see online version for colours)

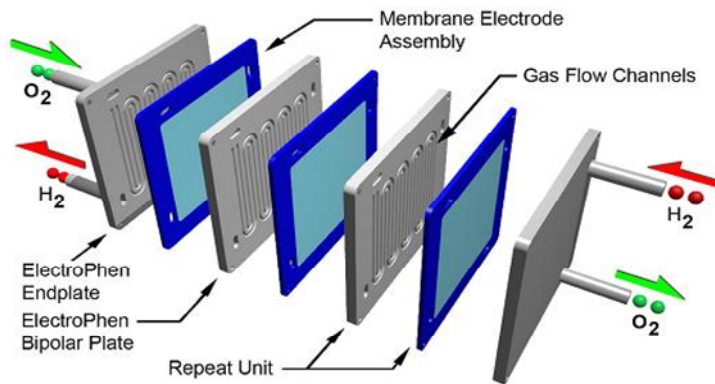
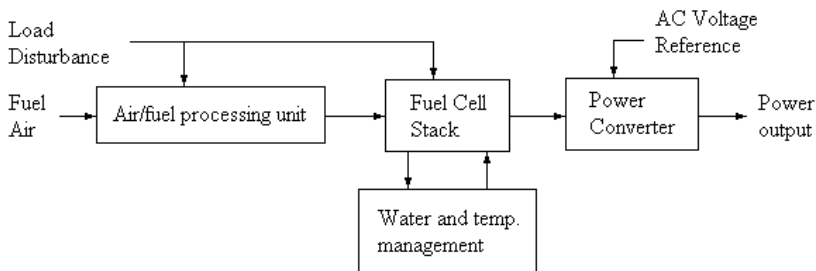


Figure 2 Diagram of a typical fuel cell power plant



2 Fuel cell power plant

The typical structure of a fuel cell power system includes fuel/air processing units, fuel cell stacks and power conditioning units (El-Sharkh et al., 2004a, 2004b). The fuel/air processing subsystem carries the function of reforming hydrogen from hydrocarbon fuel such as methane and supply properly processed (such as humidified and compressed) hydrogen and oxygen to the fuel cell stack (Min et al., 2009). The DC power has to be converted into AC power with standard voltage and frequency for commercial application using the power conditioning units. A sample illustration and the general sketch of the fuel cell power plant have been shown in Figure 2.

2.1 Air/fuel processing unit

To ensure the operational quality and efficiency of operating, the fuel cell system should be able to act promptly and accurately to the demand variation. Meanwhile, the consumption of hydrogen should be minimised and hydrogen/oxygen ratio should be adjusted to reach the optimal performance of the fuel cell (Zhang, 2008). To extend the life time of the fuel cell stack, the fuel/air processing unit also needs to deal with the oxygen starvation problem, which would cause the fuel cell stack to act as a hydrogen

pump, lead to the ‘burn-through’ effect to degrade or even destroy the membrane and also increase the hazard of explosion in the cathode (Zhang et al., 2008). As discussed above, supplying hydrogen to the fuel cell by reforming methane is a feasible alternative to avoid the problems of storing hydrogen.

Yang et al. (2008) proposed a model reference adaptive control method to regulate the voltage and current of the low-power fuel cell system, in which the parameters are estimated online. They increased the fuel efficiency by applying an on-off switch to the hydrogen valve. Hasikos et al. (2009) described the proton exchange membrane (PEM) fuel cell performance with a non-linear programming problem. They raised the efficiency of the fuel cell system by minimising the stack current, which is proportional to the consumption of hydrogen. He et al. (2008) presented the modelling and control analysis of a fuel delivery system, which increased the fuel efficiency by applying a recirculation loop to reuse the unconsumed hydrogen. Zhang et al. (2008) proposed an adaptive control method to maintain the oxygen excess ratio around an optimal value in transient level in order to avoid oxygen starvation. Dalvi and Guay (2009) applied two ultra-capacitors to improve the transient response of the fuel cell system, mitigate the affect by the load deviation and reduce the possibility of oxygen starvation. Chen et al. (2009) described a centralised multiple model predictive control that can handle power distribution and oxygen control simultaneously in order to reach global optimisation. In Hauer’s dissertation (Hauer, 2001), a second-order simple LTI model is developed for the methane to hydrogen reformer.

2.2 Fuel cell stack

Padulles et al. (2000) presented a solid oxide fuel cell (SOFC) dynamic model with the combination of the perfect gas and the Nernst’s equation. The model has been modified to represent the PEM fuel cell stacks in several works related to fuel cell power plants (Golkar and Hajizadeh, 2000; El-Sharkh et al., 2004c; Sakhare et al., 2004; Golbert and Lewin, 2004).

The operation condition is also related to the efficiency of the fuel cell system. To be specific, the excess water (flooding) may block the transportation of the hydrogen and oxygen, while a low level of vapour (drying) may degrade the performance of the membranes. Kurz et al. (2008) applied electrochemical impedance spectroscopy to detect the flooding and drying modes in fuel cell systems and designed control algorithms to prevent the flooding and drying based on the proposed sensing technique. Thermal control is also an important issue in operating fuel cell stacks since each type of fuel cell has a specific range to obtain the optimal performance. Otherwise the system might deteriorate by either the degraded catalyst or the thermal damage. Yang et al. (2009) developed a temperature predictive controller on an improved Takagi-Sugeno fuzzy model.

2.3 Power conditioning unit

The DC output of the fuel cell stack has to be converted into commercially acceptable AC electric power. Normally, the power conditioning unit includes DC/DC and DC/AC converters to conform to the standard for both stand-alone and grid-connected power plants. Since the dynamic of the power conditioning unit is typically much faster than the reformer and the stack, using simple model for the subsystem makes good sense for the

simplification of the whole system without affecting the accuracy of the response (El-Sharkh et al., 2004a). Xie et al. (2009) developed a large signal dynamic model for the full bridge DC/DC converter from the operation analysis. Hatziaioniu et al. (2002) derived a mathematical model for a DC/AC inverter from the equivalent circuit and phasor diagram of a grid-connected fuel cell plant. This model has also been modified by El-Sharkh et al. (2004a) and applied to the analysis of the active and reactive power control of a stand-alone FCPP with both conventional PI and advanced neural network control methods.

3 Mathematical models of processes in PEM fuel cells

3.1 Linear models

In El-Sharkh et al. (2004a), a linear mathematical model was derived for the PEMFC dynamics of three fundamental variables: pressures of hydrogen, oxygen, and water. The model was obtained by keeping the same state equation and slightly modifying the output equation of the mathematical model derived for the SOFC dynamics in Padulles et al. (2000). The system state space model is given by:

$$\begin{aligned}\dot{x}_1(t) &= -\frac{RTK_{H_2}}{V_A}x_1(t) + \frac{RT}{V_A}q_{H_2}^{\text{in}}(t) - \frac{2RTK_r}{V_A}I(t) \\ &= -\frac{1}{\tau_{H_2}}x_1(t) + \frac{1}{\tau_{H_2}K_{H_2}}q_{H_2}^{\text{in}}(t) - \frac{2K_r}{\tau_{H_2}K_{H_2}}I(t)\end{aligned}\quad (1)$$

$$\begin{aligned}\dot{x}_2(t) &= -\frac{RT}{V_C}K_{O_2}x_2(t) + \frac{RT}{V_C}q_{O_2}^{\text{in}}(t) - \frac{RTK_r}{V_C}I(t) \\ &= -\frac{1}{\tau_{O_2}}x_2(t) + \frac{1}{\tau_{O_2}K_{O_2}}q_{O_2}^{\text{in}}(t) - \frac{K_r}{\tau_{O_2}K_{O_2}}I(t)\end{aligned}\quad (2)$$

$$\dot{x}_3(t) = -\frac{RT}{V_C}K_{H_2O}x_3(t) - \frac{2RTK_r}{V_C}I(t) = -\frac{1}{\tau_{H_2O}}x_3(t) - \frac{2K_r}{\tau_{H_2O}K_{H_2O}}I(t)\quad (3)$$

with the state space variables representing

$$x(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^T = [p_{H_2}(t) \quad p_{O_2}(t) \quad p_{H_2O}(t)]^T\quad (4)$$

The output equation represents the measured fuel cell voltage and it is obtained using the Nernst formula and Ohms law as

$$y(t) = V(t) = N \left(E_0 + \frac{RT}{2F} \ln \left\{ \frac{x_1(t)(x_2(t))^{0.5}}{x_3(t)} \right\} \right) - B \ln(CI(t)) - R^{int}\quad (5)$$

The system inputs are molar flow rates of hydrogen and oxygen, that is, $q_{H_2}(t)$ and $q_{O_2}(t)$ that can be regulated (controlled). The stack current $I(t)$ plays a role of a disturbance. The values of the constant coefficients defined in the model equations can be found in El-Sharkh et al. (2004a). This model was used in (El-Sharkh, 2004a, 2004b) to

study residential electric power generation for ‘stand-alone’ fuel cells, without actual connections to the power grid.

3.2 Bilinear model

Na et al. (2007) developed a simple three state variable model of PEMFC using as the state space variables the most fundamental quantities of fuel cells, namely the partial pressures of hydrogen H_2 , oxygen O_2 , and water H_2O on the cathode side of the fuel cell, that is $x(t) = [p_{H_2}(t) \ p_{O_2}(t) \ p_{H_2O}(t)]^T$. The input variables are inlet flow rates of hydrogen (at the anode side), oxygen (at the cathode side), and the cell current density, that is $u(t) = [H_{2in}(t) \ O_{2in}(t) \ i(t)]^T$. As the output variable the fuel cell voltage $V(t)$ obtained equation is chosen, $y(t) = V(t)$. The corresponding bilinear (nonlinearities are only of the type of products of state and control variables) mathematical model of Na et al. (2007) is given by

$$\dot{x}_1(t) = \frac{RT}{V_A} \left(1 - \frac{x_1(t)}{P_{op}} \right) u_1(t) + \frac{RT}{V_C P_{op}} (-2K_r A_C + 2K_r A_C x_1(t)) u_3(t) \quad (6)$$

$$\dot{x}_2(t) = \frac{RT}{V_C} \left(1 - \frac{x_2(t)}{P_{op}} \right) u_2(t) + \frac{RT}{V_C P_{op}} (-K_r A_C + 2K_r A_C x_2(t)) u_3(t) \quad (7)$$

$$\dot{x}_3(t) = -\frac{RT}{V_A P_{op}} x_3(t) u_2(t) + \frac{RT}{V_C P_{op}} (2K_r A_C - 2K_r A_C x_3(t)) u_3(t) \quad (8)$$

$$y(t) = V(t) = N \left(E_0 + \frac{RT}{2F} \ln \left\{ \frac{x_1(t)(x_2(t)/P_{std})^{0.5}}{x_3(t)} \right\} - L \right) \quad (9)$$

Note that $y(t)$ represents the Nernst formula for the fuel cell thermodynamic potential. In addition to all state space model variables, the remaining quantities are constant. Their numerical values can be found in Na et al. (2007). This model and its linearised variants were also studied in Gemmen (2003), Chui (2004) and Page et al. (2007).

3.3 Nonlinear models

Na and Gou (2008) extended the previous three dimensional model include the nitrogen pressure and water pressure at the anode side and derived the following fifth-order nonlinear mathematical model for PEMFC

$$\dot{x}_1(t) = RT \lambda_{H_2} \left(\frac{k_a Y_{H_2}}{V_A} - \frac{k_a}{V_A} \frac{x_1(t)}{x_1(t) + x_2(t)} \right) u_a(t) + RT \left(-\frac{C_1}{V_A} + \frac{C_1 x_1(t)}{V_A (x_1(t) + x_2(t))} \right) I_{fc}(t) \quad (10)$$

$$\begin{aligned} \dot{x}_2(t) = & RT\lambda_{H_2} \left(\frac{k_a \phi_a P_{vs}}{V_A (x_1(t) + x_2(t) - \phi_a P_{vs})} - \frac{k_a}{V_A} \frac{x_1(t)}{x_1(t) + x_2(t)} \right) u_a(t) \\ & + RT \left(-\frac{C_1}{V_A} + \frac{C_1 x_2(t)}{V_A (x_1(t) + x_2(t))} \right) I_{fct}(t) \end{aligned} \quad (11)$$

$$\begin{aligned} \dot{x}_3(t) = & RT\lambda_{air} \left(\frac{k_c Y_{O_2}}{V_C} - \frac{k_c}{V_C} \frac{x_3(t)}{x_3(t) + x_4(t) + x_5(t)} \right) u_c(t) \\ & + RT \left(-\frac{C_1}{2V_C} + \frac{C_1 x_2(t)}{2V_C (x_3(t) + x_4(t) + x_5(t))} \right) I_{fct}(t) \end{aligned} \quad (12)$$

$$\dot{x}_4(t) = RT\lambda_{air} \left(\frac{k_c Y_{N_2}}{V_C} - \frac{k_c}{V_C} \frac{x_4(t)}{x_3(t) + x_4(t) + x_5(t)} \right) u_c(t) \quad (13)$$

$$\begin{aligned} \dot{x}_5(t) = & RT\lambda_{air} \left(\frac{k_c \phi_c P_{vs}}{V_C (x_3(t) + x_4(t) + x_5(t) - \phi_c P_{vs})} - \frac{k_c}{V_C} \frac{x_5(t)}{x_3(t) + x_4(t) + x_5(t)} \right) u_c(t) \\ & + RT \left(-\frac{C_1}{V_C} + \frac{C_1 x_5(t)}{V_C (x_3(t) + x_4(t) + x_5(t))} - \frac{C_2 x_5(t)}{V_C (x_3(t) + x_4(t) + x_5(t))} + \frac{C_2}{V_C} \right) I_{fct}(t) \end{aligned} \quad (14)$$

The state variables in this model represent respectively the pressures of hydrogen and water at the anode side and the pressures of oxygen, nitrogen, and water at the cathode side, that is $x(t) = [p_{H_2}(t) \ p_{H_2O_A}(t) \ p_{O_2}(t) \ p_{N_2}(t) \ p_{H_2O_C}(t)]^T$, with the output variables defined by $y(t) = [p_{H_2}(t) \ p_{O_2}(t)]^T$. The system control input is given by $u(t) = [u_a(t) \ u_c(t)]^T$ where $u_a(t) = \frac{1}{k_a} (H_{2in}(t) + H_2O_{Ain}(t))$. $H_{2in}(t)$ and $H_2O_{Ain}(t)$ represent inlet flow rates of the anode hydrogen and water with k_a being known constant, and $u_c(t) = \frac{1}{k_c} (O_{2in}(t) + N_{2in}(t) + H_2O_{Cin}(t))$. $O_{2in}(t)$, $N_{2in}(t)$, $H_2O_{Cin}(t)$ represent respectively inlet flow rates of the cathode oxygen, nitrogen, and water with k_c being known constant. $I_{fct}(t)$ represents the cell current density and it is considered as a disturbance.

Higher-order dimensional nonlinear models of PEMFC were obtained in the work of Pukrushpan and Stefanopoluou and their co-workers (Pukrushpan et al., 2004a, 2004b, 2006). Pukrushpan et al. (2004a) derived a ninth-order nonlinear model that in addition to dynamics of anode (two-dimensional: hydrogen and vapour (water present in vapour phase) masses) and cathode (three-dimensional: oxygen, nitrogen, and vapour masses) flow models (derived using the mass balance) equations has one dimensional cathode compressor model: rotor speed), two-dimensional cathode supply manifold model (air mass and air pressure), and one-dimensional (air pressure) return manifold models. In Pukrushpan et al. (2006) the original nonlinear fuel cell model of Pukrushpan et al. (2004a, 2004b) is extended by modelling dynamics of the fuel processor (also known as reformer, see also Pukrushpan et al. (2005) that coverts hydrocarbhone fuels (natural gas,

methanol, gasoline) into hydrogen. The obtained mathematical model for the fuel cell processor is of order 10, which after coupling the fuel cell system gives an augmented system of order 19.

3.4 Distributed parameter models (PDEs)

Golbert and Lewin (2004) developed a complex mathematical model for special dependences of the fuel cell voltage, current, molar mass flows, and temperature using partial differential equations. The obtained model is simplified by neglecting the special dependence and obtaining systems of differential and algebraic equations. For such a simplified model, the model predictive controller is designed. What is interesting in their work is that the fuel cell temperature changes are taken into account in contrast to all other mathematical models of PEMFC that assume constant temperature during the entire phase of the fuel cell operation.

4 Simulation results

The fuel cell stack model proposed in Padulles et al. (2000) has been modified and applied to several works (Golkar and Hajizadeh, 2000; Sakhare et al., 2004; Golbert, 2004). The decoupled active power control scheme introduced in El-Sharkh et al. (2004a, 2004b) has been repeated to verify the validity of the model.

4.1 Model of the reformer

El-Sharkh et al. (2004a, 2004b) referred to a reformer model developed in Hauer's dissertation (Hauer, 2001). This reformer is used to generate hydrogen through reforming methane. The reformer model can be described as

$$\frac{q_{H_2}^{in}}{q_{methane}} = \frac{CV}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + 1}. \quad (15)$$

The required molar flow of hydrogen is determined by the stack current,

$$q_{H_2}^{req} = \frac{N_0 I}{2FU_f}. \quad (16)$$

This required flow is used to control the molar flow rate of the methane. El-Sharkh et al. (2004a, 2004b) used the proportional-integral (PI) control method,

$$q_{methane} = \left(k_{pf} + \frac{k_{if}}{s}\right) (q_{H_2}^{req} - q_{H_2}^{in}). \quad (17)$$

4.2 Model of the fuel cell stack

The research work by Padulles et al. (2000) reported that the molar flow of any gas through the valve and its partial pressure inside the channel is proportional. Taking the anode as an example, the relationship can be described as

$$\frac{q_{H_2}^{out}}{p_{H_2}} = \frac{K_{an}}{\sqrt{M_{H_2}}} = K_{H_2}, \quad (18)$$

$$\frac{q_{H_2O}^{out}}{p_{H_2O}} = \frac{K_{an}}{\sqrt{M_{H_2O}}} = K_{H_2O}. \quad (19)$$

Taking hydrogen as an example of developing the equations for partial pressures, apply the perfect gas equation:

$$p_{H_2} V_{an} = n_{H_2} RT. \quad (20)$$

By definition,

$$q_{H_2} = \frac{dn_{H_2}}{dt}. \quad (21)$$

Take all the contributions to q_{H_2} into consideration,

$$q_{H_2} = q_{H_2}^{in} - q_{H_2}^{out} - q_{H_2}^r. \quad (22)$$

Take the time derivative of equation (20) and substitute with equations (21) and (22) yields

$$\frac{d}{dt} p_{H_2} = \frac{RT}{V_{an}} (q_{H_2}^{in} - q_{H_2}^{out} - q_{H_2}^r). \quad (23)$$

Due to the basic electrochemical relationships,

$$q_{H_2}^r = \frac{N_0 I}{2F} = 2K_r I. \quad (24)$$

Replace $q_{H_2}^{out}$ and $q_{H_2}^r$ in equation (9) with equations (4) and (10) respectively, then take the Laplace transform,

$$\left(s + \frac{K_{H_2} RT}{V_{an}} \right) p_{H_2} = \frac{RT}{V_{an}} (q_{H_2}^{in} - 2K_r I). \quad (25)$$

Define

$$\tau_{H_2} = \frac{V_{an}}{K_{H_2} RT}. \quad (26)$$

Then equation (11) can be reformulated as

$$p_{H_2} = \frac{1}{1 + \tau_{H_2} s} (q_{H_2}^{in} - 2K_r I). \quad (27)$$

Similarly, we can get the partial pressure representations for oxygen and water.

In the papers by Amphlett et al. (1994) and Hamelin et al. (2001), a model that describes the polarisation curve is introduced where the cell voltage can be represented as

$$V_{cell} = N_0 \left[E_0 + \frac{RT}{2F} \left(\ln \frac{P_{H_2} P_{O_2}^{0.5}}{P_{H_2O}} \right) \right] - B \ln(CI) - rI . \quad (28)$$

4.3 The power conditioning unit model

Normally this part is consisted of a DC/DC converter and a DC/AC inverter. Since the response time of the reformer and the fuel cell stack are much slower, the dynamics of the inverter can be neglected. The model of the power conditioning unit can be seen in Figure 4 and described as

$$V_{ac} = mV_{cell} \angle \delta , \quad (29)$$

$$P_{ac} = \frac{mV_{cell}V_s}{X} \sin(\delta) , \quad (30)$$

$$\delta = \frac{2FU_f X}{mV_s N_0} q_{H_2}^{req} . \quad (31)$$

4.4 The overall fuel cell power system and the simulation results

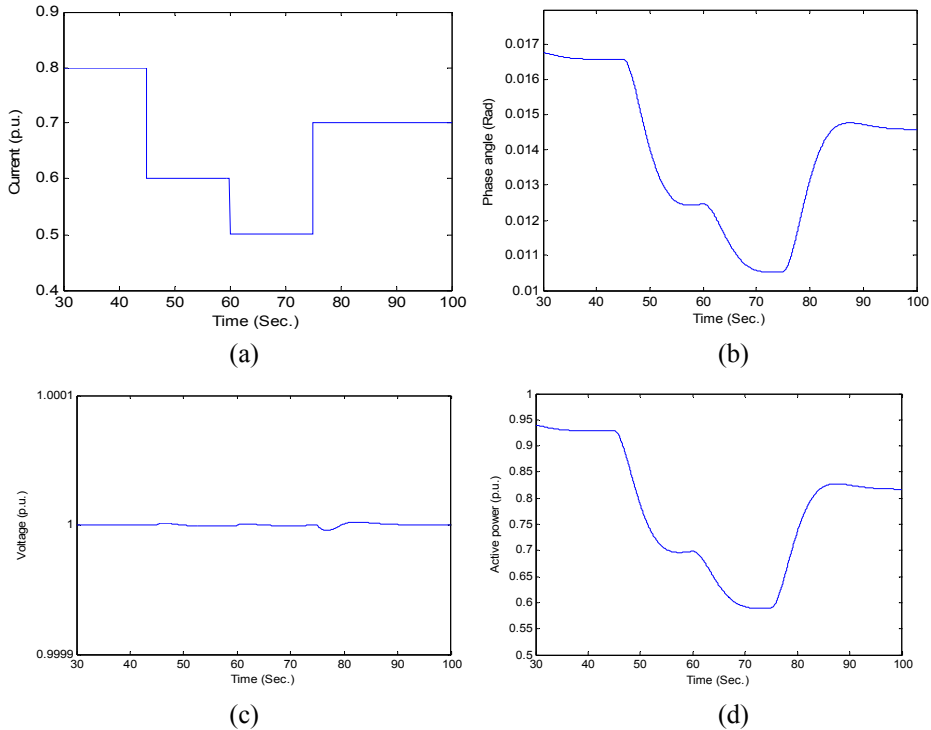
The combined fuel cell power plant with a methane reformer and a DC/AC inverter is considered. The input of the system is the fuel cell stack while the outputs are the AC voltage and the active power respectively. We have used this model to simulate system responses with Matlab Simulink software. The simulated system responses of current, phase angle, voltage and active power output have been shown in Figure 3. The responses in the first several seconds are considered as the dormant state of the system thus not shown in the figures. From the results, we can clearly see that the active power and the phase angle follow the step changes of the stack current with a certain time constant. The settling time is around 8 s. Meanwhile, the AC voltage output is almost maintained constant with the regulation of the PI controller. It should be noted that these models do not account for the system uncertainty and disturbances due to change in demand or air and hydrogen supply subsystems.

5 Improvement of the fuel cell dynamics

The objective of this task is to improve the existing dynamic model of the fuel cell power system by introducing uncertainties in demand and supply subsystems and all the necessary components. The model proposed in the preliminary results section of a stand-alone fuel cell power system has the following limitations.

- 1 It is a nominal design model without the consideration of any uncertainties.
- 2 It only regards the quality of the generated power but not considering the issues such as the operation condition, the state of health of the fuel cell stack and the fuel efficiency. Hence the fuel cell dynamics can be improved in the following aspects.

Figure 3 System responses: (a) current; (b) phase angle; (c) voltage and (d) active power output (see online version for colours)



5.1 Identifying and introducing uncertainties in the existing model

When developing the presented model, several approximations have been made for convenience, e.g., the gas is assumed ideal so that the perfect gas equation can be applied; the fuel cell stack is operated under normal condition; the pressure inside the channels is unique, each of which would cause the deviation between the model and real stack. Moreover, due to the immature manufacturing process of the bipolar plates, the produced PEM fuel cell stacks normally perform with huge variances. All of the facts raise the requirement of the uncertainties in the model for a more accurate representation.

In general, the model uncertainties are classified with the frequency level. The low frequency errors can be described in highly-structured form, while the high frequency errors require the help of the so-called less-structured representations (Zhou, 1996). Those uncertainties can be realised by adding disturbances, transfer functions and varying parameters of the model in practice.

5.2 *Considering the operation condition management in the existing model*

The temperature of the stack was assumed to remain constant in the proposed model, yet it varies with the deviation of the load demand. Thermal control plays an important role of avoiding deterioration of the catalyst and permanent damage of the membrane. It is predictable that the temperature might have a weak relationship with the hydrogen usage so that thermal control has the possibility to be decoupled from the original model in order to simplify the control scheme.

Oxygen supply can also affect the health of the fuel cell stack with the starvation problem mentioned in the background section. The air pump of the oxygen supply subsystem normally has a large time constant so that it does not have the capability to act promptly to the rapid change of the hydrogen input, which might determine the rate limit of the stack response to the load deviations. The usage of ultra capacitor has been reported (Dalvi and Guay, 2009) as an alternative to avoid this limit.

An extension to the existing fuel cell system model can be performed by adding an oxygen supply subsystem. The system model then can be tested for the changes in oxygen supply.

5.3 *Considering the hydrogen consumption in the existing model*

Currently the electrical efficiency of the stationary PEM fuel cell is around 30%. It is reported that the oxygen excess ratio can affect the fuel efficiency (Zhang et al., 2008). The hydrogen-oxygen flow ratio existed in the model was set to 1.168 as a constant. Design of strategy that optimises this ratio will reduce the consumption of hydrogen and lower the cost of electric power generated by PEM fuel cells. This ratio can be optimised.

Within the task, the valve constants, time constants, no load voltage and stack internal resistance are varied with reasonable percentage. The positions of disturbances and dynamic components representing low frequency errors in the model are determined. The fuel, air processing units and the operation management subsystems will be included to form an integrated fuel cell power plant model. This modified model can be validated with comparison of response from real-world PEM fuel cell power plant.

6 **Multi-time scale (singular perturbations) control system model**

Such techniques are suitable for complex systems composed of state space variables that have different physical nature (electrical, mechanical, chemical), different time constant, and naturally operate in different time scales. The multi-time scale property of PEMFC has been observed in the paper by Zenith and Skogestad (2009). In addition, it can be found in the works of Pukrushpan and Stefanopuluou and their coworkers that linearised systems of PEMFC at the considered operating points have the eigenvalues clustered in three groups, which clearly indicates the presence of three time scales. Including the thermal dynamics and the membrane-type humidifier dynamics, which are considered to be very slow (Pukrushpan et al., 2004; Muller et al., 2006; Karnik et al., 2009; McKay et al., 2010) most likely the complete complex dynamics of PEMFC will be adequately model using four to five time scales. Using multi-time scale modelling and corresponding subsystem based control techniques we will be able to simplify the controller design for PEMFC and at the same time to keep and better understand interactions among various

subsystems (one of important future directions in modelling and control of PEMFC as identified in Varigonda (2006)).

7 Robust controller design

After modifying the dynamic model of the system, the next task is to design and simulate a set of robust controllers to operate the PEM fuel cell power plant stably, safely and efficiently while ensuring the quality of the generated electric power. The conventional PI controller is easy to apply, yet better performance can be achieved with a well-designed advanced controller. The key to this task is to ensure the robustness of the proposed control method, since the complex structure of the fuel cell power plant with all the disturbances and uncertainties is considered.

There are several controller design strategies available in control literature applicable to different types of fuel cells. Here, we review only those relevant for this proposal. Pukrushan et al. (2004b) studied various linear controller design techniques for the nonlinear model PEMFC that and his coworkers obtained including the linear-quadratic optimal controller and observer based controller for the linearised system at corresponding operating points. Similarly, in Serra et al. (2005) first linearised the nonlinear ninth-order model of Pukrushpan et al. (2004a) and then study its controllability with different linear controller structures at several operating points. Na et al. (2007) used the exact linearisation technique to design a controller with a goal to improve both the steady state and transient performances for the third-order bilinear model presented in the previous section. In Na and Gou (2008) the fifth-order nonlinear PEMFC model is considered and the corresponding feedback linearisation control strategy is developed.

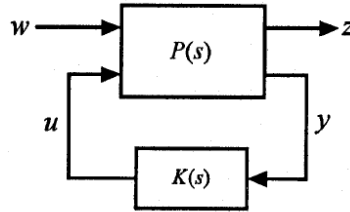
Interesting paper by Stefanopoulou and Suh (2007) studies performance limitations of different control strategies for PEMFC. Both feedforward and feedback controllers are considered in this paper. Control of integrated fuel cell and fuel processor with exhaust heat recirculation was considered in Tsourapas (2007) using a model of order nineteen. The model is first linearised and then an optimal-linear quadratic controller is designed. Kunusch et al. (2009) a sliding mode controller was designed for a sixth-order PEMFC model obtained from the original model of Pukrushpan et al. (2004) by performing a system order reduction technique. The controller appears to be efficient against uncertainties in the system. Another sliding mode control technique for air (oxygen) supply is presented in Garcia-Gabin (2010). Zenith and Skogestad (2009) have shown that a model of a PEMFC system can be divided into three subsystems corresponding to three different time constants: electrochemical subsystem operating in seconds; chemical part of the PEMFC system (energy balance and mass balance) operating in minutes, and electrical part of the PEMFC system operating in milliseconds. They designed both feedforward and feedback controllers. The paper of Karnik et al. (2009) first extends the original model of Pukrushpan et al. (2004) to include an anode water recirculation system, and then studies humidity and pressure regulation using gain-scheduled static feedback controllers. The overall model is linearised and the feedback gains are obtained either through the linear-quadratic full state feedback optimisation using the solution of the corresponding Riccati equation or through a partial output feedback linear-quadratic optimisation by solving a set of highly nonlinear algebraic equations.

Nonlinear robust control of PEMFC via state feedback exact linearisation has been recently studied in Li et al. (2010). The paper uses the fifth-order nonlinear model whose state variables the pressures of hydrogen and water at the anode side and the pressures of oxygen, nitrogen, and water at the cathode side, similar to the one of Na and Gou (2008) presented in this proposal. Integration of PEMFC into a power system was considered in several papers (El-Sharkh et al., 2004b; Paradhar et al., 2004). In Paradhar et al. (2004), a seventh-order power system linear is integrated with a seventh-order linearised model of PEMFC (Yerramalla, 2003) and the corresponding linear-quadratic optimal controller is designed for the augmented system. A nonlinear observer design for PEM fuel cell hydrogen estimation was considered in Arcak et al. (2004). Other nonlinear observers for PEMFC state variable estimates were presented in Gorgun (2005). PEM fuel cell system identification and adaptive control was presented in Yang et al. (2007). An overview of modelling and control techniques indicating progress and future opportunities was presented in Varigonda (2006).

7.1 H_∞ optimal control

A typical scheme of the H_∞ control problem is shown in Figure 4, where w is the disturbance, z is the error signal that should be minimised, u is the control output (effort) and y is the available system output measurement. The designed H_∞ controller $K(s)$ should minimise the H_∞ norm of the transfer function T_{wz} , noted as $\|T_{wz}\|_\infty$. An equivalent representation is that the designed H_∞ controller $K(s)$ should not let $\|T_{wz}\|_\infty$ exceed γ . Moreover, when such an H_∞ controller $K(s)$ exists, certain matrix inequality relationships hold. Thus the optimal H_∞ controller $K(s)$ is determined by minimising γ subject to the matrix inequalities.

Figure 4 Block diagram of H_∞ control



Source: Zhou (1996)

7.2 Active disturbance rejection control (ADRC)

The idea of ADRC is to analyse the input and the output of the plant in order to extract all the states and disturbances for the use of simplify the control problem. To be specific, a typical scheme of ADRC controlling an n th order plant is shown in Figure 5. Except the input and the highest order of the response, all the disturbances and internal dynamics have been treated as a generalised disturbance $f(t)$ (Gao, 2006), as in

$$y^{(n)}(t) = bu(t) + f(t). \quad (32)$$

With the control law,

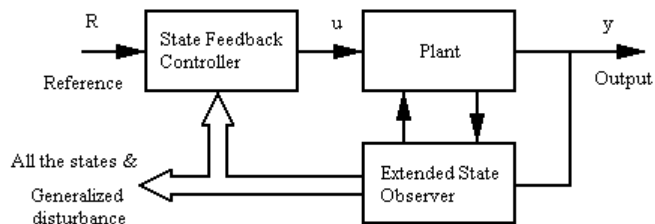
$$u(t) = \frac{u_0(t) - \hat{f}(t)}{b}, \quad (33)$$

where $\hat{f}(t)$ is obtained by the extended state observer (ESO), equation (25) can be simplified as,

$$y^{(n)}(t) = b \frac{u_0(t) - \hat{f}(t)}{b} + f(t) = u_0(t) - \hat{f}(t) + f(t) \approx u_0(t). \quad (34)$$

With the simplification process, the plant has been approximated to a pure integral plant no matter how complex it used to be. This control technique has very strong robustness since most information of the plant is not even used for controller design. Plus all the external disturbances will be evaluated by the ESO and be cancelled out from the system response. Although ADRC lacks systematic approaches of determining the control parameters, it is fairly easy to tune empirically.

Figure 5 Block diagram of ADRC



7.3 Sliding mode control technique

A sliding mode control technique will be used recently developed by one of coauthors of the this proposal and his coworkers for two-time scale singularly perturbed systems (Nguyen et al., 2010a, 2010b) to cope with system uncertainties and disturbances. It is important to emphasise that in the recent years we have witnessed many applications of sliding mode control in power electronics, mechatronics, and multidisciplinary area, including two recent papers on control of PEMFC by Kunusch et al. (2009) and Garcia-Gabin et al. (2010). Our goal is to extend the two-time-scale sliding model control of (Nguyen et al., 2010a, 2010b) to at least three and hopefully four and five time scales within the content of mathematical models developed for PEMFC cells. It should be emphasised that dealing with models given in their implicit form state space form, it is not easy to put those models into the explicit singularly perturbed, especially into multi-time explicit form, so that a lot of efforts have to be made to obtain the proper explicit multi-time models convenient for design of corresponding (first of all) sliding mode controllers. Other types of controllers will be also considered taking the advantage of independent subsystem design in corresponding time scale and studying interactions among the time scales.

8 Conclusions

The hydrogen fuel cell technology has been identified as the most promising technology in both power generation and transportation industries in the next. The application of hydrogen fuel cells has the potential to bring far reaching changes to the society and national economy by providing increased energy security, significant reduction in oil consumption and dependency, reduced greenhouse gas emissions, and improved sustainability.

This paper provides a framework to optimise control parameters of fuel cell power stations under uncertainty rising from power demand and/or air and hydrogen supply systems. The modelling analysis provided in the paper can be used on:

- 1 improving the fuel cell power plant dynamic model by considering system uncertainties
- 2 design robust control strategies to cancel the deviation of voltage and frequency while the fuel cell power system is being able to respond to the load demand uncertainty and disturbances and uncertainty in supply systems promptly.

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