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Xiping Jiang, Shihai Zhang, Haiyi Wan, Yongfu Li

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# Accurate positioning system in complex environment based on Beidou dual frequency differential positioning technology

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Xiping Jiang\* and Shihai Zhang

State Grid Chongqing Electric Power Company,  
Electric Power Science Research Institute,  
80 Huangshan Road Middle Section,  
Yubei District, Chongqing, China  
Email: 996579270@qq.com  
Email: 287333915@qq.com  
\*Corresponding author

Haiyi Wan

State Grid Chongqing Electric Power Company,  
Yongchuan Branch,  
No. 402, Xuanhua Road, Shengli Road Street,  
Yongchuan District, Chongqing, China  
Email: 272001313@qq.com

Yongfu Li

State Grid Chongqing Electric Power Company,  
Electric Power Science Research Institute,  
80 Huangshan Road Middle Section,  
Yubei District, Chongqing, China  
Email: 535897713@qq.com

**Abstract:** Due to the differences between the orbit composition and error processing methods of Beidou satellites and GPS, the original GPS technology needs to be optimised when applied to Beidou satellites. Then, this paper makes a systematic analysis of the basic principle of satellite positioning (SP) through pseudo-range single point positioning (SPP), and uses static reference stations with accurate known positions to dynamically locate mobile base stations and mobile stations. At the same time, for the mobile base station and mobile station, this paper respectively uses SPP method and dynamic positioning (DP) method to calculate the baseline length between the two stations, and carries out baseline calculation experiments for the mobile base station and mobile station in different motion scenes. The results show that the accuracy of baseline length calculated by using DP is better than that by using SPP. Therefore, the method proposed in this paper can be applied to vehicle and pedestrian navigation to improve the navigation effect.

**Keywords:** Beidou; dual frequency differential; positioning; complex environment; single point positioning; SPP.

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**Biographical notes:** Xiping Jiang is a Doctoral student in Chongqing University, and is working at State Grid Chongqing Electric Power Company Electric Power Science Research Institute. His current research interests focus on Fault diagnosis technology of electrical equipment.

Shihai Zhang is working at State Grid Chongqing Electric Power Company Electric Power Science Research Institute. His current research interests focus on Fault diagnosis technology of electrical equipment.

Haiyi Wan is working at State grid Chongqing Electric Power Company Yongchuan Branch. Her current research interests focus on electrical engineering.

Yongfu Li is working at State Grid Chongqing Electric Power Company Electric Power Science Research Institute. His current research interests focus on fault diagnosis technology of electrical equipment.

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## 1 Introduction

The accuracy, real-time (RT), continuity, reliability and global coverage of satellite navigation (SN) and positioning play a very important role in many scientific research fields and practical engineering construction. In the process of HP, the signal is generated by the satellite and transmitted in the atmosphere, and finally received by the receiver, which is affected by various error sources. In the traditional pseudo-range SPP algorithm, the basic error is corrected, and the positioning accuracy (PA) of 5–10 metres can be achieved, but it cannot meet the application requirements of more accurate positioning (Ansari et al., 2024). In engineering applications such as automatic driving, surveying and mapping, and deformation monitoring. Many experts and scholars found that the carrier phase has millimetre-level observation accuracy when studying satellite observations. Around this characteristic, scholars have successively proposed RTKinematic (RTK) algorithm and precise point positioning (PPP) algorithm based on absolute positioning technology (Bao et al., 2021). The PPP algorithm uses a single-point observation model at the receiver terminal, and does not need to rely on network difference or reference station observation data like RTK, and can realise wide-area precise positioning.

PPP has undergone more than 30 years of development since its proposal in the 1990s, and is widely used in fields such as satellite precision positioning and orbit determination, precision timing, Earth space science, and atmospheric science. In terms of positioning algorithm methods, PPP algorithm initially adopted GPS static dual frequency method, and gradually developed into multi frequency and multi system positioning. Later, in order to achieve fast positioning, research was conducted on fixed ambiguity. Now, in order to meet practical applications, RT positioning based on satellite based enhancement is being studied (Bayer et al., 2020).

The Beidou Navigation Satellite System (BDS) is a type of global navigation system that China has independently developed and maintained. Its development stage has gone from the initial Beidou-1 SN system to the most advanced Beidou-3 SN system. BDS integrates HP, communication timing, global search and rescue services, and has completed multiple advanced technological breakthroughs. In terms of satellite based enhanced precision positioning, the Beidou satellite broadcasts precise positioning and integrity information to relevant users through the Geostationary Orbit Satellite (GEO), including various precision correction parameters, and RT phase partition deviation information.

The RT PPP technology for Beidou is attracting the attention of many scientific research institutes. Due to the differences between the orbit composition and error processing methods of Beidou satellites and GPS, the original GPS technology needs to be optimised when applied to Beidou satellites, and the relevant differentiated characteristics need special treatment. At present, the relevant research is being carried out step by step, and the corresponding positioning methods and receiver terminal design also need to be continuously optimised. The purpose of this paper is to improve the precise positioning effect of Beidou system in complex environment through double-spread differential positioning technology. The contribution of this paper is to propose the accuracy of baseline length solution by using the DP method, which is better than the accuracy of baseline length solution.

This article mainly studies the dynamic positioning (DP) based on the Beidou SN system. In response to the inability of traditional differential positioning to meet certain positioning requirements, an improved differential positioning method is studied. In the case where only one receiver is used for positioning, this method utilises inter epoch difference to improve PA, and can increase PA to sub metre level without relying on other reference stations. This article also studied the mathematical model of dynamic differential positioning and conducted DP experiments to analyse the accuracy of DP baseline solution.

## **2 Related work**

The Jet Propulsion Laboratory (JPL), with the background of GPS relative positioning technology, is the first to propose the global differential GPS (GDGPS) in the networking mode. This project systematically integrates GPS positioning technology and internet technology, estimates the high-precision (HP) orbit, and broadcasts it to users through the internet, realising RT decimetre PA (Beasley et al., 2024). On the basis of the RT absolute HP positioning software (GIPSY) developed by JPL, NavCom in the USA has developed a new generation. It has gradually improved the differential HP service that JPL could only provide in LAN mode to support global absolute single point precision positioning services (Catania et al., 2020). Later, it has integrated satellite systems such as GLONASS and Galileo, further improving positioning performance. Currently, it can achieve an absolute SPP accuracy of 5 centimetres on a global scale. Fugro also developed a differential positioning system similar to GIPSY, OmniSTAR (Cui et al., 2019). This system combines satellite communication and Internet technology, integrates central station service, reference station control network, and satellite broadcasting. Through the global reference station network, it accurately calculates various errors related to GPS satellites, estimates global ionospheric correction parameters, and

broadcasts them to users in real time through communication satellites, supporting global fast absolute single point HP positioning services. Farhangian and Landry (2020) integrates differential positioning services with satellite receivers, allowing users to achieve HP positioning of OmniSTAR with simple operations. The enhanced correction information of the OmniSTAR system is broadcasted through the L-band, and the centre frequency is basically consistent with the frequency of the SN signal. The receiver hardware can share the same system with the satellite signal reception, reducing the development cost of the receiver. The OmniSTAR system supports services in multiple scenarios, and users can choose according to application requirements and cost budgets.

Gan et al. (2019) uses hundreds of wide area GPS monitoring stations to construct a data analysis network, accurately fits satellite orbits using multi day data, and estimates satellite clock deviations accurately in combination with the monitoring station network. Corresponding error sources are corrected in positioning calculations, which greatly improves PA. It has been widely applied worldwide. In PPP data processing, clock errors can be accurately estimated through a reference station network. Other significant errors such as ionospheric delay can be effectively suppressed through multi frequency observation value combinations. However, when using ionospheric free combinations, the noise in signal measurement values will be amplified, and the carrier phase ambiguity will also lose its full cycle characteristics (Gante et al., 2020). Gao (2019) developed a precision positioning software P3 for GPS data processing.

In order to achieve RT PPP, many scholars and research institutions at home and abroad have broadcast HP orbit and clock deviation information of satellites through internet technology, and carried out RT HP positioning simulation. Research has shown that when precise satellite orbit information and clock bias information are used, HP SPP algorithms can achieve a positioning deviation of better than 1 metre in tens of minutes (Gao et al., 2018). In terms of precise satellite orbit and clock bias prediction, Ge et al. (2022) found that the RT nature of satellite clock bias can affect the convergence performance of positioning. Using IGS observation station data to re estimate the satellite's clock bias and improve the RT nature of clock bias prediction can help to converge positioning bias to decimetre level accuracy in a short period of time. Hemkumar (2024) developed a precision SPP simulation software TriP that supports GPS. This software can not only perform positioning calculations, but also predict satellite clock deviations, achieving centimetre level PA. In order to improve the fast convergence ability of RT precise SPP and reduce absolute SPP deviation, the method of fixed integer ambiguity has also been gradually developed in recent years (Hu et al., 2020). In order to solve the problem that the initial carrier phase deviation of multi frequency real coefficient combination no longer has integer ambiguity, many scholars have divided the ambiguity of the combination without ionospheric delay into wide lane (WL) and narrow lane (NL) ambiguities, and constructed the Melbourne Wiibbena combination. The WL ambiguity has the characteristic of long wavelength, and the Melbourne Wtibbena combination combines the carrier phase WL combination of long wavelength and the low noise pseudo range NL combination. Combination, fully utilising the combination characteristics of carrier phase and pseudorange, can improve the search success rate of WL ambiguity (Hu et al., 2020). Jiang et al. (2021) found that when using the Melbourne Wiibbena combination in RT precision SPP. The carrier observation values without ionospheric delay combination only have NL ambiguity and uncalibrated phase delay (UPD). If UPD is not processed, it still takes about 20 minutes to fix the ambiguity. Teixeira et al. (2021) utilises multi day data and network optimisation

methods to construct multiple sets of constraint models, which can effectively estimate UPD parameters and improve the search success rate of ambiguity. Inspired by network RTK, in order to reduce the estimation parameters of RT precision SPP, atmospheric delay parameters can be pre estimated through external measurement methods. Many scholars estimate the atmospheric delay that affects precision SPP in real time based on the regional reference station network model, and correct it to the receiver end through interpolation (Kavetha et al., 2022). In precision SPP, in addition to the traditional multi frequency non-ionospheric delay combination, non-differential non-combination methods can also be used, provided that ionospheric delay can be accurately estimated. Currently, the global ionosphere maps (GIM) has high accuracy. In the non-differential non-combination precision SPP algorithm, Kolenov et al. (2020) uses an estimation model containing ionospheric delay parameters to improve the accuracy of precision SPP. Under the premise of only correcting satellite precise clock deviation and precise orbit, precise SPP requires tens of minutes of convergence time, which greatly limits the engineering application of precise SPP. On the basis of GPS dual frequency observations, Krasuski and Savchuk (2020) mixed multiple frequency points and satellite system observations to improve the initialisation time of precision SPP. However, the effect was not significant, and the first successful ambiguity search still took more than ten minutes.

In precision SPP observation models, obtaining more prior information can effectively improve the correlation between the estimated parameters. Many scholars use pre estimated atmospheric delay information to correct observation values. Through experiments, it has been found that using accurately estimated atmospheric delays can effectively separate relevant error parameters and improve the correlation of the covariance matrix of ambiguity real number solutions. On the basis of quasi RT estimation of tropospheric delay, Lagona et al. (2022) used a prediction model to accurately obtain RT tropospheric delay, and applied it to the observation model of RT precision SPP. It was found that the initial PA and initialisation time to the decimetre level were both greatly improved. Liu et al. (2021) applied the global ionospheric model and regional ionospheric model to non-differential and non-combined precision SPP, and found that the first positioning can achieve decimetre level accuracy, and the PA using the regional ionospheric model is higher, which can converge to centimetre level PA after tens of minutes.

### 3 Differential positioning technology and system model

#### 3.1 Differential positioning technology

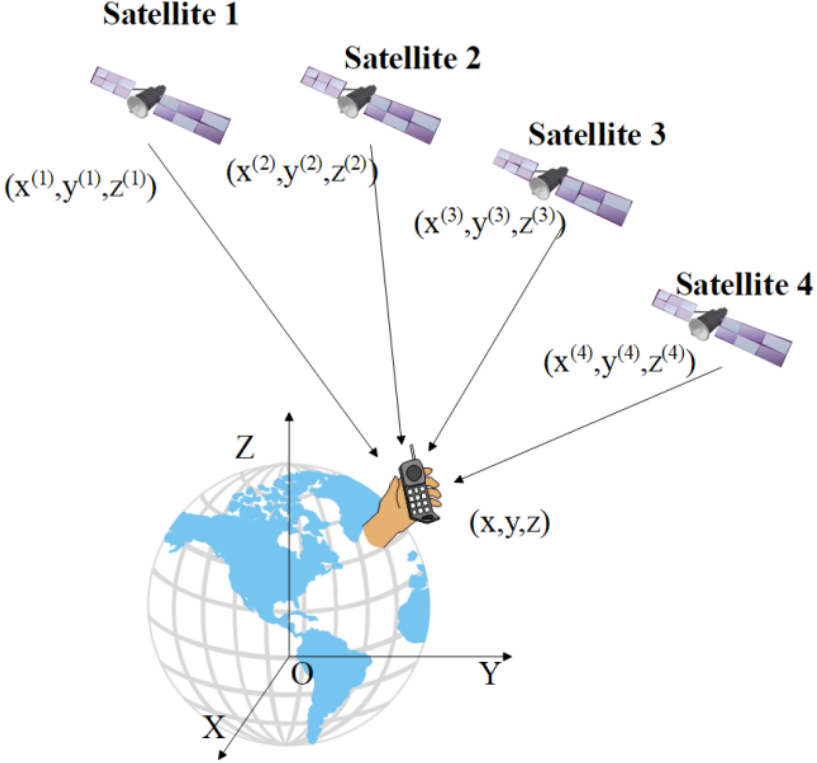
##### 3.1.1 Basis of positioning

The basic principle of HP is shown in Figure 1. The position of satellite  $n$  is obtained through calculation. The coordinates is  $(x^{(n)}, y^{(n)}, z^{(n)})$ , and the station receiver directly outputs the distance  $\rho^{(n)}$  between it and the satellite (station-satellite distance). When the coordinates of two points in space are known, the distance between these two points can be calculated according to the coordinate information, and the equation can be listed accordingly (Luo et al., 2021):

$$\sqrt{(x^{(n)} - x)^2 + (y^{(n)} - y)^2 + (z^{(n)} - z)^2} = \rho^{(n)} \quad (1)$$

Among them, the unknown  $(x, y, z)$  is the coordinate of the receiver. The clock error of the receiver is also an unknown that needs to be calculated. In HP, the positioning equation contains four unknown parameters.

**Figure 1** Principle of HP (see online version for colours)



However, the station-satellite distance obtained by the receiver is not an accurate geometric distance between the two, and there are also various errors in it.

Different from the traditional differential positioning, in DP, the reference station is not fixed in a certain place, and its position coordinates are not necessarily known accurately. In this kind of positioning, the mobile base station acts as a traditional reference station, so it is necessary to obtain the absolute position coordinates of the mobile base station, and then the mobile station performs baseline calculation with it. Compared with the traditional differential positioning, the observation equation in DP is no different. The mathematical principle of linearisation of observation equation in DP is explained.

The carrier phase single difference observation equation can be described as (Nie et al., 2020):

$$\begin{aligned} \lambda\phi_{ij}^{(p)} = & -(l_j^{(p)}dX_j - l_j^{(p)}dX_i) - (m_j^{(p)}dY_j - m_j^{(p)}dY_i) - (n_j^{(p)}dZ_j - n_j^{(p)}dZ_i) \\ & + \lambda N_{ij}^{(p)} + c\delta t_{ij} + \Delta_{ij}^{(p)} \end{aligned} \quad (2)$$

Among them,  $(l, m, n)$  represents the direction cosine, which is obtained by linearising the star distance of the station, and the approximate coordinates of the two stations  $i$  and  $j$  are  $(X_i^0, Y_i^0, Z_i^0)$  and  $(X_j^0, Y_j^0, Z_j^0)$  respectively. At the time of the signal transmission,  $(dx, dy, dz)$  is the correction to the approximate coordinate of the station.  $\Delta_{ij}^{(p)}$  is the sum of some errors, mainly including ephemeris errors, errors caused by atmospheric propagation and some noise.

If the station  $i$  is a reference station for which the coordinates are known, then the corresponding coordinate corrections should all be zero, and equation (2) can be simplified as (Nijak et al., 2024):

$$\lambda\phi_{ij}^{(p)} = -l_j^{(p)}dX_j - m_j^{(p)}dY_j - n_j^{(p)}dZ_j + \lambda N_{ij}^{(p)} + c\delta t_{ij} + \Delta_{ij}^{(p)} \quad (3)$$

By imitating equation (3), then the two equations are differential and linearised, so that:

$$\lambda\phi_{ij}^{(pq)} = -l_j^{(pq)}dX_j - m_j^{(pq)}dY_j - n_j^{(pq)}dZ_j + \lambda N_{ij}^{(pq)} + \Delta_{ij}^{(pq)} \quad (4)$$

The approximate value of the baseline vector can be obtained, that is (Prakash et al., 2021),

$$\vec{ij}_0 = \begin{bmatrix} \Delta X_{ij}^0 \\ \Delta Y_{ij}^0 \\ \Delta Z_{ij}^0 \end{bmatrix} = \begin{bmatrix} X_j^0 - X_i^0 \\ Y_j^0 - Y_i^0 \\ Z_j^0 - Z_i^0 \end{bmatrix} \quad (5)$$

It is not difficult to see that there are two ways to express the coordinates of a station  $j$ : one is to add a coordinate correction  $(dX_j, dY_j, dZ_j)$  on the basis of its own approximate coordinates, and the other is to express it by adding the baseline vector between the two stations through the position of the station  $i$ , that is (Rychlicki et al., 2020),

$$(6)$$

Among them,  $(d\Delta X_{ij}, d\Delta Y_{ij}, d\Delta Z_{ij})$  is the correction of the baseline vector. Equation (4) can be expressed as:

$$\lambda\phi_{ij}^{(pq)} = -l_j^{(pq)}dX_{ij} - m_j^{(pq)}dY_{ij} - n_j^{(pq)}dZ_{ij} + \lambda N_{ij}^{(pq)} + \Delta_{ij}^{(pq)} \quad (7)$$

Compared with the usual filtering methods, due to the difference of motion models, in order to make the connection between the epochs and the epochs, some constraints are needed, and the baseline vector correction method takes the baseline vector as this condition. According to the above derivation process, it can be seen that the two correction methods are equivalent, and different linearisation methods can be used for relative positioning in different scenarios (Schmid, 2009).

### 3.1.2 DP solution algorithm

The optimal candidate vectors of integer least squares must be found by searching, but in the process of fast ambiguity resolution, because of the strong correlation between solutions, the search space becomes longer and the search efficiency becomes lower. It is necessary to reduce the relevance of the solution before searching. LAMBDA algorithm is currently the most perfect method in GNSS ambiguity resolution. It is worth noting



that the process of reducing correlation only improves the search efficiency and does not change the least square search results (Sun et al., 2020).

For relative positioning (Uradziński and Bakuła, 2020):

$$y = A(\Delta b_{ur}) + BN \quad (8)$$

Among them,  $y$  is given directly by the receiver,  $A$  and  $B$  are coefficient matrices with constant elements,  $\Delta b_{ur}$  is the baseline between two base stations, and  $N$  is the integer ambiguity after the double difference. These values are all in the form of vectors. An initial value may be needed in the solution process, and the selection of the initial value can directly use the rounded value of the floating point solution.

The objective function for solving is (Wang et al., 2021):

$$\min_{\Delta b_{ur}, N} \|y - A(\Delta b_{ur}) - BN\|_C^2 = \min_{\Delta b_{ur}, N} \|y - A(\Delta b_{ur}) - BN\|^T C (y - A(\Delta b_{ur}) - BN) \quad (9)$$

If the condition that the ambiguity must be integer is ignored first, the weighted least squares is used to solve the problem, and two solutions  $\widehat{\Delta b_{ur}}$  and  $\widehat{N}$  are obtained. Then, the new objective function is (Wanninger and Heßelbarth, 2020):

$$\min_N \|N - \widehat{N}\|_{Q_N^{-1}}^2 \quad (10)$$

Among them,  $Q_N^{-1}$  is the covariance matrix of the floating point solution  $\widehat{N}$ . The integer solution satisfying equation (10) is the optimal solution, denoted as  $\check{N}$ .

Taking an appropriate threshold  $T$ , the search space (Weng et al., 2020):

$$\|N - \widehat{N}\|_{Q_N^{-1}}^2 < T \quad (11)$$

If  $M - \widehat{M} = Z(N - \widehat{N})$ , then formula (10) can be rewritten as

$$\min_N \|N - \widehat{N}\|_{Q_N^{-1}}^2 = \min_M \|M - \widehat{M}\|_{Z^T Q_N^{-1} Z}^2 \quad (12)$$

Among them,  $Z$  and  $Z - 1$  should satisfy the following conditions: in order to ensure that the transformation is a one-to-one map, the matrix elements should be integers, and in order to keep the volume of the search space unchanged before and after the transformation, the determinant of the matrix should be 1. In this case, the search space is an approximate sphere space, and the relationship between the optimal solutions is as follows:

$$\check{N} = Z^{-1} \check{M} \quad (13)$$

By substituting the calculated results  $\check{N}$  into equation (8), the optimal solution  $\check{\Delta b_{ur}}$  of the baseline vector or the baseline vector correction can be obtained. Regardless of whether the receiver is static or dynamic, the data processing is RT or afterwards, the LAMBDA algorithm can quickly and reliably calculate the integer ambiguity (Xiao et al., 2018).

Kalman filtering (KF) is an algorithm for estimating the state of a system. Through KF, some noises can be filtered out. Simple KF application scenarios are limited, and the filtering system must conform to a normal distribution.

Extended Kalman filter (EKF) measurement update: it can be described as follows:

According to the double difference formula of carrier phase and pseudo-range, the measurement model vector  $h(x)$ , partial derivative matrix  $H(x)$  and measurement error covariance matrix  $R$  can be expressed as (Yang and Qin, 2021; Yang et al., 2020; Yu et al., 2020; Yuan et al., 2021; Zekavat et al., 2021; Zeybek, 2021):

$$h(x) = (h_{\phi,1}^T, h_{\phi,2}^T, h_{P,1}^T, h_{P,2}^T)^T \quad (14)$$

$$H(x) = \frac{\partial h(x)}{\partial x} = \begin{bmatrix} -DE & 0 & \lambda_1 D & 0 \\ -DE & 0 & 0 & \lambda_2 D \\ -DE & 0 & 0 & 0 \\ -DE & 0 & 0 & 0 \end{bmatrix} \quad (15)$$

$$R = \text{diag}(DR_{\phi,1}D^T, DR_{\phi,2}D^T, DR_{P,1}D^T, DR_{P,1}D^T) \quad (16)$$

$$h_{\phi,i} = \begin{bmatrix} \rho_{ur}^{12} + \lambda_i (B_{ur}^1 - B_{ur}^2) \\ \rho_{ur}^{12} + \lambda_i (B_{ur}^1 - B_{ur}^2) \\ \dots \\ \rho_{ur}^{1m} + \lambda_i (B_{ur}^1 - B_{ur}^m) \end{bmatrix} \quad (17)$$

$$h_{P,i} = (\rho_{ur}^{12}, \rho_{ur}^{13}, \dots, \rho_{ur}^{1m}) \quad (18)$$

$$D = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 1 & 0 & -1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots & -1 \end{bmatrix} \quad (19)$$

$$E = (e_r^1, e_r^2, \dots, e_r^m)^T \quad (20)$$

$$R_{\phi,i} = \text{diag}(2\sigma_{\phi,i}^{1,2}, 2\sigma_{\phi,i}^{2,2}, \dots, 2\sigma_{\phi,i}^{m,2}) \quad (21)$$

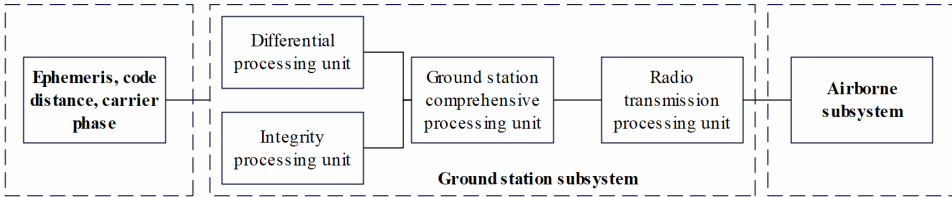
$$R_{P,i} = \text{diag}(2\sigma_{P,i}^{1,2}, 2\sigma_{P,i}^{2,2}, \dots, 2\sigma_{P,i}^{m,2}) \quad (22)$$

$e_r$  is the line-of-sight vector,  $\sigma_{\phi,i}$  and  $\sigma_{P,i}$  are the carrier phase and pseudo-range measurement error standard deviation of the  $L_i$  frequency, respectively. Based on the updated solution (14) of the measured values, the position coordinates of the rover antenna, the velocity and the floating point single difference carrier phase at time  $t_k$  can be estimated.

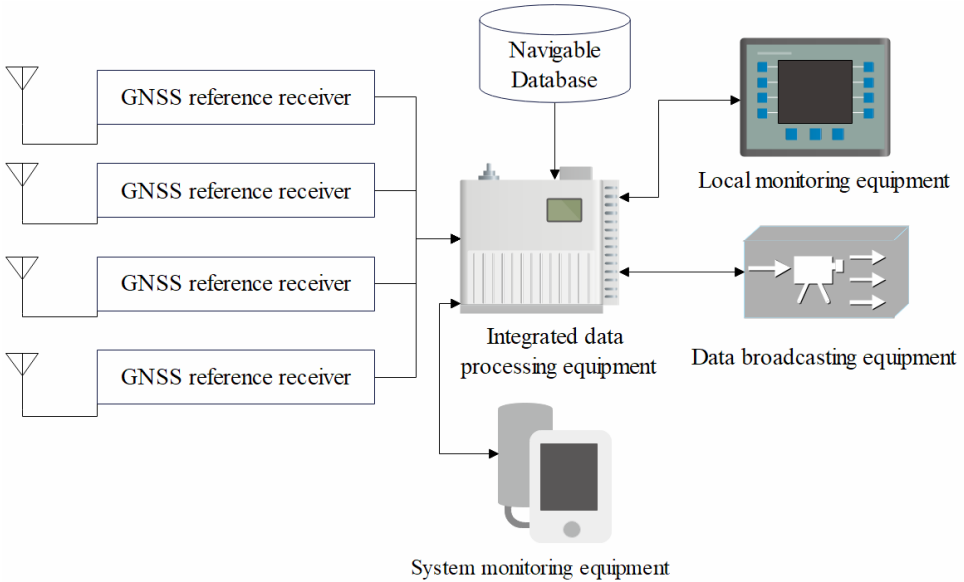
### 3.2 Beidou positioning system

The ground station subsystem of the ground augmentation system based on Beidou mainly needs to complete the generation of differential information and integrity monitoring, and generate differential messages and send them through the radio. The system consists of a differential processing unit, an integrity monitoring processing unit, a ground station comprehensive simulation unit and a radio simulation transmission simulation unit, as shown in Figure 2. This paper mainly studies the generation of differential information and integrity monitoring.

**Figure 2** Ground subsystem module



**Figure 3** Composition of the ground station system (see online version for colours)



The differential processing unit performs differential enhancement information processing according to the input observation data, navigation message data and reference position data. The integrity monitoring processing unit completes the calculation of integrity monitoring parameters and the generation of message data. The comprehensive processing unit of the ground station extracts the message parameters of each SN system, and generates observations such as pseudo-range and carrier phase according to the accurate position and environmental data of multiple receivers of the reference station. The radio transmitting unit demodulates the message data and processes

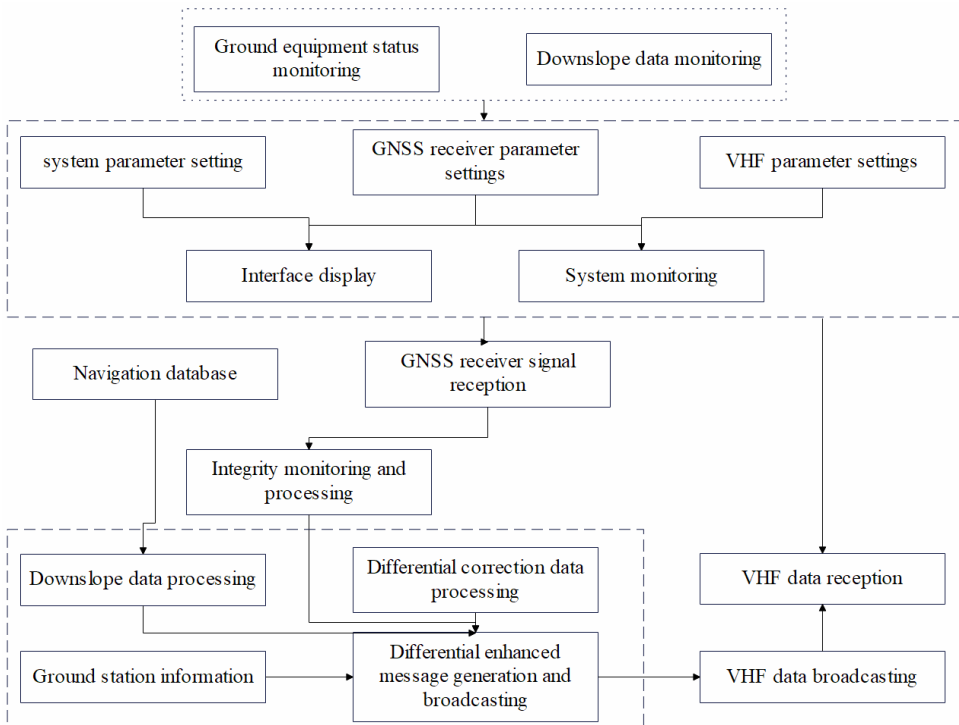
the message data. This paper mainly studies the differential processing unit and the integrity monitoring unit.

The system consists of GNSS reference receiver, integrated data processing equipment, data broadcasting equipment, local monitoring equipment, system monitoring equipment, navigation database and ATC monitoring equipment, as shown in Figure 3.

The ground station system in this article receives data through a receiver and transmits it to a comprehensive data processing device. The comprehensive data processing device not only needs to process the data received from the receiver, but also needs to process navigation data and system data, and interact with monitoring equipment and broadcasting equipment for feedback

The working flow of the ground station subsystem is shown in Figure 4.

**Figure 4** System workflow chart



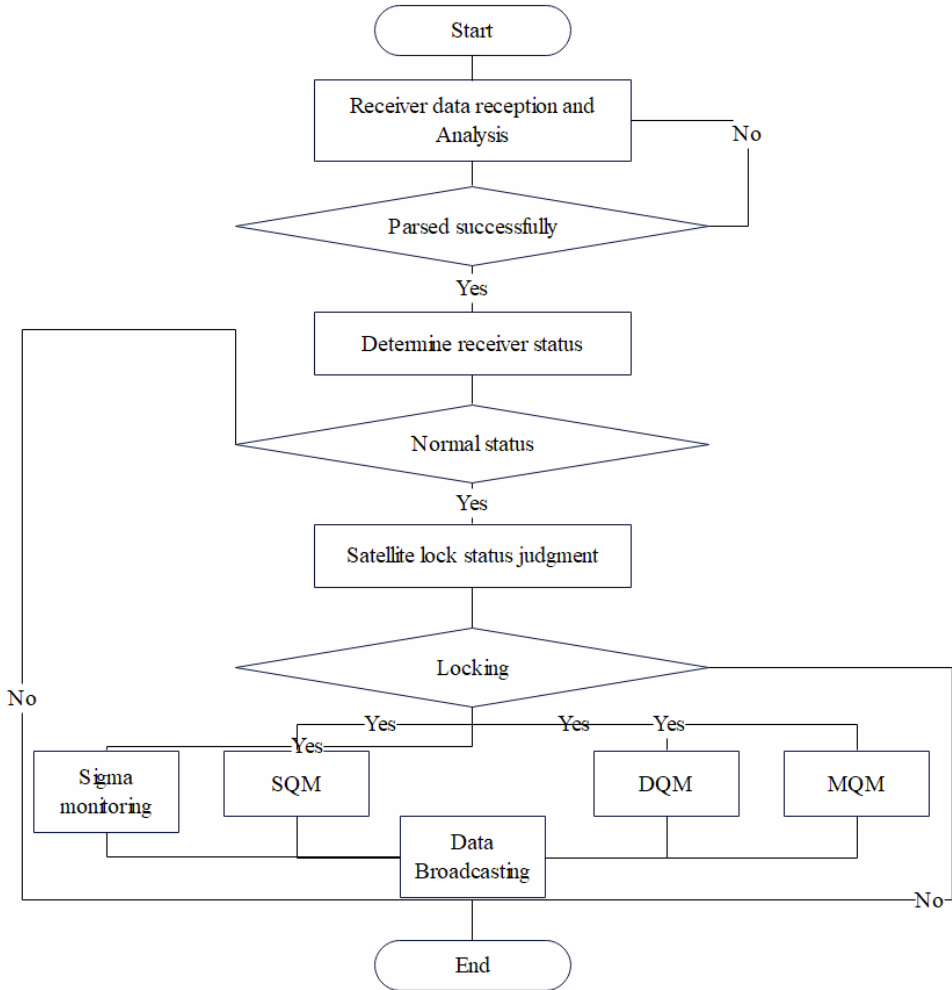
The system monitors the data through ground detection equipment and transmits it to the parameter processing module, which can set parameters and visualise management. After processing, the information is transmitted to the ground information processing centre for interaction, and the data is fed back to the ground information monitoring centre through broadcasting and wireless transmission.

The overall processing flow of the system is shown in Figure 5.

First, the receiver receives information from the satellite and analyses it. If the analysis is successful, the state of the receiver is judged, but if it fails, it returns to re-analysis. If the receiver state is normal, the system enters the satellite lock state judgment, but if the receiver state is not normal, the system exits and ends the process. If the

satellite is locked, the system performs Sigma, SQM, MQM, and DQM algorithm monitoring, but if the satellite is not locked, the system exits and ends the process. If there is no abnormality in the algorithm monitoring, MRCC verification is carried out, and finally it is sent to the airborne receiver through VHF broadcast to end the process.

**Figure 5** Overall processing flow chart of the system



## 4 Experimental research

### 4.1 Experimental environment

The static single-station receiver is used for data acquisition, and the surveying and mapping coordinates of the station are accurately known. Moreover, this paper uses it to collect SN messages and receiver observation data for a day and the sampling rate is 30 s data, and there are 2,878 epochs in total.

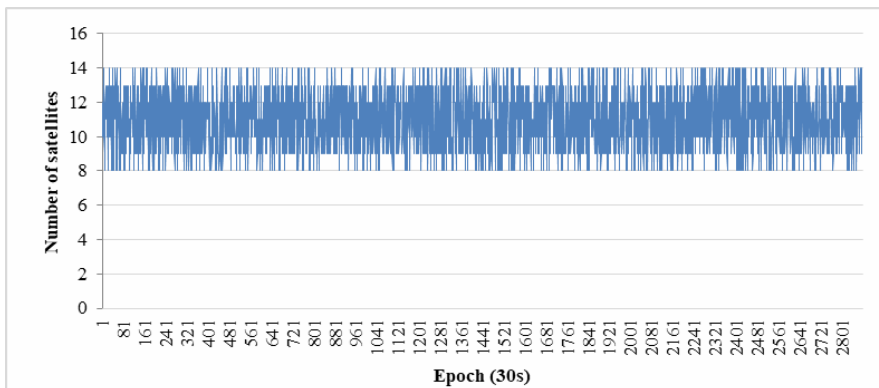
In order to explore whether the high-precision pseudorange measurement positioning method can effectively estimate the position coordinates of the user receiver, this paper designs to use the simulation platform MATLAB software to analyse the real Beidou observation data files and ephemeris data files downloaded from the IGS website, and implement the positioning method according to the design process. Observation data pre-processing is the preparation work for precision positioning and provides relatively clean ‘raw materials’ for it. Among them, carrier phase cycle slip detection and invalid data removal play a crucial role in precision positioning. Firstly, incomplete three frequency data are removed, and the observation values are detected for cycle slips using the dual frequency geometric independence algorithm. For those with large residual values, the carrier phase observation is corrected using the multi epoch averaging method.

Unlike static positioning, it is not possible to accurately obtain the true coordinates of the receiver at each positioning moment when it is in motion. Therefore, this paper will use static reference stations to perform differential positioning on mobile base stations and mobile stations, and evaluate the accuracy of DP by treating the positioning coordinates obtained by this method as the true coordinates of mobile base stations and mobile stations. The static reference station is located on the roof of the information building, and its position coordinates are precisely known. Both the mobile base station and the mobile station are positioned within a range of 1–2 km from the static reference station antenna. Under the condition of short baseline, the accuracy of differential positioning can be guaranteed, so the positioning results of mobile base stations and mobile stations in this positioning scenario can be used as a reference for DP.

## 4.2 Results

The number of visible satellites in the receiver is shown in Figure 6, and the number of satellites observed by the receiver is always greater than 4, which meets the positioning requirements. The satellites observed at each moment are shown in Figure 7.

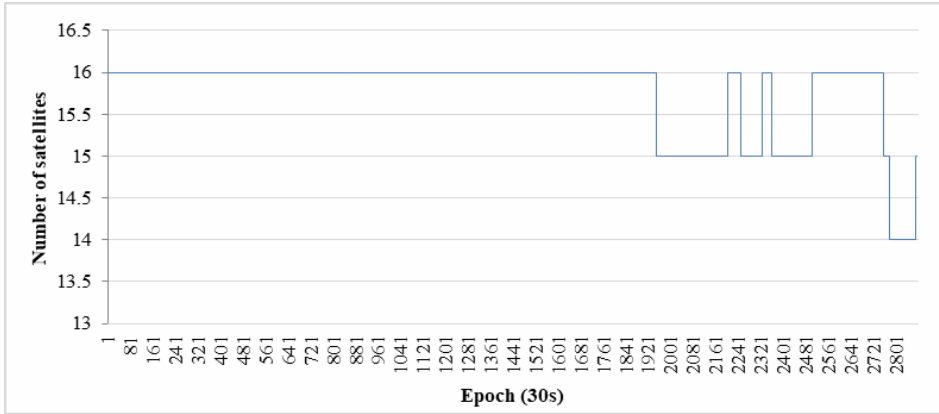
**Figure 6** Number of visible satellites (see online version for colours)



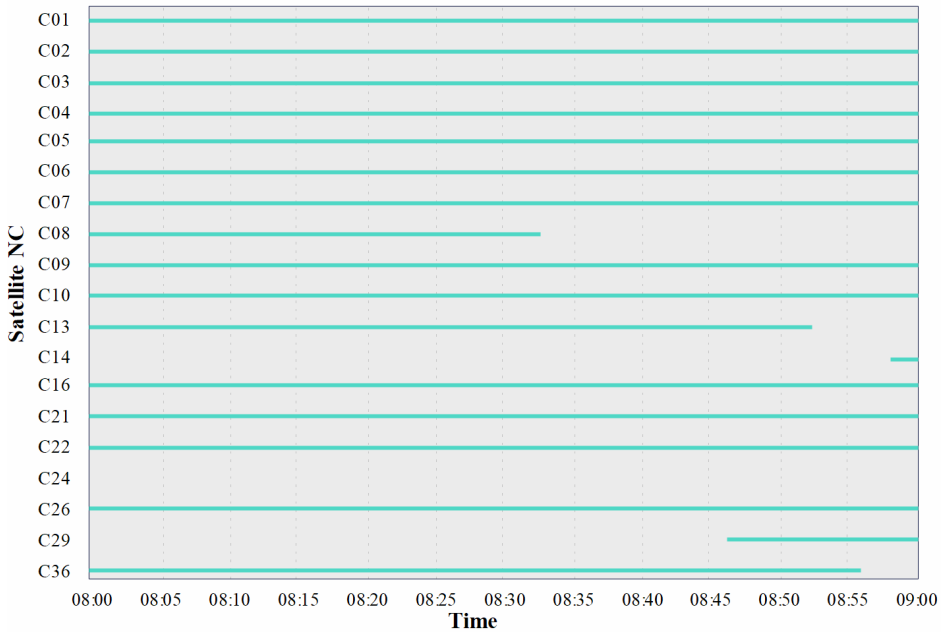
Data are collected for an hour at a sampling rate of 1 s per epoch at a station with fixed antennas and precise surveying and mapping positions. SPP and carrier phase differential positioning between epochs are carried out on the collected data, and the PA of the two

positioning methods is compared. The number and specifics of satellites observed by the station receiver are shown in Figures 7 and 8.

**Figure 7** Number of visible satellites (see online version for colours)



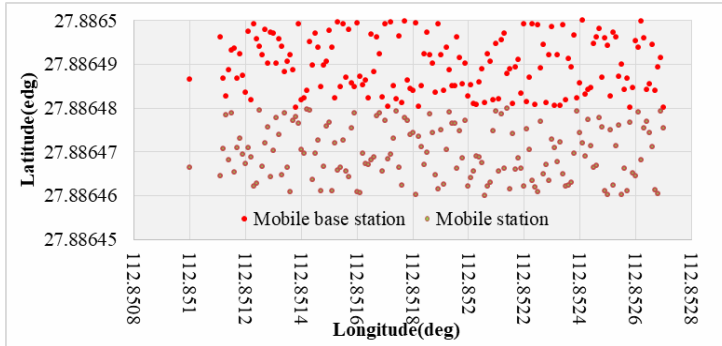
**Figure 8** Specific observed satellites (see online version for colours)



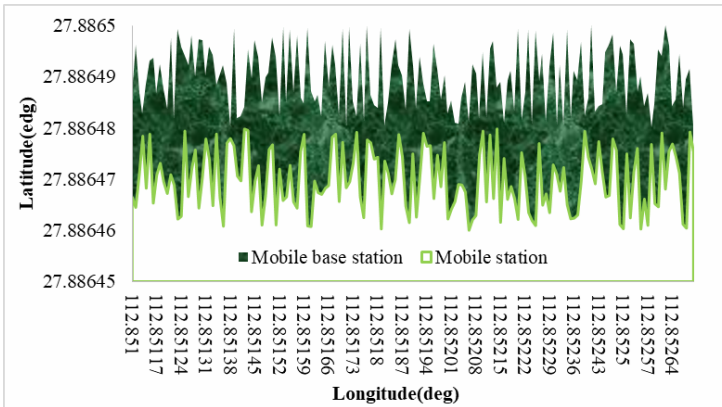
The experiment of DP involves three receivers: static reference station, mobile base station and mobile station. Static reference station is fixed and its position is known accurately, and its function is to provide differential positioning service to mobile base station and mobile station receiver, and obtain more accurate position coordinates of mobile base station and mobile station. The mobile base station acts as a reference station in the mobile location, and the absolute position of the mobile base station can be obtained by pseudo-range SPP.

The motion trajectory of the mobile base station and the mobile station is shown in Figure 9, the two stations travel in the same direction, and the motion direction is shown by the arrow in the figure. The baseline between the mobile base station and the mobile station is shown in Figure 10. The black circle in the figure represents the position of each epoch of the mobile base station, and the blue circle represents the position of each epoch of the mobile station. Moreover, the line between the positions of the mobile base station and the mobile station at the same epoch is the baseline, and the red line in the figure represents the baseline of each epoch.

**Figure 9** Motion trajectory diagram (see online version for colours)



**Figure 10** Schematic diagram of baseline (see online version for colours)

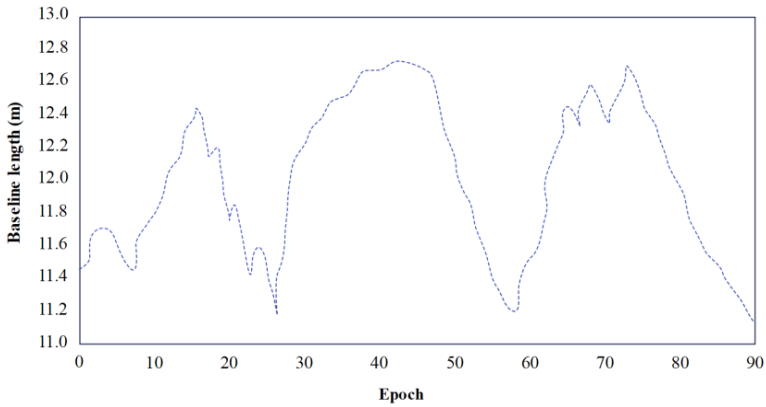


The static reference station is used to locate the mobile base station and the mobile station respectively, and the more accurate positioning results are obtained, and the baseline length is obtained. The baseline length changes as shown in Figure 11.

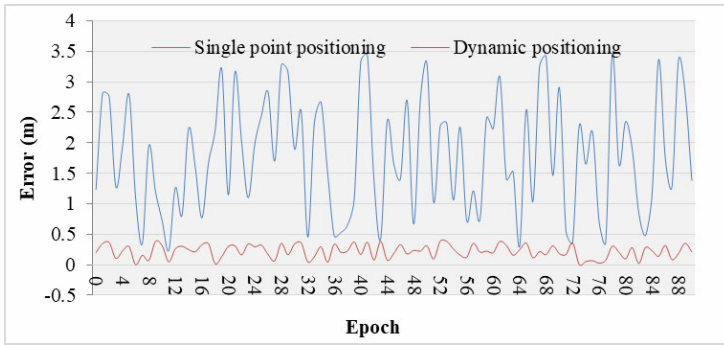
The mobile base station and mobile station are located separately, and the absolute position coordinates of the two stations are obtained, and the baseline between the two stations is solved. Pseudo-range single-point location for mobile base station is carried out, and then the baseline vector is solved by using the DP model. Comparing the baseline length obtained by the two positioning methods with the baseline length of the previous differential positioning, the baseline calculation errors of the two different positioning methods are obtained. The specific error results are shown in Figure 12.



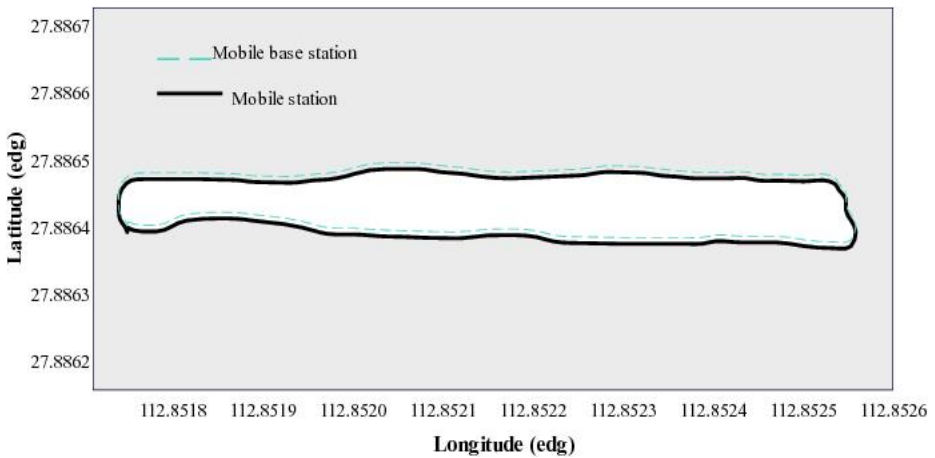
**Figure 11** Variation of baseline length (see online version for colours)



**Figure 12** Baseline solution error (see online version for colours)



**Figure 13** Motion trajectory diagram (see online version for colours)

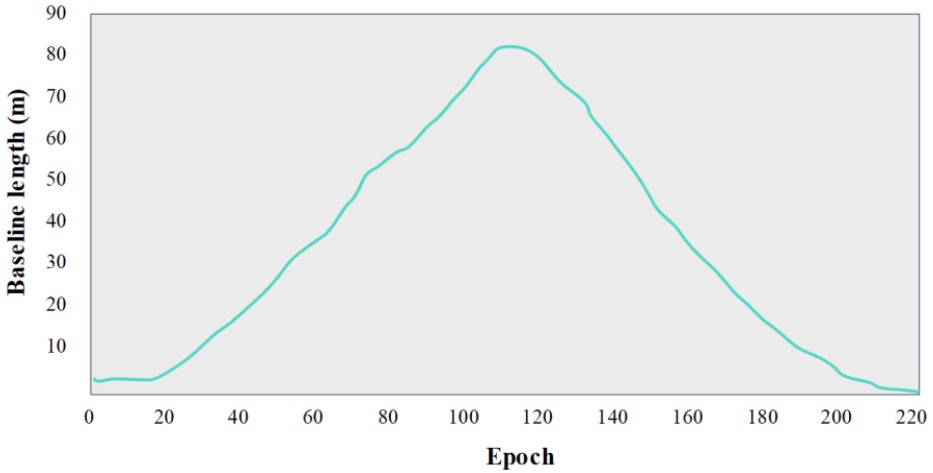


On the basis of the above, the test process is improved and the second type of test is carried out. The duration of the experiment is 3 minutes and 54 seconds, with a total of

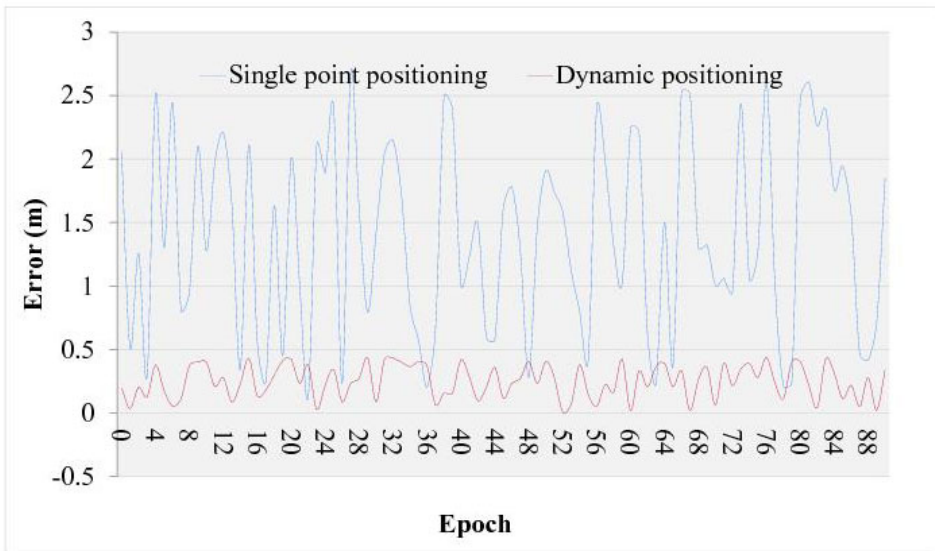
234 epochs. The running trajectory of the mobile base station and the mobile station is shown in Figure 13. The mobile base station is stationary at a certain place, the mobile station moves according to a preset rectangular route, and the moving direction of the mobile station is shown by the arrow in the figure.

The static reference station is used to carry out differential positioning of the mobile base station and the mobile station, and the more accurate positioning results are obtained and the baseline length change between the two stations is calculated. The results are shown in Figure 14.

**Figure 14** Variation of baseline length (see online version for colours)



**Figure 15** Baseline resolution error (see online version for colours)



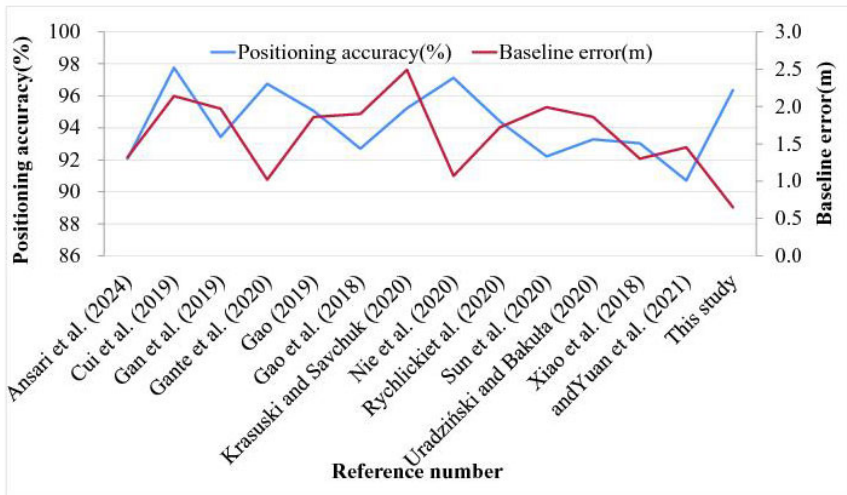
Then, the baseline is solved by SPP and DP respectively, and the baseline solution results are compared with the baseline solution results obtained by differential positioning. The baseline solution error is shown in Figure 15.

To further verify the effectiveness of the model proposed in this paper, the Beidou positioning method was compared with the most widely used GPS technology. The methods in Ansari et al. (2024), Cui et al. (2019), Gan et al. (2019), Gante et al. (2020), Gao (2019), Gao et al. (2018), Krasuski and Savchuk (2020), Nie et al. (2020), Rychlicki et al. (2020), Sun et al. (2020), Uradziński and Bakula (2020), Xiao et al. (2018), and Yuan et al. (2021) were selected as controls. The PA and baseline calculation error of these models were statistically analysed, and the experimental results shown in Table 1 and Figure 16 were obtained.

**Table 1** Model comparison results

<i>Literature</i>	<i>Positioning accuracy (%)</i>	<i>Baseline error(m)</i>	<i>Literature</i>	<i>Positioning accuracy (%)</i>	<i>Baseline error(m)</i>
Ansari et al. (2024)	92.07	1.32	Nie et al. (2020)	97.12	1.07
Cui et al. (2019)	97.76	2.14	Rychlicki et al. (2020)	94.39	1.72
Gan et al. (2019)	93.43	1.97	Sun et al. (2020)	92.21	1.99
Gante et al. (2020)	96.75	1.02	Uradziński and Bakula (2020)	93.28	1.86
Gao (2019)	95.06	1.86	Xiao et al. (2018)	93.03	1.30
Gao et al. (2018)	92.70	1.90	Yuan et al. (2021)	90.71	1.42
Krasuski and Savchuk (2020)	95.22	2.49	This paper	96.35	0.65

**Figure 16** Statistical chart of model localisation results (see online version for colours)



To further verify the positioning effect of the model proposed in this paper on moving objects, ten vehicles were selected as the research objects and allowed to move freely on the test site. The positioning error was calculated using the model proposed in this paper, and the results are shown in Table 2.

**Table 2** Test results of PA for mobile vehicles

<i>Vehicle</i>	<i>Error(m)</i>	<i>Vehicle</i>	<i>Error(m)</i>
1	1.67	6	1.11
2	1.82	7	1.83
3	1.26	8	1.57
4	1.01	9	1.69
5	1.48	10	1.70

### 4.3 Analysis and discussion

Not all the satellites observed by the receiver are in a good position, and the spatial position of the satellite will also have a certain impact on the PA, so some indicators need to be used to judge the quality of the satellite. Dilution of precision (DOP) is a parameter specially used to judge the mass of satellites. DOP values include four types: geometric dilution of precision (GDOP), position dilution of precision (PDOP), horizontal dilution of precision (HDOP) and vertical dilution of precision (VDOP). Through these precision factors, the quality of the satellite can be judged intuitively. On the premise of the same observation conditions and HP errors, the smaller the DOP value, the smaller the error caused by the satellite. As can be seen from Figure 6 and Figure 7, the DOP value of the satellite is within a range of smaller values most of the time.

It can be seen from Figures 8 and 9 that during this one-hour observation time, the number of visible satellites is basically maintained at about 16 satellites. According to the specific observation satellite situation, it can be seen that most of the satellites have been observed all the time, except satellite C08 was not observed in the second half, and satellites C14 and C29 were observed near the end of the experiment. The observed satellites remain basically unchanged to ensure that the satellites observed between two adjacent epochs will not change much, that is, the number of satellites jointly observed between epochs is guaranteed, so that there are enough inter-epoch carrier phase difference observations for position resolution.

As can be seen from Figure 10 and Figure 11, the baseline calculation error by using the DP method is less than the error of calculating the baseline length after obtaining the absolute position of the station by using the SPP method, wherein the average error of the baseline result obtained by the SPP method is 1.02 m, and the average error of the baseline result calculated by the DP method is 0.16 m.

As can be seen from Figures 12–14, since the mobile base station is stationary, the change of the baseline length between the two stations depends entirely on the rover station. At the beginning, the rover station is in a state far away from the mobile base station, and the baseline length gradually becomes longer. When the rover station reaches the vertex of the rectangle, the distance between the two stations begins to shorten, and the baseline length also becomes shorter.

According to the experimental results in Figure 15, the average baseline solution error using SPP is 0.62 m, while the average baseline solution error using DP is 0.09 m. Combined with the baseline solution error map, it can be concluded that when the mobile base station position is fixed, the error of baseline solution using DP is better than that using SPP to obtain absolute position for baseline solution. In addition, because the mobile base station is fixed and only the mobile station moves, this positioning scene is very similar to the traditional differential positioning, except that the position of the mobile base station is not accurately known like the static reference station. At this time, the average error of baseline calculation using DP is not much different from the baseline calculation result of traditional differential positioning.

From Table 1, it can be seen that the positioning method proposed in this paper, although not reaching the highest accuracy compared to GPS, is still at a relatively high level. Moreover, the model proposed in this paper can minimise system errors to the greatest extent possible. Overall, the comprehensive performance of the algorithm system in this article is higher than other models. In the subsequent research of this paper, the positioning algorithm will be further improved to further enhance the PA of this paper, which can effectively improve the practical effect of the system proposed in this paper.

From the experimental results, it can be concluded that the error of baseline calculation using the DP method is smaller than that using the SPP method. Different motion scenes also affect the error of baseline solution. In experiment 2, the mobile base station is stationary while the mobile station moves along a circular route, which is similar to the situation of traditional differential positioning, so the error of baseline solution is the smallest in all experiments.

From the experimental results, it can be concluded that the error of baseline calculation using DP method is smaller than that using single point positioning (SPP) method. Different motion scenarios also have an impact on the error of baseline calculation, and the motion speed of mobile base stations and mobile stations does not exceed 2 m/s. Generally speaking, the movement speed of the receiver does not generate new positioning errors. However, when the receiver moves at high speed, the distance between the receiver positions between two epochs increases, and the propagation errors of ionospheric delay, tropospheric delay, and other signals related to the height of the geometric position change too much. This may introduce significant errors when performing differential positioning. The experimental results show that the accuracy of using DP method to solve the baseline length is better than that of using SPP method to solve the baseline length.

From the above analysis, it can be seen that the precise positioning system based on Beidou dual splicing differential positioning technology proposed in this article not only has high reliability, but also high accuracy, and has good positioning performance in many aspects. This also verifies that the model in this article meets the PA requirements, can meet practical needs, and can play an important role in subsequent practical applications.

In the DP experiment, since both the mobile base station and the mobile station are in motion, it is difficult to determine the RT accurate position coordinates of the two measurement stations. In this experiment, the result of differential positioning of the two measurement stations by the static reference station is used as the real coordinates to evaluate the accuracy of the baseline solution. Due to the small amount of error in the differential positioning results between mobile base stations and mobile stations, there is also a certain impact on the evaluation of baseline solutions. When planning the motion

route for mobile base stations and mobile stations, when the motion route is a straight line, the precise position coordinates of the motion route endpoints can be determined in advance, and the position coordinates in the motion route can be calculated through mathematical models. In the case of complex motion routes between mobile base stations and mobile stations, important points in the motion routes can be calibrated in advance, and the baseline solution accuracy can be evaluated when the two measuring stations move to the calibration points.

From the results in Table 2, it can be seen that the PA of mobile vehicles in this paper is below 2 metres. Therefore, it can be concluded that the model proposed in this paper also has certain effectiveness in vehicle positioning, indicating that the model proposed in this paper has certain practical effects.

## **5 Conclusions**

In this paper, the precise positioning technology of Beidou dual frequency differential positioning technology in complex environment is studied. The dynamic differential positioning of mobile base stations and mobile stations is carried out by using static reference stations with precise and known positions. Moreover, the positioning results obtained by this positioning method are regarded as accurate, and are used as a reference for subsequent baseline calculation.

In the experiment, the mobile base station and mobile station are calculated by using SPP method and DP method respectively. Among them, the SPP method refers to using pseudo-range SPP to solve the absolute position coordinates of the mobile base station and mobile station to obtain the baseline length between the two stations. The DP method refers to using pseudo-range SPP to solve the absolute coordinates of the mobile base station first, and then using the position coordinate correction to solve the position coordinates of the mobile station relative to the mobile base station to obtain the baseline length between the two stations. The experimental results show that the accuracy of baseline length calculated by DP method is better than that calculated by SPP method.

In this experiment, the accuracy of baseline calculation between mobile base stations and mobile stations was evaluated based on the baseline length. However, in the actual positioning environment, the baseline vectors of the two stations in three-dimensional space also contain directional information, which is limited by the experimental hardware equipment and environment, making it impossible to measure the directional angle between the two stations. If there are conditions to measure the relative angle and direction between two measuring stations, the accuracy evaluation of baseline calculation will also be more comprehensive.

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