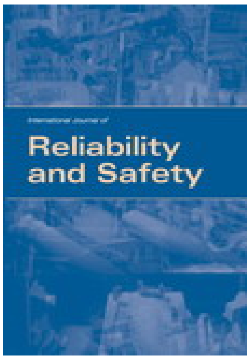

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Growth model for detection and removal of faults having different severity with single change point and imperfect debugging

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Abstract: Throughout the last decades, researchers have modelled a variety of software reliability growth models for estimating measures of reliability. In the present paper, we have classified faults into four divergent types as per their easiness and hardness in detection and removal. Also, variations in fault detection and correction rates can be because of the testing strategy, changing testing environment, motivation, proficiency and organisation of the debugging and testing teams, etc. In the present paper, a change point has been applied to four types of faults along with imperfect debugging during the correction of faults. This paper comprises two proposed software reliability growth models, which are compared on the basis of rate of detection as well as correction. All the model parameters are evaluated by the method of least squares. These models are assessed using various comparison measures like SSE, MSE, RMSE, Bias, variance and RMSPE.

Keywords: software reliability; software reliability growth model; change point; fault detection process; non-homogeneous Poisson process; sum of squared error; mean squared error; root mean square error; root mean square prediction error.

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1 Introduction

The Software Reliability Growth Model (SRGM) is the best means for estimation of software reliability, which is used to assess the performance of the software available (Downs and Scott, 1992). In recent eras, various software reliability models have been formulated for the estimation of failure rate, fault content, as well as the rate of fault removal in software. These models are also utilised to calculate the reliability of software at the time of release. Almost all are illustrated using the Mean Value Function (MVF) derived using a non-homogeneous Poisson process and make use of the past failure data. Firstly, Goel and Okumoto (1979) modelled an non-homogeneous Poisson process (NHPP)-based growth model which was time reliant. In such formulation, they considered that the failure strength is relative to the total faults endured in the system. Such formulation is extremely easy which explains curves of exponential failure. In common all accessible SRGM illustrates either an s-shaped or exponential failure trend Further, Ohba (1984) modified the previous model by considering that the detection, as well as removal rate of fault, varies through time and two kinds of faults exist in the software. Further Bittanti (1988), Kapur et al. (1999) proposed SRGMs that are analogous to Ohba (1984) but are formulated beneath a diverse set of postulations. Roy et al. (2013) proposed an SRGM based on NHPP by considering imperfect debugging and fault content function and s-shaped detection rate. Fault content increases rapidly at the beginning of testing process while it grows gradually at the end of testing process due to increasing efficiency of testing team with testing time Bittanti (1988) modelled an SRGM by employing the fault correction rate throughout the complete period of testing. However, Kapur et al. (1999) discussed a fault correction phenomenon, in which they assumed that throughout the fault removal process, many other faults may be removed exclusive of such faults causing any other failure. Such models can depict together the S-shaped and exponential growth curves and hence are considered flexible models.

All such models were formulated beneath the postulation that analogous testing strategies and testing effort is needed for correcting every fault. Though, such consideration might not be correct in reality. Diverse faults may involve different testing strategies and different amounts of testing effort for their correction. Hence faults are classified as of diverse kinds and are explored individually. Yamada et al. (1985) intended a modified exponential type SRGM by considering two types of faults. Pham (1993) formulated an SRGM with numerous types of failure. Afterward, Kapur et al. (1995, 2006) proposed a reliability model which is known as the generalised Erlang SRGM by categorising the faults as simple, hard, and complex. This is also considered that the delay in time in the study of failure and its succeeding correction signifies the complexity of faults. Hence, this is preferred to investigate the debugging and testing process of every kind of fault independently. MVF of SRGM is illustrated through the combined effect of the kind of faults. This strategy may incarcerate the unpredictability in the growth curve because of the faults of multiple severities relying on the surrounding of testing.

The models considered are consequent beneath the postulation that the rate of fault detection, as well as removal, stays steady throughout the complete period of testing. But throughout the testing phase, it is found that these rates may not be steady and hence can modify once the testing advances. The changes in the rates of fault detection and removal can be found because of the variations in the surrounding of testing, size, and complexity of the functions beneath testing, testing strategy, proficiency and organisation of the debugging and testing team, etc. The variation in the rate of fault detection and removal can be considered by employing the 'change point concept'. After the change point, the total testing period is divided into multiple sub-intervals and considers that throughout an individual sub-interval, the testing environment as well as strategy are almost alike and are somewhat diverse from further sub-intervals. The rate of fault detection or removal is considered to be either steady or a function of testing time throughout every sub-interval but alters from the additional sub-intervals.

In the present paper, we propose a reliability growth model assuming four kinds of faults which are categorised based on easiness and hardness throughout debugging of faults by integrating the change point idea. The broad structure of the model is formulated under the consideration that many new faults are introduced all through the correction of faults. The proposed model is authenticated on actual data sets.

2 Literature review

The view of change point was first initiated by Zhao (1993), who first formulated the change point methodology in software as well as hardware reliability. Shyur (2003) and Kapur et al. (2004, 2008) further assisted in this field. Shyur (2003) modelled an SRGM intended for several kinds of faults introducing the idea of a change point by considering that the rate of fault detection is steady and diverse for diverse kinds of faults. Huang and Lyu (2011) prposed different SRGMs based on NHPP with considerations of two types of errors, change point and imperfect debugging. They found that the influence of increasing new errors during the testing period will be gradually unremarkable as the testing time elapses. Chatterjee and Shukla (2019) formulated the SRGM under imperfect debugging situation. They applied the concept of change point using single type of errors. Khurshid et al. (2021) integrated the concept of change point, fault reduction factor and error generation under imperfect debugging situation. Kumar (2021) proposed a model by considering the desired performance and reliability of Open Source Software. The concept of change point was used to estimate the optimal time. Zhang et al. (2022) proposed two types of imperfect debugging models by considering S-type FDR function. They found a fault reduction function that can better describe the fault detection process. Inoue et al. (2022) proposed a framework that extends error-correction process by considering the software application characteristic as a parameter. They incorporated the change point scenario to give the real time edge to the problem. Huang et al. (2022) proposed a software reliability growth model under real and imperfect debugging situation. They used the concept of multiple change points and analysed the results by estimating several parameters. Pradhan et al. (2022) formulated the SRGM using integration of both testing effort and change point. They analysed the model using multiple change point. Huang et al. (2022) developed a SRGM under imperfect debugging situation. They concluded that fault detection and correction is affected due to severity of faults. They assumed simple and complex faults and applied the concept of multiple change points.

Table 1 Table for comparison of past models

Model	*Imperfect debugging*	*Types of faults*	*Change point*
Zhao et al. (1993)	No	One	Single
Shyur et al. (2003)	Yes	More than one	Single
Kapur et al. (2004)	No	One	Single
Kapur et al. (2008)	No	More than one	Single
Huang et al. (2011)	Yes	Two	Multiple
Chatterjee et al. (2019)	Yes	One	Single
Khurshid et al. (2021)	Yes	One	Single
Kumar (2021)	Yes	One	Single
Zhang et al. (2022)	Yes	Two	Single
Inoue et al. (2022)	No	One	Single
Huang et al. (2022)	Yes	Two	Multiple
Pradhan et al. (2022)	No	One	Multiple
Huang et al. (2022)	Yes	One	Single

3 Framework for modelling proposed SRGMs

3.1 Assumptions

According to model consideration, all faults in software are of different types. For originating the software reliability formulation furthermore up to accuracy generally, four categories of faults are considered: (1) faults whose detection as well as correction is easy (2) faults which are easily detected but their correction is difficult (3) faults which are difficult to detect although corrected easily (4) faults whose detection as well as correction is difficult (Zhang et al., 2022).

For a specified data set the overall counting of faults is signified by 'a'. The proportion of easily removable faults is taken as 'q' while the percentage of easily detected faults is taken as 'p'. As easily detected faults are not always easily removed, this is assumed that the easiness of correction and detection are not equally dependent. Accordingly, the predictable number of faults that are easily detected and corrected is apq, though the predictable number of faults which are difficult to detected but easily corrected faults is $(1-p)aq$ and the probable count of easily detected but corrected hardly faults is $ap(1-q)$. In such an assumption, the expected number of faults whose detection as well as correction is difficult is $a(1-q)(1-p)$ (Chatterjee and Shukla, 2019). The detection rate of easily detected faults is b_1, while the detection rate of faults whose detection is difficult is b_2 such that $b_2 < b_1$.

It is concerned that the rate of correction for easily corrected faults is c_1, while the correction rate for faults whose correction is hard is c_2 such that $c_2 < c_1$. The modelling of growth for the proposed model is essentially signified by employing the mean value function that is assumed like the anticipated counts of detected and corrected faults up to time t. Therefore, the mean value functions for detection in addition to removal are signified by $m_{di}(t)$ and $m_{ci}(t)$. Here '*d*' emphasises 'detection', '*c*' signifies 'correction' and $i = 1, 2, 3, 4$ indicates the kind of faults. Where $i = 1$ represents the type of faults whose detection and correction is easy, $i = 2$ indicates the type of faults whose detection is easy but correction is hard, $i = 3$ indicates the type of fault whose detection is hard but correction is easy while $i = 4$ indicates the type of faults whose detection as well as correction is difficult. The basic assumptions are-

1 The failure observation/fault removal phenomenon is modelled by NHPP.

2 Software failures take place during implementation that is raised because of available faults.

3 At each moment once a failure occurs, direct debugging begins for establishing the precise reason for that failure to correct it.

4 During the correction of faults, new faults are introduced through a consistent probability β whether any fault is corrected successfully or not.

5 The removal rate of fault (every kind of fault) varies at the change point.

3.2 Notations

Table 2 Notations used in current modelling

Symbol	*Description*
a	Total faults accessible in the software
p	Percentage of faults which are easily detected.
q	Percentage of faults which are easily corrected.
b_1	The detection rate of faults that are easily detected
b_2	The detection rate of faults that are detected hardly.
c_1	Rate of correction of faults that are easily corrected.
c_2	Rate of correction of faults that are corrected hardly.
b_{11}	Rate of detection for easily detected faults before the change point
b_{12}	Rate of detection for easily detected faults after the change point.
b_{21}	Rate of detection for faults which are detected hardly before the change point
b_{22}	Rate of detection for faults which are detected hardly after the change point
c_{11}	Rate of detection for easily corrected faults before change point occurs.
c_{12}	Rate of correction for easily corrected faults after change point occurs.
c_{21}	Rate of correction for faults which are corrected hardly before change point.
c_{22}	Rate of correction for faults which are corrected hardly after change point.
$m_{di}(t)$	Mean value function of detected faults, here *i* indicates types of fault.
$m_{ci}(t)$	Mean value function of corrected faults, here *i* indicates types of fault.
β	Probability of fault introduction throughout the correction of hard faults.
τ	Change point at which the rate of detection and correction varies

4 Proposed software reliability growth model

4.1 Fault detection process (FDP)

This model is formulated for each kind of fault using NHPP, while the total faults observed in phase t as well as $(t + \Delta t)$ are associated with the number of undetected faults on any moment t. So, one gets (Tiwari and Sharma, 2021):

$$\frac{dm_{d1}(t)}{dt} = \left(apq - m_{d1}(t)\right)b_1 \tag{1}$$

$$\frac{dm_{d2}(t)}{dt} = \left((1-q)\,ap - m_{d2}(t)\right)b_1 \tag{2}$$

$$\frac{dm_{d3}(t)}{dt} = \left(aq\,(1-p) - m_{d3}(t)\right)b_2 \tag{3}$$

$$\frac{dm_{d4}(t)}{dt} = \left((1-q)(1-p)\,a - m_{d4}(t)\right)b_2 \tag{4}$$

Adding the above equations (1), (2), (3) and (4) beneath the postulation $m_{di}(t) = 0$ while $t = 0$.

$$m_{d1}(t) = \left(1 - e^{-b_1 t}\right)apq \tag{5}$$

$$m_{d2}(t) = \left(1 - e^{-b_1 t}\right)(1-q)\,ap \tag{6}$$

$$m_{d3}(t) = \left(1 - e^{-b_2 t}\right)(1-p)\,aq \tag{7}$$

$$m_{d4}(t) = (1-q)(1-p)\left(1 - e^{-b_2 t}\right)a \tag{8}$$

Adding equations (5), (6), (7) and (8) for each category of faults, MVF for detection of fault is formulated as (Tiwari and Sharma, 2021):

$$m_d(t) = \left(1 - e^{-b_1 t}\right)ap + a\left(1 - e^{-b_2 t}\right)(1-p) \tag{9}$$

The fault detection rate varies at a change point τ and it can be assumed like:

$$b_1 = \begin{cases} b_{11}, 0 \le t \le \tau \\ b_{12}, t > \tau \end{cases}$$

$$b_2 = \begin{cases} b_{21}, 0 \le t \le \tau \\ b_{22}, t > \tau \end{cases}$$

Case 1: For $(0 \le t \le \tau)$:

$$m_{d(1)}(t) = a\left(1 - e^{-b_{11} t}\right)p + a\,(1-p)\left(1 - e^{-b_{21} t}\right) \tag{10}$$

Case 2: For $(t > \tau)$:

$$m_{d(2)}(t) = a\left(1 - e^{-(b_{11}\tau + b_{12})(t-\tau))}\right)p + a(1-p)\left(1 - e^{-(b_{21}\tau + b_{22})(t-\tau)}\right) \tag{11}$$

4.2 Fault correction process (FCP)

The fault removal process is also formulated for every fault. Since faults of types 1 and 3 are easily corrected, we model all those jointly. Similarly, faults of types 2 and 4 are modelled commonly. Furthermore, several novel faults are entering throughout the removal of types 2 and 4 faults. We formulate the following equations (Tiwari and Sharma, 2021):

$$\frac{d\left(m_{c1}(t) + m_{c3}(t)\right)}{dt} = \left(m_{d1}(t) + m_{d3}(t) - m_{c1}(t) - m_{c3}(t)\right)C_1 \tag{12}$$

$$\frac{d\left(m_{c2}(t) + m_{c4}(t)\right)}{dt} = \left(m_{d2}(t) + m_{c2}\beta + m_{d4}(t) + m_{c4}\beta - m_{c2}(t) - m_{c4}(t)\right)c_2 \tag{13}$$

Putting equations (5), (6), (7) and (8) in equations (12) and (13)

$$\begin{aligned} m_{c_1}(t) + m_{c_3}(t) = {} & \frac{ac_1}{c_1 - b_1}\left(e^{-c_1 t} - e^{-b_1 t}\right)pq \\ & \frac{+\left(e^{-c_1 t} - e^{-b_1 t}\right)a(1-p)qc_1}{c_1 - b_2} + \left(1 - e^{-c_1 t}\right)aq \end{aligned} \tag{14}$$

$$\begin{aligned} m_{c_2}(t) + m_{c_4}(t) = {} & \frac{(1-q)\,pc_2 a}{\left[c_2(1-\beta) - b_1\right]}\left(e^{-c_2 t(1-\beta)} - e^{-b_1 t}\right) \\ & + \frac{(1-q)(1-p)\,c_1 a}{\left[c_2(1-\beta) - b_2\right]}\left(e^{-c_2 t(1-\beta)} - e^{-b_2 t}\right) \\ & + (1-q)\,a\left(1 - e^{-c_2 t(1-\beta)}\right) \end{aligned} \tag{15}$$

Adding equations (14) and (15), we get

$$\begin{aligned} m_c(t) = {} & \left(e^{-c_1 t} - e^{-b_1 t}\right)\frac{ac_1 pq}{\left(c_1 - b_1\right)} + \frac{\left(e^{-c_1 t} - e^{-b_2 t}\right)aq(1-p)c_1}{\left(c_1 - b_2\right)} \\ & + a(1 - e^{-c_1 t})q + (e^{-c_2 t(1-\beta)} - e^{-b_2 t})\frac{a(1-q)\,pc_2}{\left[c_2(1-\beta) - b_1\right]} \\ & + \left(e^{-c_2 t(1-\beta)} - e^{-b_2 t}\right)\frac{a(1-q)(1-p)\,c_1}{\left[c_2(1-\beta) - b_2\right]} \\ & + \left(1 - e^{-C_2(1-\beta)t}\right)a(1-q) \end{aligned} \tag{16}$$

The fault correction rate varies at the change point τ and it can be assumed like

$$c_1 = \begin{cases} c_{11}, 0 \le t \le \tau \\ c_{12}, t > \tau \end{cases} \qquad c_2 = \begin{cases} c_{21}, 0 \le t \le \tau \\ c_{22}, t > \tau \end{cases}$$

Case: 1. For $(0 \le t \le \tau)$

$$\begin{aligned} m_c(t) = {} & \frac{apqc_{11}}{(c_{11}-b_{11})}\left(e^{-c_{11}t}-e^{-b_{11}t}\right) \\ & +\frac{a(1-p)qc_{11}\left(e^{-c_{11}t}-e^{-b_{21}t}\right)}{(c_{11}-b_{21})}+a\left(1-e^{-c_{11}t}\right)q \\ & +\frac{ap(1-q)c_{21}}{\left[c_{21}(1-\beta)-b_{11}\right]}\left(e^{-c_{21}(1-\beta)t}-e^{-b_{11}t}\right) \\ & +\frac{a(1-p)(1-q)c_{11}}{\left[c_{21}(1-\beta)-b_{21}\right]}\left(e^{-c_{21}(1-\beta)t}-e^{-b_{21}t}\right) \\ & +(1-q)a\left(1-e^{-C_{21}(1-\beta)t}\right) \end{aligned} \tag{17}$$

Case: 2. For $(t > \tau)$

$$\begin{aligned} m_c(t) = {} & \frac{apqc_{12}}{(c_{12}-b_{12})}\left(e^{-\left(c_{11}\tau+C_{12}(t-\tau)\right)}-e^{-\left(b_{11}\tau+b_{12}(t-\tau)\right)}\right) \\ & +\frac{a(1-p)qc_{12}\left(e^{-\left(c_{11}\tau+C_{12}(t-\tau)\right)}-e^{-\left(b_{21}\tau+b_{22}(t-\tau)\right)}\right)}{(c_{12}-b_{22})}+aq\left(1-e^{-\left(c_{11}\tau+C_{12}(t-\tau)\right)}\right) \\ & +\frac{ap(1-q)c_{22}}{\left[c_{22}(1-\beta)-b_{12}\right]}\left(e^{-\left(c_{21}(1-\beta)\tau+C_{22}(1-\beta)(t-\tau)\right)}-e^{-\left(b_{11}\tau+b_{12}(t-\tau)\right)}\right) \\ & +\frac{a(1-p)(1-q)c_{12}}{\left[c_{22}(1-\beta)-b_{22}\right]}\left(e^{-\left(c_{21}(1-\beta)\tau+C_{22}(1-\beta)(t-\tau)\right)}-e^{-(b_{21}\tau+b_{22}(t-\tau)}\right) \\ & +(1-q)a\left(1-e^{-\left(C_{21}(1-\beta)\tau+C_{22}(1-\beta)(t-\tau)\right)}\right) \end{aligned} \tag{18}$$

Equations (11) and (18) are modelled as proposed SRGM 1 under conditions $b_{11} > b_{21}$, $c_{11} > c_{21}$ $b_{12}>b_{22}$, and $c_{12} > c_{22}$. To illustrate the results of considering four kinds of faults, one can assess the proposed models by assuming two diverse kinds of faults having similar detection rates and somewhat diverse rates of correction, i.e., $b_{11}=b_{21}=b_1$, and $b_{12}=b_{22}=b_2$, $p = 1$, $c_{11} > c_{21}$, $c_{12} > c_{22}$.

$$m_d(t) = \left(1-e^{-(b_1\tau+b_2)(t-\tau)}\right)a \tag{19}$$

$$\begin{aligned} m_c(t) = {} & \frac{aqc_{12}}{(c_{12}-b_2)}\left(e^{-\left(c_{11}\tau+C_{12}(t-\tau)\right)}-e^{-\left(b_1\tau+b_2(t-\tau)\right)}\right) \\ & +aq\left(1-e^{-\left(c_{11}\tau+C_{12}(t-\tau)\right)}\right)+\frac{a(1-q)c_{22}}{\left[c_{22}(1-\beta)-b_2\right]} \\ & \left(e^{-\left(c_{21}(1-\beta)\tau+C_{22}(1-\beta)(t-\tau)\right)}-e^{-\left(b_1\tau+b_2(t-\tau)\right)}\right) \\ & +a(1-q)\left(1-e^{-\left(c_{21}(1-\beta)\tau+C_{12}(t-\tau)(1-\beta)\right)}\right) \end{aligned} \tag{20}$$

Equations (19) and (20) are considered proposed SRGM 2.

5 Parameter estimation

The data set considered for the analysis of the above-proposed models is from the data set of Rome Air Development Centre (RADC) (Littlewood et al., 1957) as shown in Table 3. This comprises fault detection and removal data. It contains the increasing counts of definitely detected faults along with actually corrected faults throughout 21 weeks. As the derived equations of the proposed models are non-linear so the estimation of the parameters is done by using the Method of Least Squares (Non-Linear Regression method).

Table 3 Data set T1 of Rome Air Development Centre (RADC) (Littlewood et al., 1957)

Weeks	*Time (in CPU hrs)*	*Cumulative number of faults detected (m_d)*	*Cumulative number of faults corrected (m_c)*
1	4	2	1
2	8.3	2	2
3	10.3	2	2
4	10.9	3	3
5	13.2	4	4
6	14.8	6	4
7	16.6	7	5
8	31.3	16	7
9	56.4	29	13
10	60.9	31	17
11	70.4	42	18
12	78.9	44	32
13	108.4	55	37
14	130.4	69	56
15	169.9	87	75
16	195.9	99	85
17	220.9	111	97
18	252.3	126	117
19	282.3	132	129
20	295.1	135	131
21	300.1	136	136

5.1 Comparison criterion for proposed SRGMs

The goodness of growth models is measured by their capability to fit the precedent faulty data.

5.1.1 Sum of squared error (SSE)

It is calculated as the summation of divergences between the computed value and the mean of the reliant variable. It is calculated as

$$SSE = \frac{1}{n}\sum_{j=1}^{n}\left[\left(m_d\left(t_j\right) - m_{d,i}\right) + \left(m_c\left(t_j\right) - m_{c,i}\right)\right] \tag{21}$$

The least SSE provides less fitting error so get a better fitting on the curve (Kapur et al., 1999).

5.1.2 Mean square error (MSE)

It is characterised as the difference between the expected data and the investigational data. In this situation, it is assessed as

$$MSE = \frac{1}{2n}\sum_{j=1}^{n}\left[\left(m_c\left(t_j\right) - m_{c,i}\right)^2 + \left(m_d\left(t_j\right) - m_{d,i}\right)^2\right] \tag{22}$$

Here, m_{di} and m_{ci} are the cumulative number of detected and removed faults at any moment $t_j\left(j = 1, 2, \ldots, n\right)$. Its small value gives lesser errors in fitting (Kapur et al., 1999).

5.1.3 Bias

The deviation under the studies and predictions of the counting of failures at any instantaneous extent of time is recognised as prediction error and the mean value of prediction error is termed as bias. A lower value of bias gives better goodness of fit (Peng et al., 2018).

$$Bias = \frac{\sum_{j=1}^{k}\left(m\left(t_j\right) m_i\right)}{k} \tag{23}$$

5.1.4 Variance

This is assumed as the standard deviation of prediction. Its lesser value indicates better goodness of fit (Armstrong and Collopy, 1992).

$$Variance = \sqrt{\frac{\sum_{j=1}^{k}\left(m_i - m\left(t_j\right) Bias\right)^2}{k-1}} \tag{24}$$

5.1.5 Root mean square prediction error

It is defined as a review of the proximity during which a model predicts the study

$$RMSPE = \sqrt{(Variance)^2 + (Bias)^2} \tag{25}$$

Its lesser value indicates the best goodness of fit (Kapur et al., 1999).

5.1.6 Root mean square error (RMSE)

It is considered as the proximity during which the model calculation and the study occur. The smallest value of RMSE gives the best fitting (Kapur et al., 1999).

$$RMSE = \sqrt{(MSE)} \tag{26}$$

5.2 Data analysis

The assessment of parameters for the data set of RADC and infers of assessment standard for proposed SRGMs are shown in Tables 4 and 5. The appropriate curve of proposed SRGMs to the concerned data set is demonstrated graphically in Figures 1(a) and 1(b), 2(a) and 2(b). Proposed SRGM 2 shows an excellent fitting through the exact values of the data set while Proposed SRGM 1 exhibits a divested fitting along with actual values of the assumed data set.

Table 4 Estimated model parameters

Parameters	*Parameters for data set (RADC)*			
	Proposed SRGM 1		*Proposed SRGM 2*	
	Detection	*Correction*	*Detection*	*Correction*
a	220	252	380	215
P	0.325	1.090	1	1
b_{11}	0.530	0.00165	0.0015	0.0130
b_{12}	0.000314	0.0359	0.0203	0.0064
b_{21}	0.00734	0.0042	0.0015	0.0130
b_{22}	0.00214	0.0053	0.0015	0.0064
q	–	0.769	–	0.777
c_{11}	–	0.0358	0.0045	0.0128
c_{12}	–	0.0359	0.0034	0.0073
c_{21}	–	0.001	–	0.254
c_{22}	–-	0.0021	–	0.0043
β	–	0.9899	–	0.857

Table 5 Results of model comparison

Comparison criteria	*Model comparison*	
	Proposed SRGM 1	*Proposed SRGM 2*
SSE	7656.7	131.3
MSE	364.3	6.25
RMSE	14.68	2.49
Bias	12.43	0.453
Variance	6.60	2.47
RMSPE	14.76	2.55

The following conclusions have been drawn from the results of inference of parameter and assessment criteria:

1 As novel faults are introduced all through the removal process of a hard type of faults, the total number of initial faults (a) detected has been raised. The percentage of easily detected faults (p) ranges to about unity according to the consideration. It is found that fault detection rate (b), fault correction rate (c) and fault introduction probability (β) all are fewer than unity and remain unvarying with time.

2 Every one of the calculated parameters for the data set is according to the above-considered assumptions. The fault introduction rate increases quickly as some novel faults are brought in and a state of imperfect debugging is attained throughout the removal of hard faults. The state of perfect debugging holds as examined by Peng et al. (2018) throughout the detection and removal of easily detected faults.

3 In both proposed SRGMs detection and correction rates vary as change point occurs. The infer emphasise that proposed SRGM 2 exhibits the best fit beneath the presumed assessment criteria. These two suggested models are evaluated through accessible models in 'Table 6'. The lower values of statistical measures such as SSE, MSE, Bias and Variance indicate the best performance of proposed SRGM 2.

4 From the results of the assessment criteria given in Figures 1(a), 1(b), 2(a) and 2(b) for the data set, it is analysed that the detection and correction rates vary as the change point occurs at $\tau = 16$. Values of low value-based parameters like SSE, MSE, RMSE and Bias are decreased with time. Figures 3 to 6 gives the enhanced results of proposed SRGM 2 in contrast to proposed SRGM 1 for the above-assumed data set.

Table 6 Comparison with existing models

SRGMs	*Results*	
	SSE	*MSE*
GO Model with single change point (Zhao and Wang, 2007)	2939.89	139.9
Shyur (2003)	527.31	23.11
Proposed SRGM 1	7656.7	364.3
Proposed SRGM 2	131.3	6.25

Figure 1 (a) Detection curve using SRGM (b) Correction curve using SRGM 1 (see online version for colours)

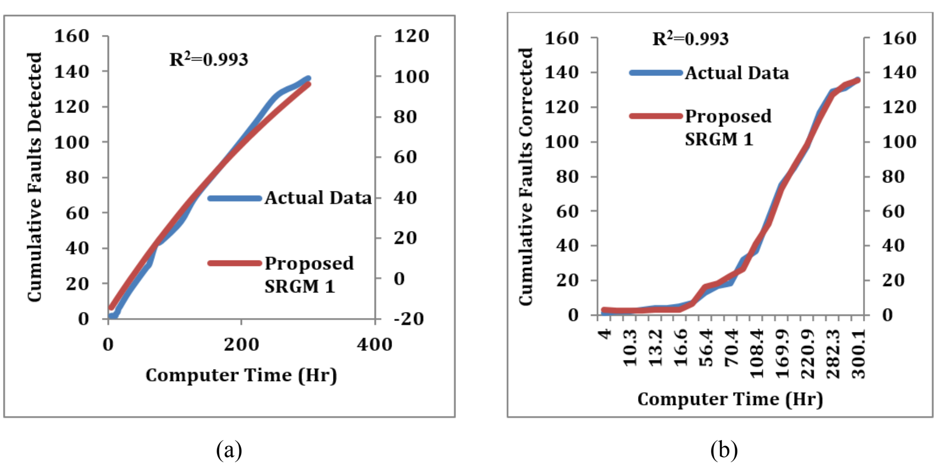
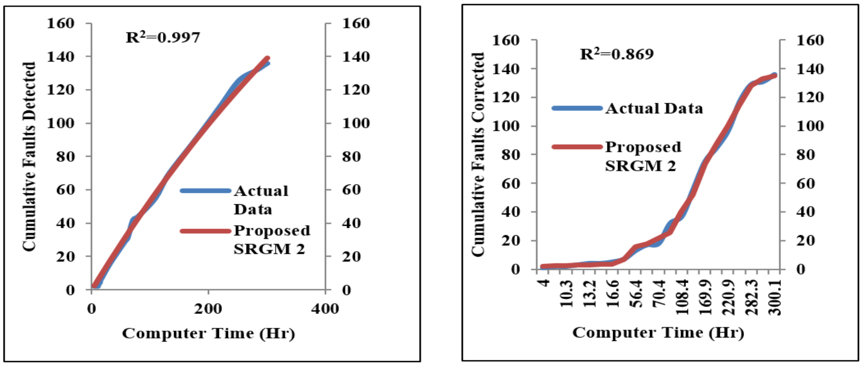


Figure 2 (a) Detection curve using SRGM 2 (b) Correction curve using SRGM 2 (see online version for colours)



(a) (b)

Figure 3 SE curve for proposed SRGMs (see online version for colours)

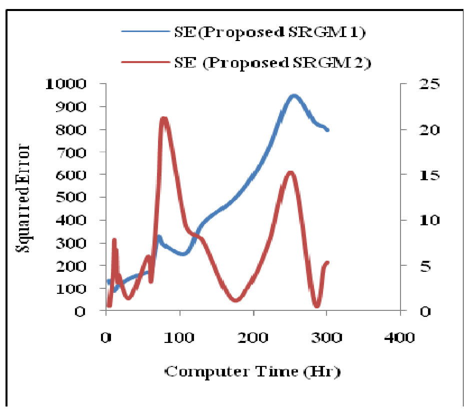


Figure 4 MSE curve for proposed SRGMs (see online version for colours)

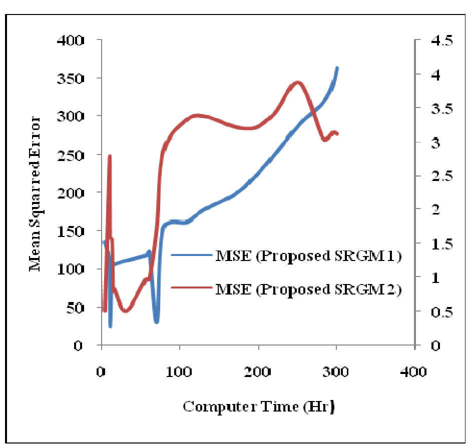


Figure 5 Bias curve for proposed SRGMs (see online version for colours)

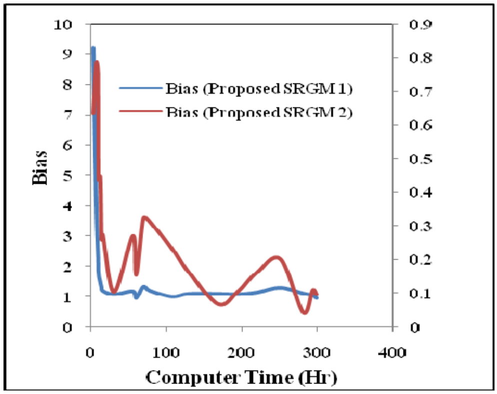
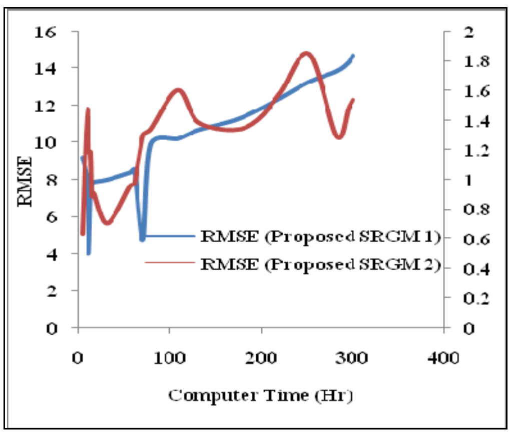


Figure 6 RMSE curve for proposed SRGMs (see online version for colours)



6 Conclusions

The framework presented in this paper is targeted to extend the change point phenomenon with imperfect debugging situation during correction of faults. This paper incorporates a single change point framework where the fault detection, as well as, correction is characterised by distinct SRGMs with change points. The above situation is applied for four types of faults. It is observed from the data collected through real data set that both SRGMs present the conventional reliability computes. Using this inclusive formulation, a new insight for imperfect debugging and change point is modelled. In both SRGMs detection and correction rates vary as change point occurs. The outcomes indicate that SRGM 2 provides enhanced fitting beneath the above-considered assessment criterion. These two formulated SRGMs are contrasted through other accessible models in Table 6. The upcoming research can be fairly promoting by introducing the multiple change point and fault dependence among easy and hard faults.

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