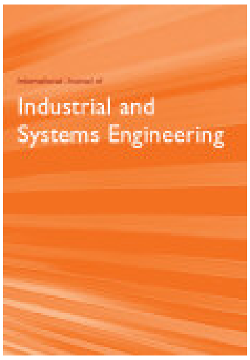

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Conditions for viable horizontal collaborative transport: insights from a stylised model

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Abstract: Recent developments such as increased volatility and sustainability requirements, lead to added pressure on supply chain performance. This is frequently considered a reason for firms to engage in collaboration. However, in practice the level of collaboration lags behind the expectations. To contribute to closing this gap between theory and practice, in this paper, the conditions for establishing a viable horizontal transport collaboration are studied. To this end, a stylised quantitative model of two supply chains, each comprising a single buyer and a single supplier located in different geographical regions is modelled. A horizontal logistics collaboration (HLC) scenario is investigated. It is demonstrated that due to hidden coordination costs in terms of inventory and warehousing costs, forming a viable HLC is not straightforward. This adds to the literature in the sense that in evaluating the viability of HLCs hardly ever trade-offs between different organisational functions are addressed.

Keywords: emission tax policy; greenhouse gas emissions; horizontal logistics collaboration; HLC; joint inventory; transport planning.

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1 Introduction

Recent developments such as increased volatility due to geopolitical tensions and inflation together with growing sustainability requirements place added pressure on supply chain performance, especially concerning logistics activities such as road transportation. Considering such developments, three features of road transportation are worth noticing. First, the road transportation function has recently encountered higher operational costs. The recent global energy market disruption, exacerbated by the armed conflict in Europe, has taken energy prices to a historical spike in 2022, and such volatility is probably not an incident. In addition, the continuing challenge of a truck driver shortage has increased truck drivers' wages substantially, as the imbalance between inland transportation demand and supply capacity continues (Duggan and McMurtrey, 2021).

Second, international regulatory standards (e.g., the Paris Agreement and the EU Fit for 55) and customers' increased awareness of environmental issues have propelled firms to reduce their contribution to greenhouse gases (GHGs) that cause climate disruptions (Peng et al., 2020; Toptal and Çetinkaya, 2017). Road transportation is one of the main contributors to GHG emissions generated by the entire transportation sector (United State Environmental Protection Agency, 2023) and it has been reported that the CO_2 emissions from road transport amount to 11.9% of the total (Ritchie et al., 2020).

Third, it is well-known that the road transportation sector as a whole is not operating very efficiently. For instance, it is reported that 20% of all road freight kilometres in the EU are run with empty trucks, and that percentage goes to 24% when it comes to only national transport (European Commission, 2021). When looking at the load factors (i.e., utilisation as a percentage of capacity), the numbers are in the range of 40%–70% (European Environment Agency, 2021).

To cope with the three challenges described above, frequently it is suggested that collaboration between firms on the various supply chain activities might substantially improve the situation (Pan et al., 2019; Simatupang and Sridharan, 2002; Verdonck et al., 2013). Given the increasing pressure to reduce cost and increase efficiency (e.g., through improved truck utilisation) and at the same time reduce GHG emissions, it is only natural to consider collaborative transport or, more generally, a so-called horizontal logistics collaboration (HLC) between firms. Within HLC, different firms can combine their transport or other logistics activities and thereby improve their triple-bottom-line performance that integrates economic, social and environmental aspects (Kumar Dadsena et al., 2019).

Despite its intuitive attractiveness, it turns out that the HLC benefits in practice are not realised to the level that is expected from the theoretical benefits (Badraoui et al., 2023; Barratt, 2004; Ferrell et al., 2020; Sabath and Fontanella, 2002). One reason for this gap between theory and practice could be that the underlying trade-offs upon entering an HLC is not well understood. That is, the cost reduction and GHG emissions upon forming an HLC might be offset by (visible or hidden) coordination costs, and it is not always immediately clear how the distinct factors influence each other.

To get a better understanding of the conditions under which an HLC can be mutually beneficial or viable, in this study a practical yet stylised quantitative model is analysed. More specifically, a situation is modelled with two independent buyers located in two different geographical regions. Each buyer is supplied by a supplier in the other buyer's region. The buyers replenish their inventory from their corresponding supplier based on the economic order quantity (EOQ) control policy. Each buyer's objective is to minimise the total annual transportation, warehousing, and GHG emissions costs. The latter is modelled by including a carbon tax.

The government rules on GHG reduction are becoming stronger rapidly, e.g., the Corporate Sustainability Reporting Directive (CSRD) initiative, city-distribution limitations, and several other laws that are expected to be coming up soon. In the model presented in this paper, it is assumed that GHG emissions are regulated by a tax policy in which the government levies a tax in terms of the amount of carbon emission.

Although the model described above is stylised, it is firmly rooted in practice. In 2017, the Netherlands-based association evofenedex[1] initiated a project to encourage horizontal collaboration among shippers. As part of this effort, they developed a match-making tool called Compose, which helped to find a match for shippers who were interested in forming an HLC (Kant et al., 2021). The situation modelled in this study was one of the most prevalent in Compose, where two shippers were located in different regions and were transporting return shipments from each other's region. For instance, around 2014, the retailer of household goods HEMA decided to open stores in Paris to expand its business outside the Netherlands, where it had previously solely operated. This required shipping some items from the HEMA distribution centre located in the Netherlands to stores in Paris, which required travelling roughly 470 kilometres in one direction. At the same time, the truck manufacturer DAF was transporting parts from France to its assembly plant in the Netherlands. Both firms faced the problem that their reverse trucks were largely empty. In response to this issue, HEMA suggested an HLC with DAF, which would be executed by a transportation company and could reduce the transportation cost to Paris by some 25%.

Within the model setting, two scenarios are considered, namely stand-alone and HLC. In the stand-alone scenario, buyers replenish their inventory independently of each other and use a dedicated transport vehicle for transportation from their supplier. Alternatively, in the HLC scenario, both buyers enter an HLC and synchronise their inventory replenishments and deliveries using the same transport vehicle on roundtrips. By doing so, the truck kilometres and thereby the transportation costs and the GHG emissions (i.e., the carbon tax costs) are substantially reduced for both buyers. From a transportation perspective, this implies that getting into the HLC scenario rather than operating stand-alone would be a no-brainer as it, obviously, results in substantially lower transportation costs and less GHG emissions at the same time.

However, even though from a pure transportation perspective this might not be immediately visible, the HLC might negatively impact the buyers' warehousing costs because both need to use the same ordering frequency which might be non-optimal for the individual buyer, leading to higher warehousing costs. As the cost arises in warehousing and not transportation, this can be considered as a hidden coordination cost. With on the one hand the lower transportation cost and on the other hand the increased warehousing cost, it is not straightforward how exactly such trade-offs have an impact on the economic and environmental feasibility of the HLC, especially regarding the impact of the geographical distances and the cost of the carbon tax. The analysis in this paper will therefore provide an answer to the research problem: within the context of the stylised model, under what conditions is an HLC the preferred option above working stand-alone?

While practical problems might be (far) more complex, it makes sense to review the stylised model for a better understanding of the conditions for firms to engage in an HLC. Keeping the model relatively simple, yet rich enough to incorporate the most relevant parameters, allows for the mathematical derivation of conditions without losing sight of the key practicalities. In doing so, the main practical contribution of this study comes in the form of providing better managerial insights into the trade-offs of the HLC scenario. More specifically, in this paper, it is derived:

a the conditions under which an HLC is viable

b the conditions under which a higher emission tax rate would lead to emission reduction.

Such findings can be useful to companies aiming to initiate an HLC and to public policymakers in understanding the impact of the tax rate.

The remainder of this paper is organised as follows. First, the relevant literature is shortly reviewed in Section 2. Next, Section 3 describes the configuration of the model at hand. The overall cost minimisation problem in stand-alone and HLC scenarios is formulated and analysed in Section 4. Subsequently, Section 5 discusses the allocation rule for the HLC cost savings and the required conditions to form a viable HLC. In Section 6, sensitivity analysis is provided. The paper is concluded by summarising the key findings and contribution to the literature and suggesting future research directions in Section 7. To increase readability, all proofs are deferred to Appendix.

2 Literature review

The topic of horizontal collaboration in logistics has received ample interest in the literature (e.g., Eirinakis et al., 2022; Argyropoulou et al., 2023; Hacardiaux et al., 2022; Badraoui et al., 2023). The focus of this section will be on positioning the approach of this paper in the overall body of knowledge and is to demonstrate where value is added. As the model analysed in this paper considers joint inventory and transport planning within a horizontal collaboration context, two streams of literature appear to be relevant, namely first the HLC literature (reviewed in Section 2.1) and second the joint inventory, transport, and emission management literature (Section 2.2). This review will be closed in Section 2.3, in which the contribution of this paper to the literature will be outlined.

2.1 HLC models

Given the clear benefits of an HLC, yet its lacking number of practical applications, an interesting question is: under what conditions is an HLC viable? Or stated otherwise, what are the pros (i.e., benefits and opportunities) and cons (i.e., impediments and barriers) of HLCs?

Even though HLC among rival firms is counterintuitive, it can lead to mutual benefits if implemented successfully (Ramjaun et al., 2024). Over the last decade, the positive impact of HLC on environmental sustainability has been recognised as an additional advantage of HLCs (Aloui et al., 2021). Furthermore, HLC can be utilised as a mitigation tool to hedge against disruptions (Hosseinnezhad et al., 2023). Therefore, the benefits of HLC can be modelled as a multi-objective function (Golmohammadi et al., 2024).

In Cruijssen et al. (2007), the pros of horizontal collaboration are given as improvements in cost-reduction and productivity, customer service, and market position while information sharing, incentive alignment, relationship management, contracts, and information technology are seen as facilitators of horizontal collaboration in transportation.

Additionally, Cruijssen et al. (2007) mention barriers from partner-matching, gain-sharing, negotiations, coordination and ICT. Similarly, Basso et al. (2019) address 16 subcategories of impediments to horizontal collaboration clustered into four main categories, namely design, planning and operations, business/market and behaviours. Furthermore, Karam et al. (2021) distinguish 31 barriers to establishing so-called collaborative transport networks. These barriers are clustered into five categories, namely business model, information sharing, collaborative decision support systems, market and human factors.

When reviewing the above-mentioned papers, it is clear that some categories of impediments are well-covered in the literature, yet others received only limited attention. The one in the latter category that triggered this research relates to operational decision-making. In Cruijssen et al. (2007), the relevant impediment is classified as the subcategory with high hidden coordination costs due to differences in operating procedures which falls under the main impediment category partner-matching. In Basso et al. (2019), the subcategory of interest is fulfilment and standards falling under the impediment planning and operations and it is remarked: "Companies with the best fulfilment may not be interested in collaborating with those in worse situations, even though a cost reduction could be obtained in theory." In Karam et al. (2021), the relevant subcategory is unbalanced freight flows which is part of the main category market.

The above observations were a motivation to research in this paper to which extent hidden operational coordination costs could be a barrier to implementing an HLC. Noting that decisions such as joint transportation and inventory replenishment, and demand or capacity allocation decisions are made at the operational level (Amer and Eltawil, 2015), the focus of this study is on the analytical modelling of operational decisions with a supply chain configuration of two pairs of a single supplier and a single buyer facing deterministic demand using an unconstrained optimisation model.

There are only a limited number of other papers that address the viability of HLCs by modelling operational decisions. Some of the more relevant are highlighted below. In Lozano et al. (2013), a linear model is developed to analyse a match-up with various possible collaboration partners so that based on, e.g., transportation volumes, the most profitable options can be selected. In Vanovermeire and Sörensen (2014) and Vanovermeire et al. (2014), it is investigated how flexibility in delivery dates, flexibility in order sizes, and order splitting rules allowed by collaboration participants can improve cost savings. Wang et al. (2014) studied the operational planning in road transportation of freight forwarding companies. They integrated subcontracting in vertical cooperations and through request exchange in horizontal coalitions and proposed approaches for realising the potential cost savings. Palhazi Cuervo et al. (2016) perform a simulation study of the horizontal collaboration benefits that two firms can have based on their respective characteristics such as the number of orders, the average order size, and the maximum number of days an order can be delayed. Verdonck et al. (2019) report on an empirical study on favourable horizontal collaboration coalitions for sharing orders based on organisations' characteristics such as order sizes, geographical coverage, demand volume, and time windows for ordering. Yuan et al. (2019) studied the value of HLC for the case of the Dutch horticultural supply chain and suggested an HLC as a solution to decrease transportation and pollution costs on the one hand and increase service level and asset utilisation on product delivery on the other. Numa-Navarro et al. (2023) studied empty container repositioning within a Colombian logistics network. It was demonstrated that to achieve the potential efficiency gains from collaboration, it is important to consider the incentives of all the actors. Finally, Stoop et al. (2023) found that an HLC among road carriers can reduce average traffic time by 13%, leading to decreased road congestion and a possible boost in capacity.

This paper will add to the aforementioned literature by focusing on diverse cost categories such as transportation cost and warehousing costs. Although, this might appear straightforward from a modelling perspective, when studying transportation improvement efforts from an HLC such internal cost trade-offs are typically overlooked. Where most of the literature focuses on methods to share the gain from an HLC (e.g., Lozano et al., 2013; Vanovermeire et al., 2014; Hezarkhani et al., 2016; Mrabti et al., 2023), this paper is more focused on decision making from a total cost point of view.

2.2 Joint inventory, transportation and emission management models

Turning to the second stream of relevant literature, some of the relevant articles in joint inventory, transportation, and emission management models will be discussed below. Jaber et al. (2013) analysed a two-echelon supply chain consisting of a single vendor and a single buyer, considering GHG emissions as a function of the vendor's production rate. They aimed to find the optimal joint lot sizing policy that minimises supply chain and emission costs under the carbon tax and carbon cap emissions trading schemes. The same supply chain configuration under similar supply chain costs and emissions trading schemes was studied by Zanoni et al. (2014). They presented a joint-economic lot size under the vendor-managed inventory with a consignment stock agreement to determine the vendor's production lot size, the number of shipments in a cycle, and the production rate.

Toptal and Çetinkaya (2017) investigated the impact of centralised and decentralised replenishment decisions on total carbon emissions under cap-and-trade and tax schemes using a two-echelon supply chain. They found a coordination mechanism that decreases supply chain costs without violating emission regulations. However, they emphasised that the cost-minimising policy may lead to an increase in carbon emissions under some conditions.

Wahab et al. (2011) studied a two-echelon supply chain with forward and backward logistics. They considered the carbon tax regulation scheme and calculated emission costs in terms of fixed and variable components for transporting shipments. Tiwari et al. (2018) investigated the optimal decisions for an integrated two-echelon supply chain considering emissions for keeping imperfect-quality items in addition to transporting and warehousing activities. They determined the optimal replenishment frequency, quantity level, and inventory level such that the supply chain costs and the emission costs under the tax scheme are minimised.

A similar study was conducted by Rout et al. (2020). They calculated emissions from the disposal of deteriorated items as well as emissions from manufacturing, transportation and warehousing. They considered different emission schemes such as carbon tax, carbon cap-and-offset and cap-and-trade.

Hacardiaux and Tancrez (2020) used a location-inventory model for minimising facility opening, transportation, cycle inventory, ordering, safety stock cost, and CO_2 emissions to execute a vast number of experiments to determine which configurations based on market and firm characteristics such as capacity, demand variability and the number of partners.

From the above, it can be observed that there are three key aspects considered by scholars in joint inventory, transportation, and emission models. First, the type of decision variables such as order quantity, number of shipments, reorder point and production rate. Second, the type of GHG emissions sources such as production, inventory, and transportation activities. Third, the type of emission legislation such as cap-and-trade, cap-and-offset, carbon tax and strict carbon cap (Castellano et al., 2019; Jain et al., 2023). In Marchi and Zanoni (2023), a two-echelon supply chain was examined and mathematically modelled as an integrated inventory management system, taking into account carbon emissions from production, transportation and storage processes. The paper explored the consignment inventory approach, wherein the upstream inventory is shifted downstream, in the context of three carbon emissions policies: limited total carbon emissions, carbon taxation and cap-and-trade. It was posited that this model could enable firms to identify optimal integrated decisions that minimise supply chain costs.

As such, the above literature leads to the objective criteria in the model addressed in this paper.

2.3 Contribution of this paper

This paper adds to the literature in the following ways. A model of an HLC between two buyers that plan joint inventory replenishments and a transport plan that minimises warehousing and carbon emission costs is discussed. As such the paper bridges perspectives from considering transportation distances, carbon regulatory schemes, inventory and transportation policies, and HLC adoption. Although the model might be stylised from a practical point of view, it is rich enough to include all relevant trade-offs while allowing for the derivation of analytical results.

Within the context of the proposed model, this study contributes to the limited stream of analytical papers that address operational issues in forming an HLC. Moreover, this study contributes to the existing research on horizontal collaboration by shedding light on hidden (internal cross-functional) coordination costs in transportation, warehousing, and the associated GHG emissions as a potential barrier for viable HLC even when it is optimised between two firms. Although many factors need to be considered when forming a successful HLC, underestimating such hidden coordination costs might be one of the reasons why the development of HLCs in practice is still lacking.

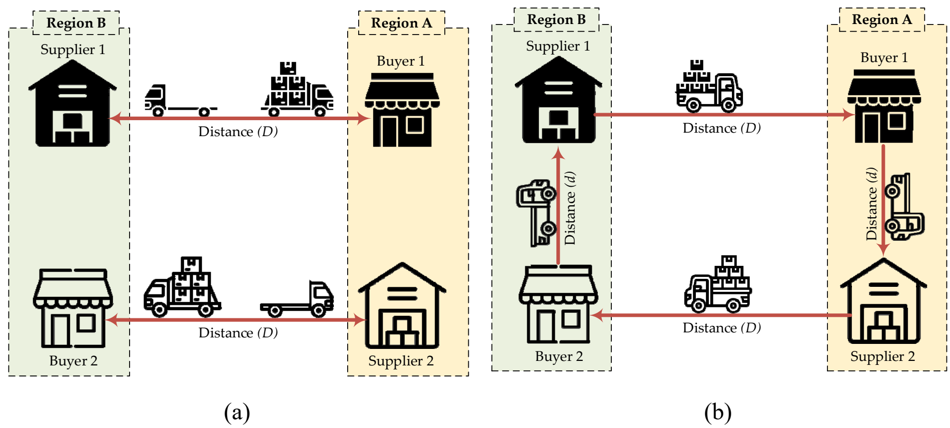
3 Model and problem description

Motivated by the practical situation where an HLC could be desired (e.g., for HEMA and DAF as mentioned in Section 1), two supply chains are considered, both comprised of a single supplier (e.g., a wholesaler) and a single buyer (e.g., a retailer), see Figure 1. In both supply chains, the buyer and the supplier are located in two different geographical regions with a significant travelling distance D from each other. In each region, there are one supplier and one buyer with a travelling distance of d from each other. Given the setting, it is fair to assume that the travelling distance within the regions is substantially shorter than the travel distance between the regions (i.e., $d < D$).

Let subscript i indicate buyer i (i = 1, 2). Buyer i aims to fully satisfy the deterministic annual demand of λ_i without having shortages. To do so, buyer i controls its inventory based on the EOQ policy and pays the annual holding cost of h_i' per stock unit. Also, each buyer places its replenishment orders with the supplier located in the other buyer's region. For each replenishment order, buyers internalise inbound transportation for which buyer i pays a fixed ordering cost of a_i per order and transportation cost of t

per travel distance unit. As depicted in Figure 1, two scenarios are considered: the stand-alone and HLC.

Figure 1 An illustration of the stand-alone and HLC scenarios, (a) stand-alone scenario (b) HLC scenario (see online version for colours)



Source: The image icons extracted and modified from 'Flaticon.com'

The stand-alone scenario refers to a decentralised control system in which each buyer plans its inventory replenishments and deliveries independently of the other buyer and uses a dedicated transport vehicle for its deliveries [see Figure 1(a)]. The HLC scenario refers to a centralised control system in which both buyers synchronise their inventory replenishment and deliveries such that they use the same transport vehicle for their deliveries as roundtrips [see Figure 1(b)]. In other words, in the HLC scenario, the buyers take advantage of reducing the empty kilometres to reduce their transportation and GHG emission costs.

In the model, three costs are considered, namely transportation costs that are dependent on the distance travelled, warehousing costs related to the amount of inventory kept, and carbon tax cost related to GHG emissions. The objective of both firms is to minimise total cost. The choice for either a stand-alone or an HLC is also based on this criterion.

It can be observed that the above model is stylised in the sense that the travel distances between and within regions are supposed to be the same for both regions. Clearly, in reality, this probably is not the case. However, this assumption helps in keeping the model mathematically tractable while not much is lost in determining the key factors that impact the decision to engage in an HLC.

Similarly, the model does not include several factors that might make it more practical. For example, in reality, the buyers might want to consider a combination of stand-alone and combined roundtrips. Clearly, including this option would make the model far more complex while at the same time, the trade-off between transportation cost and hidden coordination cost would not fundamentally change. Yet other factors such as stochastic demand, travel time diversity, weight and size of goods transported, time window requirements, etc. are not included for similar reasons. To summarise, while the model presented is stylised and kept simple for reasons of tractability, it is still rich enough to be able to derive meaningful managerial insights. As such its alleged simplicity can be seen as a virtue as it can help managers (and students) as a tutorial on making HLC decisions.

4 Scenario analysis

In this section, first, the cost-oriented objective functions of the buyers in the stand-alone and HLC scenarios are formulated. In the stand-alone scenario, each buyer aims to find its optimal annual order frequency under which their warehousing, transportation, and emission costs are minimised. In the HLC scenario, buyers aim to find the optimal annual roundtrip frequency that minimises their joint warehousing and emission costs. Below the optimisation problem is formulated and analysed within the stand-alone (Section 4.1) and the HLC scenario (Section 4.2), respectively.

4.1 Stand-alone scenario

Under a stand-alone scenario, each buyer decides on their inventory replenishment frequency (or, equivalently, order quantity) and uses a dedicated transport vehicle for its deliveries. Consequently, per each inventory replenishment cycle, in total D vehicle kilometres are running empty for each of the buyers. Buyer i aims at minimising its total annual cost consisting of warehousing (including transportation) and carbon emission costs. Let $W_i(F_i)$ represents the total annual warehousing and transportation costs of buyer i when its annual order frequency is equal to F_i. Then, the total annual warehousing and transportation costs of buyer i consist of fixed ordering costs, inventory holding costs, and transportation costs can be formulated as

$$W_i(F_i) = \underbrace{a_i F_i}_{\substack{\text{ordering}\\ \text{cost}}} + \underbrace{\frac{h_i' \lambda_i}{2F_i}}_{\substack{\text{holding}\\ \text{cost}}} + \underbrace{2tDF_i}_{\substack{\text{transportation}\\ \text{cost}}} = (a_i + 2tD) F_i + \frac{h_i' \lambda_i}{2F_i} \tag{1}$$

Emissions are associated with transportation and inventory storage. Let α and β_i be carbon emissions associated per distance unit and per stock unit per time unit, respectively. Then, the total annual carbon emissions from warehousing and transportation for buyer i can be calculated as follows:

$$E_i(F_i) = \underbrace{\alpha(2D)F_i}_{\substack{\text{transportation}\\ \text{emissions}}} + \underbrace{\frac{\beta_i \lambda_i}{2F_i}}_{\substack{\text{warehousing}\\ \text{emissions}}} . \tag{2}$$

Buyer i intends to minimise its total annual cost consisting of warehousing, transportation, and GHG emissions costs. Let C_i be the total annual cost of buyer i and let η denote the tax rate per carbon unit imposed by the government. Then, the total annual cost of buyer i in terms of warehousing, transportation, and GHG emissions costs can be expressed as a linear combination of expressions (1) and (2):

$$
\begin{aligned}
C_i(F_i) &= W_i(F_i) + \eta E_i(F_i) \\
&= (a_i + 2tD)F_i + h_i' \frac{\lambda_i}{2F_i} + \eta\left(2\alpha D F_i + \frac{\beta_i \lambda_i}{2F_i}\right) \\
&= (a_i + 2(t+\alpha\eta)D)F_i + \frac{(h_i' + \eta\beta_i)\lambda_i}{2F_i} \\
&= k_i F_i + \frac{h_i \lambda_i}{2F_i}
\end{aligned} \tag{3}
$$

where $k_i = a_i + 2(t + \alpha\eta)D$ and $h_i = h_i' + \eta\beta_i$.

From the EOQ model, it is known that $C_i(F_i)$ is a convex function in F_i. Therefore, the optimisation problem of $\min_{F_i} C_i(F_i)$ has a unique optimal annual order frequency

$$
F_i^* = \sqrt{\frac{h_i \lambda_i}{2k_i}} \tag{4}
$$

and the optimal total annual cost equals

$$
C_i(F_i^*) = \sqrt{2k_i h_i \lambda_i} = 2k_i F_i^*. \tag{5}
$$

As a result, in the stand-alone scenario, each buyer minimises its total annual cost by placing F_i^* orders a year. Depending on the warehousing, transportation, and GHG emissions cost parameters, the optimal annual order frequency of each buyer behaves differently as summarised in Proposition 1.

Proposition 1: For buyer i, the optimal annual order frequency F_i^* is

1 increasing in η, when $\frac{1}{\alpha}\left(\frac{a_i}{2D} + t\right) > \frac{h_i'}{\beta_i}$

2 decreasing in η, when $\frac{1}{\alpha}\left(\frac{a_i}{2D} + t\right) < \frac{h_i'}{\beta_i}$

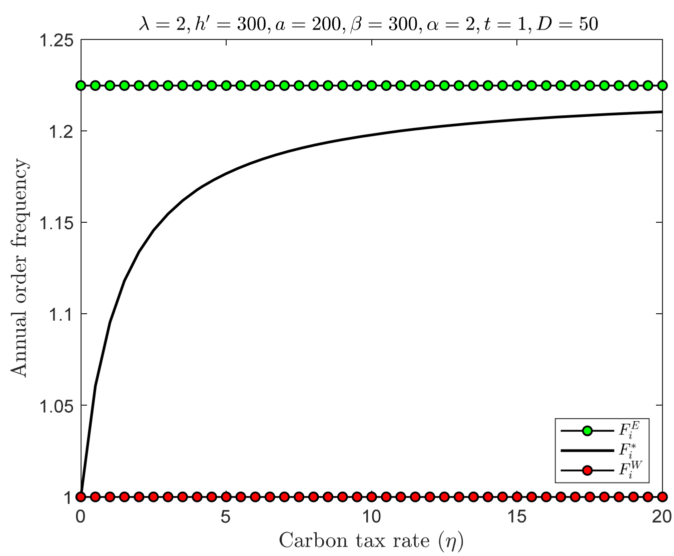
3 constant in η, when $\frac{1}{\alpha}\left(\frac{a_i}{2D} + t\right) = \frac{h_i'}{\beta_i}$.

See Appendix A1 for the proof. An immediate consequence of Proposition 1 is that if the condition in Proposition 1.3 is not fulfilled, a higher carbon tax rate always leads to lower GHG emissions and when the condition is fulfilled, the amount of GHG emissions is independent of the carbon tax rate. This can be better understood as follows. Let F_i^E and F_i^W represent the optimal order frequency under which the total annual carbon emission (E) and total annual warehousing and transportation (W) costs of buyer i are minimised, respectively. From expressions (1) and (2), it can be obtained that

$$
F_i^W = \sqrt{\frac{h_i' \lambda_i}{2(a_i + 2tD)}} \text{ and } F_i^E = \sqrt{\frac{\beta_i \lambda_i}{4\alpha D}}.
$$

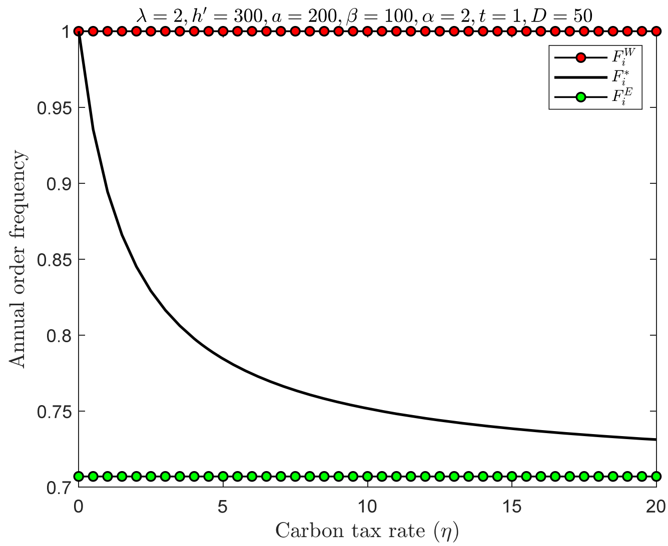
Since the optimal annual order frequency (F_i^*) is the optimal solution of a linear combination of the two convex functions, it follows $\min(F_i^W, F_i^E) \le F_i^* \le \max(F_i^W, F_i^E)$. It is obvious that when there is no carbon tax rate ($\eta = 0$), $F_i^* = F_i^W$ and for large enough carbon tax rates, $F_i^* = F_i^E$. The inequality $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right) > \frac{h_i'}{\beta_i}$ implies that $F_i^W < F_i^E$ and $F_i^W \le F_i^* \le F_i^E$. Therefore, by increasing the carbon tax rate, F_i^* increases such that for a very large carbon tax rate, it converges to F_i^E [see Figure 2(a)]. Similarly, $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right) < \frac{h_i'}{\beta_i}$ implies that $F_i^E < F_i^W$ and $F_i^E \le F_i^* \le F_i^W$. Therefore, by increasing the carbon tax rate, F_i^* decreases such that for a very large carbon tax rate it converges to F_i^E [see Figure 2(b)]. Furthermore, $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right) = \frac{h_i'}{\beta_i}$ implies that $F_i^W = F_i^E$ and $F_i^W = F_i^* = F_i^E$. Therefore, by increasing the carbon tax rate, F_i^* stays constant and equals F_i^E and F_i^W [see Figure 2(c)].

Figure 2 The behaviour of the optimal annual order frequency (F_i^*) in terms of the carbon tax rate (η), (a) $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right) = 1.5 > \frac{h_i'}{\beta_i} = 1$ (b) $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right) = 1.5 < \frac{h_i'}{\beta_i} = 3$ (c) $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right) = 1.5 = \frac{h_i'}{\beta_i} = 1.5$ (see online version for colours)

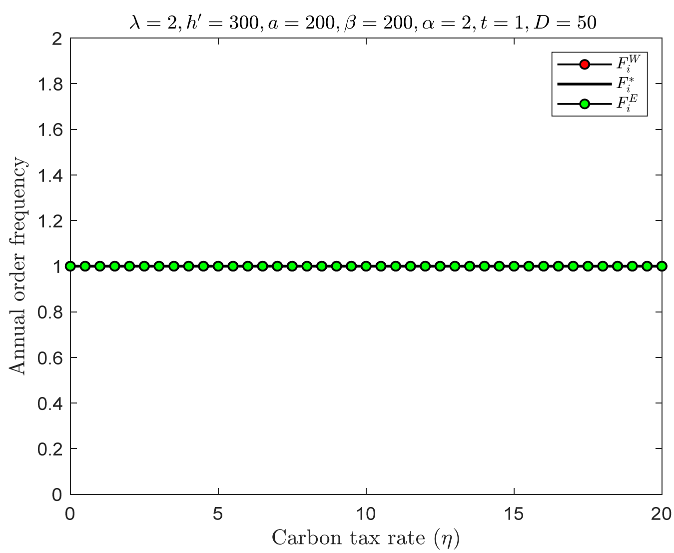


(a)

Figure 2 The behaviour of the optimal annual order frequency (F_i^*) in terms of the carbon tax rate (η), (a) $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right)=1.5>\frac{h_i'}{\beta_i}=1$ (b) $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right)=1.5<\frac{h_i'}{\beta_i}=3$ (c) $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right)=1.5=\frac{h_i'}{\beta_i}=1.5$ (continued) (see online version for colours)



(b)



(c)

4.2 The HLC scenario

It is obvious that when both buyers decide to work together by synchronising their inventory replenishments and deliveries, they can no longer determine their order frequencies independently. Instead, they need to jointly plan the annual number of round trips. Let F_{12} represents the annual number of joint replenishments through roundtrips. Then, satisfying the whole annual demand by both buyers implies $F_{12} = \frac{\lambda_1 + \lambda_2}{Q_{12}}$, where Q_{12} is the total order quantity in a roundtrip. Consequently, $2d$ vehicle kilometres are running empty per round trip. Then, the total annual warehousing and transportation costs of the HLC scenario, denoted by $W_{12}(F_{12})$, consists of the fixed ordering costs, inventory holding costs, and transportation costs of both buyers can be derived as:

$$W_{12}(F_{12}) = (a_1 + a_2 + 2t(D+d))F_{12} + \frac{1}{2F_{12}}\left(\sum_{i=1}^{2} h_i'\lambda_i\right). \tag{6}$$

Similar to the stand-alone scenario, the joint total annual GHG emission of both buyers can be calculated as:

$$E_{12}(F_{12}) = 2\alpha(D+d)F_{12} + \frac{1}{2}\sum_{i=1}^{2}\beta_i\frac{\lambda_i}{F_{12}}. \tag{7}$$

Consequently, the total annual cost of the HLC scenario, C_{12}, can be written as a linear combination of expressions (6) and (7) as

$$\begin{aligned}
C_{12}(F_{12}) &= W_{12}(F_{12}) + \eta E_{12}(F_{12}) \\
&= (a_1 + a_2 + 2t(D+d))F_{12} + \frac{1}{2}\left(\sum_{i=1}^{2} h_i'\frac{\lambda_i}{F_{12}}\right) \\
&\quad + \eta\left(2\alpha(D+d)F_{12} + \frac{1}{2}\sum_{i=1}^{2}\beta_i\frac{\lambda_i}{F_{12}}\right) \\
&= (a_1 + a_2 + 2(t+\alpha\eta)(D+d))F_{12} + \left(\frac{1}{2}\sum_{i=1}^{2}(h_i' + \eta\beta_i)\lambda_i\right)\frac{1}{F_{12}} \\
&= k_{12}F_{12} + \left(\frac{1}{2}\sum_{i=1}^{2} h_i\lambda_i\right)\frac{1}{F_{12}}
\end{aligned} \tag{8}$$

where $k_{12} = a_1 + a_2 + 2(t + \alpha\eta)(D + d)$. Then, the optimal joint annual order frequency F_{12}^* is equal to

$$F_{12}^* = \sqrt{\frac{1}{2k_{12}}\sum_{i=1}^{2} h_i\lambda_i} \tag{9}$$

and its corresponding optimal joint total annual cost is equal to

$$C_{12}\left(F_{12}^{*}\right)=\sqrt{2k_{12}\sum_{i=1}^{2}h_i\lambda_i}=2k_{12}F_{12}^{*}. \tag{10}$$

Note that k_{12} can be rewritten in terms of k_1 and k_2 as $k_{12} = k_1 + k_2 - \Delta$, where $\Delta: = 2(t + \alpha\eta)(D - d) > 0$. In essence, Δ represents the cost reduction of the HLC associated with emission and transportation costs. For the sake of simplicity, we refer to $C_i(F_i^*)$ and $C_{12}(F_{12}^*)$ as C_i^* and C_{12}^*, respectively. Depending on the warehousing and emission costs parameters of both buyers, the optimal joint order frequency behaves differently as summarised in Proposition 2.

Proposition 2: The optimal joint annual order frequency of the buyers F_{12}^* in the HLC scenario is

1 increasing in η, when $\dfrac{1}{\alpha}\left(\dfrac{a_1+a_2}{2(D+d)}+t\right)>\dfrac{\lambda_1 h_1'+\lambda_2 h_2'}{\lambda_1\beta_1+\lambda_2\beta_2}$

2 decreasing in η, when $\dfrac{1}{\alpha}\left(\dfrac{a_1+a_2}{2(D+d)}+t\right)<\dfrac{\lambda_1 h_1'+\lambda_2 h_2'}{\lambda_1\beta_1+\lambda_2\beta_2}$

3 constant in η, when $\dfrac{1}{\alpha}\left(\dfrac{a_1+a_2}{2(D+d)}+t\right)=\dfrac{\lambda_1 h_1'+\lambda_2 h_2'}{\lambda_1\beta_1+\lambda_2\beta_2}.$

See Appendix A2 for the proof. Similar to what has been observed for the stand-alone scenario after Proposition 1, an immediate consequence of Proposition 2 is that, unless the condition in Proposition 2.3 holds, a higher carbon tax rate always leads to lower emissions (and when the condition holds, the amount of emission is independent of the carbon tax rate). The underlying logic is, mutatis mutandis, the same as in the stand-alone scenario and therefore omitted here.

5 Potential gains from the HLC

In this section, the results from the HLC scenario are compared to the stand-alone scenario. In Section 5.1, the overall joint potential gains (if any) from engaging in an HLC are analysed. This is followed by Section 5.2 in which cost allocation rules and the conditions for a mutually beneficial HLC are discussed. After this, a numerical example is given in Section 5.3.

5.1 Conditions on gains through HLC

In this subsection, the gains from an HLC when compared to the stand-alone scenario are analysed. Let $\mathcal{C}^*$ represent the gains from the HLC. The gains can be measured in terms of joint cost-saving as $\mathcal{C}^* = C_1^* + C_2^* - C_{12}^*$. Clearly, $\mathcal{C}^* > 0$ implies that the HLC outperforms the stand-alone scenario financially, $\mathcal{C}^* < 0$ implies the other way around, and $\mathcal{C}^* = 0$ represents an indifferent situation. The following proposition analyses the behaviour of $\mathcal{C}^*$ respect to travelling distances within and between the regions, transportation cost unit, and fixed ordering cost parameters.

Proposition 3: Let $\mathcal{C}^*$ be the gains from the HLC when compared to stand-alone, then

1. $\mathcal{C}^*$ is decreasing in travelling distance within regions (d) and increasing in travelling distance between regions (D).
2. If $\frac{F_1^* + F_2^*}{F_{12}^*} \geq \frac{D+d}{D}$, then $\mathcal{C}^*$ is increasing in transportation cost unit (t); otherwise, it is decreasing.
3. If $F_i^* \geq F_{12}^*$, then $\mathcal{C}^*$ is increasing in fixed ordering cost per order of buyer i (a_i); otherwise, it is decreasing.

See Appendix A3 for the corresponding proof. Partly, Proposition 3.2 is a counter-intuitive result, namely where it states that not always an increase in transportation cost unit results in more cost-saving by moving from the stand-alone scenario to the HLC scenario. The reason is that when for a small transportation cost unit there is no cost-saving, i.e., $\mathcal{C}^* < 0$, then for the larger transportation cost unit, the cost-saving becomes increasingly negative. To further analyse the sign of potential gains from the HLC in terms of parameters, the following lemma is useful.

Lemma 1: Let LB and UB be defined as $LB = 1 - \frac{k_1 F_1^*}{k_{12} F_{12}^*}$ and $UB = \frac{k_2 F_2^*}{k_{12} F_{12}^*}$. Then, $0 < LB < 1$ and $0 < UB < 1$.

See Appendix A4 for the corresponding proof. We use Lemma 1 as a stepping stone to analyse the joint potential gains from the HLC and see when it is positive and when it turns negative.

Proposition 4: The joint potential gain from the HLC $(\mathcal{C}^*)$ is positive iff $LB < UB$, negative iff $LB > UB$, and zero iff $LB = UB$.

See Appendix A5 for the corresponding proof. In essence, Proposition 4 is based on the intuitive notion that when the two optimal order frequencies of the buyers under the stand-alone scenario are quite different, engaging in an HLC with a single order frequency might be less beneficial as this would require both buyers to deviate too far from their optimal own situation. The precise trade-off is given in the condition of Proposition 4.

5.2 *The HLC total cost allocation rule*

Even when there is a cost advantage in the HLC scenario when compared to the stand-alone scenario, it does not necessarily imply that the HLC is beneficial to both buyers. An HLC can only be successful when both buyers profit from the collaboration, i.e., reduce their costs when compared to the stand-alone situation (Dai and Chen, 2012). To split the gain jointly established from an HLC, so-called cost allocation methods can be used. Different allocation methods are proposed in the literature (Amiri and Farvaresh, 2023). One popular way to split the gains is the so-called proportional method (Audy et al., 2012a; Hezarkhani et al., 2016; Massol and Tchung-Ming, 2010). Another possible criterion would be to define each player's share of the joint gain based on their cost in the stand-alone scenario (Audy et al., 2012b; Özener, 2014).

As it helps to keep the model tractable, due to its simplicity and practicality, here the proportional method similar to the one proposed in Guajardo and Rönnqvist (2016) is used to divide the gain for engaging in an HLC between the buyers. In essence, the proportional method is a cost allocation rule by which the share of each player $\mathcal{S}_i$ is a portion of the total cost C_{12}^*, that is, $\mathcal{S}_i = \gamma_i C_{12}^*, i = 1, 2$ and $\gamma_1 + \gamma_2 = 1$.

Without loss of generality, for all γ, $0 < \gamma < 1$, define $\mathcal{S}_1(\gamma) = (1-\gamma)C_{12}^*$ and $\mathcal{S}_2(\gamma) = \gamma C_{12}^*$. In this setting, γ is referred to as the cost-sharing agreement factor. It means that both buyers should agree upon a single γ by which they can share the joint total cost in the HLC. Clearly, an HLC is only feasible in practice when there is a win-win situation for both players, which is therefore called a viable HLC. More specifically, a viable HLC refers to a collaboration situation in which both buyers incur less cost compared to their cost in the stand-alone scenario.

The following lemma defines a feasible domain for the cost-sharing agreement factor by which a viable HLC can be formed.

Lemma 2: If $LB < UB$, then any value of $\gamma \in (LB, UB)$ leads to a viable HLC.

See Appendix A6 for the corresponding proof. Lemma 2 provides necessary and sufficient conditions under which a viable HLC is guaranteed. To analyse the total cost paid by each buyer concerning different cost-sharing agreement factors and the stability of the HLC, this paper's key result is given by the following proposition.

Proposition 5: Let $\mathcal{S}_1(\gamma)$ and $\mathcal{S}_2(\gamma)$ be the cost share of the buyers in the HLC scenario. Then, the following collaboration payment schemes exist:

1. If $LB < UB$, for all $\gamma \in (LB, UB)$ the HLC is beneficial to both buyers, i.e., $\forall\, \gamma \in (LB, UB)$, $\mathcal{S}_1(\gamma) < C_1^*$ and $\mathcal{S}_2(\gamma) < C_2^*$.
2. For all $\gamma \in (\max(LB, UB), 1)$, the HLC is just beneficial to buyer 1, i.e., $\forall\, \gamma \in (\max(LB, UB), 1)$, $\mathcal{S}_1(\gamma) < C_1^*$ and $\mathcal{S}_2(\gamma) > C_2^*$.
3. For all $\gamma \in (0, \min(LB, UB))$, the HLC is just beneficial to buyer 2, i.e., $\forall\, \gamma \in (0, \min(LB, UB))$, $\mathcal{S}_1(\gamma) > C_1^*$ and $\mathcal{S}_2(\gamma) < C_2^*$.
4. If $UB < LB$, for all $\gamma \in (UB, LB)$, the HLC is not beneficial to either buyer, i.e., $\forall\, \gamma \in (UB, LB)$, $\mathcal{S}_1(\gamma) > C_1^*$ and $\mathcal{S}_2(\gamma) > C_2^*$.

See Appendix A7 for the corresponding proof. As a complement to Proposition 5, it can be observed that when $\gamma = LB$, then $\mathcal{S}_1(\gamma) = C_1^*$ and if $\gamma = UB$, then $\mathcal{S}_2(\gamma) = C_2^*$. Moreover, when $\gamma = LB = UB$, then $\mathcal{S}_1(\gamma) = C_1^*$ and $\mathcal{S}_2(\gamma) = C_2^*$. This can be referred to as an indifferent situation because $\mathcal{C}^* = 0$. Whether the HLC is viable or not also depends on the transportation distances d and D. Clearly, the overall assumption $d < D$ itself does not necessarily mean that a viable HLC is guaranteed. The following proposition determines the domain of d for which this is the case.

Proposition 6: There exists a threshold $\bar{d}$, with $0 < \bar{d} < D$ so that for all $d < \bar{d}$ and $\gamma \in (LB, UB)$, a viable HLC exists, where

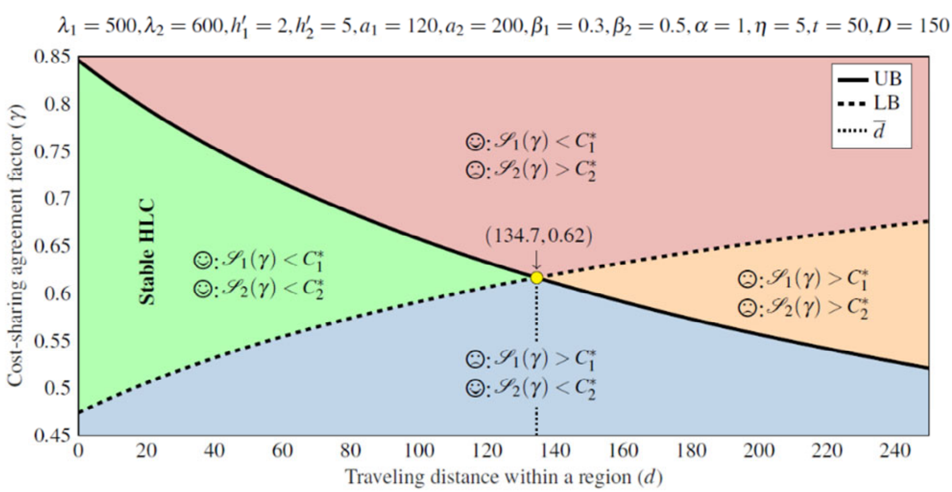
$$\bar{d} = \frac{1}{2(t+\alpha\eta)}\left(\frac{2\left(k_1F_1^* + k_2F_2^*\right)^2}{h_1\lambda_1 + h_2\lambda_2} - \left(a_1 + a_2\right)\right) - D.$$

See Appendix A8 for the corresponding proof. Although the threshold value as provided in Proposition 6 is far from trivial, the key result is intuitive; only when the distance the truck needs to drive empty within the region is limited when compared to the distance between regions, a viable HLC can be formed.

5.3 Numerical example

In this section, a numerical example is provided. Consider the parameter setting of $\lambda_1 = 500$, $\lambda_2 = 600$, $h_1' = 2$, $h_1' = 5$, $a_1 = 120$, $a_2 = 200$, $\beta_1 = 0.3$, $\beta_2 = 0.5$, $\alpha = 1$, $\eta = 5$, $t = 50$ and $D = 150$. As can be seen from Figure 3, $\bar{d} = 134.7$. For $d < \bar{d}, \mathcal{C}^* > 0$ and for $d > \bar{d}, \mathcal{C}^* < 0$. Note that when $\bar{d} < d < D$, then $\mathcal{C}^* < 0$ and no viable HLC is possible. Depending on the value of γ and d, four areas are distinguished by dedicated colours. The green area represents combinations of (d, γ) that result in a viable HLC. In contrast, the orange area shows conditions under which both buyers prefer to operate alone. The blue and red areas show the conditions under which just one of the buyers is willing to join the collaboration while the other one is not (i.e., buyers 1 and 2 are not willing to join the collaboration in the blue and red areas, respectively). Figure 3 clearly demonstrates that, in contrast to the everyday pure transportation perspective, a viable HLC is far from being straightforward. Only when the parameter settings are favourable, both parties benefit from the collaboration.

Figure 3 Feasible region for establishing a stable HLC in terms of d (see online version for colours)



Based on $\bar{d} = 134.7$, we look at d = 80 and d = 200, i.e., two situations where in one situation d is less than $\bar{d}$ and the other situation d is more than $\bar{d}$. Corresponding to each value of d, we consider three values for γ, namely $\gamma \in \{0.5, 0.6, 0.8\}$. As presented in Table 1, among the six combinations of (d, γ) with different potential gains for the buyers, just one setting, namely (80, 0.6), results in a viable HLC.

Table 1 Numerical results based on the parameter setting of Figure 3 (see online version for colours)

d	γ	LB	UB	C_1^*	C_2^*	$S_1(\gamma)$	$S_2(\gamma)$	C_{12}^*	*Potential gains*
80	0.5	0.57	0.69	7,626.9	12,259.7	8,947.8	8,947.8	17,895.5	$\mathcal{C}^* > 0$ $S_1(\gamma) > C_1^*$ and $S_2(\gamma) < C_2^*$
	0.6	0.57	0.69	7,626.9	12,259.7	7,158.2	10,737.3	17,895.5	$\mathcal{C}^* > 0$ $S_1(\gamma) < C_1^*$ and $S_2(\gamma) < C_2^*$
	0.8	0.57	0.69	7,626.9	12,259.7	3,579.1	14,316.4	17,895.5	$\mathcal{C}^* > 0$ $S_1(\gamma) < C_1^*$ and $S_2(\gamma) > C_2^*$
120	0.5	0.65	0.56	7,626.9	12,259.7	11,014.2	11,014.2	22,028.4	$\mathcal{C}^* < 0$ $S_1(\gamma) > C_1^*$ and $S_2(\gamma) < C_2^*$
	0.6	0.64	0.56	7,626.9	12,259.7	8,811.4	13,217.0	22,028.4	$\mathcal{C}^* < 0$ $S_1(\gamma) > C_1^*$ and $S_2(\gamma) > C_2^*$
	0.8	0.65	0.56	7,626.9	12,259.7	4,405.7	17,622.7	22,028.4	$\mathcal{C}^* < 0$ S_1^* and $S_2(\gamma) > C_2^*$

6 Sensitivity analysis of the viable HLC

In this section, the impact of various parameters on the viable HLC is investigated by analysing the behaviour of the threshold $\bar{d}$ in terms of one parameter while the other parameters are fixed. To make the analyses tractable, we consider different simplifying assumptions.

Corollary 1: Suppose both buyers have the same ordering cost factors (i.e., $a_1 = a_2 = a$), then

$$\bar{d} = \frac{a}{2(t+\alpha\eta)}\left(G(\boldsymbol{h}, \boldsymbol{\lambda}) - 1\right) + DG(\boldsymbol{h}, \boldsymbol{\lambda})$$

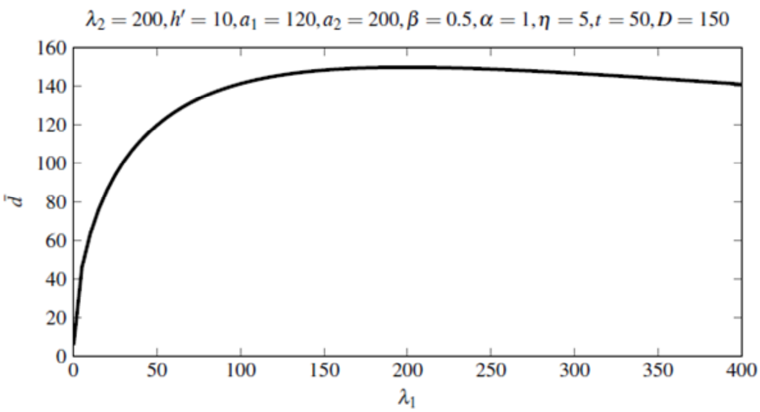
where $G(\boldsymbol{h}, \boldsymbol{\lambda}) = \frac{2\sqrt{h_1 h_2 \lambda_1 \lambda_2}}{h_1\lambda_1 + h_2\lambda_2}$, $\boldsymbol{h} = (h_1, h_2)$ and $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)$.

Corollary 1 can be calculated by substituting a_1 and a_2 with a in Proposition 5. Knowing $G(\boldsymbol{h}, \boldsymbol{\lambda}) \leq 1$ leads to $G(\boldsymbol{h}, \boldsymbol{\lambda}) - 1 \leq 0$ and $D \times G(\boldsymbol{h}, \boldsymbol{\lambda}) \leq D$. Therefore, $\bar{d} \leq D$ which is in line with Proposition 7. Particularly, when we have $h_1\lambda_1 = h_2\lambda_2$ (i.e., $h_1' = h_2'$, $\beta_1 = \beta_2$ and $\lambda_1 = \lambda_2$), then $G(\boldsymbol{h}, \boldsymbol{\lambda}) = 1$ and $\bar{d} = D$, Otherwise, $\bar{d} < D$. This leads to the following result.

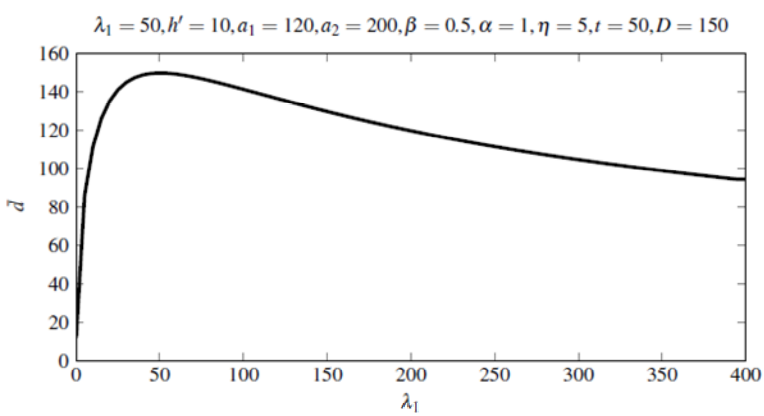
Proposition 7: Suppose both buyers have the same ordering cost factors (i.e., $a_1 = a_2 = a$), then the threshold $\bar{d}$ is decreasing in a and increasing in t, α and D.

See Appendix A9 for the corresponding proof. Proposition 7 provides insights into the impact of various parameters on the chance of an existing viable HLC between the two buyers. When all the parameters are fixed and just the fixed ordering cost factor of both buyers increases, the feasible area of the viable HLC (see the example in Figure 3) shrinks. In other words, the viable HLC exists only for shorter travelling distances within the regions. Furthermore, when the transportation cost per travel distance unit increases, a viable HLC exists for longer travelling distances within the regions. This makes sense since when transportation is expensive, having an HLC will be more economical. The same logic can be applied to explain why when the carbon emission per distance unit or travelling distance between regions increases, the viable HLC exists for longer travelling distances within the regions.

Figure 4 An illustration of $\bar{d}$ behaviour in terms of annual demand rates, (a) the behaviour of $\bar{d}$ in terms of λ_1 (b) the behaviour of $\bar{d}$ in terms of λ_1



(a)



(b)

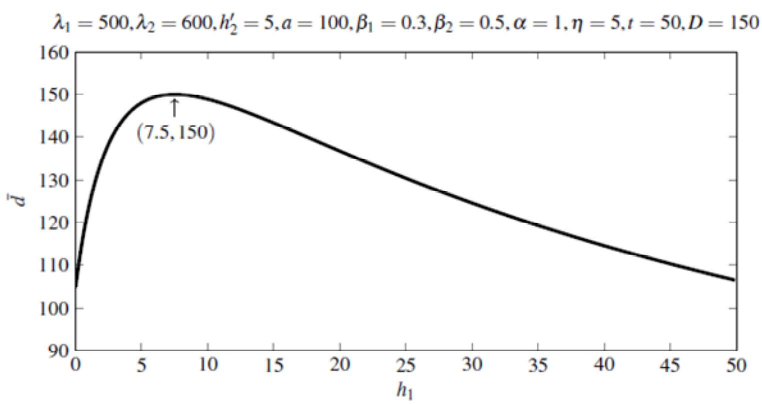
Proposition 8: Suppose both buyers have the same annual holding cost factors (i.e., $h_1' = h_2' = h'$) and $\beta_1 = \beta_2 = \beta$. Then:

1 the threshold $\bar{d}$ is increasing in η

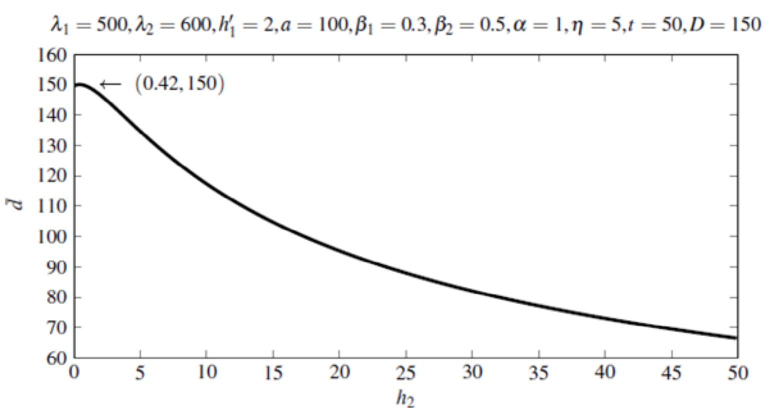
2 for $i = 1, 2$, the threshold $\bar{d}$ is increasing in λ_i when $\lambda_i < \lambda_{3-i}$, decreasing in λ_i when $\lambda_i > \lambda_{3-i}$, and constant in λ_i when $\lambda_i = \lambda_{3-i}$.

See Appendix A10 for the corresponding proof. It is quite intuitive that when the tax rate per carbon emission unit increases, a viable HLC exists for longer travelling distances within the regions since a higher tax rate per carbon emission unit has a positive effect on making an HLC economical. For the annual demand rates, the story is different as it depends on the imbalance of the buyers' annual demand rates (and thereby on the hidden coordination cost when entering an HLC). These relations are further detailed in Figure 4. For instance, when the annual demand rate of buyer 1 is lower than the annual demand rate of buyer 2 then the viable HLC can be achieved for longer travelling distances within the regions since the annual demand rate of buyer 2 is big enough such that it can result in a viable HLC, see Figure 4(a). The same logic holds for the behaviour presented in Figure 4(b).

Figure 5 An illustration of $\bar{d}$ behaviour in terms of annual inventory holding cost factors, (a) behaviour of $\bar{d}$ in terms of h_1' (b) behaviour of $\bar{d}$ in terms of h_2'



(a)



(b)

It is worth noting that when both buyers are selling the same products they have the same annual holding cost factors (i.e., $h_1' = h_2' = h'$) and $\beta_1 = \beta_2 = \beta$, then $G(\boldsymbol{h}, \boldsymbol{\lambda})$ just depends on the annual demand rates vector $\boldsymbol{\lambda}$ $\left(\text{i.e., } G(\boldsymbol{\lambda}) = \frac{2\sqrt{\lambda_1 \lambda_2}}{\lambda_1 + \lambda_2}\right)$. This implies that $\bar{d}$ will be independent of h' and β. By relaxing the condition of Proposition 7, we can see the impact of annual inventory-holding cost factors on the threshold $\bar{d}$.

As can be seen from Figure 5, when $h_i'\lambda_i$ and $h'_{3-i}\lambda_{3-i}$ are not equal, $\bar{d}$ may increase or decrease. For instance, in Figure 5(a), when h_1' is small enough such that $h_1'\lambda_1 < h_2'\lambda_2$, $\bar{d}$ increases in h_1' until it takes 7.5 at which $h_1'\lambda_1 = h_2'\lambda_2$, and $\bar{d} = D$. If h_1' continues then $h_1'\lambda_1 > h_2'\lambda_2$ and $\bar{d}$ decreases in h_1'. Figure 5(b) can be explained similarly.

7 Conclusions

Today's highly volatile road freight market with increasing transportation costs and its substantial contribution to the climate change risks have increased the attention to the use of HLC to improve their efficiency while at the same time decreasing their GHG emissions. Despite its clear benefits, surprisingly, HLCs are not implemented to the extent that could be expected from the underlying theoretical findings. To this, various impediments to HLCs are distinguished in the literature. However, it can be observed that model-based analysis on operational decisions pertaining to implementing an HLC is relatively underdeveloped.

This paper aims to contribute to the knowledge base of HLCs in terms of gaining a better understanding of the operational conditions under which an HLC would be a viable option. More specifically, this paper analyses a stylised model that has clear roots in practice. In the situation modelled, there are two supply chains each comprised of a single buyer (retailer) and a single supplier (wholesaler) located in different geographical regions. It is assumed that each buyer's supplier is located in the same region as the other buyer and that travelling distances between the two regions are substantially longer than the travelling distance within the regions. The objective of both buyers is to minimise the total annual cost consisting of warehousing (i.e., ordering and inventory holding) costs, transportation costs and GHG emissions costs. The emissions are associated with both transportation and holding inventory and the costs are determined by a government tax policy.

The unique contribution of the paper is that it is demonstrated that, within the setting of the model, the benefits of an HLC when it comes to reduced transportation costs and GHG emissions can be offset by hidden coordination costs stemming from higher warehousing costs because both buyers need to use a joint, for themselves non-optimal, order frequency. In other words, there is a trade-off situation, so that under some conditions (parameter settings) an HLC can be viable, i.e., beneficial to both buyers, yet in other conditions, this is not the case. Through a detailed analysis, it is demonstrated what exactly such conditions are and how sensitive these are to the distances between and within the regions and the level of the government-determined emission tax. In doing so, the conditions under which a higher emission tax rate would lead to emission reduction are specified.

The results in this paper might be useful to the government and companies to gain a better understanding of to what extent carbon taxation might lead to viable horizontal collaboration. Theoretical contributions highlight the fact that before engaging in an HLC a realistic trade-off needs to be considered and how the decision depends on the parameter settings. This adds to the literature where typically advantages and impediments of HLCs are considered, yet such considerations hardly ever address trade-offs between distinct functions (such as transportation and warehousing) as highlighted here. It is demonstrated that, next to focussing on external gain distribution between the cooperating parties, such internal debates are crucial in deciding on the viability of the HLC.

From a managerial perspective, it is found that although from a pure transportation perspective, an HLC appears as a no-brainer, from a total cost perspective forming an HLC is far from being straightforward. The reason is that there can be substantial hidden coordination costs involved in coordinating inventory replenishment decisions. In other words, the decision of whether to form an HLC cannot be left to the transportation department alone but needs to be considered from an integrative logistics or supply chain perspective. Additionally, when considering an HLC, there must be a good match between the two parties, not only in terms of the within and between region distances but also with the difference in the stand-alone optimal order frequencies. In some cases, even when transportation costs and GHG emissions become very high, an HLC may not be profitable without such a good match.

Clearly, the approach in this paper comes with several limitations. The model studied is stylised to make it tractable while including the most important trade-offs. However, this implies that many practical features are not included in the model. These include the option to have a combination of stand-alone and combined roundtrips, incorporate stochastic demand, consider travel time diversity or the weight and size of goods transported, and involve time window requirements.

Possible future research directions include the extension of this study along multiple dimensions. One immediate extension could be to include other sustainability objectives, viz., social dimensions such as labour safety conditions in the context of inventory and transport planning. In the model analysed, the various objectives are all translated to commensurable cost; an extension could be to instead formulate multiple criteria decision-making models. A further potential extension is to address the same research context under different supply chain structures (such as the number and location of players) and different supply chain policies (i.e., inventory and transport policies). Yet another interesting line of research could be to address the same research context by studying how the necessary condition of information sharing (Raweewan and Ferrell, 2018; Yuan et al., 2019) can be fulfilled under the emerging technology environment such as artificial intelligence (AI), blockchain, and internet of things (IoT) that can connect transport vehicles with dynamic vehicle load allocation and integration (Koh et al., 2020; Pournader et al., 2020).

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Notes

1 An association representing the interests of around 10,000 companies in the Netherlands that are involved in exporting, importing, and transporting goods on their own account or subcontract through a professional transport company.

Appendix

A1 Proof of Proposition 1

Based on expression (4), we have

$$\frac{\partial F_i^*}{\partial \eta} = \frac{\lambda_i}{2}\left(\frac{\beta_i\left(\frac{a_i}{2}+D(t+\alpha\eta)\right)-D\alpha\left(h_i'+\eta\beta_i\right)}{\left(a_i+2(t+\alpha\eta)D\right)^2}\right)\left(\frac{\lambda_i\left(h_i'+\eta\beta_i\right)}{2\left(a_i+2D(t+\alpha\eta)\right)}\right)^{-\frac{1}{2}}$$

Since $\lambda_i > 0$, $(a_i + 2(t + \alpha\eta)D)^2 > 0$ and $\left(\frac{\lambda_i\left(h_i'+\eta\beta_i\right)}{2\left(a_i+2D(t+\alpha\eta)\right)}\right)^{-\frac{1}{2}} > 0,$ then the sign of $\frac{\partial F_i^*}{\partial \eta}$ depends on the sign of the expression $\beta_i\left(\frac{a_i}{2}+D(t+\alpha\eta)\right)-D\alpha\left(h_i'+\eta\beta_i\right).$ By using basic algebra, we have we can rewrite $\beta_i\left(\frac{a_i}{2}+D(t+\alpha\eta)\right)-D\alpha\left(h_i'+\eta\beta_i\right)$ as $\alpha\beta_i D\left(\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right)-\frac{h_i'}{\beta_i}\right).$ Therefore, $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right)>\frac{h_i'}{\beta_i}$ implies $\frac{\partial F_i^*}{\partial \eta}>0,$ $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right) < \frac{h_i'}{\beta_i}$ implies $\frac{\partial F_i^*}{\partial \eta}<0,$ and $\frac{1}{\alpha}\left(\frac{a_i}{2D}+t\right)=\frac{h_i'}{\beta_i}$ implies $\frac{\partial F_i^*}{\partial \eta}=0.$

A2 Proof of Proposition 2

By extending expression (9), we have

$$\frac{\partial F_{12}^*}{\partial \eta}=\frac{1}{2}\left(\frac{(\beta_1\lambda_1+\beta_2\lambda_2)(2k_{12})-(h_1\lambda_1+h_2\lambda_2)(4\alpha(D+d))}{(2k_{12})^2}\right)(F_{12}^*)^{-\frac{1}{2}}$$

By further expansion and applying basic algebra, we get

$$\frac{\partial F_{12}^*}{\partial \eta}=\left(\alpha(D+d)(\beta_1\lambda_1+\beta_2\lambda_2)\right)\left(\frac{\frac{1}{\alpha}\left(\frac{a_1+a_2}{2(D+d)}-t\right)-\frac{h_1'\lambda_1+h_2'\lambda_2}{\beta_1\lambda_1+\beta_2\lambda_2}}{(k_{12})^2}\right)(F_{12}^*)^{-\frac{1}{2}}$$

Since the first and last terms are positive, i.e., $\alpha(D+d)(\beta_1\lambda_1+\beta_2\lambda_2)>0$ and $(F_{12}^*)^{-\frac{1}{2}}>0$, the sign of $\frac{\partial F_{12}^*}{\partial \eta}$ depends on the sign of the expression $\frac{1}{\alpha}\left(\frac{a_1+a_2}{2(D+d)}-t\right)-\frac{h_1'\lambda_1+h_2'\lambda_2}{\beta_1\lambda_1+\beta_2\lambda_2}$. Therefore, $\frac{1}{\alpha}\left(\frac{a_1+a_2}{2(D+d)}-t\right)>\frac{h_1'\lambda_1+h_2'\lambda_2}{\beta_1\lambda_1+\beta_2\lambda_2}$ implies $\frac{\partial F_{12}^*}{\partial \eta}>0$, $\frac{1}{\alpha}\left(\frac{a_1+a_2}{2(D+d)}-t\right)<\frac{h_1'\lambda_1+h_2'\lambda_2}{\beta_1\lambda_1+\beta_2\lambda_2}$ implies $\frac{\partial F_{12}^*}{\partial \eta}<0$, and $\frac{1}{\alpha}\left(\frac{a_1+a_2}{2(D+d)}-t\right)=\frac{h_1'\lambda_1+h_2'\lambda_2}{\beta_1\lambda_1+\beta_2\lambda_2}$ implies $\frac{\partial F_{12}^*}{\partial \eta}=0$.

A3 *Proof of Proposition 3*

For proving the proposition, we use $\mathcal{C}^*=C_1^*+C_2^*-C_{12}^*$, where

$$C_1^*=\sqrt{2\left(a_1+2(t+\alpha\eta)D\right)(h_1'+\eta\beta_1)\lambda_1}$$

$$C_2^*=\sqrt{2\left(a_2+2(t+\alpha\eta)D\right)(h_2'+\eta\beta_2)\lambda_2}$$

$$C_{12}^*=\sqrt{2\left(a_1+a_2+2(t+\alpha\eta)(D+d)\right)\left((h_1'+\eta\beta_1)\lambda_1+(h_2'+\eta\beta_2)\lambda_2\right)}.$$

A3.1 *Proof of Proposition 3.1*

$$\frac{\partial \mathcal{C}^*}{\partial d}=-\frac{\partial C_{12}^*}{\partial d}=-\frac{2(t+\alpha\eta)(h_1\lambda_1+h_2\lambda_2)}{C_{12}^*}<0$$

Therefore, $\mathcal{C}^*$ is always decreasing in *d*.

$$\left.\frac{\partial \mathcal{C}^*}{\partial D}\right|=\frac{2(t+\alpha\eta)(h_1'+\eta\beta_1)\lambda_1}{C_1^*}+\frac{2(t+\alpha\eta)(h_2'+\eta\beta_2)\lambda_2}{C_2^*}-\frac{2(t+\alpha\eta)\left((h_1'+\eta\beta_1)\lambda_1+(h_2'+\eta\beta_2)\lambda_2\right)}{C_{12}^{*2}}$$

$$=\left|2(t+\alpha\eta)\left(\frac{(h_1'+\eta\beta_1)\lambda_1}{C_1^*}+\frac{(h_2'+\eta\beta_2)\lambda_2}{C_2^*}-\frac{\left((h_1'+\eta\beta_1)\lambda_1+(h_2'+\eta\beta_2)\lambda_2\right)}{C_{12}^*}\right)\right.$$

$$=\left|2(t+\alpha\eta)\left(F_1^*+F_2^*-F_{12}^*\right)\right.$$

Based on Proposition 1, we have $F_1^* + F_1^* > F_{12}^*$. Then, $t + \alpha\eta > 0$ implies $\frac{\partial \mathcal{C}^*}{\partial D} > 0$. Therefore, $\mathcal{C}^*$ is increasing in *D*.

A3.2 Proof of Proposition 3.2

$$\left.\frac{\partial \mathcal{C}^*}{\partial t}\right| = \frac{2D(h_1' + \eta\beta_1)\lambda_1}{C_1^*} + \frac{2D(h_2' + \eta\beta_2)\lambda_2}{C_2^*} - \frac{2(D+d)\left((h_1' + \eta\beta_1)\lambda_1 + (h_2' + \eta\beta_2)\lambda_2\right)}{C_{12}^*}$$

$$= \left| 2D\left(\frac{(h_1' + \eta\beta_1)\lambda_1}{C_1^*} + \frac{(h_2' + \eta\beta_2)\lambda_2}{C_2^*}\right) - 2(D+d)\left(\frac{(h_1' + \eta\beta_1)\lambda_1 + (h_2' + \eta\beta_2)\lambda_2}{C_{12}^*}\right)\right.$$

$$= \left| 2\left(D(F_1^* + F_2^*) - (D+d)F_{12}^*\right)\right.$$

Then, we have

$$\frac{\partial \mathcal{C}^*}{\partial t} \geq 0 \Leftrightarrow D(F_1^* + F_2^*) - (D+d)F_{12}^* \geq 0 \Leftrightarrow \frac{F_1^* + F_2^*}{F_{12}^*} \geq \frac{D+d}{D}$$

and

$$\frac{\partial \mathcal{C}^*}{\partial t} \leq 0 \Leftrightarrow D(F_1^* + F_2^*) - (D+d)F_{12}^* \leq 0 \Leftrightarrow \frac{F_1^* + F_2^*}{F_{12}^*} \leq \frac{D+d}{D}.$$

A3.3 Proof of Proposition 3.3

$$\frac{\partial \mathcal{C}^*}{\partial a_i} = \frac{(h_1' + \eta\beta_i)\lambda_i}{C_i^*} - \frac{(h_1' + \eta\beta_1)\lambda_1 + (h_2' + \eta\beta_2)\lambda_2}{C_2^*} = F_i^* - F_{12}^*.$$

Then, we have

$$\frac{\partial \mathcal{C}^*}{\partial a_i} \geq 0 \Leftrightarrow F_i^* - F_{12}^* \geq 0 \Leftrightarrow F_i^* \geq F_{12}^*$$

and

$$\frac{\partial \mathcal{C}^*}{\partial a_i} \leq 0 \Leftrightarrow F_i^* - F_{12}^* \leq 0 \Leftrightarrow F_i^* \leq F_{12}^*.$$

A4 Proof of Lemma 1

We know that $\Delta - k_2 = 2(t + \alpha\eta)(D - d) - (a_2 + 2(t + \alpha\eta)D) = -(2(t + \alpha\eta)d + a_2) < 0$. Then, $(\Delta - k_2)(h_1\lambda_1 + h_2\lambda_2) < 0$. Knowing that $k_1h_2\lambda_2 > 0$, we can write

$$k_1h_2\lambda_2 > (\Delta - k_2)(h_1\lambda_1 + h_2\lambda_2)$$
$$k_1h_2\lambda_2 - (\Delta - k_2)h_2\lambda_2 > (\Delta - k_2)h_1\lambda_1$$
$$(k_1 + k_2 - \Delta)h_2\lambda_2 > (\Delta - k_2)h_1\lambda_1$$
$$k_2h_2\lambda_2 + k_2h_1\lambda_1 + k_1h_2\lambda_2 > \Delta(h_1\lambda_1 + h_2\lambda_2)$$
$$k_2h_2\lambda_2 + k_2h_1\lambda_1 + k_1h_2\lambda_2 - \Delta(h_1\lambda_1 + h_2\lambda_2) > 0.$$

By adding $k_1h_1\lambda_1$ to both sides of the inequality, we have

$$k_1h_1\lambda_1 + k_2h_2\lambda_2 + k_2h_1\lambda_1 + k_1h_2\lambda_2 - \Delta(h_1\lambda_1 + h_2\lambda_2) > k_1h_1\lambda_1$$
$$(k_1 + k_2 - \Delta)(h_1\lambda_1 + h_2\lambda_2) > k_1h_1\lambda_1$$
$$k_{12}(h_1\lambda_1 + h_2\lambda_2) > k_1h_1\lambda_1.$$

By dividing both sides of the inequality by $k_{12}(h_1\lambda_1 + h_2\lambda_2) > 0$, we have

$$0 < \frac{k_1h_1\lambda_1}{k_{12}(h_1\lambda_1 + h_2\lambda_2)} < 1 \Rightarrow 0 < \frac{k_1}{k_{12}}\sqrt{\frac{\frac{h_1\lambda_1}{2k_1}}{\frac{h_1\lambda_1 + h_2\lambda_2}{2k_{12}}}} < 1 \Rightarrow 0 < \frac{k_1F_1^*}{k_{12}F_{12}^*} < 1$$

$$\Rightarrow 0 < -\frac{k_1F_1^*}{k_{12}F_{12}^*} > 0.$$

Therefore, $0 < LB < 1$. Similarly, we can prove $0 < UB < 1$.

A5 Proof of Proposition 4

To prove the theorem, we use $C_i^* = k_iF_i^*$ and $C_{12}^* = 2k_{12}F_{12}^*$. Knowing $\mathcal{C}^* = C_1^* + C_2^* - C_{12}^*$, then $\mathcal{C}^* = 2(k_1F_1^* + k_2F_2^* - k_{12}F_{12}^*)$:

1 If $\mathcal{C}^* > 0 \Leftrightarrow LB < UB$:

$$\mathcal{C}^* > 0 \Leftrightarrow 2(k_1F_1^* + k_2F_2^* - k_{12}F_{12}^*) > 0 \Leftrightarrow k_1F_1^* + k_2F_2^* > k_{12}F_{12}^*$$

since $k_{12}F_{12}^* > 0$, then

$$\Leftrightarrow k_1F_1^* + k_2F_2^* > k_{12}F_{12}^* \Leftrightarrow \frac{k_1F_1^*}{k_{12}F_{12}^*} + \frac{k_2F_2^*}{k_{12}F_{12}^*} > 1 \Leftrightarrow 1 - \frac{k_1F_1^*}{k_{12}F_{12}^*} < \frac{k_2F_2^*}{k_{12}F_{12}^*} \Leftrightarrow LB < UB.$$

2 If $\mathcal{C}^* < 0 \Leftrightarrow LB > UB$:

$$\mathcal{C}^* < 0 \Leftrightarrow 2(k_1F_1^* + k_2F_2^* - k_{12}F_{12}^*) < 0 \Leftrightarrow k_1F_1^* + k_2F_2^* < k_{12}F_{12}^*$$

since $k_{12}F_{12}^* > 0$, then

$$\Leftrightarrow k_1F_1^* + k_2F_2^* < k_{12}F_{12}^* \Leftrightarrow \frac{k_1F_1^*}{k_{12}F_{12}^*} + \frac{k_2F_2^*}{k_{12}F_{12}^*} < 1 \Leftrightarrow 1 - \frac{k_1F_1^*}{k_{12}F_{12}^*} > \frac{k_2F_2^*}{k_{12}F_{12}^*} \Leftrightarrow LB > UB.$$

3 If $\mathcal{C}^* = 0 \Leftrightarrow LB = UB$:

$$\mathcal{C}^* = 0 \Leftrightarrow 2\left(k_1F_1^* + k_2F_2^* - k_{12}F_{12}^*\right) = 0 \Leftrightarrow k_1F_1^* + k_2F_2^* = k_{12}F_{12}^*$$

since $k_{12}F_{12}^* > 0$, then

$$\Leftrightarrow k_1F_1^* + k_2F_2^* = k_{12}F_{12}^* \Leftrightarrow \frac{k_1F_1^*}{k_{12}F_{12}^*} + \frac{k_2F_2^*}{k_{12}F_{12}^*} = 1 \Leftrightarrow 1 - \frac{k_1F_1^*}{k_{12}F_{12}^*} = \frac{k_2F_2^*}{k_{12}F_{12}^*} \Leftrightarrow LB = UB.$$

A6 Proof of Lemma 2

For buyer 1 to benefit from the collaboration, it should hold that

$$\mathcal{S}_1(\gamma) = (1-\gamma)C_{12}^* < C_1^*$$
$$(1-\gamma)2k_{12}F_{12}^* < 2k_1F_1^*$$
$$\gamma > 1 - \frac{k_1F_1^*}{k_{12}F_{12}^*}$$

Similarly, for buyer 2 it should hold that

$$\mathcal{S}_2(\gamma) = \gamma C_{12}^* \le C_2^*$$
$$2\gamma k_{12}F_{12}^* < 2k_2F_2^*$$
$$\gamma < \frac{k_2H_2}{k_{12}H_{12}}$$

Therefore, if $\frac{k_2H_2}{k_{12}H_{12}} > 1 - \frac{k_1H_1}{k_{12}H_{12}}$, then there exists a $\gamma_0 \in \left(1 - \frac{k_1H_1}{k_{12}H_{12}}, \frac{k_2H_2}{k_{12}H_{12}}\right)$ such that both buyers are willing to stay in the collaboration.

A7 Proof of Proposition 5

A7.1 Proof of Proposition 5.1

Please refer to the proof of Lemma 2.

A7.2 Proof of Proposition 5.2

$\gamma \in$ (max(LB, UB), 1) implies $\gamma > LB$ and $\gamma > UB$. Based on Appendix A6, we know that if $\gamma > LB$, then $\mathcal{S}_1(\gamma) \le C_1^*$. Similarly, if $\gamma > UB$, then $\mathcal{S}_2(\gamma) > C_2^*$.

A7.3 Proof of Proposition 5.3

$\gamma \in$ (0, min(LB, UB)) implies $\gamma < LB$ and $\gamma < UB$. Based on Appendix A6, we know that if $\gamma < LB$, then $\mathcal{S}_1(\gamma) > C_1^*$. Similarly, if $\gamma < UB$, then $\mathcal{S}_2(\gamma) \le C_2^*$.

A7.4 Proof of Proposition 5.4

If $LB > UB$, then $\gamma \in$ (UB, LB) implies $\gamma < LB$ and $\gamma > UB$. Based on Appendix A6, we know that if $\gamma < LB$, then $\mathcal{S}_1(\gamma) > C_1^*$. Similarly, if $\gamma > UB$, then $\mathcal{S}_2(\gamma) > C_2^*$.

A8 Proof of Proposition 6

Since for $d = 0$, $LB < UB$, then

$$1-\frac{k_1F_1^*}{k_{12}F_{12}^*}<\frac{k_2F_2^*}{k_{12}F_{12}^*}$$
$$k_{12}F_{12}^*<k_1F_1^*+k_2F_2^*$$
$$\sqrt{k_{12}\left(h_1\lambda_1+h_2\lambda_2\right)}<\sqrt{2}\left(k_1F_1^*+k_2F_2^*\right)$$
$$k_{12}<\frac{2\left(k_1F_1^*+k_2F_2^*\right)^2}{h_1\lambda_1+h_2\lambda_2}$$
$$a_1+a_2+2(t+\alpha\eta)(D+d)<\frac{2\left(k_1F_1^*+k_2F_2^*\right)^2}{h_1\lambda_1+h_2\lambda_2}$$
$$d<\frac{1}{2(t+\alpha\eta)}\left(\frac{2\left(k_1F_1^*+k_2F_2^*\right)^2}{h_1\lambda_1+h_2\lambda_2}-\left(a_1+a_2\right)\right)-D$$

Therefore, $d < \bar{d}$, where

$$\bar{d}=\frac{1}{2(t+\alpha\eta)}\left(\frac{2\left(k_1F_1^*+k_2F_2^*\right)^2}{h_1\lambda_1+h_2\lambda_2}-\left(a_1+a_2\right)\right)-D$$

First, we prove the following lemma and use it to prove the existence of $\bar{d}$ and $\bar{d} < D$ afterward.

Lemma 3: If $\alpha, \beta, X, Y \geq 0$, then $\sqrt{(\alpha+\beta)(X+Y)} \geq \sqrt{\alpha X}+\sqrt{\beta Y}$ holds.

Proof of Lemma 3: It is obvious that $(\alpha Y - \beta X)^2 \geq 0$. By expanding the expression, we get $\alpha^2Y^2 - 2\alpha\beta XY + \beta^2X^2 \geq 0$. By adding $4\alpha\beta XY$ to both sides of the inequality, then

$$\alpha^2Y^2+2\alpha\beta XY+\beta^2X^2\geq 4\alpha\beta XY$$
$$(\alpha Y+\beta X)^2\geq 4\alpha\beta XY.$$

By taking both sides of inequality to the power of $\frac{1}{2}$, we have

$$\alpha Y+\beta X\geq 2\sqrt{\alpha\beta XY}.$$

By adding $\alpha X + \beta Y$ to both sides of the inequality, then

$$\alpha Y+\beta X+\alpha Y+\beta X\geq \alpha X+\beta Y+2\sqrt{\alpha\beta XY}$$
$$(\alpha+\beta)(X+Y)\geq\left(\sqrt{\alpha X}+\sqrt{\beta Y}\right)^2.$$

By taking both sides of inequality to the power of $\frac{1}{2}$, we have

$$\sqrt{(\alpha+\beta)(X+Y)}\geq\sqrt{\alpha X}+\sqrt{\beta Y},$$

and proof of Lemma 3 is complete.

To prove the existence of $\bar{d}$, we need to show that at $d = 0$, a viable HLC exists. In other words, we need to show that for $d = 0$, $LB < UB$.

$$LB < UB \Leftrightarrow 1 - \frac{k_1 F_1^*}{k_{12} F_{12}^*} < \frac{k_2 F_2^*}{k_{12} F_{12}^*} \Leftrightarrow \frac{\sqrt{k_1 h_1 \lambda_1}}{\sqrt{k_{12}\left(h_1\lambda_1 + h_2\lambda_2\right)}} + \frac{\sqrt{k_2 h_2 \lambda_2}}{\sqrt{k_{12}\left(h_1\lambda_1 + h_2\lambda_2\right)}} > 1$$

Based on Lemma 3, we can write

$$\sqrt{\frac{k_1 + k_2}{k_{12}}} = \frac{\sqrt{\left(k_1 + k_2\right)\left(h_1\lambda_1 + h_2\lambda_2\right)}}{\sqrt{k_{12}\left(h_1\lambda_1 + h_2\lambda_2\right)}} > \frac{\sqrt{k_1 h_1 \lambda_1}}{\sqrt{k_{12}\left(h_1\lambda_1 + h_2\lambda_2\right)}} + \frac{\sqrt{k_2 h_2 \lambda_2}}{\sqrt{k_{12}\left(h_1\lambda_1 + h_2\lambda_2\right)}} > 1$$

since for $d = 0$, $\Delta = 2(t + \alpha\eta)D > 0$. Then,

$$\sqrt{\frac{k_1 + k_2}{k_1 + k_2 - \Delta}} > 1$$

Therefore, for $d = 0$, $LB < UB$.

Now, we use contradiction to prove $\bar{d} < D$. Suppose $\bar{d} \geq D$, then

$$\frac{\left(\sqrt{k_1 h_1 \lambda_1} + \sqrt{k_2 h_2 \lambda_2}\right)^2}{h_1\lambda_1 + h_2\lambda_2} - \left(a_1 + a_2\right) - 4(t + \alpha\eta)D \geq 0$$

Based on Lemma 3, we can write

$$\frac{\left(\sqrt{\left(k_1 + k_2\right)\left(h_1\lambda_1 + h_2\lambda_2\right)}\right)^2}{h_1\lambda_1 + h_2\lambda_2} - \left(a_1 + a_2\right) - 4(t + \alpha\eta)D$$
$$> \frac{\left(\sqrt{k_1 h_1 \lambda_1} + \sqrt{k_2 h_2 \lambda_2}\right)^2}{h_1\lambda_1 + h_2\lambda_2} - \left(a_1 + a_2\right) - 4(t + \alpha\eta)D \geq 0$$

Then

$$\frac{\left(\sqrt{\left(k_1 + k_2\right)\left(h_1\lambda_1 + h_2\lambda_2\right)}\right)^2}{h_1\lambda_1 + h_2\lambda_2} - \left(a_1 + a_2\right) - 4(t + \alpha\eta)D > 0$$
$$\frac{\left(k_1 + k_2\right)\left(h_1\lambda_1 + h_2\lambda_2\right)}{h_1\lambda_1 + h_2\lambda_2} - \left(a_1 + a_2\right) - 4(t + \alpha\eta)D > 0$$
$$k_1 + k_2 - \left(a_1 + a_2\right) - 4(t + \alpha\eta)D > 0$$
$$0 > 0$$

This is a contradiction. Therefore, $\bar{d} < D$.

A9 Proof of Proposition 7

We show that the partial derivative of $\bar{d}$ with respect to a is non-positive and with respect to t, α, and D are non-negative. From Corollary 1, we see that $G(\boldsymbol{h}, \boldsymbol{\lambda})$ does not depend on a, t, α and D. Let us rewrite $G(\boldsymbol{h}, \boldsymbol{\lambda})$ as follows:

$$\bar{d} = \frac{\left(G(\boldsymbol{h}, \boldsymbol{\lambda}) - 1\right)}{2} a(t + \alpha\eta)^{-1} + G(\boldsymbol{h}, \boldsymbol{\lambda}) D$$

Then,

$$\frac{\partial \bar{d}}{\partial a} = \frac{\left(G(\boldsymbol{h}, \boldsymbol{\lambda}) - 1\right)}{2(t + \alpha\eta)}$$

Since $G(\boldsymbol{h}, \boldsymbol{\lambda}) \leq 1$; then $\frac{\partial d}{\partial a} = \frac{\left(G(\boldsymbol{h}, \boldsymbol{\lambda}) - 1\right)}{2(t + \alpha\eta)} \leq 0.$ Therefore, the threshold $\bar{d}$ is decreasing in a.

$$\frac{\partial \bar{d}}{\partial t} = \frac{\left(G(\boldsymbol{h}, \boldsymbol{\lambda}) - 1\right)}{2} a(-1)(t + \alpha\eta)^{-2} = \frac{\left(1 - G(\boldsymbol{h}, \boldsymbol{\lambda})\right) a}{2(t + \alpha\eta)^2}$$

Since $G(\boldsymbol{h}, \boldsymbol{\lambda}) \leq 1$; then $\frac{\partial d}{\partial t} = \frac{\left(1 - G(\boldsymbol{h}, \boldsymbol{\lambda})\right) a}{2(t + \alpha\eta)^2} \geq 0.$ Therefore, the threshold $\bar{d}$ is increasing in t.

$$\frac{\partial \bar{d}}{\partial \alpha} = \frac{\left(G(\boldsymbol{h}, \boldsymbol{\lambda}) - 1\right)}{2} a(-\eta)(t + \alpha\eta)^{-2} = \frac{\left(1 - G(\boldsymbol{h}, \boldsymbol{\lambda})\right) a\eta}{2(t + \alpha\eta)^2}$$

Since $G(\boldsymbol{h}, \boldsymbol{\lambda}) \leq 1$; then $\frac{\partial d}{\partial \alpha} = \frac{\left(1 - G(\boldsymbol{h}, \boldsymbol{\lambda})\right) a\eta}{2(t + \alpha\eta)^2} \geq 0.$ Therefore, the threshold $\bar{d}$ is increasing in α.

$$\frac{\partial \bar{d}}{\partial D} = G(\boldsymbol{h}, \boldsymbol{\lambda})$$

Since $G(\boldsymbol{h}, \boldsymbol{\lambda}) \geq 0$; then $\frac{\partial d}{\partial D} = G(\boldsymbol{h}, \boldsymbol{\lambda}) \geq 0.$ Therefore, the threshold $\bar{d}$ is increasing in D.

A10 Proof of Proposition 8

By assuming $h_1' = h_2' = h'$ and $\beta_1 = \beta_2 = \beta$, the function $G(\boldsymbol{h}, \boldsymbol{\lambda})$ can be rewritten as $G(\boldsymbol{\lambda}) = \frac{2\sqrt{\lambda_i \lambda_{3-i}}}{\lambda_i + \lambda_{3-i}}$. Hence,

$$d = \frac{a}{2(t + \alpha\eta)}\left(G(\boldsymbol{\lambda}) - 1\right) + DG(\boldsymbol{\lambda})$$

Then,

$$\frac{\partial d}{\partial \eta} = \frac{a\alpha}{2(t+\alpha\eta)}(1-G(\boldsymbol{\lambda}))$$

Since $G(\boldsymbol{h}, \boldsymbol{\lambda}) \leq 1$; then $\frac{\partial d}{\partial \eta} =\geq 0$. Therefore, the threshold d is increasing in η.

Also, for i = 1, 2, we can write

$$\frac{\partial G(\boldsymbol{\lambda})}{\partial \lambda_i} = \frac{\sqrt{\lambda_{3-i}}}{\sqrt{\lambda_i}\,(\lambda_i + \lambda_{3-i})^2}(\lambda_{3-i} - \lambda_i)$$

Then,

$$\frac{\partial \overline{d}}{\partial \lambda_i} = \frac{a}{2(t+\alpha\eta)}\left(\frac{\partial G(\lambda)}{\partial \lambda_i}\right) + D\left(\frac{\partial G(\lambda)}{\partial \lambda_i}\right)$$

$$= \left(\frac{\sqrt{\lambda_{3-i}}}{\sqrt{\lambda_i}\,(\lambda_i + \lambda_{3-i})^2}\right)\left(\frac{a}{2(t+\alpha\eta)} + D\right)(\lambda_{3-i} - \lambda_i)$$

When $\lambda_i < \lambda_{3-i}$, then $\frac{\partial d}{\partial \lambda_i} > 0$. On the other hand, when $\lambda_i > \lambda_{3-i}$, then $\frac{\partial d}{\partial \lambda_i} < 0$. Also, when $\lambda_i = \lambda_{3-i}$, then $\frac{\partial d}{\partial \lambda_i} = 0$.